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# The 2-level in series capacitated lot-sizing problem with identical capacities

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**Mots-clés** : *capacitated lot sizing, two-echelon, dynamic programming*

## 1 Introduction

We consider a natural extension of the capacitated lot-sizing problem (CLSP) to two production levels in series. We call the first level *the manufacturer* and the second level *the retailer*, see Figure 1. The retailer has to satisfy a deterministic demand of finished goods at each period over a finite time horizon of length  $T$ . Demands have to be satisfied on time, without backlogging. Each unit of demand has to be processed on both levels, first at the manufacturer and then at the retailer. At each level, the production of an amount  $x$  in period  $t$  incurs a setup cost  $K_t^i$  and a variable production cost  $p_t^i x$ . In addition, the amount  $x$  of unit produced can not exceed a given capacity. It is also possible to store some units at each level from one period to another to serve a future demand. Units carried in stock at level  $i$  at the end of period  $t$  incurs a per unit holding cost  $h_t^i$ . The objective is to find a production planning minimizing the total cost of the system, that is, the sum of setup, production and holding costs paid at both levels. We assume in this paper that the capacities at both level are stationary and identical, equal to  $C$ . To develop an efficient algorithm, we also assume that the production costs are stationary at the manufacturer and non-speculative at the retailer.

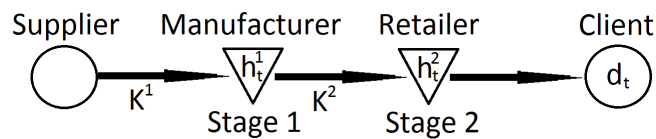


FIG. 1 – Serial supply chain with two production levels

The capacitated lot-sizing problem (CLSP) was introduced by Florian and Klein [1]. They proposed a dynamic programming algorithm to solve this problem in time  $O(T^4)$ , assuming a stationary capacity, and established that the problem is NP-hard for time-varying capacities. Van Hoesel and Wagelmans [4] developed an efficient algorithm in  $O(T^3)$  for CLSP with a stationary capacity, concave production costs and linear holding costs. Multi-level lot-sizing problems with capacity has been studied more recently. Van Hoesel et al. [3] developed an algorithm for the multi-level capacitated lot-sizing problem with general concave cost functions, assuming a finite capacity only at the first stage. The algorithm is polynomial for a fixed number of levels. Using the novel concept of basis path, Hwang et al. [2] developed the first polynomial algorithm with general concave costs when the number of levels is part of the input. However, they also consider a limited capacity only at the first level of the chain, while we consider in this paper a capacity at both levels.

## 2 Dynamic algorithm

A solution of the lot-sizing problem can be decomposed into a set of connected components. A classical way to find an optimal solution of a given instance is to evaluate the minimal cost of all the possible connected components, and then to construct the optimal solution by combining the best possible set of connected components with a dynamic algorithm.

A retailer subplan is a set of consecutive periods of production at the retailer level, bounded by two regeneration periods (periods with no entering stock). To find an optimal policy for a given connected component, the principle of our algorithm is to decompose it in its turn into a sequence of retailer subplans.

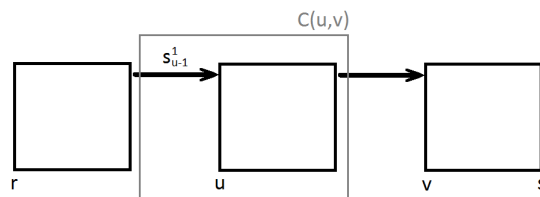


FIG. 2 – Schematic illustration of the cost of a retailer subplan  $(u, v)$  in a connected component  $(r, s)$

## 3 Structure of an extreme point solution

Multi-level lot-sizing problems can be seen as fixed-charge network flow problems. In such a network, an arc is said to be free if it is not saturated and its flow is positive. In our problem, it represents period with a production which is not at full capacity. It is well known that the graph induced by the free arcs of an extreme point is acyclic. Our assumptions on the cost structure allow us to identify two additional dominance properties : (i) There exists an optimal policy which is nested, that is, the retailer produces in each period the manufacturer produces. (ii) There exists an optimal policy such that each full production period at the retailer is also a full production period at the manufacturer.

From these dominances, we are able to characterize quite precisely the structure of a retailer subplan. This structure allows us to compute the optimal solution of a retailer subplan by solving a single level capacitated lot-sizing problem. This reduction is based on the fact that the production periods at both stages are nearly synchronous, except for the fractional production periods. Using the algorithm of [4], the evaluation of a subplan can be performed in time  $O(T^2)$ . We state the following result :

**Theorem 1** *The 2-level in series capacitated lot-sizing problem with identical capacities and non-speculative motives can be solved in time  $O(T^5)$*

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