# Comment on "An experimental study of symmetry lowering of analcime" 

Massimo Nespolo

## To cite this version:

Massimo Nespolo. Comment on "An experimental study of symmetry lowering of analcime". Physics and Chemistry of Minerals, 2018, 45 (4), pp.391-393. 10.1007/s00269-018-0951-4 . hal-01719808

HAL Id: hal-01719808<br>https://hal.univ-lorraine.fr/hal-01719808

Submitted on 28 Feb 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Massimo Nespolo
CRM2 UMR - CNRS 7036, Institut Jean Barriol, FR 2843, Université de Lorraine, Faculté des
Sciences et Technologies, BP 70239, Boulevard des Aiguillettes, F54506 Vandoeuvre-lès-Nancy
cedex France
massimo.nespolo@,univ-lorraine.fr
Tel. +33-(0)372-745646, Fax $+33-(0) 372-745218$
ORCID ID: 0000-0003-2530-5399
Manuscript prepared with LibreOffice Writer version 5.4.3.2 under MS Windows 10 version 1709

Massimo Nespolo, Université de Lorraine, CRNS, CRM2, Nancy, France

massimo.nespolo@univ-lorraine.fr


#### Abstract

In a recent article (Sugano and Kyono, 2018), analcime hydrothermally synthesized from a gel and then reheated was reported to have undergone a loss of symmetry to Ibca. We show that the reheated samples, reported as different, are identical to each other and actually tetragonal. The fact that the tetragonal axis was oriented along the $\mathbf{a}$ or $\mathbf{b}$ vector in the two reheated samples instead of the $\mathbf{c}$ vector as in the conventional setting seems to have been the cause of the misinterpretation of the diffraction pattern.


## Introduction

The symmetry of a crystal structure is expressed through its space group, which can be described in a variety of settings. The most convenient setting is called standard: in this setting, the symmetry operations are represented by unimodular matrices and particularly simple to read off. A number of alternative settings are also in use, which are called unconventional or non-standard, depending on whether or not the metric of the alternative unit cell still corresponds to the same crystal family. These alternative settings are useful in diverse cases, like the study of the phase transitions and derivative structures, the analysis of twinned crystals, or in presence of a high degree of pseudosymmetry (Nespolo and Aroyo, 2016). In some cases, standard and alternative settings differ just by a permutation of the axes, and one orientation is taken as standard by convention. This is typically the case of crystals with a unique axis (monoclinic, tetragonal, trigonal and hexagonal), for which the orientation of the unique axis can actually be along any of the basis vectors, but one of them has been taken as standard by convention. When experimental data of a crystal with a unique axis are expressed in a setting in which this axis is not along the standard direction, the transformation to the standard setting is just a matter of permutation of the basis vectors, and of the corresponding coordinates. Unfortunately, sometimes the unusual orientation of the unique axis may escape to the investigators, who may then be trapped in the search for a subgroup, whereas their structure is actually more symmetric. This is what happened in the experimental investigation of analcime in a recent article by Sugano and Kyono (2018).
Analcime hydrothermally synthesized from a gel at $200^{\circ} \mathrm{C}$ for 24 hours was reported in the usual space group $I a \overline{3} d$. Two samples were then reheated for 24 and 48 hours respectively, and reported to have undergone a loss of symmetry to Ibca , following the $\mathrm{Si} / \mathrm{Al}$ ordering but without changes in the extra framework site occupancy, where Na is hosted. Unfortunately, the authors have overestimated the loss of symmetry; not only the two reheated samples are practically identical to each other, but they are actually tetragonal, with space group $I 4_{1} / a c d$. Both samples have cell parameters indicative of a tetragonal lattice, although with a non-standard orientation. For the sample 24h, reported cell parameters are $a=13.727(2) \AA, b=c=13.707(2) \AA$, with a fourfold axis for the lattice along the a basis vector. For the sample 48h, reported cell parameters are $a=13.705(2) \AA, b=13.717(2) \AA, c=$ $13.705(6) \AA$, with a fourfold axis for the lattice along the $\mathbf{b}$ basis vector (the difference between the $a$ and $c$ cell parameters, within one standard uncertainty, is not statistically significant); moreover, the cell parameters of the 48 h sample actually fell within the cubic limits for three standard uncertainties: $13.699-13.711 \AA$ for $a, 13.711-13.723 \AA$ for $b, 13.700-13.712 \AA$ for $c$. A metric specialisation (symmetry of the lattice higher than the symmetry of the structure) is far from being a rare phenomenon (Janner, 2004a,b), but should nevertheless prompt the investigators to check their model for any possible overlooked symmetry. The fact that refinement converges in Ibca does not exclude that the real symmetry of the structure be $I 4_{1} / a c d$; indeed, the reported structural models are actually compatible with the $I 4_{1} /$ acd symmetry within two standard uncertainties and the structure
factors calculated from the published model also agree with the higher symmetry. The results shown below have been obtained with the Bilbao Crystallographic Server (Aroyo et al. 2006).

## Test for higher symmetry in direct space

The structural models 24 h and 48 h published in Table 3 of Suygano and Kyono (2018) have been checked for pseudosymmetry in the minimal translationengleiche supergroup $I 4_{1} /$ acd . The transformations from the published settings of 24 h and from 48 h to $I 4_{1} /$ acd are abc $\rightarrow$ bca and abc $\rightarrow$ cab respectively. Tables S1 and S2 give, for the two structures, the original fractional atomic coordinates, the idealized fractional coordinates in $I 4_{1} /$ acd expressed in the subgroup setting, the corresponding shifts $u_{x}, u_{y}, u_{z}$, and the ratio $u / \sigma$ of these shifts with respect to the published standard uncertainties. All ratios are smaller than 3 (in most cases even smaller than 2), showing that the published structures are actually compatible with $I 4_{1} / a c d$ within three standard uncertainties. Table S3 gives the structural model expressed in $I 4_{1} / a c d$. The very close result suggests that the two structural models reported in the original publication are actually practically the same, just expressed in a different axial setting. The larger difference in the cell parameters is between $a$ of the 24 h sample and the $b$ of the 48 h sample, i.e. the tetragonal axis that has been missed by the authors. The difference is $0.0010 \AA$, again within three standard uncertainties, which are $0.0002 \AA$ for both models (13.721-13.733 $\AA$ vs. 13.711-13.723 $\AA$ ). The fractional atomic coordinates are practically identical, once expressed in the same reference. The space group of both models has affine normalizer $\operatorname{Pm} \overline{3} n(1 / 2 \mathbf{a}, 1 / 2 \mathbf{b}, 1 / 2 \mathbf{c})^{1}$; if we apply to the fractional atomic coordinates of the 48 h model a three-fold rotation of the normalizer we obtain an alternative description of the model, which is identical to the 24 model within two standard uncertainties (Table S4).
Even the tetragonal structural model is actually very strongly pseudosymmetric with respect to $I a \overline{3} d$. If we neglect the small difference in cell parameters of the tetragonal structure (for the 48 sample, the cell is actually cubic within three standard uncertainties, as shown in the previous section), we can apply the same pseudosymmetry test, this time towards Ia3d. The result for the 48 h sample is shown in Table S5, where we see that fractional atomic coordinates do fit the cubic supergroup within two standard uncertainties. The reported structures are therefore tetragonal, strongly pseudo-cubic.

## Test for higher symmetry in reciprocal space

In discussing the diffraction pattern of both reheated samples, the authors state that "weak reflections of type $00 l$ with $l=2 n$ became observed, which correspond to the reflection conditions of orthorhombic $I b c a$ ". As we have seen, the reheated samples have the $4_{1}$ axis along the a vector ( 24 h sample) or the $\mathbf{b}$ vector ( 48 h sample), so that nobody could expect any influence on the 00 l reflections. The statement above is therefore incorrect.
We have simulated the single-crystal X-ray diffraction pattern from the published cell parameters, fractional atomic coordinates and atomic distribution on both samples with the software package Jana2006 (Petricek et al. 2014). The computed diffraction patterns do not show any violation of reflection conditions of $I 4_{1} / a c d$. We have also performed a space group test on the reflection files computed in Ibca and the result confirms the higher symmetry $I 4_{1} /$ acd (see the deposited supplementary material). No indication of $I a \overline{3} d$ was however obtained.

[^0]
## Discussion

The structure models of the analcime reheated samples as published by Sugano and Kyono (2018) are actually more symmetric than what the authors stated. Both the fractional atomic coordinates and the calculated diffraction pattern from the published structures perfectly fit $I 4_{1} /$ acd, a translationengleiche supergroup of the Ibca group reported in the article. The overlook was probably originated by the orientation of the unique axis along the $\mathbf{a}$ or $\mathbf{b}$ basis vector, instead than the conventional orientation along the $\mathbf{c}$ basis vector. As a consequence, the conclusions about the $\mathrm{Si} / \mathrm{Al}$ ordered distribution have to be reconsidered. In fact, the model proposed by Sugano and Kyono (2018) shows a significant difference between the sites T11 and T12, whereas T12 and T2 would have almost the same $\mathrm{Si} / \mathrm{Al}$ ratio. In the $I 4_{1} /$ acd model, T 11 and T 2 merge into a single site ( $32 g$ ) whereas T 12 remains independent (16f) and this invalidates the $\mathrm{Si} / \mathrm{Al}$ ordering scheme presented.
The presence of twin domains is extremely probable because of the pronounced cubic pseudosymmetry of tetragonal analcime. As pointed out by Mazzi and Galli (1978), if the size of these domains is small enough to simulate diffraction from a single crystal, then, depending on the relative proportions of the twin domain states, both the apparent symmetry and the apparent ordering of the sample may be affected. In the case of the samples studied Sugano and Kyono (2018) the symmetry was clearly tetragonal. No conclusions can however be drawn about the possible $\mathrm{Si} / \mathrm{Al}$ distribution without a careful reinvestigation of the experimental data.
After reading the demonstration above, the authors have performed another structure refinement on an analcime sample reheated for 24 hours and again persisted with orthorhombic symmetry, Ibca. I have analysed these new data and I show the results in Table S6. Once again, the results confirm that the sample is tetragonal, $I 4_{1} / a c d$, within three standard uncertainties. The authors use reflections 002,0010 and 442 as "proof" of an alleged violation of systematic absences for $I 4_{1} / a c d$. As detailed in the are evidently unaware that they have collected data in an unconventional setting of the space group, with the tetragonal axis along a instead than along $\mathbf{c}$. When the diffraction pattern is reindexed in the standard (c-unique) setting, these reflections become 020,0100 and 424 , showing no violation of systematic absences for $I 4_{1} / a c d$. Their sample is therefore definitely tetragonal and the discussion about $\mathrm{Si} / \mathrm{Al}$ ordering is based on a misinterpretation of the diffraction pattern.

## References

Aroyo MI, Perez-Mato JM, Capillas C, Kroumova E, Ivantchev S, Madariaga G, Kirov A, Wondratschek H (2006) Bilbao Crystallographic Server I: Databases and crystallographic computing programs. Z Kristallogr 221:15-27.
Janner A (2004a) Integral lattices. Acta Crystallogr A60:198-200.
Janner A (2004b) Zones and sublattices of integral lattices. Acta Crystallogr A60:611-620.
Koch E, Fischer W (2006) Normalizers of space groups: A useful tool in crystal-structure description, comparison and determination. Z Kristallogr. 221:1-14.
Mazzi F, Galli E (1978). Is each analcime different? Am Mineral 63:448-460.
Nespolo M. (2015) The same once again please: a pedagogical introduction to normalizers. J Appl Crystallogr 48:1985-1997.
Nespolo M., Aroyo MI (2016) The crystallographic chameleon: when space groups change skin. Acta Crystallogr A72:523-538.
Petricek V, Dusek M, Palatinus L (2014) Crystallographic Computing System JANA2006: General features. Z Kristallogr 229:345-352.
Sugano N, Kyono A (2018) An experimental study of symmetry lowering of analcime. Phys Chem Minerals in press DOI 10.1007/s00269-017-0922-1
Table S1. Fractional atomic coordinates of the sample 24 h in the setting of $I b c a$, as published (in parentheses the standard uncertainty $\sigma$ ) and idealized to fit the $I 4_{1} /$ acd symmetry. $u$ is the absolute value of the shift between the published and idealized coordinates, $u / \sigma$ is the ratio between this shift and the standard uncertainty of the corresponding coordinate. This ratio shows that the two models are identical within three standard uncertainties.
Table S2. Fractional atomic coordinates of the sample 48h in the setting of $I b c a$, as published and idealized to fit the $I 4_{1} / a c d$ symmetry. Same conventions and in Table 1. The two models are identical within two standard uncertainties.

| Site | Wyckoff position | Structural model in Ibca |  |  | Idealized structurein $I 4_{1} /$ acd |  |  | $u_{x}$ | $u_{x} / \sigma_{x}$ | $u_{y}$ | $u_{y} / \sigma_{y}$ | $u_{z}$ | $u_{z} / \sigma_{z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $x$ | $y$ | $z$ | $x$ | $y$ | $z$ |  |  |  |  |  |  |
| T11 | $16 f$ | 0.12498(3) | 0.16223(3) | 0.41224(3) | 0.12500 | 0.16220 | 0.41220 | 0.00002 | 0.67 | 0.00003 | 1.00 | 0.00004 | 1.33 |
| T12 | $16 f$ | 0.41224(3) | 0.12505(3) | 0.16226(3) | 0.41230 | 0.12500 | 0.16220 | 0.00006 | 2.00 | 0.00005 | 1.67 | 0.00006 | 2.00 |
| T2 | $16 f$ | 0.16225(3) | 0.41224(3) | 0.12501(3) | 0.16220 | 0.41220 | 0.12500 | 0.00005 | 1.67 | 0.00004 | 1.33 | 0.00001 | 0.33 |
| Na11 | $8 c$ | 0.12486(17) | 0 | 0.25 | 0.12490 | 0 | 0.25 | 0.00004 | 0.24 | 0 |  | 0 | 0.00 |
| Na12 | $8 d$ | 0.25 | 0.12516(17) | 0 | 0.25 | 0.12500 | 0 | 0 | 0.00 | 0.00016 | 0.94 | 0 |  |
| Na 2 | $8 e$ | 0 | 0.25 | 0.12512(17) | 0 | 0.25 | 0.1251 | 0 |  | 0 | 0.00 | 0.00002 | 0.12 |
| O11 | $16 f$ | 0.10450(11) | 0.36577(12) | $0.21947(10)$ | 0.10450 | 0.36590 | 0.21950 | 0.00000 | 0.00 | 0.00013 | 1.08 | 0.00003 | 0.30 |
| O12 | $16 f$ | $0.38419(12)$ | $0.14557(11)$ | 0.46950(10) | 0.38420 | 0.14550 | 0.46950 | 0.00001 | 0.08 | 0.00007 | 0.64 | 0.00000 | 0.00 |
| O21 | $16 f$ | $0.21946(10)$ | $0.10455(12)$ | $0.36571(12)$ | 0.21950 | 0.10450 | 0.36580 | 0.00004 | 0.40 | 0.00005 | 0.42 | 0.00009 | 0.75 |
| O22 | $16 f$ | $0.14539(12)$ | $0.46948(10)$ | $0.38407(12)$ | 0.14540 | 0.46950 | 0.38410 | 0.00001 | 0.08 | 0.00002 | 0.20 | 0.00003 | 0.25 |
| O31 | $16 f$ | 0.36590 (12) | $0.21952(10)$ | $0.10453(11)$ | 0.36590 | 0.21950 | 0.10460 | 0.00000 | 0.00 | 0.00002 | 0.20 | 0.00007 | 0.64 |
| O32 | $16 f$ | 0.46952(10) | $0.38396(12)$ | $0.14549(11)$ | 0.46950 | 0.38410 | 0.14550 | 0.00002 | 0.20 | 0.00014 | 1.17 | 0.00001 | 0.09 |
| W | $16 f$ | 0.1250(2) | 0.1254(2) | 0.1251(2) | 0.1250 | 0.1250 | 0.1250 | 0 | 0.00 | 0.0004 | 2.00 | 0.0001 | 0.50 |

Table S3. Fractional atomic coordinates of the two reheated samples expressed in the $I 4_{1} / a c d$ translationengleiche supergroup of $I b c a$. Cell parameters are $a=13.707(2), c=13.727(2)$, and $a=13.705(2) c=13.717(2)$ respectively

|  |  | Structural model 24h |  |  | Structural model 48h |  |  |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Site | Wyckoff position | $x$ | $y$ | $z$ | $x$ | $y$ | $z$ |
| T1 | $16 f$ | 0.16226 | 0.41226 | 0.125 | 0.16225 | 0.412250 | 0.125 |
| T2 | $32 g$ | 0.12499 | 0.16224 | 0.41225 | 0.12502 | 0.16224 | 0.41223 |
| Na 1 | $8 b$ | 0. | 0.25 | 0.125 | 0 | 0.25 | 0.125 |
| Na 2 | $16 e$ | 0.12494 | 0 | 0.25 | 0.12512 | 0 | 0.25 |
| O 1 | $32 g$ | 0.36602 | 0.21957 | 0.10454 | 0.36576 | 0.21948 | 0.10449 |
| O 2 | $32 g$ | 0.14547 | 0.46951 | 0.38399 | 0.14549 | 0.46949 | 0.38410 |
| O 3 | $32 g$ | 0.10453 | 0.36593 | 0.21952 | 0.10457 | 0.36591 | 0.21950 |
| $W$ | $16 f$ | 0.1249 | 0.1251 | 0.125 | 0.1251 | 0.1250 | 0.125 |

Table S4. Fractional atomic coordinates of the two reheated samples in the setting of the sample 24 h . Same conventions as in Table 1 . The values $u / \sigma$ are reported with respect to both samples when the $\sigma$ differs. The two models are identical within two standard uncertainties.

| Structural model 24h |  |  |  | Structural model 48h in the setting of 24h |  |  |  | $u_{x}$ | $u_{v} / \sigma_{x}$ | $u_{y}$ | $u_{y} / \sigma_{y}$ | $u_{z}$ | $u_{2} / \sigma_{z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sis | $x$ | $y$ | $z$ | Site | $x$ | $y$ | $z$ |  |  |  |  |  |  |
| T11 | $0.12507(4)$ | 0.16228 (3) | 0.41224(3) | T2 | 12501(3) | $0.16225(3)$ | $0.41224(3)$ | 0.00006 | 1.50/2.00 | 0.00003 | 1.00 | 0 | 0.00 |
| T12 | $0.41228(3)$ | $0.12497(4)$ | $0.16222(3)$ | T11 | 0.41224(3) | 0.12498 (3) | $0.16223(3)$ | 0.00004 | 1.33 | 0.00001 | 0.25/0.33 | 0.00001 | 0.3 |
| T2 | $0.16222(3)$ | $0.41225(3)$ | 0.12500 (4) | T12 | $0.16226(3)$ | $0.41224(3)$ | $0.12505(3)$ | 0.00004 | 1.33 | 0.00001 | 0.33 | 0.00005 | 1.25/1.6 |
| Na11 | 0.12520(19) | 0 | 0.25 | Na 2 | 0.12512(17) | 0 | 0.25 | 0.00008 | 0.42/0.47 | 0 |  | 0 |  |
| Na12 | 0.25 | 0.12492(18) | 0 | Na 11 | 0.25 | $0.12486(17)$ | 0 | 0 | -- | 0.00006 | 0.33/0.35 | 0 |  |
| Na 2 | 0 | 0.25 | 0.12504(19) | Na 12 | 0 | 0.25 | $0.12516(17)$ | 0 | -- | 0 | -- | 0.00012 | 0.63/0.7 |
| O | 0.10456(12) | $0.36608(12)$ | 0.21960(10) | O 31 | 0453(11) | 0.36590(12) | 0.21952(10) | 0.00003 | 0.25/0.27 | 0.00018 | 1.50 | 0.00008 |  |
| O12 | 0.38389(12) | $0.14538(12)$ | 0.46953(10) | O 22 | 0.38407(12) | $0.14539(12)$ | 0.46948(10) | 0.00018 | 1.50 | 0.00001 | 0.08 | 0.00005 |  |
| 021 | $0.21966(11)$ | $0.10453(12)$ | $0.36596(12)$ | O 11 | 0.21947(10) | $0.10450(11)$ | 0.36577(12) | 0.00019 | 1.73/1.90 | 0.00003 | 0.25/0.27 | 0.00019 | 1.58 |
| 022 | 0.14549(12) | $0.46954(11)$ | 0.38404(12) | O 32 | 0.14549(11) | 0.46952(10) | $0.38396(12)$ | 0 | 0.00 | 0.00002 | 0.18/0.20 | 0.00008 |  |
| O31 | 0.36591(12) | $0.21948(11)$ | $0.10445(12)$ | O 21 | 0.36571(12) | 0.21946(10) | $0.10455(12)$ | 0.00020 | 1.67 | 0.00002 | 0.18/0.20 | 0.00010 | 0.83 |
| O32 | 0.46938(11) | 0.38411(12) | $0.14547(12)$ | O 2 | 0.46950(10) | 0.38419(12) | $0.14557(11)$ | 0.00012 | 1.09/1.20 | 0.00008 | 0.67 | 0.00010 | 0.83/0.9 |
| W | 0.1252(2) | 0.1249(2) | 0.1251(3) | W | 0.1251(2) | 0.1250(2) | 0.1254(2) | 0.0001 | 0.50 | 0.0001 | 0.50 | 0.0003 | 1.00/1. |

Table S5. Fractional atomic coordinates of the sample 48h in the setting of Ibca, as published and idealized to fit the Ia3d symmetry. Same conventions and in Table 1. The two models are identical within two standard uncertainties.

| Site | Wyckoff position | Structural model in Ibca |  |  | Idealized structure in $I a \overline{3} d$ |  |  | $u_{x}$ | $u_{x} / \sigma_{x}$ | $u_{y}$ | $u_{y} / \sigma_{y}$ | $u_{z}$ | $u_{z} / \sigma_{z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $x$ | $y$ | $z$ | $x$ | $y$ | $z$ |  |  |  |  |  |  |
| T11 | $16 f$ | 0.12498 (3) | 0.16223(3) | $0.41224(3)$ | 0.12500 | 0.16224 | 0.41224 | 0.00002 | 0.67 | 0.00001 | 0.33 | 0 | 0 |
| T12 | $16 f$ | $0.41224(3)$ | $0.12505(3)$ | $0.16226(3)$ | 0.41224 | 0.125 | 0.16224 | 0 | 0 | 0.00005 | 1.67 | 0.00002 | 0.67 |
| T2 | $16 f$ | $0.16225(3)$ | $0.41224(3)$ | $0.12501(3)$ | 0.16224 | 0.41224 | 0.125 | 0.00001 | 0.33 | 0 | 0 | 0.00001 | 0.33 |
| Na11 | $8 c$ | $0.12486(17)$ | 0 | 0.25 | 0.12500 | 0 | 0.25 | 0.00014 | 0.82 | 0 | 0 | 0 | 0 |
| Nal2 | $8 d$ | 0.25 | 0.12516 (17) | 0 | 0.25 | 0.125 | 0 | 0 | 0 | 0.00016 | 0.94 | 0 | 0 |
| Na2 | $8 e$ | 0 | 0.25 | 0.12512(17) | 0 | 0.25 | 0.125 | 0 | 0 | 0 | 0 | 0.00012 | 0.71 |
| O11 | $16 f$ | 0.10450(11) | $0.36577(12)$ | 0.21947(10) | 0.10452 | 0.36586 | 0.21949 | 0.00002 | 0.18 | 0.00009 | 0.75 | 0.00002 | 0.20 |
| O12 | $16 f$ | 0.38419(12) | $0.14557(11)$ | 0.46950 (10) | 0.38414 | 0.14548 | 0.46949 | 0.00005 | 0.42 | 0.00009 | 0.81 | 0.00001 | 0.10 |
| 021 | $16 f$ | $0.21946(10)$ | $0.10455(12)$ | $0.36571(12)$ | 0.21949 | 0.10452 | 0.36586 | 0.00003 | 0.30 | 0.00003 | 0.25 | 0.00015 | 1.25 |
| 022 | $16 f$ | 0.14539(12) | $0.46948(10)$ | $0.38407(12)$ | 0.14548 | 0.46949 | 0.38414 | 0.00009 | 0.75 | 0.00001 | 0.10 | 0.00007 | 0.58 |
| 031 | $16 f$ | 0.36590 (12) | $0.21952(10)$ | 0.10453(11) | 0.36586 | 0.21949 | 0.10452 | 0.00004 | 0.33 | 0.00003 | 0.30 | 0.00001 | 0.09 |
| O32 | $16 f$ | 0.46952(10) | $0.38396(12)$ | 0.14549(11) | 0.46949 | 0.38414 | 0.14548 | 0.00003 | 0.30 | 0.00018 | 1.5 | 0.00001 | 0.09 |
| W | $16 f$ | 0.1250(2) | 0.1254(2) | 0.1251(2) | 0.125 | 0.125 | 0.125 | 0 | 0 | 0.0004 | 2.0 | 0.0001 | 0.50 |

Table S6. Fractional atomic coordinates of the sample reheated for 24 given in the authors' reply, expressed in the setting of Ibca, as published (in parentheses the standard uncertainty $\sigma$ ) and idealized to fit the $I 4_{1} / a c d$ symmetry. $u$ is the absolute value of the shift between the published and dealized coordinates, $u / \sigma$ is the ratio between this shift and the standard uncertainty of the corresponding coordinate. This ratio shows that the two models are identical within three standard uncertainties.

| Site | Wyckoff position | Structural model in Ibca |  |  | Idealized structure in $I 4_{1} /$ acd |  |  | $u_{x}$ | $u_{x} / \sigma_{x}$ | $u_{y}$ | $u_{y} / \sigma_{y}$ | $u_{z}$ | $u_{z} / \sigma_{z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $x$ | $y$ | $z$ | $x$ | $y$ | $z$ |  |  |  |  |  |  |
| T11 | $16 f$ | 0.12499 | 0.16226 | 0.41226 | 0.12500 | 0.16230 | 0.41230 | 0.00001 | 0.50 | 0.00004 | 2.00 | 0.00004 | 2.00 |
| T12 | $16 f$ | 0.41225 | 0.12501 | 0.16226 | 0.41230 | 0.12500 | 0.16220 | 0.00005 | 2.50 | 0.00001 | 0.50 | 0.00006 | 3.00 |
| T2 | $16 f$ | 0.16226 | 0.41224 | 0.12500 | 0.16230 | 0.41220 | 0.12500 | 0.00004 | 2.00 | 0.00004 | 2.00 | 0.00000 | 0.00 |
| Na11 | 8 c | 0.12498 | 0 | 0.25 | 0.125 | 0 | 0.25 | 0.00002 | 0.15 | --- | --- | --- | --- |
| Na12 | $8 d$ | 0.25 | 0.12497 | 0 | 0.25 | 0.12490 | 0 | --- | --- | 0.00007 | 0.54 | --- | --- |
| Na2 | $8 e$ | 0 | 0.25 | 0.12495 | 0 | 0.25 | 0.12510 | 0.00000 |  | 0.00000 |  | 0.00015 |  |
| O11 | $16 f$ | 0.10449 | 0.36575 | 0.21943 | 0.10450 | 0.3660 | 0.21960 | 0.00001 | 0.12 | 0.00025 | 2.78 | 0.00017 | 2.43 |
| O12 | $16 f$ | 0.38422 | 0.14552 | 0.46942 | 0.38400 | 0.1455 | 0.46950 | 0.00022 | 2.44 | 0.00002 | 0.25 | 0.00008 | 1.14 |
| O21 | $16 f$ | 0.21946 | 0.10452 | 0.3657 | 0.21950 | 0.1045 | 0.36590 | 0.00004 | 0.57 | 0.00002 | 0.25 | 0.00020 | 2.50 |
| O22 | $16 f$ | 0.14555 | 0.46941 | 0.38408 | 0.14550 | 0.4696 | 0.38400 | 0.00005 | 0.62 | 0.00019 | 2.71 | 0.00008 | 1.00 |
| O31 | $16 f$ | 0.36586 | 0.21945 | 0.10445 | 0.36600 | 0.2195 | 0.10450 | 0.00014 | 1.56 | 0.00005 | 0.71 | 0.00005 | 0.62 |
| O32 | $16 f$ | 0.46942 | 0.38416 | 0.14572 | 0.46950 | 0.3841 | 0.14550 | 0.00008 | 1.14 | 0.00006 | 0.67 | 0.00022 | 2.75 |
| W | $16 f$ | 0.12487 | 0.12483 | 0.12486 | 0.12500 | 0.1249 | 0.12510 | 0.00013 | 0.76 | 0.00007 | 0.41 | 0.00024 | 1.50 |


[^0]:    1 The operations of a normalizer map different but equivalent descriptions of a crystal structure. In this case, because of the different orientation of the tetragonal axis in the 24 h and 48 h samples, we need to consider the affine normalizer, not just the Euclidean one. Details about the normalizers of space groups can be found in Koch and Fischer (2006) and a recent discussion in Nespolo (2015).

