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# Fully electrical modeling of Thermoelectric Generators with contact thermal resistance under different operating conditions

SAIMA SIOUANE · SLAVIŠA JOVANOVIĆ · PHILIPPE POURE

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**Abstract** The Seebeck effect has been used in Thermoelectric Generators (TEGs) to supply electronic circuits by converting the waste thermal into electrical energy. This generated electrical power is directly proportional to the temperature difference between the TEG module's hot and cold sides. Depending on the applications, TEGs can be used either under constant temperature gradient between heat reservoirs or constant heat flow conditions. Moreover, the generated electrical power of a TEG depends not only on these operating conditions but also on the contact thermal resistance. The influence of the contact thermal resistance on the generated TEG's electrical power have already been extensively reported in the literature. However, as reported in [1,30], while designing TEG-powered circuit

and systems, a TEG module is mostly modeled with a Thévenin equivalent circuit whose resistance is constant and voltage proportional to the temperature gradient applied to the TEG's terminals. This widely used simplified electrical TEG model is inaccurate and not suitable under constant heat flow conditions or when the contact thermal resistance is considered. Moreover, it does not provide realistic behaviour corresponding to the physical phenomena taking place in a TEG. Therefore, from the circuit designer's point of view, faithful and fully electrical TEG models under different operating conditions are needed. Such models are mainly necessary to design and evaluate the power conditioning electronic stages and the Maximum Power Point Tracking (MPPT) algorithms of a TEG power supply. In this study, these fully electrical models with the contact thermal resistance taken into account are presented

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Institut Jean Lamour (UMR7198), Université de Lorraine,  
Vandoeuvre lès Nancy, France

E-mail: first\_name.name@univ-lorraine.fr

and the analytical expressions of the Thévenin equivalent circuit parameters are provided.

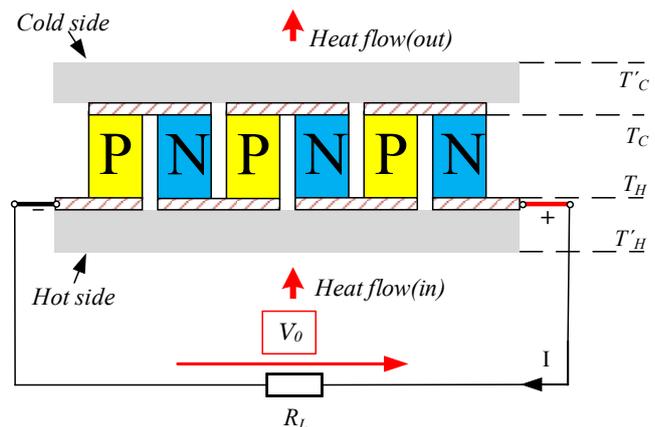
**Keywords** Thermoelectric Generator (TEG), constant temperature gradient, constant heat flow, fully electrical modeling, contact thermal resistance

## 1 Introduction

In the recent years, energy harvesting technologies appeared as a solution to satisfy the power-supply requirements of a large variety of devices such as wireless sensors, biomedical devices, etc [1]. Free availability of heat makes harvesting energy from it as one of the most viable sources of electricity [2, 3]. Thermoelectric Generators (TEGs) directly convert heat into electrical energy based on the Seebeck effect [4-6]. TEGs have gained considerable attention [7] due to their noiseless operation, stability and absence of moving parts thus providing high reliability and facilitating their implementation [8, 9]. Therefore, TEGs are used to recover waste heat in a wide range of applications, ranging from low power (autonomous wireless sensors [10] and battery charging [11]) via medium power (automotive [12], stoves [13], and hybrid photovoltaic-TEG systems [14]) up to high power (geothermal power generation [15]).

A TEG module is made of two dissimilar semiconductors: p- and n-type, which are electrically connected

in series, to generate a higher voltage and a higher electrical power, and thermally in parallel to keep each semiconductor element at the same temperature difference. These semiconductor elements are usually arranged in a planar array and fastened between two ceramic hot and cold plates [16]. The basic structure of a TEG formed by the association of 3 p-n junctions of semiconductors with the electrical load connected to its terminals is presented in Fig. 1. For a given temperature difference between the hot and the cold sides, heat flows through the TEG module as presented in Fig. 1 [17]. The circulation of heat flow through the semiconductor elements causes a displacement of electrons and holes in opposite directions and therefore the generation of an electrical current  $I$  passing through the connected load  $R_L$  [18].



**Fig. 1** General Structure of a TEG module.

The physical effects that appear in a TEG module when subjected to a temperature difference are the

thermal conduction and the Seebeck, Peltier, and Joule effects [16]. The Thomson effect is commonly neglected when all considered physical properties of a TEG module are temperature independent [19, 20, 29]. The heat flow rates through the hot side  $Q_H$  and the cold side  $Q_C$  of a TEG module, are expressed respectively by:

$$Q_H = \frac{T_H - T_C}{\theta_m} + \alpha T_H I - \frac{1}{2} R_E I^2. \quad (1)$$

$$Q_C = \frac{T_H - T_C}{\theta_m} + \alpha T_C I + \frac{1}{2} R_E I^2. \quad (2)$$

where  $R_E$  and  $\theta_m$  are the electrical and thermal resistances of  $N$  pairs of p-n elements respectively, and  $\alpha$  is the Seebeck coefficient of the TEG module.  $T_H$  and  $T_C$  denote the internal hot and cold side temperatures at the terminals of the p and n elements.  $T'_H$  and  $T'_C$  in Fig. 1 are the external temperatures applied on both hot and cold sides of the TEG module.

Depending on the applications, TEGs can be used either under constant temperature gradient or constant heat flow conditions. In the first case, the temperature difference  $\Delta T'$  across a TEG module is considered constant over time, even though its value slowly evolves with variable environmental conditions. On the other hand, the constant heat flow conditions imply that the input heat flow  $Q_H$  is considered constant over time. Moreover, the generated electrical power of a TEG depends not only on the working conditions (constant temperature gradient or heat flow) but also on

the contact thermal resistance. The influence of these contact thermal resistances on the TEG's generated electrical power and its efficiency in different operating conditions have already been discussed in the literature [25–28]. Indeed, the thermal contact plates placed on the both sides of the TEG present an interface between the surrounding environment and its semiconductor elements and have a finite thermal resistance. If its value is high or not negligible, the semiconductor elements can perceive a different temperature gradient or a heat flow than the ones really applied on the both sides of the TEG module. However, while designing TEG-powered systems, the TEG module is mostly modeled with a simplified Thévenin equivalent circuit whose resistance is constant and equal to the TEG's electrical resistance and voltage is proportional to the temperature gradient applied to the TEG's terminals, where the coefficient of proportionality is the Seebeck coefficient. This widely used simplified electrical model of TEG provides approximated, inaccurate and often unrealistic behaviour because the parameters of the thermal environment and those of TEG are not fully taken into account. On the other hand, the recent studies on TEG modelling provide accurate and realistic behaviours through electro-thermal models [27, 28]. Therefore, from the circuit designer's point of view, a faithful and fully electrical model under dif-

ferent working conditions (constant temperature gradient or heat flow) is needed. In this study, these fully electrical models of the TEG module with the contact thermal resistance taken into account are presented and the analytical expressions of the Thévenin equivalent circuit parameters are provided. By providing a realistic fully electrical TEG model reflecting different working and environmental conditions, the benefits are twofold: first, the design of TEG-powered circuits and systems is simplified and more realistic (system modeling and simulation), and second, the harvested thermal energy can be maximized by associating to the models the most adapted Maximum Power Point Tracking (MPPT) techniques, whose efficiency relies on the faithfulness of the used electrical models.

The main contributions of the present work can be summarized as follows:

- The unified approach allowing to model a TEG module with or without contact thermal resistance, regardless the working conditions, as a Thévenin equivalent circuit whose parameters are function of these working conditions and the TEG’s module properties;
- The fully electrical model of the TEG module under constant temperature gradient conditions. Although some recent studies ([27, 28]) have already addressed the electrical behaviour of the TEG mod-

ule under constant temperature gradient presenting partially the equivalent electrical model, the model proposed in this work is fully electrical and fits to the unified approach described above;

- The fully electrical model of the TEG module under constant heat flow conditions. This is the major and most original contribution of this work. To the best of our knowledge, a Thévenin fully electrical TEG model under constant heat flow has never been presented before in the literature. Montecucco et al. in [25] discovered experimentally the polynomial relationship between the load voltage and the TEG’s terminals temperature gradient without identifying analytically this relationship in function of the TEG’s parameters and working conditions.

This paper is organized as follows. The fully electrical TEG models under constant temperature gradient and constant heat flow conditions, without and with contact thermal resistance taken into account, are presented in Section 2 and 3 respectively. In Section 4, the presented models are discussed and validated through simulation. Finally, Section 5 concludes this paper giving some perspectives for future works.

## 2 TEG electrical modeling under constant temperature gradient conditions

In this section, an equivalent fully electrical Thévenin model of a TEG module without contact thermal resistance is first described. Next, considering the contact thermal resistance, the modified fully electrical TEG model is presented.

### 2.1 TEG electrical modeling without contact thermal resistance

As the contact thermal resistance is not considered, the thermal gradient  $\Delta T'$  applied on both hot and cold sides of a TEG module is the same as the thermal gradient  $\Delta T$  at the terminals of its p and n elements (see Fig. 1), consequently:

$$\Delta T' = (T'_H - T'_C) = \Delta T = (T_H - T_C). \quad (3)$$

The open circuit voltage of the TEG module is proportional to the temperature gradient across it. The proportionality factor is the Seebeck coefficient  $\alpha$  of the TEG module:

$$V_{OC} \Big|_{\Delta T' = \text{const}, \theta_c = 0} = \alpha \Delta T. \quad (4)$$

where  $\alpha = \alpha_p - \alpha_n$ ,  $\alpha_p$  and  $\alpha_n$  are the Seebeck coefficients of p and n semiconductor elements, respectively.

The output voltage of the TEG module is defined with:

$$V_O \Big|_{\Delta T' = \text{const}, \theta_c = 0} = \alpha \Delta T - R_E I. \quad (5)$$

The electrical power supplied to the load  $R_L$  is the difference between the input and output heat flow, which can be also defined as the product of the output voltage of the TEG module  $V_O$  and the load current  $I$ :

$$P \Big|_{\Delta T' = \text{const}, \theta_c = 0} = R_L I^2 = Q_H - Q_C = \alpha \Delta T I - R_E I^2. \quad (6)$$

Consequently, under constant temperature gradient conditions, the TEG module without contact thermal resistance  $\theta_c$  can be electrically modeled by an equivalent constant voltage source  $V_{eq}$  in series with an equivalent internal resistance  $R_{eq}$ :

$$V_{eq} \Big|_{\Delta T' = \text{const}, \theta_c = 0} = \alpha \Delta T = \alpha \Delta T'. \quad (7)$$

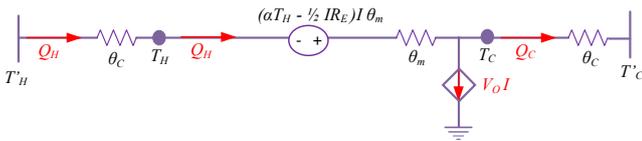
$$R_{eq} \Big|_{\Delta T' = \text{const}, \theta_c = 0} = R_E. \quad (8)$$

The equivalent Thévenin circuit with the parameters  $V_{eq}$  and  $R_{eq}$  presented with Eqs. 7 and 8 respectively is the electrical TEG model commonly used in the literature in the TEG-powered electronics circuit and system designs [1, 30]. As it can be seen from the previous demonstration, this electrical TEG model is only valid under constant temperature gradient conditions and does not consider contact thermal resistances, thus greatly approximating the realistic behaviour of the TEG module.

## 2.2 TEG electrical modeling with contact thermal resistance

In prior research works on thermoelectric power generation systems, the temperature gradient across the thermoelectric module is considered constant independently of electrical current levels drawn by the connected load  $R_L$ . In fact, the waste heat is transferred from the environment to the semiconductor elements of the TEG module through the contact plates, being at the origin of the thermal contact resistance  $\theta_c$ . Due to this contact thermal resistance  $\theta_c$ , the thermal gradient  $\Delta T' = (T'_H - T'_C)$  applied on both hot and cold sides of the TEG module is "seen" as  $\Delta T = (T_H - T_C) < \Delta T'$  at the terminals of the p and n semiconductor elements.

Using the duality between electrical and thermal phenomena, a thermal modeling of the TEG module with contact thermal resistance can be represented as in Fig. 2 [29]. This duality allows to represent the temperature by an electrical voltage, the heat flow by an electrical current, and the thermal resistance by an electrical resistance.



**Fig. 2** Thermal modeling of the TEG module with contact thermal resistance

The main goal of using the above mentioned electro-thermal duality is to model the TEG module as an equivalent Thévenin circuit (fully electrical model). To achieve this, the same approach is used throughout the paper: the electrical power of the TEG module is derived and presented as more as possible under the following form  $P = V_{eq}I - R_{eq}I^2$ , where  $V_{eq}$  is the equivalent Thévenin generator and  $R_{eq}$  is the corresponding equivalent resistance respectively. In the further discussion,  $\theta_m$  is the thermal resistance of N pairs of p and n semiconductor elements whereas  $\theta_c$  is the contact thermal resistance of contact plates on the top and the bottom of the p and n elements. Each junction formed by p and n elements is connected to two metal contacts from the top and bottom (see Fig. 1), which can be modeled by two series resistors with the value of  $\theta_c/N$  [22].

From Fig. 2, the expression of the heat flow through the metal contacts on the hot side of the TEG module can be written as:

$$Q_H = \frac{T'_H - T_H}{\theta_c}. \quad (9)$$

This heat flow is equal to the one flowing through the TEG module (see Eq. 1), which allows writing:

$$\frac{T_H + \alpha \theta_m T_H I - \frac{1}{2} R_E \theta_m I^2 - T_C}{\theta_m} = \frac{T'_H - T_H}{\theta_c}. \quad (10)$$

The difference between  $Q_H$  and  $Q_C$  (Eqs. 1 and 2) provides the relationship between the heat flow difference and the power transmitted to the load  $R_L$ :

$$Q_H - Q_C = I \cdot V_O \Big|_{\theta_c \neq 0}. \quad (11)$$

where  $V_O \Big|_{\theta_c \neq 0}$  is the output voltage at the terminals of the load  $R_L$ .

When considering the contact thermal resistance, and following the same analysis to determine the expression of the heat flow through the metal contacts on the cold side of the TEG module, it can be deduced from Fig. 2:

$$\frac{T'_H - T_H}{\theta_c} - \frac{T_C - T'_C}{\theta_c} = V_O \Big|_{\theta_c \neq 0} I. \quad (12)$$

Knowing that  $T_H = \Delta T + T_C$ , the temperature applied at the cold side of the p and n semiconductor elements is:

$$T_C = - \frac{\theta_c V_O \Big|_{\theta_c \neq 0} I + \Delta T - T'_H - T'_C}{2}. \quad (13)$$

By replacing  $T_C$  by its expression in Eq. 10, the expression of the thermal gradient at the terminals of the p and n elements with contact thermal resistance can be given by:

$$\Delta T \Big|_{\Delta T' = cnst, \theta_c \neq 0} = \left( \frac{1}{\alpha \theta_c \theta_m I + \theta_m + 2 \theta_c} \right) \times \{ \theta_c \theta_m [(\alpha \theta_c V_O \Big|_{\theta_c \neq 0} + R_E) I^2 + (V_O \Big|_{\theta_c \neq 0} - \alpha (T'_H + T'_C)) I] + \theta_m \Delta T' \}. \quad (14)$$

It should be noted that  $\Delta T \Big|_{\Delta T' = cnst, \theta_c \neq 0}$  is load current-dependent.

The output voltage across the load  $R_L$  can be defined by the following expression:

$$V_O \Big|_{\theta_c \neq 0} = \alpha \Delta T \Big|_{\Delta T' = cnst, \theta_c \neq 0} - R_E I. \quad (15)$$

By substituting Eq. 15 in Eq. 14, the thermal gradient at the terminals of the p and n elements becomes:

$$\Delta T \Big|_{\Delta T' = cnst, \theta_c \neq 0} = \left( \frac{1}{\alpha^2 \theta_c^2 \theta_m I^2 - \theta_m - 2 \theta_c} \right) \times \{ \alpha R_E \theta_c^2 \theta_m I^3 + \alpha \theta_c \theta_m (T'_H + T'_C) I - \theta_m \Delta T' \}. \quad (16)$$

Therefore, the open circuit voltage generated by the TEG module is expressed with:

$$V_{OC} \Big|_{\Delta T' = cnst, \theta_c \neq 0} = \alpha \Delta T \Big|_{\Delta T' = cnst, \theta_c \neq 0} (I = 0) = \frac{\theta_m}{\theta_m + 2 \theta_c} \alpha \Delta T'. \quad (17)$$

The output voltage across the load  $R_L$  can be expressed by:

$$V_O \Big|_{\Delta T' = cnst, \theta_c \neq 0} = \frac{\alpha R_L \Delta T \Big|_{\Delta T' = cnst, \theta_c \neq 0}}{R_L + R_E} = \frac{\alpha R_L \theta_m [\alpha \theta_c I (R_E \theta_c I^2 + T'_H + T'_C) - \Delta T']}{(R_L + R_E) (\alpha^2 \theta_c^2 \theta_m I^2 - \theta_m - 2 \theta_c)}. \quad (18)$$

with:

$$R_L = \frac{V_O \Big|_{\Delta T' = cnst, \theta_c \neq 0}}{I}. \quad (19)$$

By substituting Eq. 19 in Eq. 18, the resulting expression of the output voltage as a function of the load current  $I$  is derived:

$$V_O \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} = \left( \frac{1}{\alpha^2 \theta_c^2 \theta_m I^2 - \theta_m - 2\theta_c} \right) \times \{ [\alpha^2 \theta_c \theta_m (T'_H + T'_C) + R_E (\theta_m + 2\theta_c)] I - \alpha \theta_m \Delta T' \}. \quad (20)$$

The electrical power generated by the TEG module is defined as the product of the output voltage  $V_O$  with the load current  $I$ , and can be expressed with the following expression:

$$P \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} = \left( \frac{1}{\alpha^2 \theta_c^2 \theta_m I^2 - \theta_m - 2\theta_c} \right) \times \{ [\alpha^2 \theta_c \theta_m (T'_H + T'_C) + R_E (\theta_m + 2\theta_c)] I^2 - \alpha \theta_m \Delta T' I \}. \quad (21)$$

As expected,

$$\lim_{\theta_c \rightarrow 0} P \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} = P \Big|_{\Delta T' = \text{cnst}, \theta_c = 0}. \quad (22)$$

Knowing that when the contact thermal resistance converges towards zero ( $\theta_c \rightarrow 0$ ), the thermal gradient  $\Delta T'$  is equal to  $\Delta T$  (Eq. 3).

Eq. 21 does not allow to express the electrical power generated by the TEG module in the desired form with equivalent voltage and resistance, as it was the case without contact thermal resistance (Eq. 6). For these reasons, an approximate expression of the electrical power can be proposed. This can be achieved by neglecting the term  $\alpha^2 \theta_c^2 \theta_m I^2$  behind  $\theta_m + 2\theta_c$ . The V-I characteristic of the TEG module with the contact thermal

resistance is nonlinear, due to the term  $\alpha^2 \theta_c^2 \theta_m I^2$ . Analytically, this non-linearity occurs for large values of the load current greater few orders of magnitude than the short-current value  $I_{SC}$ , which explains why this term can be neglected. Moreover, the V-I characteristic is linear for positive values of the load current,  $I > 0$ . This approximation is also justified by the order of magnitude of  $\alpha^2 \approx 10^{-4}$  whereas the order of magnitudes of other parameters is about unity. By applying this approximation to Eq. 21, it becomes:

$$P \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} \approx \frac{\theta_m}{\theta_m + 2\theta_c} \alpha \Delta T' I - [R_E + \frac{\alpha^2 \theta_c \theta_m}{\theta_m + 2\theta_c} (T'_H + T'_C)] I^2. \quad (23)$$

If we apply the same approximation to Eq. 20, the output voltage  $V_O$  of the TEG module becomes:

$$V_O \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} \approx \frac{\theta_m}{\theta_m + 2\theta_c} \alpha \Delta T' - [R_E + \frac{\alpha^2 \theta_c \theta_m}{\theta_m + 2\theta_c} (T'_H + T'_C)] I. \quad (24)$$

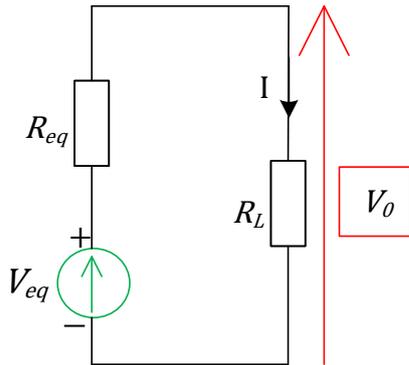
Consequently, under constant temperature gradient conditions, the TEG module with contact thermal resistance  $\theta_c$  can also be modeled by an equivalent constant voltage source  $V_{eq}$  in series with an equivalent internal resistance  $R_{eq}$  (Thévenin equivalent circuit):

$$V_{eq} \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} = \alpha \Delta T' \frac{\theta_m}{\theta_m + 2\theta_c}. \quad (25)$$

$$R_{eq} \Big|_{\Delta T' = \text{cnst}, \theta_c \neq 0} \approx R_E + \frac{\alpha^2 \theta_c \theta_m (T'_H + T'_C)}{\theta_m + 2\theta_c}. \quad (26)$$

### 2.3 Conclusion

To conclude this section, one can summarize the results of the proposed unified approach to model the TEG module as a Thévenin equivalent circuit under constant temperature gradient conditions. The equivalent electrical model of the TEG module under constant temperature gradient conditions can be represented as in Fig. 3, where  $V_{eq}$  and  $R_{eq}$  are given by Eqs. 7 and 8 for the model without contact thermal resistance, and by Eqs. 25 and 26 for the model with contact thermal resistance.



**Fig. 3** Equivalent electrical model of the TEG under constant temperature gradient conditions.

### 3 TEG electrical modeling under constant heat flow conditions

In most practical applications, as in the case of thermal energy recovery systems for automobile exhaust gas, the TEG module is subject to a constant thermal input flow

instead of a constant temperature gradient. This is in literature designated as "constant heat" operation conditions. Kumar et al. in [23] observed a significant difference between the electric power generated by a TEG module with an exhaust gas flow rate and with a temperature gradient. Even if the available input thermal heat flow change over time, its rate of variation will be few order of magnitude slower than the dynamic electrical response of the TEG module [24].

In this section, by applying the same approach used in the previous sections for the TEG module under constant gradient conditions, the equivalent fully electrical models of the TEG module without and with contact thermal resistance under constant heat flow conditions are presented. In both cases, the input thermal power  $Q_H$  flowing through the TEG module and the cold source temperature  $T'_C$  are assumed constant throughout this discussion. In most TEG systems, the cold side temperature remains almost constant with relatively small changes in thermal power flowing into the cold side [25]. Moreover, to the best of our knowledge, no fully electrical TEG model under constant heat flow conditions has been previously presented in the literature.

### 3.1 TEG electrical modeling without contact thermal resistance

Given that the contact thermal resistance  $\theta_c$  is not taken into account, Eq. 3 is verified. Moreover, from Eq. 1 it can be obtained:

$$\Delta T \Big|_{Q_H=cnst, \theta_c=0} = \frac{\frac{1}{2}R_E\theta_m I^2 - \alpha\theta_m T'_C I + \theta_m Q_H}{1 + \alpha\theta_m I}. \quad (27)$$

The open circuit voltage  $V_{OC}$  of the TEG module is expressed with:

$$\begin{aligned} V_{OC} \Big|_{Q_H=cnst, \theta_c=0} &= \alpha \Delta T \Big|_{Q_H=cnst, \theta_c=0, I=0} \\ &= \alpha\theta_m Q_H. \end{aligned} \quad (28)$$

The electrical power delivered by the TEG module is defined by:

$$\begin{aligned} P \Big|_{Q_H=cnst, \theta_c=0} &= \alpha \Delta T \Big|_{Q_H=cnst, \theta_c=0} I - R_E I^2 \\ &= \frac{-\frac{1}{2}\alpha\theta_m R_E I^3 - (R_E + \alpha^2\theta_m T'_C) I^2 + \alpha\theta_m Q_H I}{1 + \alpha\theta_m I}. \end{aligned} \quad (29)$$

An approximate expression of this electrical power can be proposed by using the development of Maclaurin up to the 4th order:

$$\begin{aligned} P \Big|_{Q_H=cnst, \theta_c=0} &= \alpha Q_H \theta_m I - \{R_E + \alpha^2 Q_H \theta_m^2 \\ &+ \alpha^2 \theta_m T'_C - [\frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C + \alpha^3 \theta_m^3 Q_H] I \\ &+ [\alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 + \frac{\alpha^2 R_E \theta_m^2}{2}] I^2\} I^2. \end{aligned} \quad (30)$$

Consequently, under constant heat flow conditions, the TEG module without contact thermal resistance can also be modeled by an equivalent constant voltage source

$V_{eq}$  in series with a variable equivalent internal resistance  $R_{eq}$  (Thévenin equivalent circuit):

$$V_{eq} \Big|_{Q_H=cnst, \theta_c=0} = \alpha\theta_m Q_H. \quad (31)$$

$$\begin{aligned} R_{eq} \Big|_{Q_H=cnst, \theta_c=0} &\approx R_E + \alpha^2 Q_H \theta_m^2 + \alpha^2 \theta_m T'_C \\ &- [\frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C + \alpha^3 \theta_m^3 Q_H] I \\ &+ [\alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 + \frac{\alpha^2 R_E \theta_m^2}{2}] I^2. \end{aligned} \quad (32)$$

### 3.2 TEG electrical modeling with contact thermal resistance

Using the expression of the thermal gradient at the terminals of the p and n elements under constant heat flow conditions given by Eq. 27, and knowing that  $T'_C$  is different from  $T_C$  when considering contact thermal resistance, the thermal gradient at the terminals of the p and n elements with contact thermal resistance can be expressed as:

$$\Delta T \Big|_{Q_H=cnst, \theta_c \neq 0} = \frac{\frac{1}{2}R_E\theta_m I^2 - \alpha\theta_m T_C I + Q_H\theta_m}{1 + \alpha\theta_m I}. \quad (33)$$

By substituting the expression of  $T_C$  given by Eq. 13 in Eq. 33, we obtain:

$$\begin{aligned} \Delta T \Big|_{Q_H=cnst, \theta_c \neq 0} &= \left( \frac{1}{\alpha\theta_m I + 2} \right) \times \\ &\{(\theta_m (R_E + \alpha\theta_c V_O \Big|_{\theta_c \neq 0})) I^2 - \alpha\theta_m (T'_H + T'_C) I \\ &+ 2Q_H \theta_m\}. \end{aligned} \quad (34)$$

However, this thermal gradient must be expressed only as a function of  $Q_H$  and  $T'_C$ . From Eq. 9 we can write:

$$T'_H = T_H + Q_H \theta_c = \Delta T + T_C + Q_H \theta_c. \quad (35)$$

and from Eq. 12:

$$T_C = -\theta_c V_O \Big|_{\theta_c \neq 0} I + T'_C + Q_H \theta_c. \quad (36)$$

By substituting Eq. 36 in Eq. 35, and then by substituting the result in Eq. 34, the thermal gradient across the p and n elements becomes:

$$\begin{aligned} \Delta T \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} &= \left( \frac{1}{1 + \alpha \theta_m I} \right) \times \\ &\{ \theta_m \left( \frac{1}{2} R_E + \alpha \theta_c V_O \Big|_{\theta_c \neq 0} \right) I^2 - \alpha \theta_m (T'_C + Q_H \theta_c) I \\ &\quad + Q_H \theta_m \}. \end{aligned} \quad (37)$$

As expected, under constant heat flow conditions:

$$\lim_{\theta_c \rightarrow 0} \Delta T \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} = \Delta T \Big|_{Q_H = \text{cnst}, \theta_c = 0}. \quad (38)$$

The open circuit voltage of the TEG module is given

by:

$$\begin{aligned} V_{OC} \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} &= \alpha \Delta T \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} (I = 0) \\ &= \alpha \theta_m Q_H. \end{aligned} \quad (39)$$

By following the same analysis used for the determination of the output voltage in Section 2.2, the output voltage of the TEG module can be obtained:

$$\begin{aligned} V_O \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} &= \left( \frac{1}{\alpha^2 \theta_c \theta_m I^2 - \alpha \theta_m I - 1} \right) \times \\ &\{ \frac{1}{2} \alpha R_E \theta_m I^2 + (R_E + \alpha^2 \theta_m T'_C + \alpha^2 Q_H \theta_c \theta_m) I \\ &\quad - \alpha Q_H \theta_m \}. \end{aligned} \quad (40)$$

By the same, the electrical power generated by the TEG module is given by:

$$P \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} = \left( \frac{1}{\alpha^2 \theta_c \theta_m I^2 - \alpha \theta_m I - 1} \right) \times$$

$$\begin{aligned} &\{ \frac{1}{2} \alpha R_E \theta_m I^3 + (R_E + \alpha^2 \theta_m T'_C + \alpha^2 Q_H \theta_c \theta_m) I^2 \\ &\quad - \alpha Q_H \theta_m I \}. \end{aligned} \quad (41)$$

Similarly, using the development of Maclaurin up to the 4th order, the expression of the electrical power can be approximated with:

$$\begin{aligned} P \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} &\approx \alpha Q_H \theta_m I - \{ R_E + \alpha^2 \theta_c \theta_m Q_H \\ &\quad + \alpha^2 \theta_m^2 Q_H + \alpha^2 \theta_m T'_C - [ \frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C \\ &\quad + \alpha^3 \theta_m^3 Q_H + 2 \alpha^3 \theta_m^2 \theta_c Q_H ] I + [ \alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 \\ &\quad + \frac{\alpha^2 R_E \theta_m^2}{2} + \alpha^2 R_E \theta_m \theta_c + \alpha^4 \theta_m^2 \theta_c Q_H (\theta_c + 3 \theta_m) \\ &\quad + \alpha^4 \theta_m^3 \theta_c^2 T'_C ] I^2 \} I^2. \end{aligned} \quad (42)$$

The choice to develop only up to the 4th order using Maclaurin series can be justified by the results reported by Montecucco et al. in [25] where a quadratic relationship between the TEG's output voltage and the temperature gradient has been observed experimentally. If this nonlinear voltage relationship is transposed to the power, the four order polynomial current dependence is obtained.

As expected, under constant heat flow conditions:

$$\lim_{\theta_c \rightarrow 0} P \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} = P \Big|_{Q_H = \text{cnst}, \theta_c = 0}. \quad (43)$$

Applying the same development of Maclaurin to Eq. 40, the output voltage of the TEG module becomes:

$$\begin{aligned} V_O \Big|_{Q_H = \text{cnst}, \theta_c \neq 0} &\approx \alpha Q_H \theta_m - \{ R_E + \alpha^2 \theta_c \theta_m Q_H \\ &\quad + \alpha^2 \theta_m^2 Q_H + \alpha^2 \theta_m T'_C - [ \frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C \\ &\quad + \alpha^3 \theta_m^3 Q_H + 2 \alpha^3 \theta_m^2 \theta_c Q_H ] I + [ \alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 \end{aligned}$$

$$+\frac{\alpha^2 R_E \theta_m^2}{2} + \alpha^2 R_E \theta_m \theta_c + \alpha^4 \theta_m^2 \theta_c Q_H (\theta_c + 3\theta_m) + \alpha^4 \theta_m^3 \theta_c^2 T'_C \} I^2 \} I. (44)$$

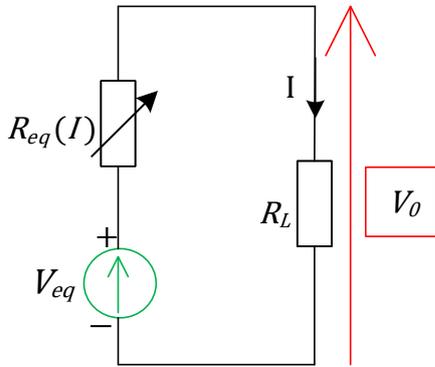
Consequently, under constant heat flow conditions, the TEG module with contact thermal resistance can also be modeled with an equivalent constant voltage source  $V_{eq}$  in series with a variable equivalent internal resistance  $R_{eq}$  (Thévenin equivalent circuit):

$$V_{eq} \Big|_{Q_H=const, \theta_c \neq 0} = \alpha \theta_m Q_H. (45)$$

$$R_{eq} \Big|_{Q_H=const, \theta_c \neq 0} \approx R_E + \alpha^2 \theta_c \theta_m Q_H + \alpha^2 \theta_m^2 Q_H + \alpha^2 \theta_m T'_C - \left[ \frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C + \alpha^3 \theta_m^3 Q_H + 2\alpha^3 \theta_m^2 \theta_c Q_H \right] I + \left[ \alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 + \frac{\alpha^2 R_E \theta_m^2}{2} + \alpha^2 R_E \theta_m \theta_c + \alpha^4 \theta_m^2 \theta_c Q_H (\theta_c + 3\theta_m) + \alpha^4 \theta_m^3 \theta_c^2 T'_C \right] I^2. (46)$$

### 3.3 Conclusion

To conclude this section, it is important to underline one of the major contributions of this paper: a TEG



**Fig. 4** Equivalent electrical model of the TEG module under constant heat flow conditions.

module under constant heat flow conditions can also be represented as in Fig. 4, by an equivalent Thévenin electrical model where  $V_{eq}$  and  $R_{eq}$  are given by Eqs. 31 and 32 for the TEG module without contact thermal resistance, and by Eqs. 45 and 46 for the TEG module with contact thermal resistance.

## 4 Results and discussion

Table 1 summarizes the analytical expressions of the parameters for the fully electrical TEG models (Thévenin equivalent circuit) detailed previously. It includes the expressions of the equivalent open circuit voltage and the equivalent internal resistance of the TEG module under constant temperature gradient, and constant heat flow conditions with  $\theta_c = 0$  and with  $\theta_c \neq 0$ .

Under constant temperature gradient conditions, as it can be seen from Table 1, the equivalent open-circuit voltage of the TEG module with the contact thermal resistance taken into account is lower than the one in the absence of the latter. Apertet et al. observed in [28] a decrease in the open-circuit voltage due to an additional term depending on the load current and TEG's parameters including the contact thermal resistance  $\theta_c$ . However, from the electrical modeling's point of view, the equivalent Thévenin generator which represents the open-circuit voltage of the TEG module in the absence of the load current, should not be load current dependent.

**Table 1** Summary of electrical modeling of the TEG module under constant temperature gradient and heat conditions

	Constant $\Delta T'$		Constant $Q_H$	
	$V_{eq}$	$R_{eq}$	$V_{eq}$	$R_{eq}$
$\theta_c = 0$	$\alpha\Delta T'$	$R_E$	$\alpha\theta_m Q_H$	$R_E + \alpha^2 Q_H \theta_m^2 + \alpha^2 \theta_m T'_C$ $-[\frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C + \alpha^3 \theta_m^3 Q_H]I$ $+[\alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 + \frac{\alpha^2 R_E \theta_m^2}{2}]I^2$
$\theta_c \neq 0$	$\frac{\theta_m}{\theta_m + 2\theta_c} \alpha\Delta T'$	$R_E + \frac{\alpha^2 \theta_c \theta_m (T'_H + T'_C)}{\theta_m + 2\theta_c}$	$\alpha\theta_m Q_H$	$R_E + \alpha^2 \theta_c \theta_m Q_H + \alpha^2 \theta_m T'_C + \alpha^2 \theta_m^2 Q_H$ $-[\frac{\alpha R_E \theta_m}{2} + \alpha^3 \theta_m^2 T'_C + \alpha^3 \theta_m^3 Q_H$ $+ 2\alpha^3 \theta_m^2 \theta_c Q_H]I + [\alpha^4 \theta_m^3 T'_C + \alpha^4 Q_H \theta_m^4 + \frac{\alpha^2 R_E \theta_m^2}{2}$ $+ \alpha^2 R_E \theta_m \theta_c + \alpha^4 \theta_m^2 \theta_c Q_H (\theta_c + 3\theta_m)$ $+ \alpha^4 \theta_m^3 \theta_c^2 T'_C]I^2$

dent as it was presented in [28] (Eq. 11 in [28]). In the presented fully electrical TEG model, the equivalent voltage source is constant for a given temperature gradient and is valid regardless the value of the load current. On the other hand, the equivalent electrical resistance of the TEG module with the contact thermal resistance taken into account is larger than without it. The additional term presented in Table 1 depends on the thermal resistances, Seebeck coefficient and operating conditions of the TEG module. This term has also been identified by Apertet et *al.* in [28] in terms of the figure of merit  $Z\bar{T}$ , electrical resistance  $R_E$  and thermal conductances.

Under constant heat flow conditions, the results presented in Table 1 show that there is no difference between the equivalent open-circuit voltage of the TEG module with and without the contact thermal resistance taken into account. However, the relation of the TEG's equivalent electrical resistance (see Table 1) is non-linear and it depends on both the TEG's parameters and the load current. However, this resistance is composed of constant load current independent (first line of the equivalent resistance expression in Table 1) and load current dependent terms (the rest of the expression). It can be noticed that this equivalent electrical resistance in open-circuit conditions ( $I = 0$ ) without considering the contact thermal resistance is greater

than the  $R_E$ , which is the equivalent resistance under constant temperature gradient conditions. Therefore, from the presented electrical model it can be also deduced that the output voltage delivered to the load is a non-linear function of the load current and the TEG's parameters. Montecucco et *al.* observed in [25] this non-linear variation of the TEG's output voltage and expressed it as a function of the load current, temperature gradient and constant polynomial coefficients. Moreover, these constant polynomial coefficients were determined experimentally for the tested module, and no relation was established between them and the TEG's parameters. However, in the presented fully electrical model of the TEG module, all coefficients of this non-linearity are clearly identified in terms of the TEG's parameters and load current.

In order to appreciate the faithfulness of the presented fully electrical TEG models under different operation conditions, the analytical expressions of Table 1 were validated through simulation using Matlab software. The used physical parameters of the TEG module are presented in Table 2. The percentage ratio  $\beta$  between the contact thermal resistance  $\theta_c$  and the thermal resistance  $\theta_m$  have been varied from 0% (case of the TEG module with  $\theta_c = 0$ ) to 100% (case of the TEG module with a value of  $\theta_c$  equal to its thermal resistance

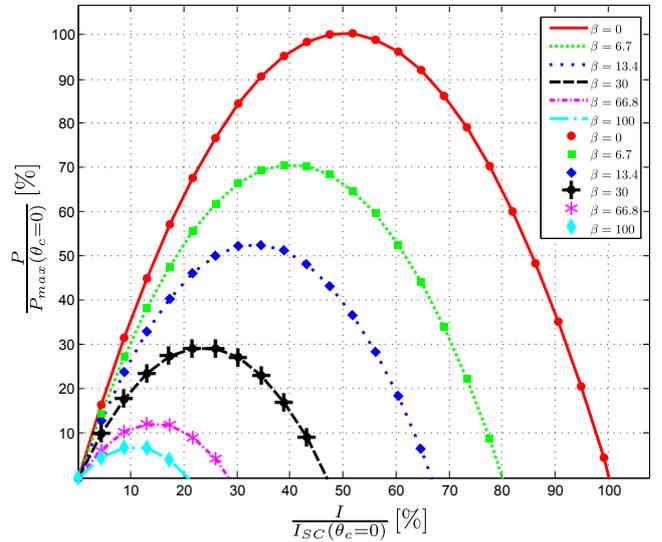
$\theta_m$ ). For all values of  $\theta_c$ , the electrical power and the output voltage of the TEG module were observed.

Fig. 5 shows the TEG's electrical power versus load current for different values of the contact thermal resistance  $\theta_c$  under constant temperature gradient conditions. The parabolic curves in solid line result from the analytical expression of the electrical power without approximation (Eq. 21), and the curves in dotted line result from the approximate one (Eq. 23). It can be seen from Fig. 5 that the electric power of the TEG module obtained with the proposed approximated expression is close to the exact one. As expected from the physical modeling studies, the electrical power decreases as  $\theta_c$  increases [27,28]. This electrical power reduction results from the decrease of the effective thermal gradient

**Table 2** Numerical values of the TEG used in simulations

Parameter	Value	Description
N	127	Number of p-n pellets
$\alpha$ [V/K]	0.0531876	Seebeck coefficient
$R_E$ [ $\Omega$ ]	1.6	Electrical resistance
$\theta_m$ [K/W]	1.498	Thermal resistance
$\theta_c$ [K/W]	0 to 1.5	Contact thermal resistance on the top / bottom of TEG
$T'_H$ [K]	368	Temperature at the hot side
$T'_C$ [K]	298	Temperature at the cold side
$Q_H$ [W]	70	Heat flow

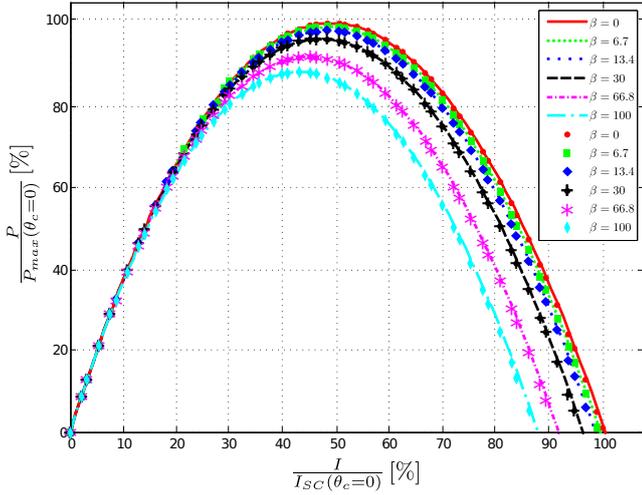
applied to the TEG module, which moves from  $\Delta T'$  to  $\Delta T$  ( $\Delta T < \Delta T'$ ), essentially caused by the presence of the contact thermal resistance  $\theta_c$ .



**Fig. 5**  $P = f(I, \theta_c)$  under constant temperature gradient conditions, where  $I_{SC}$  is the short-circuit current.

Fig. 6 shows the TEG's electrical power versus load current for different values of the contact thermal resistance  $\theta_c$  under constant heat flow conditions. Similarly as in the constant temperature gradient case, the parabolic curves in solid line result from the exact analytical expression of the electrical power (Eq. 41), whereas the curves in dotted line result from the approximate expression obtained by using the development of Maclaurin (Eq. 42). As in the previous case, the electrical power decreases as  $\theta_c$  increases. However, this decrease is smaller compared to the one obtained in the case of the TEG module under constant temperature gradient

conditions. It does mean that the electrical modeling of the TEG under constant heat flow conditions is very little influenced by the presence of  $\theta_c$  compared to the one under constant temperature gradient conditions.

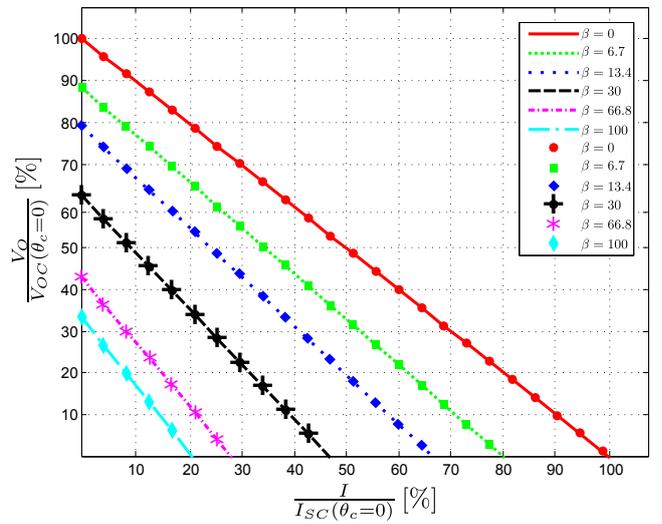


**Fig. 6**  $P = f(I, \theta_c)$  under constant heat flow conditions.

Fig. 7 shows the TEG's output voltage  $V_O$  versus load current  $I$  for different values of the contact thermal resistance  $\theta_c$  under constant temperature gradient conditions. It can be seen that the exact analytical expression of the output voltage of the TEG module (Eq. 20) presented with solid lines for different values of the contact thermal resistance are superposed to the dotted lines used to present the approximated expression given by Eq. 24. This justifies the validity of the used approximation. The equivalent internal resistance of the TEG module  $R_{eq}$  determines the slope of the linear V-I characteristic. As expected from the physical modeling studies, Fig. 7 illustrates that more  $\theta_c$  increases, more the

output voltage of the TEG module decreases and the slope of the V-I characteristic becomes steeper [27, 28].

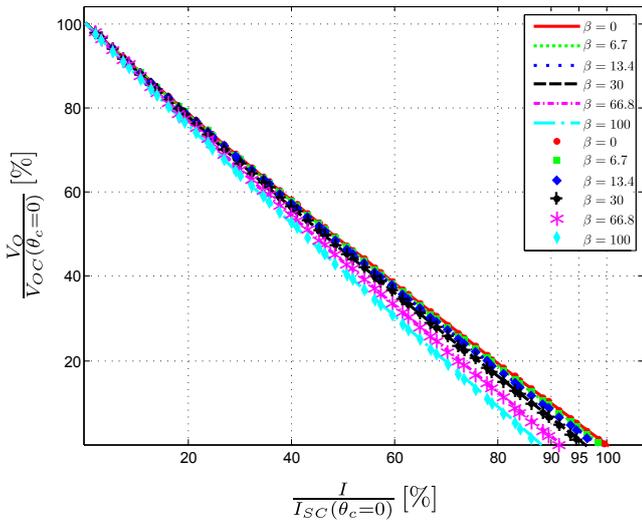
This can be interpreted by the increase of the equivalent internal electrical resistance of the TEG module "seen" by the connected load  $R_L$ , as represented with Eq. 26. Notice that the open circuit voltage of the TEG module decreases as  $\theta_c$  increases.



**Fig. 7**  $V_O = f(I, \theta_c)$  under constant temperature gradient conditions.

Fig. 8 shows the TEG's output voltage  $V_O$  versus load current  $I$  for different values of the contact thermal resistance  $\theta_c$  under constant heat flow conditions. Similarly, the solid lines used to represent the exact analytical equation of the output voltage of the TEG module (Eq. 40), are superposed to the dotted lines used to represent the approximated expression given by Eq. 44. This result justifies the validity of the approximation made with the development of Maclaurin.

Unlike the constant temperature gradient conditions, the TEG module under constant heat flow conditions has the same open circuit voltage  $V_{OC}$ , whatever the value of the contact thermal resistance  $\theta_c$ . The slope of the V-I characteristic in this case is slightly modified with  $\theta_c$ . This can be explained by comparing the equivalent internal electrical resistance of the TEG module with  $\theta_c = 0$  and  $\theta_c \neq 0$  given by Eqs. 32 and 46 respectively, which are almost equal. The main difference lies in the additional terms present in Eq. 46 whose contribution is low since they are multiplied either by  $\alpha^2$  or  $\alpha^4$ .



**Fig. 8**  $V_O = f(I, \theta_c)$  under constant heat flow conditions.

## 5 Conclusion

In this study, the fully electrical modeling of the TEG module taking into account its contact thermal resis-

tance under different operation conditions is presented and discussed. An unified fully electrical TEG modeling approach based on a Thévenin equivalent circuit were proposed and the analytical expressions of the parameters  $V_{eq}$  and  $R_{eq}$  were identified without and with the contact thermal resistance: first under constant temperature gradient, and second under constant heat flow conditions. The electrical model of the TEG module under constant temperature gradient conditions is linear and depends only on the TEG's electrical and thermal parameters. On the other hand, the electrical model of the TEG module under constant heat flow conditions is non linear and load current dependent. The proposed fully electrical TEG models were validated through simulation. The obtained simulation results fit into the already reported physical behaviour of the TEG modules presented in recent research studies [25,27,28]. The proposed fully electrical models of the TEG module can be used in the design of the TEG powered electronics circuits and systems in simulation and modeling phases as well in the development of the most adapted MPPT techniques to maximize the harvesting of the available waste thermal energy [1, 30].

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