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# 2D Adaptive Mesh with Movement

Stéphane Dufour, Gérard Vinsard, and Bernard Laporte

**Abstract**—Adaptive meshes of electrical machines have to take into account the movement of the rotor. To combine accuracy and fast computation, nodes are added and cancelled for any variation of the rotor position.

**Index Terms**—Magnetostatics, mesh generation, finite element methods, rotating machines, reluctance motors.

## I. INTRODUCTION

In order to calculate an electrical machine, the coenergy has to be computed for different rotor positions, and the mesh has to be modified for each position.

For a constant high density (the average number of elements around a point for a given surface) of the mesh, the value of the coenergy will be accurate, but it costs computation time.

The mesh density can also be variable: by starting from a basic mesh, nodes can be added where they are necessary (where the curvature of the field lines is important) to increase the accuracy [1]. But a mesh that is adapted for one rotor position will not be necessarily adapted for another position.

To take into account the movement of the rotor, the elements of the airgap and the rotor have to be modified at each rotor position.

A dynamical procedure that combines the mesh adaptation with the modification of the mesh required by the rotor movement is described.

## II. MESH AND ROTOR MOVEMENT

The synchronous-type machine, for example a Switched Reluctance Machine (Fig. 1), is entirely described from the electromagnetical point of view by the coenergy function  $\overline{W}$ : it depends only of the current of the supplied phase  $I$ , and of the position of the rotor  $\theta$ . As the basis of the study is the bidimensional linear magnetostatic

model, the coenergy functional is:

$$\overline{W}(I, \theta, a) = \left[ \int_D j a - \frac{1}{2} \nu (\nabla a)^2 \right] \quad (1)$$

where  $a$  is the magnetic vector potential (MVP),  $j$  the current density in the conductors, and  $\nu$  the reluctivity function. The coenergy function is the maximum of the coenergy functional, reached for the MVP  $a^\infty$  (among the MVP which have the square of their gradient integrable)

$$\overline{W}(I, \theta) = \max_a \overline{W}(I, \theta, a) = \overline{W}(I, \theta, a^\infty) \quad (2)$$

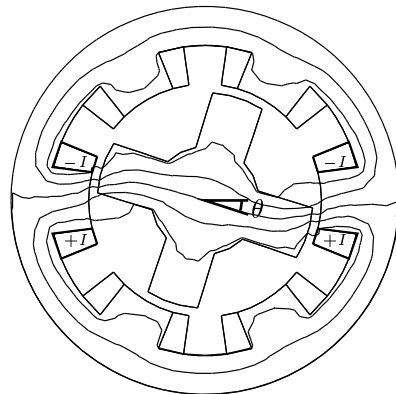


Fig. 1. 3-phase 6/4 Switched Reluctance Machine (SRM)

A mesh is characterized by the position of its nodes and the edge connections between the nodes. The finite element approximation consists in taking the MVP as a linear function inside each element and then parameterizing the MVP by its values on the nodes of the mesh. Then the maximization of the coenergy functional on the subspace of finite dimension constituted by the values of the MVP of the nodes ( $a_1, \dots, a_N$ ) gives an approximation of the coenergy function.

To start the movement of the rotor, a given mesh is done with a constant density, for the first position. In order to move the rotor of an angle  $\theta$ , the mesh must be modified. The nodes of the stator keep their positions, the nodes of the rotor move of the angle  $\theta$ , and the elements of the airgap are modified to ensure the connections between the nodes of the stator and of the rotor.

The next question is the adaptation of the mesh to the solution, for a given position.

If a node is added to the existing mesh, it will add one more degree of freedom to the maximization of the coenergy functional: for any position of the node, the value of

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the maximum of the coenergy functional will be higher.

$$\max_{(a_1, \dots, a_N, a_{N+1})} \overline{W}(I, \theta, a) \geq \max_{(a_1, \dots, a_N)} \overline{W}(I, \theta, a) \quad (3)$$

On the contrary if a node is cancelled from the existing mesh, with one degree less of freedom, the maximum value will decrease. A local approach is searched for a local meshing and de-meshing [2]. The goal is to have a good approximation of the coenergy function, with the least possible number of nodes.

The last point is to combine both operations: the rotor movement with the mesh adaptation for any rotor position. The nodes of the existing mesh are rotated of an angle  $\theta$ . Then the mesh has to be adapted to the new rotor position: nodes are cancelled or added to the mesh where they are necessary.

### III. NODE CANCELLATION

In a mesh, the presence of a particular node may not bring a significant contribution to the value of  $\overline{W}(I, \theta)$ . Thus, the utility of each node has to be tested separately.

The influence of the node on the value of the coenergy functional could have been tested with the use of the global mesh; for each node two global computations are done basically: one with the node, and one without.

A local test is searched here: even if the resulting solution will be under-optimal, the influence of the node is tested with the use of a reduced part of the mesh.

Before checking the contribution of the node, several conditions should be met. The mesh without the node should preserve the geometry of the machine.

The particular case of the nodes belonging to the border has been treated, but is avoided in the presentation below for reasons of simplicity. The mesh of the source domains is kept constant: it is not investigated for the de-meshing.

First of all, a global computation is made to obtain the value of the MVP at each node.

At the scale of the node  $n$ , the polygon  $P$  formed by the  $m$  elements containing the node  $n$  is considered (Fig. 2).

The influence of the node is studied by setting the MVP as constant on the boundary of the polygon. Then, the contribution of this polygon  $P$  to the coenergy is computed. It corresponds to the opposite of the energy  $-W(\vec{x}_n, a_n)$ , where  $\vec{x}_n = (x_n, y_n)$  is the position of the node  $n$ . The energy of the polygon with the node is:

$$W_0 = \sum_{j=1}^m W_{\text{el}_j}(a_n, a_j, a_{j+1}) \quad (4)$$

$$W_{\text{el}_j}(a_n, a_j, a_{j+1}) = \frac{1}{2} \int_{\text{el}_j} \nu_j(a_n, a_j, a_{j+1}) R \begin{pmatrix} a_n \\ a_j \\ a_{j+1} \end{pmatrix} \quad (5)$$

where  $R$  is the contribution of the element  $\text{el}_j$  to the stiffness matrix.

When the node  $n$  is cancelled, the elements of the polygon have their connections modified, and two of them are

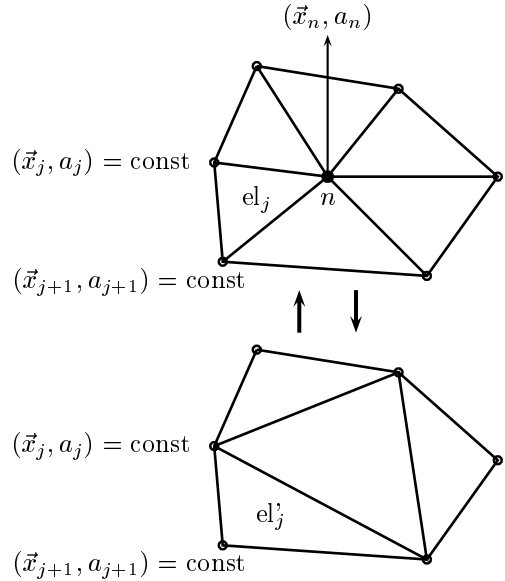


Fig. 2. Cancellation of the node  $n$

cancelled (Euler relations on planar graphs). The energy of the polygon without the node is (Fig. 2):

$$W_1 = \sum_{j=1}^{m-2} W_{\text{el}'_j} \quad (6)$$

The connections that minimize the increase of energy between  $W_0$  and  $W_1$  are chosen. If the difference between both contributions is lower than a threshold value, practically proportional to the global energy divided by the number of elements  $N_{\text{be}}$ , then the node is effectively cancelled:

$$W_1 - W_0 < \alpha \frac{W_{\text{total}}}{N_{\text{be}}} \quad (7)$$

Otherwise, the node is kept as its contribution to the coenergy functional is significant. This local procedure is carried out component by component, even if the values of the MVP slightly change with the cancellations. But it is efficient to decrease the number of nodes (and the computation time) without changing too much the value of the coenergy.

### IV. NODE ADDITION

Node addition is just seen as the inverse operation of the node cancellation. It is carried out in order to get a better accuracy on the coenergy functional.

Two ways can be used to include a new node in the mesh. This node can be introduced:

- in only one element of the mesh, without modifying the existing connections.
- in a polygon made of several elements, by modifying the existing connections.

The value of the MVP of the nodes of the polygon are fixed, as in the previous section. The contribution of the polygon to the coenergy is compared to the one with the

node added, instead of using estimator errors such as the measurement of the energy sensitivity of the nodal position [3], or the expression of the Hessian matrix [4].

The polygon which is used with this method is made out of two adjacent elements (Fig. 3): it is chosen to preserve the rapidity of the method (if the node was introduced in an element, then the solution would be linear, and there would be no variation of coenergy).

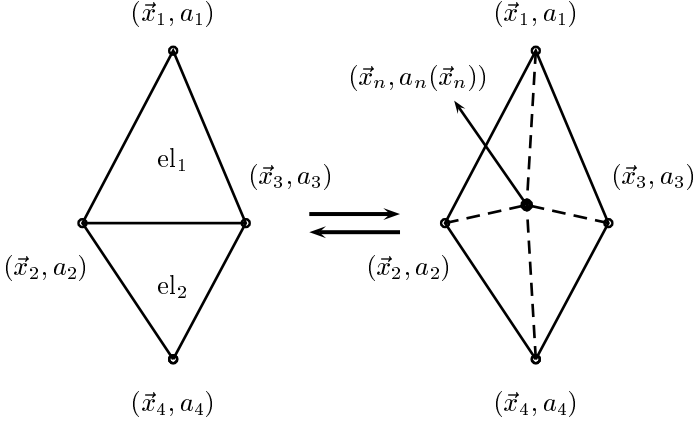


Fig. 3. Node addition in a polygon of 2 elements

Just as in the previous section, the values of the MVP on the boundary of the polygon are fixed, and the positions of the nodes on the boundary are also fixed.

The energy of the polygon will decrease with the introduction of another node  $n$ , and this fall depends on the position of the node  $n$ . The position which minimizes the value of the energy is searched. Instead of approximating the gradient and the Hessian of the energy function by using finite differences [2], the derivatives are effectively calculated. The value of the MVP  $a_n$  is considered as a function of the node position  $\vec{x}_n$ .

The value  $a_n^\infty$  that minimizes  $W(\vec{x}_n, a_n)$  verifies for the zero order:

$$\frac{\partial W}{\partial a_n}(\vec{x}_n, a_n^\infty) = 0 \quad (8)$$

The derivatives of  $a_n^\infty$  with respect to the position  $\nabla_{\vec{x}} a_n^\infty$  verify for the first order:

$$\nabla_{\vec{x}} \left( \frac{\partial W}{\partial a_n}(\vec{x}_n, a_n^\infty) \right) + \left( \frac{\partial^2 W}{\partial a_n^2}(\vec{x}_n, a_n^\infty) \right) \nabla_{\vec{x}} a_n^\infty = 0 \quad (9)$$

The Taylor expansion of the energy at the second order can be expressed when using (8),(9) as:

$$\begin{aligned} W(\vec{x}_n + \delta\vec{x}, a_n(\vec{x}_n + \delta\vec{x})) &= W(\vec{x}_n, a_n(\vec{x}_n)) \\ &\quad + \nabla_{\vec{x}} W(\vec{x}_n, a_n(\vec{x}_n)) \cdot \delta\vec{x} \\ &\quad + \frac{1}{2} \delta\vec{x}^t \left[ \nabla_{\vec{x}}^2 W - \left( \frac{\partial^2 W}{\partial a_n^2} \right) (\nabla_{\vec{x}} a_n^\infty)(\nabla_{\vec{x}} a_n^\infty)^t \right] \delta\vec{x} \end{aligned} \quad (10)$$

The new coordinates  $\vec{x}_n$  are computed from this minimization of the energy, and the new value of the MVP  $a_n$  is deduced. The new contribution to the coenergy of

the polygon  $W_1$  is then compared to the previous one  $W_0$ . If the variation is higher than a threshold value, then the node is added:

$$W_0 - W_1 > \beta \frac{W_{\text{total}}}{N_{\text{be}}} \quad (11)$$

## V. RESULTS

A 3-phase switched reluctance motor (SRM) with 6 stator teeth and 4 rotor teeth is taken as example (Fig. 1). The supply is considered as fixed, as one phase is supplied at the same time. When the rotor moves, the reluctivity of the elements is a function of the rotor position: most of the mesh variation will be concentrated between the rotor slots.

The mesh is evolving globally with the rotor movement: the total number of elements  $N$  is chosen at the start. Depending on the ratios  $\alpha$  and  $\beta$ ,  $N$  will increase or decrease, and will converge rapidly to an end mean value (Fig. 4).

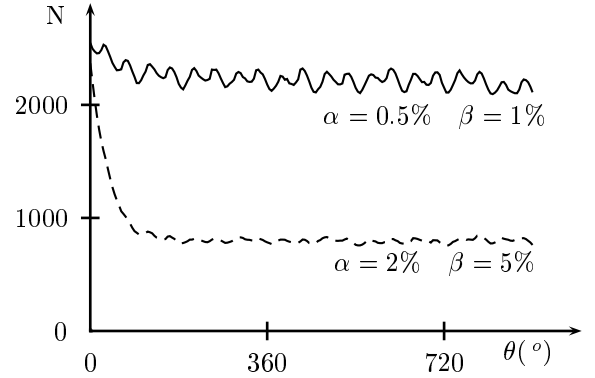


Fig. 4. Total number of nodes  $N$  as a function of the position

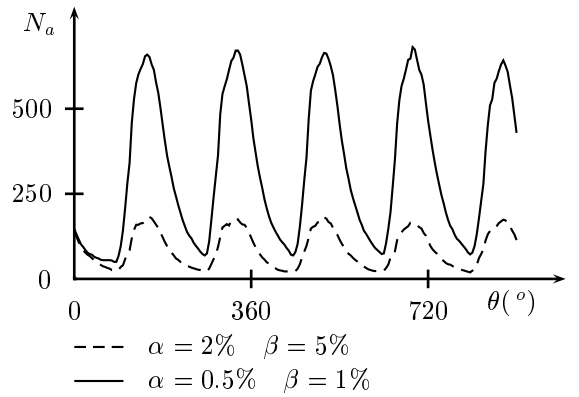


Fig. 5. Number of nodes  $N_a$  of the fourth of the airgap region as a function of the position

To show the mesh variation, one fourth of the airgap (part of the air-domain between two consecutive rotor slots) is considered (the boundary of this domain is in bold on Fig. 6). The number of nodes of this domain  $N_a$  significantly varies with the rotor position. For the unaligned rotor position (Fig. 6 bottom), the domain is in front of the supplied phase: the number of nodes  $N_a$  is

maximum. For the aligned rotor position,  $N_a$  is minimum (Fig. 5).

The mesh variation for a chosen reduced number of elements, is shown on Fig. 6 to see the regions which need to be meshed or not. The regions where the flux lines are nearly straight need not to be highly meshed (stator and rotor core), when compared with the teeth of the supplied phase. When the rotor is moving from the aligned to the un-aligned position, the number of nodes  $N_a$  increases, as more flux lines are in this region.

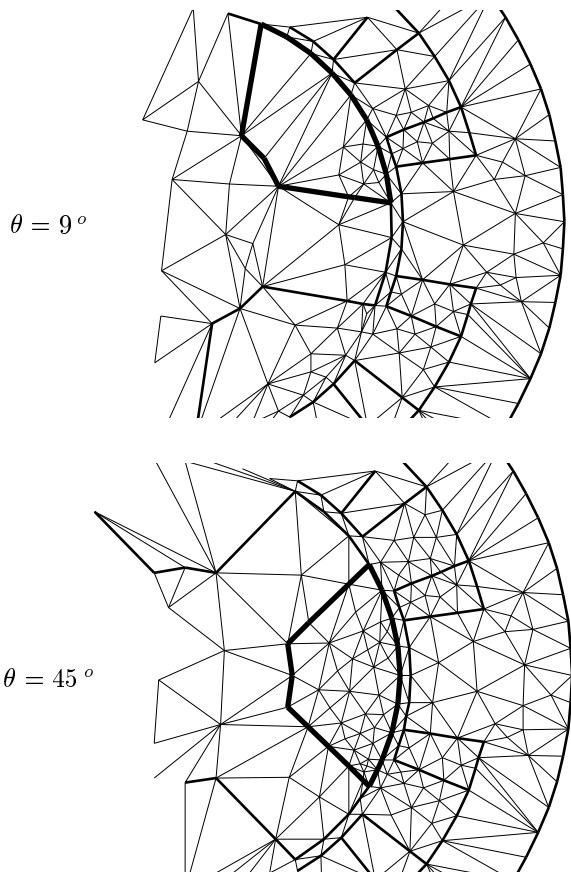


Fig. 6. Detail of mesh variation from the aligned rotor position to the unaligned

The mesh variation was implemented and tested for various rotor positions. Table I compares for two different rotor positions the results given by using a homogeneous mesh of constant density (high or low) with these of the adapted mesh. The results for the adapted mesh show a better accuracy than these for the constant low density mesh, especially in un-aligned rotor positions.

A stiff test to verify the accuracy of the solution is to consider the variation of the coenergy function with respect to the position (to get an estimation of the torque). When the values of  $\overline{W}(\theta)$  are not accurate enough, the numerical derivation leads to erroneous values (Fig. 7 dotted lines). A solution could be then to increase the number of nodes in the homogeneous mesh (dashed lines). In order to keep the number of nodes constant, an adaptive mesh

is used: it is as accurate as increasing the number of nodes in the homogeneous mesh.

TABLE I  
COMPARISON OF MESHES

	Nodes	$\overline{W}(\theta = 0^\circ)$	Nodes	$\overline{W}(\theta = 27^\circ)$
homogeneous	11000	1.456	11000	0.584
homogeneous	4000	1.447	4000	0.551
Error (%)		0.6		5.8
adaptive	3400	1.450	3400	0.581
Error (%)		0.4		0.4
adaptive	2000	1.446	2400	0.573
Error (%)		0.6		2.3

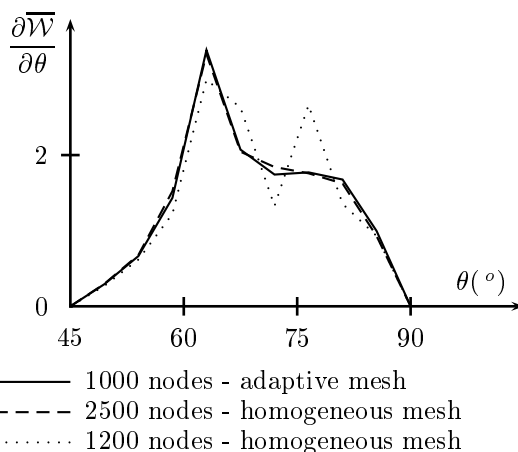


Fig. 7. Torque as a function of the rotor position for several meshes

## VI. CONCLUSION

A method to take into account simultaneously the mesh adaptation and the rotor movement of an electrical machine has been presented.

The mesh adaptation is carried out with local de-meshing and meshing techniques. From a starting position with a mesh of constant density, the mesh rapidly changes with the rotor position. This evolving mesh fits to any variation of the solution and shows a good accuracy on the coenergy function and its derivatives for a reduced number of nodes.

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