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Modeling of the Coupled Heat and Mass Transfer during the Drying of the Tropical Woods Coming From Cameroon: The Case of Lotofa and Sapelle

Dr. Merlin Simo Tagne¹, Dr. Romain Remond², Pr. Yann Rogaume³, Pr. André Zoulalian⁴

¹Douala Higher Institute of Technology (DIIT), Douala, Cameroon, PO Box 1623
²,³,⁴University of Lorraine, LERMAB,ENSTIB, Epinal, France, PO Box 1041, 88051 Epinal cedex 9

Abstract—We are modeling the drying of tropical woods in this work. Experimental measures are made on sapelle (Entandrophragma Cylindricum) and lotofa (Sterculia Rhinopetala) coming from Cameroon forest. A part of thermophysical parameters used is experimental obtained on each wood species and another is taking in the literature. Numerical simulations results of the drying of our wood samples give a good description of experimental points. The present model can to be taken to explain the drying of the other tropical woods, but a determination of thermophysical parameters of each wood to dry is necessary.

Keywords—Drying, Heat and mass transfer, Modeling, Tropical woods.

I. INTRODUCTION

Tropical countries have vast forests that contain many species of wood [1-3]. These varieties enable the majority the principal European and Asian importing these woods to be very interested. Unfortunately, the majority of wood is exported from Africa in rough timber, as many African countries as Cameroon and Gabon require that the first transformation must be done locally [4]. In effect, local transformation of wood enables job creation, promotes employment relative to the transformation of the wood and to struggle against deterioration of the exchange terms because, when the wood is sold in Africa, there are to be comes very expensive. Though, economies of the tropical countries do independ of the exterior market and the effects as the recent worldly financial crisis. Cameroon has many forests and surfaces mine potentials to become one of the rich African countries [3]. Then it is very important to develop a technology and science of tropical countries in order to answer the immediate needs as preservation of wood by thermal drying. Thus, the utilization of the forest is optimized and ecological advantages as fixation of carbon, regeneration of oxygen, preservation of animal diversity, of grass and trees can be preserved.

In this paper, we applied one model which explains the drying of tropical woods [5,6] to describe the drying of sapelle (Entandrophragma Cylindricum) and lotofa (Sterculia Rhinopetala). These woods are abundant in the region of Central Africa and are highly demanded in the interior and exterior markets of Cameroon [7].

II. MATERIALS AND METHODS

A. Modeling

A present model is developed in the literature and has been validated during a numerical simulation of the drying of ayous and ebony woods in other conditions [5,6]. It is very difficult to use all drying models without simplification because a drying process shows multidimensional characteristics and the samples to dry present many thermophysical parameters unknown[8]. Therefore, we are taking in this work the following hypothesis:

- A size of sample to dry is constant, homogeneous and it is chemically inert;
- As soon as the drying begins, water that comes from wood is made of vapor and free water, bound water is extracted when the fiber saturation point is obtained;
- We have a symmetric drying, therefore a median of the wood plank is an extreme temperature and water content;
- A heat transfer by convection takes place only at the level of thermal layer limit who to be at the borders of samples;
- We have neglected the transfer on the lateral faces. Transfers which take place are controlled by the thickness of the planks, which is smaller than width and length;
- Gravity effect and hydraulic conductivity of wood are neglected;
- The losses inherent in the dryer are not respected. All the other values are supposed to be transferred to the wood;
- Air and water are supposed to be incompressible.
We obtained the following equations (1) below [5,6,9]:

\[
\frac{\partial H}{\partial t} = D_{HH} \frac{\partial^2 H}{\partial x^2} + D_{HT} \frac{\partial^2 T}{\partial x^2} + D_{TT} \frac{\partial^2 T}{\partial x^2} (1a)
\]

\[
\rho_e C_p \frac{\partial T}{\partial t} = (\lambda + D_{TT}) \frac{\partial^2 T}{\partial x^2} + D_{TH} \frac{\partial^2 H}{\partial x^2} (1b)
\]

Between wood and air, we have:

\[
\lambda \frac{\partial T}{\partial x} = h_c (T_{air} - T) + \rho_e LD_{HH} \frac{\partial H}{\partial x} (1c)
\]

\[
D_{HH} \frac{\partial H}{\partial x} = h_m \left( X_{eq} - H \right) (1d)
\]

With:

- H: Water content of wood (kg/kg)
- T: Temperature of wood (K)
- t: Drying time (s)
- \( h_c, h_m \): Global coefficient transfer respectively of the heat (W/(m².K)) and the mass (m/s)
- \( \rho_e \): Density of the anhydride wood (kg/m³)
- \( X_{eq} \): Equilibrium water content (kg/kg)
- \( \lambda \): Thermal conductivity of wet wood (W/(m.K))

At the median plane, the fluxes diffusion of humidity and heat are neglected.

Diffusions coefficients are such as:

\[
D_{HH} = D_H (2)
\]

\[
D_{HT} = \alpha D_{HH} (3)
\]

\[
D_{TH} = \rho (E + L) D_H (4)
\]

\[
D_{TT} = \alpha D_{TH} (5)
\]

With :

- \( D_H, D_T \): diffusion coefficients of bound water respectively at a gradient humidity (m²/s) or at a gradient temperature (m²/(K.s))
- \( E \): heat desorption of the water absorbed (J/kg); \( L \): latent heat of the vaporization of liquid water (J/kg);
- \( \alpha \): thermomigration coefficient (K⁻¹).

In the space order, terms of second member of the equations (4) and (5) shows the energies contributions utilized to extract water in vapor state or liquid state.

**B. Thermophysical parameters**

*Thermal conductivity of wood (W/(mK)) [10]:

\[
\lambda = \frac{\rho}{\rho_l} (0.2003 + 0.00548H) + 0.02378 (6)
\]

H in %

*Mass heat of wood (kJ/(kg.K)) [10] :

\[
C_p = \frac{C_{pw} + 0.01HC_{pw}}{1+0.01H} + H(-0.06191+2.36*10^{-4}T-1.33*10^{-2}H) (7.1)
\]

H in %

\[
C_{pw}=0.1031+0.003867T \quad \text{and} \quad C_{pw}=4.19kJ/(kg.K) (7.2)
\]

T in Kelvin (K). Second term of second member of (7a) is equal to zero in a non hygroscopic domain, domain that is limit by the water content at the fibers saturation point given by the equation[9]:

\[
H_e=0.3161-1.327x10^{-3}T (8)
\]

*Latent heat of vaporization (J/kg) [11]:

\[
L=(3335-2.91T)*10^3 (9)
\]

L in J/kg, H in % and T in K

*Desorption heat of the water adsorbed (J/kg) [12]:

\[
E=1170.4x10^3exp(-0.14H) (10)
\]

E in J/kg and H in %.

*Diffusion coefficient of vapor of the water in the wood (m²/s):

We have experimentally obtained the diffusion coefficients of our samples by the vaporimeter method[13].

Sapelle: \( D_{exp}=8.38614x10^{-11}m^2/s \) \hspace{1cm} (11.a)

Lotofa: \( D_{exp}=7.1131x10^{-11}m^2/s \) \hspace{1cm} (11.b)

*Density of the water (kg/m³) [14]:

\[
\rho_w = -0.0038T^2 - 0.0505T + 1002.6 (12)
\]

*Activation energy of the wood (J/mol) [15]:

\[
E_p = 4.189200 - 7000X_{eq} (13)
\]

*Equilibrium water content (kg/kg) [16]:

We have obtained these equilibrium water content expressions by a saturated saline solution method.

\[
X_{eq} = \left[ -\frac{\ln(1-HR)}{A} \right]^{1/b} (14)
\]

with :

\[
A(T)=aT^4+bT^3+cT^2+dTe \hspace{1cm} (15.a)
\]

where : \( a=-0.000453942, b=0.558790272, c=-257.5534793, d=5267.406693 \) and \( e=-4032498.705 \)

\[
B(T)=-0.016384722T+7.33776532 (15.b)
\]
*Effect sorbent coefficients (K⁻¹) [17]:

\[ \frac{dX_{aq}}{dt} = \frac{E_{HR} \frac{\partial X_{aq}}{\partial HR}}{RT^2 \rho HR} \]  
\[ (16) \]

The precedent expression gives [18]:

\[ \alpha = \frac{HR.E_b}{R.T^2.A.B.(1-\text{HR})} \left( \frac{1-B}{b} \right) \]  
\[ (17) \]

\( E_b \): activation energy (J/mol)

HR: relative humidity of air (-)

R: Constance of the perfect gases (8.314/(mol.K))

*Mass transfer coefficient (m/s) [19]:

\[ h_m = 9.454 \times 10^{-3} V_a^{0.5003} \]  
\[ (18) \]

Where \( V_a \) is the air velocity in the drying-cupboard equal to 0.25 m/s.

*Heat transfer coefficient [20,21]:

\[ h_i = 11.2 \text{ W/(m}^2\cdot\text{K)} \]  
\[ (19) \]

*Density of wet wood:

We have experimentally determined this parameter. We have obtained:

Sapelle: \( \rho(H) = 3.771 \text{ H(\%)} + 707.5 \) (kg/m³) (20.a)

Lotofa: \( \rho(H) = 2.167 \text{ H(\%)} + 567.2 \) (kg/m³) (20.b)

C. Numerical method of the resolution

To find numerical resolution of the analytical equations obtained, we have utilized the finite difference method. This method allows to approximate the values of continued functions and continually derivable by the development of Taylor’s series. Implicit form is adapted at the resolution of differential equations less strongly coupled and to have the coefficients that less change. Coupling of these differential equations cannot envisage an analytical resolution. Wood plank is supposed to be a thick layer divided a 2N+2 series of thin layers, with in a half thickness the spaces orders j goes to 1 at N+1. Forms of mass and heat discrete equations are given by:

\[
\begin{align*}
-AH_{j+1}^{l+1} + (1 + 2A)H_j^{l+1} - AH_{j-1}^{l+1} &= H_{j}^{l+1} - 2DT_j^{l+1} + DT_{j-1}^{l+1} + DT_{j+1}^{l+1} \\
BT_{j+1}^{l+1} + (2B+1)T_j^{l+1} - BT_{j-1}^{l+1} &= T_j^{l+1} - 2CH_j^{l+1} + CH_{j-1}^{l+1} + CH_{j+1}^{l+1}
\end{align*}
\]  
\[ (21.a) \]

With:

\[ A = \frac{D_m \Delta t}{(\Delta x)^2} \]
\[ B = \frac{\lambda + D_{rt} \Delta t}{\rho c_p (\Delta x)^2} \]
\[ C = \frac{D_{tt} \Delta t}{\rho c_p (\Delta x)^2} \]
\[ D = \frac{D_m \Delta t}{(\Delta x)^2} \]  
\[ (21.b) \]

A, B, C and D are given at the knots (i,j) with:

\[ x_j = x_0 + jh, \quad t_i = t_0 + il \]  
\[ (22) \]

We have put the system (21.a) in the following matrix form:

\[ \begin{bmatrix} E_j \end{bmatrix} \begin{bmatrix} G_j^{l+1} \end{bmatrix} + \begin{bmatrix} F_j \end{bmatrix} \begin{bmatrix} G_j^{l+1} + G_j^{l+1} \end{bmatrix} = \begin{bmatrix} H_j \end{bmatrix} \]  
\[ (23.a) \]

With:

\[ \begin{bmatrix} E_j \end{bmatrix} = \begin{bmatrix} 2A+1 & 0 \\ 0 & 2B+1 \end{bmatrix}, \quad \begin{bmatrix} F_j \end{bmatrix} = \begin{bmatrix} -A & 0 \\ 0 & -B \end{bmatrix} \]

\[ \begin{bmatrix} H_j \end{bmatrix} = \begin{bmatrix} H_j^{l+1} + DT_{j-1}^{l+1} - 2DT_j^{l+1} + DT_{j+1}^{l+1} \\ T_j^{l+1} + CH_j^{l+1} - 2CH_{j-1}^{l+1} + CH_{j+1}^{l+1} \end{bmatrix}, \quad \begin{bmatrix} G_j^{l+1} \end{bmatrix} = \begin{bmatrix} H_j^{l+1} \\ T_j^{l+1} \end{bmatrix} \]  
\[ (23.b) \]

In order to have a recursive solution, we have considered that:

\[ \begin{bmatrix} G_j^{l+1} \end{bmatrix} = \begin{bmatrix} \gamma_j \\ \beta_j \end{bmatrix}, \quad \begin{bmatrix} G_j^{l+1} \end{bmatrix} = \begin{bmatrix} F_j \end{bmatrix} \begin{bmatrix} \gamma_j \\ \beta_j \end{bmatrix} \]  
\[ (24.a) \]

With:

\[ \begin{bmatrix} \gamma_j \\ \beta_j \end{bmatrix} = \begin{bmatrix} H_j^{l+1} - \begin{bmatrix} F_j \end{bmatrix} \begin{bmatrix} \gamma_j \\ \beta_j \end{bmatrix} \\ \begin{bmatrix} E_j \end{bmatrix} - \begin{bmatrix} F_j \end{bmatrix} \begin{bmatrix} \gamma_j \\ \beta_j \end{bmatrix} \end{bmatrix} \]  
\[ (24.b) \]

Results can be progressively obtained. We have translated our program in the class 77 of fortran to generate all numerical results. Excel permitted us to draw our curves. The time step and space step are respectively 1700s and 6.25x10⁻³ m. Initial temperature of samples is 10°C. In each drying time, from local values of humidity and the temperature of wood, average values are evaluated through relations 25.a and 25.b.

\[ T_i = \frac{\sum_{j=1}^{N+1} T_j^i}{N+1} \]  
\[ (25.a) \]

\[ H_i = \frac{\sum_{j=1}^{N+1} H_j^i}{N+1} \]  
\[ (25.b) \]

D. Experimental Protocol

Table I below, presents the physical characteristics of wood samples utilized in this study to determine the drying curves and the diffusion coefficients. The size of boards and their sawing pattern are depicted in the table 1. The diffusion coefficient is determined on samples from the same board in order to prevent sample variability.
The sample geometry used to determine the diffusion coefficient was cylindrical samples of 72 mm in diameter and thicknesses of approximately 10 mm were cut for this measure. The mass diffusivity is measured under steady state conditions (cup method with salt solutions) with the devices designed in Zohoun and Perré 1997 [22] (figure 1). In the climatic chamber, wet temperature, dry temperature and air relative humidity are respectively fixed at 25°C, 33.5°C and 59%. The samples used for the cup method and boards used for the determination of the drying curves are both placed in the climatic chamber (Figure 1).

Table 1: Woods physical characteristics

<table>
<thead>
<tr>
<th>Species</th>
<th>size (cm)</th>
<th>annual growth rings</th>
<th>material direction for transfers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sapelle</td>
<td>43x8.8x2.2</td>
<td>Quartersawn</td>
<td>Tangential</td>
</tr>
<tr>
<td>Lotofa</td>
<td>43x7x2.2</td>
<td>Quartersawn</td>
<td>Tangential</td>
</tr>
</tbody>
</table>

Fig.1. drying-cupboard

Fig.2a. profile view of the samples

Fig.2b.face view of the samples

Fig.2. Disposition of the samples in the drying-cupboard during the experimental drying
III. RESULTS AND DISCUSSION

In the case these woods, we are used a diffusion coefficient that satisfied the following relationship:

\[ D_{\text{used}} = 1.5D_{\text{experiment}} \]  

(26)

In effect, all experimental diffusion coefficient that we are obtained are appropriate to use in a hygroscopic domain. In the start of the drying process, mass diffusion coefficient is much important that \( D_{\text{experiment}} \). A complement mass diffusion coefficient explains a contribution of a gas pressure in the movement of water.

Figures 3 and 4 depict the average moisture content variations in the thickness at function of the drying time. The model gives a good description of these experimental data. But some differences can be observed on the simulated and measured drying kinetics after 150h of the drying. Due to the strong anisotropy of mass transfer coefficient and the short length of our samples, the moisture flow at the end board could be relatively important in comparison with the moisture flow in transverse directions. In this case the mono-dimensional transfer hypothesis is not true and it introduces some error in the mass loss estimation. Additionally, the validity of the global coefficient transfer in these configurations can be discussed. Finally, the extrapolation of the diffusion coefficient measured on a thinner sample (about 10mm) to the board level (about 40mm) could introduce some error, more particularly for riftsawn and flatsawn boards.

Fig.3. Experiment and simulate drying curves (sapelle wood).

Fig.4. Experiment and simulate drying curves (lotofa wood).

Fig.5. Evolution of temperature of the samples, for the conditions: air wet temperature=25°C; air dry temperature=33.5°C; air relative humidity=59%; initial water content of wood=75% and thickness of the wood plank=30mm.

Fig.6. Evolution of water content of the samples, for the conditions: air wet temperature=25°C; air dry temperature=33.5°C; air relative humidity=59%; initial water content of wood=75% and thickness of the wood plank=30mm.
Figure 5 shows that, more the wood is dense, more heat transfer is easy because, thermal conductivity of wood increases with the density. But more the wood is dense, more his drying is difficult (figure 6).

IV. CONCLUSION

The drying model established in the present paper describes with satisfaction experimental drying curves of the tropical woods studied. Nevertheless, we saw a little difference between experimental and theoretical curves. For this reason, we are taken \( D_{\text{used}} = 1.5 D_{\text{experiment}} \).

It is possible that the value of global coefficient transfer must be adapted and, this adjustment explains a contribution of gas pressure in the movement of water during the process. Also, it is also possible that many simplifications utilized in this work explain this difference. However, this model can be utilized to simulate the drying of all tropical woods after the determination of physical parameters relative to each wood species.

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Corresponding author: E-mail address: simotagne2002@yahoo.fr