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# Polynomial time algorithm for multi-level in series capacitated lot-sizing problems with constant batch size

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## Abstract

We consider a multi-level lot-sizing problem in a serial distribution network. Customers demands are to be satisfied at the last level of the network. At each level, units can be carried in stock, and the inventory is replenished from the upstream level using batch deliveries, where a fixed cost is incurred for each batch ordered. In addition, we also consider a limitation on the number of batches that can be ordered at any period. As far as we know, the status of this multi-level lot-sizing problem with batch deliveries and capacities on the size of the orders is open. Assuming stationary and identical batch sizes and capacities for all the levels, we derive some structural properties on the optimal policy, and propose a polynomial time algorithm based on a decomposition of the problem into induced connected components. Our algorithm is polynomial both in the length of the planning horizon and in the number of levels.

## 1 Introduction

We consider the distribution of a single item through a logistics network constituted of  $N$  levels in series. In the following, levels are numbered from 1 to  $N$ , where level 1 is the most upstream level and level  $N$  is the most downstream level of the network. The network is facing a deterministic demand at the last level, over a time horizon divided into  $T$  periods. At each period  $t$ , demand  $d_t$  must be satisfied on time, that is, neither lost sale nor backlogging are allowed. Level  $N$  can satisfy the demands from its stock or/and by ordering some units from level  $(N - 1)$ , which in its turn

can order from its upstream level ( $N - 2$ ), and so on. We consider that level 1 orders the units from an external supplier. The objective is to serve all the demands at the minimum cost. There are two kind of costs in the system: An unit holding cost  $h_t^i$  is paid to carry one item in stock from period  $t$  to  $t + 1$  at level  $i$ , while a procurement cost is paid for to order units from upstream. We consider in this paper that orders are delivered by batch of size  $C$ : In addition to a unit ordering cost  $p_t^i$ , a fixed cost  $k_t^i$  is charged for each batch. The cost of ordering  $x$  units at level  $i$  in period  $t$  is thus equal to  $p_t^i x + \lceil x/C \rceil k_t^i$ . We also consider a limitation  $m$  on the number of batches of an order, that is, at most  $mC$  units can be ordered at any period and any level. Decision variables  $x_t^i$  represents the amount ordered at period  $t$  at level  $i$  and  $s_t^i$  is the stock level at the end of period  $t$  at level  $i$ .

The multi-level lot-sizing problem with a serial structure and identical and stationary batch capacities at each level can be formulated as follows:

$$\min \sum_{t=1}^T \sum_{i=1}^N (\lceil x_t^i / C \rceil k_t^i + p_t^i x_t^i + s_t^i h_t^i) \quad (\text{P})$$

subject to

$$x_t^i + s_{t-1}^i = x_t^{i+1} + s_t^i, \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N - 1\}, \quad (1)$$

$$x_t^N + s_{t-1}^N = d_t + s_t^N, \quad \forall t \in \{1, \dots, T\}, \quad (2)$$

$$x_t^i \leq mC, \quad \forall t \in \{1, \dots, T\}, i \in \{1, \dots, N\}, \quad (3)$$

$$s_0^i = 0, \quad \forall i \in \{1, \dots, N\}, \quad (4)$$

$$x_t^i \geq 0, \quad \forall t \in \{1, \dots, T\}, \forall i \in \{1, \dots, N\} \quad (5)$$

$$s_t^i \geq 0, \quad \forall t \in \{1, \dots, T\}, \forall i \in \{1, \dots, N\} \quad (6)$$

Notice that this multi-level lot-sizing problem with batch deliveries captures two important special cases. In the case where  $m = 1$ , we have a multi-level capacitated lot-sizing problem (CLSP), which can model a production line of  $N$  machines in series, each one with a hard production capacity  $C$  and where the fixed cost per batch can be identified with a set-up of production. In the case where  $m$  is set to a large integer ( $m \geq d_1 + \dots + d_T$ ), we have a multi-level uncapacitated lot-sizing problem with batch deliveries, which can model a supply chain where items are transported using identical shipping containers or trucks. As far as we know, the status of both problems is open when the number  $N$  of levels is part of the inputs.

We establish that this multi-level lot-sizing problem can be solved in polynomial time assuming that: (i) unit ordering and holding costs follow a *non-speculative motives echelon cost structure*. At the first level the assumption is similar to that used in single-level problems:  $p_t^1 + h_t^1 \geq p_{t+1}^1$  must hold for any period  $t \in \{1, \dots, T - 1\}$ . At the other levels, the formula is slightly modified to take into account the holding

cost saved at the previous level:  $p_t^i + (h_t^i - h_t^{i-1}) \geq p_{t+1}^i$ , for all  $i \in \{2, \dots, N\}$ ,  $t \in \{1, \dots, T-1\}$  (ii) at each level, the fixed costs per batch are non-increasing with time, except for the last level for which they are unrestricted.

## 2 Literature review

The single-level lot-sizing problem with batch deliveries has been studied by Van Vyve (2007)[1] in the case of constant batch size and a time-varying limit on the number of batches which can be ordered at each period.

The multi-level capacitated problem (MCLSP) has been investigated quite recently. Kaminsky and Simchi-Levi (2003) [2] considered a 2-level lot-sizing with an intermediary transportation level problem having capacities at both levels. Sargut and Romeijn (2007) [4] provided different polynomial algorithms for the 2-level capacitated lot-sizing problem integrating backlogging and subcontracting decisions, with stationary production capacities only at the first level. Van Hoesel et al. (2005)[3] proposed an algorithm for the multi-level lot-sizing problem with production capacities at the first level (MLSP-PC), with general concave cost functions, polynomial with a fixed number of levels  $L$ , but whose complexity grows exponentially with  $L$ . Considering more restrictive cost structures, they were able to propose an algorithm that stays polynomial even if  $L$  is part of the input. Finally, using the novel concept of basis path, Hwang et al. (2013)[5] developed the first polynomial algorithm for MLSP-PC with general concave costs and the number of levels being part of the inputs.

To the best of our knowledge, we are the first to propose an algorithm which is both polynomial in time and in the number of levels, for the multi-level lot-sizing problem with capacities (hard or soft) at each level.

## 3 Decomposition of a solution

Our algorithm is based on a set of dominant properties, which allow to highlight a particular structure of extreme point. More precisely, we show that each full batch ordered at a period  $t$  and a level  $i$  is also ordered in  $t$  at the higher levels  $(1, \dots, i-1)$ . Similarly, if a fractional batch is ordered at a period  $t$  and a level  $i$ , every lower levels will also order a fractional batch.

We then define the key structure of our algorithm, called a Box. A Box denoted  $B^i(r, s)$  is a connected component induced by levels  $(i, i+1, \dots, N)$  and such that all its nodes are located between periods  $r$  and  $s$ . We show that two consecutive boxes on the same levels don't overlap. We are able to evaluate the amount of the flows in a Box  $B^i(r, s)$  independently for given values of  $i$ ,  $r$  and  $s$ . This can be done by observing the decomposition of  $B^i(r, s)$  into a set of boxes on levels  $(i+1, \dots, N)$ .

Using the concept of basis path introduced by Hwang et al. (2013)[5], the flow over level  $i$  can be determined as well as the amount of the fractional orders at level  $i + 1$  between periods  $r$  and  $s$ . In other words, a Box  $B^i(r, s)$  can be decomposed into a set of boxes having one less level than  $B^i(r, s)$ , and if the amount stored and ordered at level  $i$  results from this decomposition. The same process is thus performed for each Box on levels  $(i + 1, \dots, N)$  which are in turn decomposed into boxes on level  $(i + 2, \dots, N)$ . The process is repeated until the last level is reached.

**Theorem 1** *The multi-level lot-sizing problem with our cost structure can be solved in time complexity  $O(NT^4)$ .*

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