

Vlasov -Maxwell models for laser-plasma interaction in the relativistic regime

A. Ghizzo, P. Bertrand, N. Besse,

B. T. Réveillé, D. DelSarto

LPMIA

UHP-Nancy-1, CNRS

Collaborators: E. Sonnendrücker et al

(CALVI -INRIA project)

T. Johnston, INRS, Varennes, Canada

Topics

1. Vlasov plasmas
2. Modeling parametric-like instabilities
3. Applications I: Relativistic Parametric Instability
4. Application II: Overdense plasmas
 - 4A: Conventional Hole Boring scenario
 - 4B: Anomalous Electron Heating
5. Conclusions



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Vlasov plasmas

- Vlasov models have long been used to study short pulse high intensity laser-plasma interaction where collisions can be ignored:
- Parametric instabilities, beat wave, Raman and/or Brillouin scattering at moderate laser intensities

$$2a_0^2 = \frac{I(Wcm^{-2})\lambda_0^2(\mu m)}{1.368 \times 10^{18}} \ll 1$$

- *Questions:*
- What happens in the relativistic regime?

$$a_0 \geq 1$$

Vlasov plasmas: characteristic scales

	Collisional interactions	Collective interactions
Characteristic length	Mean free path $\ell \approx n^{-1} (e^2 / \epsilon_0 T)^{-2}$	Debye length $\lambda_D = (T/m)^{1/2} / \omega_p$
	$\ell \approx (n\lambda_D^3) \lambda_D$	
Characteristic time	Collision frequency $\nu \approx (T/m)^{1/2} / \ell$	Plasma frequency $\omega_p = (ne^2 / m\epsilon_0)^{1/2}$
	$\nu \approx \left(\frac{1}{n\lambda_D^3} \right) \omega_p$	

Vlasov plasmas: collective effects

A dichotomy experiment: $(e, m) \rightarrow 2(e/2, m/2) \rightarrow 4(e/4, m/4) \rightarrow \text{etc...}$

$$\begin{aligned} e &\rightarrow 0 \\ m &\rightarrow 0 \\ n &\rightarrow \infty \\ T &\rightarrow 0 \end{aligned}$$

$$\begin{aligned} \omega_p^2 &= \frac{1}{\epsilon_0} (ne) \left(\frac{e}{m}\right) = \text{const} & \nu &\rightarrow 0 \\ \lambda_D &= \text{const} & \ell &\rightarrow \infty \end{aligned}$$

$$\begin{aligned} e/m &= \text{const} \\ T/m &= \text{const} \\ ne &= \text{const} \end{aligned}$$

$$\begin{aligned} n\lambda_D^3 &\rightarrow \infty \\ g &= \frac{1}{n\lambda_D^3} \rightarrow 0 \end{aligned}$$

g = dimensionless parameter,
 - divided by 2 at each dichotomy
 - « graininess parameter »

The Vlasov-Maxwell equations

Relativistic laser-plasma interaction, with short pulses
needs to solve:

Vlasov equation for electrons or ions

$$\frac{\partial f}{\partial t} + [H; f] = 0$$

Distribution function $f = f(x, y, z, p_x, p_y, p_z, t)$

Hamiltonian $H = mc^2(\gamma - 1) + e\phi$

+ Maxwell's equations

Comparison PIC-Vlasov (1)

Sampling the x-space needs $\approx (L/\lambda_D)^{D_x}$

D_x : real space dimension

PIC Codes:

$$\begin{aligned}
 N_{part} &= n_0 (L)^{D_x} \\
 &\approx n_0 (\lambda_D)^{D_x} \left(\frac{L}{\lambda_D} \right)^{D_x} \\
 &\approx \frac{1}{g_{PIC}} (L/\lambda_D)^{D_x}
 \end{aligned}$$

$$g_{PIC} = \left(n_0 (\lambda_D)^{D_x} \right)^{-1}$$

is the graininess due to particles

Vlasov Codes:

Real space X momentum space

$$\begin{aligned}
 N_{Vlasov} &\approx (L/\lambda_D)^{D_x} N_p^{D_p} \\
 &\approx g_{PIC} N_{part} N_p^{D_p}
 \end{aligned}$$

D_p : momentum space dimension

N_p : sampling of momentum space in each direction

Comparison PIC-Vlasov (2)

Assume the same CPU time

- to push a particle (PIC)
- to move a phase space mesh point (Vlasov)

$$\frac{CPU_{Vlasov}}{CPU_{PIC}} \approx g_{PIC} N_p^{D_p}$$

The ratio of the computational effort between Vlasov and PIC depends on

- PIC graininess (must be as low as possible)
- Sampling of momentum space (must be as high as possible)

Comparison PIC-Vlasov (3)

$\frac{CPU_{Vlasov}}{CPU_{PIC}}$	$D_p=1$	$D_p=2$	$D_p=3$
$g_{PIC}=10^{-2}$	1	10 ²	10 ⁴
$g_{PIC}=10^{-4}$	10 ⁻²	1	10 ²
$g_{PIC}=10^{-6}$	10 ⁻⁴	10 ⁻²	1

Prefer PIC



Prefer Vlasov



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Modeling parametric-like instabilities

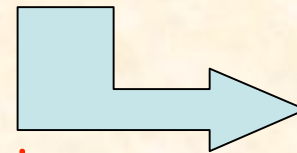
Plane wave propagation

Vlasov equation for the electronic distribution function F

$$\frac{\partial F}{\partial t} + \frac{p_x}{m\gamma} \frac{\partial F}{\partial x} + e \left(\mathbf{E} + \frac{\mathbf{p} \times \mathbf{B}}{m\gamma} \right) \cdot \frac{\partial F}{\partial \mathbf{p}} = 0$$

$$\gamma = \left(\frac{p_x^2 + \mathbf{p}_\perp^2}{m^2 c^2} \right)^{1/2}$$

$$F = F(x, p_x, \mathbf{p}_\perp, t) \Rightarrow D_x = 1 \text{ but } D_p = 3$$



Does not point
to Vlasov Codes!

Par chance we have an invariant!

Particle motion (1)

Let us come back to particle motion and consider the Hamiltonian of an electron in an electromagnetic field

$$H = mc^2 \left(1 + \frac{(\mathbf{P}_c - e\mathbf{A}_\perp)^2}{m^2 c^2} \right)^{1/2} + e\phi(x, t)$$

Using Coulomb gauge $\nabla \cdot \mathbf{A} = 0 \Rightarrow \mathbf{A} = \mathbf{A}_\perp$

$\mathbf{P}_c = \mathbf{p} + e\mathbf{A}_\perp$ is the canonical momentum

$$\text{Hamilton equation} \quad \begin{cases} \frac{dP_{cx}}{dt} = -\frac{\partial H}{\partial x} \\ \frac{d\mathbf{P}_{c\perp}}{dt} = -\nabla_\perp H = 0 \Rightarrow \mathbf{P}_{c\perp} = \text{const} \end{cases}$$

Particle motion (2)

Now let us divide the electron population into a **finite number** j_{max} of groups j

Each group has the **same perpendicular constant canonical momentum** \mathbf{C}_j

The hamiltonian of the j -particles is

$$H_j = mc^2(\gamma_j - 1) + e\phi(x, t)$$

With the Lorentz factor

$$\gamma_j(x, p_x, t) = \left[1 + \frac{p_x^2 + (\mathbf{C}_j - e\mathbf{A}_\perp(x, t))^2}{m^2 c^2} \right]^{1/2}$$

Vlasov equation for j -particles (1)

Now we have to solve j_{max} Vlasov equations
in the 2D phase space (x, p_x)

$$\frac{df_j}{dt} = \frac{\partial f_j}{\partial t} + \frac{\partial H_j}{\partial p_x} \frac{\partial f_j}{\partial x} - \frac{\partial H_j}{\partial x} \frac{\partial f_j}{\partial p_x} = 0 \quad f_j = f_j(x, p_x, t)$$

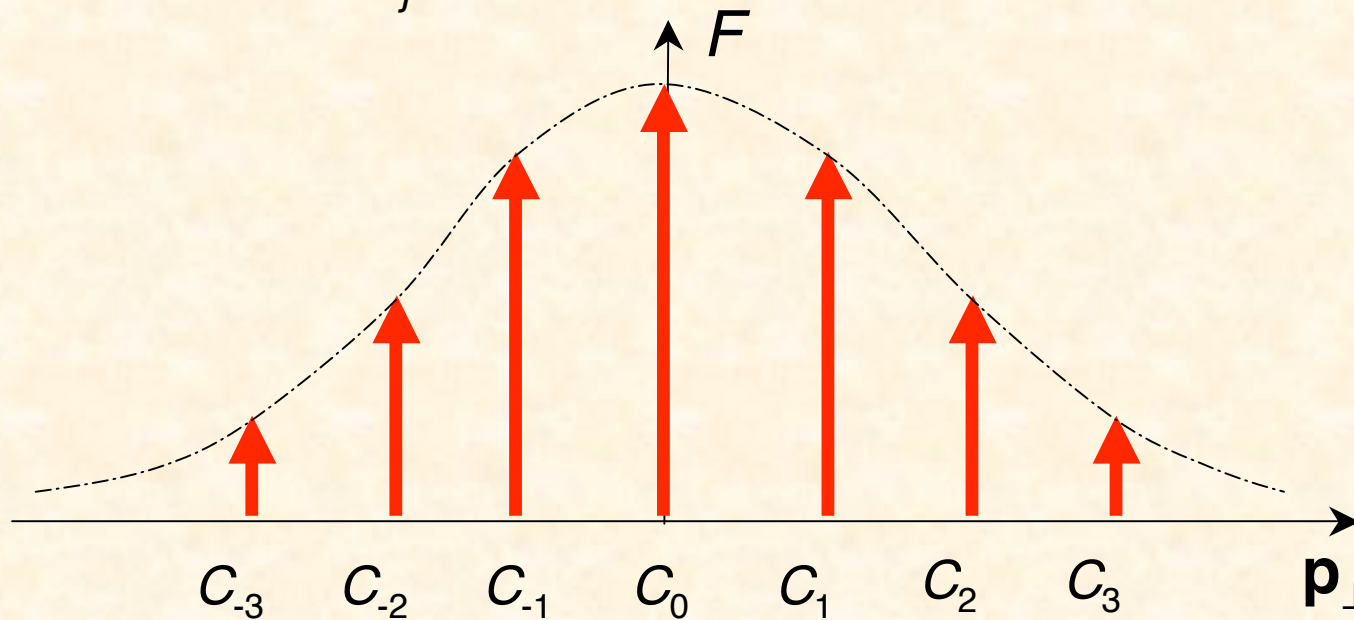
$$\frac{\partial f_j}{\partial t} + \frac{p_x}{m\gamma_j} \frac{\partial f_j}{\partial x} - \left[e \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x} \left(\mathbf{C}_j - e\mathbf{A}_\perp \right)^2 \right] \frac{\partial f_j}{\partial p_x} = 0$$

The f_j 's are coupled by Maxwell's equations for scalar and vector potential

Vlasov equation for j -particles (2)

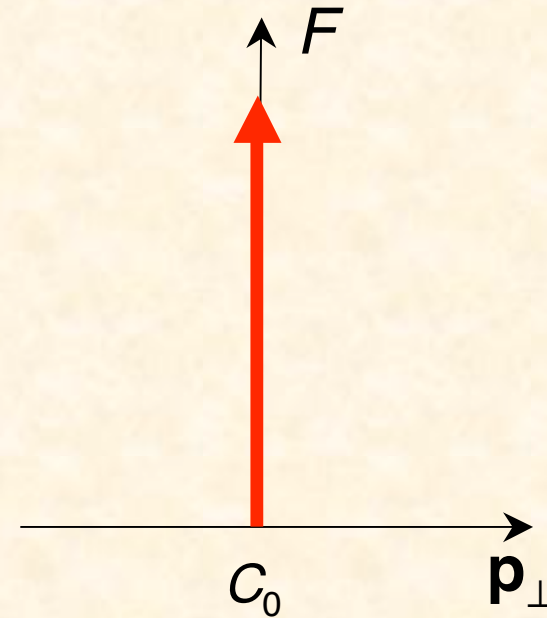
The full distribution function can be written as a sum of Dirac delta masses

$$F(x, p_x, \mathbf{p}_\perp, t) = \sum_j f_j(x, p_x, t) \delta[\mathbf{p}_\perp - (\mathbf{C}_j - e \mathbf{A}_\perp(x, t))]$$



Vlasov equation for j -particles (3)

- Advantage: solve j_{max} Vlasov equations in a 2D space
- Complexity= $(j_{max}) (N_x) (N_p)$ instead of $(N_x) (N_p)^3$.
- $j_{max} \ll (N_p)^2$ is expected
- If the perpendicular temperature remains weak (longitudinal heating only)
- Take $j_{max}=1$ and $C_0=0$



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The Relativistic Parametric Instability (1)

Dispersion relation for a cold plasma is known
(see S. Guerin et al PoP 7, 2807 - 1995):

$$D_+ D_- = \frac{\omega_p^2 a_0^2}{2\gamma_0^3} \left(\frac{k^2 c^2}{D_p} - 1 \right) (D_+ + D_-)$$

$\gamma_0 = \sqrt{1 + a_0^2}$ is the Lorentz factor

$$D_p = \omega^2 - \frac{\omega_p^2}{\gamma_0}$$

Dispersion relation for an EPW in
the cold case

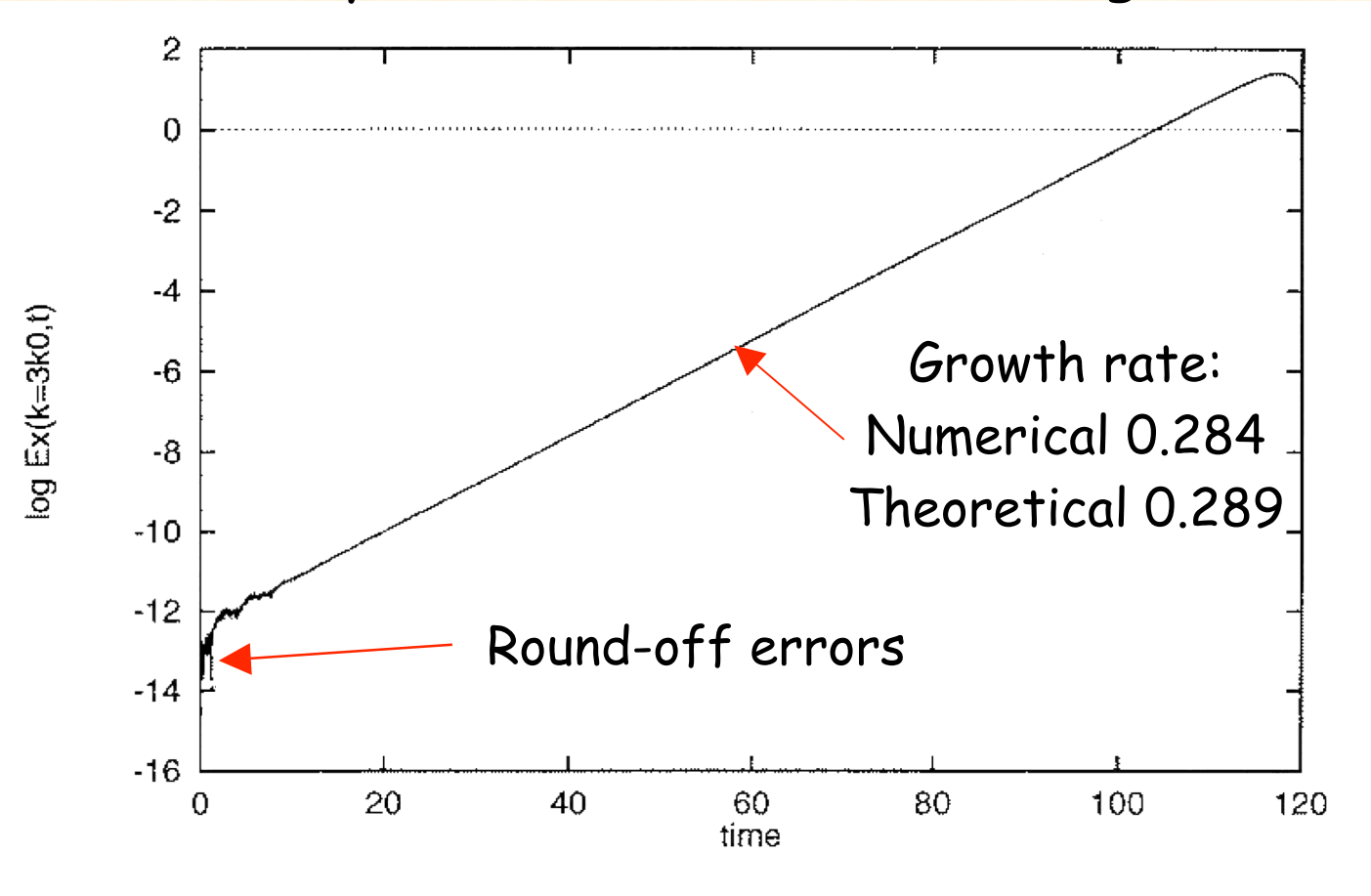
$$D_{\pm} = (\omega \pm \omega_0)^2 - (k \pm k_0)^2 c^2 - \frac{\omega_p^2}{\gamma_0}$$

Dispersion relation for an
electromagnetic wave inside the
cold plasma

The Relativistic Parametric Instability (2)

Propagation of Circularly Polarized wave in a homogeneous plasma:

Physical Parameters
 $a_0=0.75$
 $n/n_c=0.55$



The Relativistic Parametric Instability (3)

- Phase space behavior

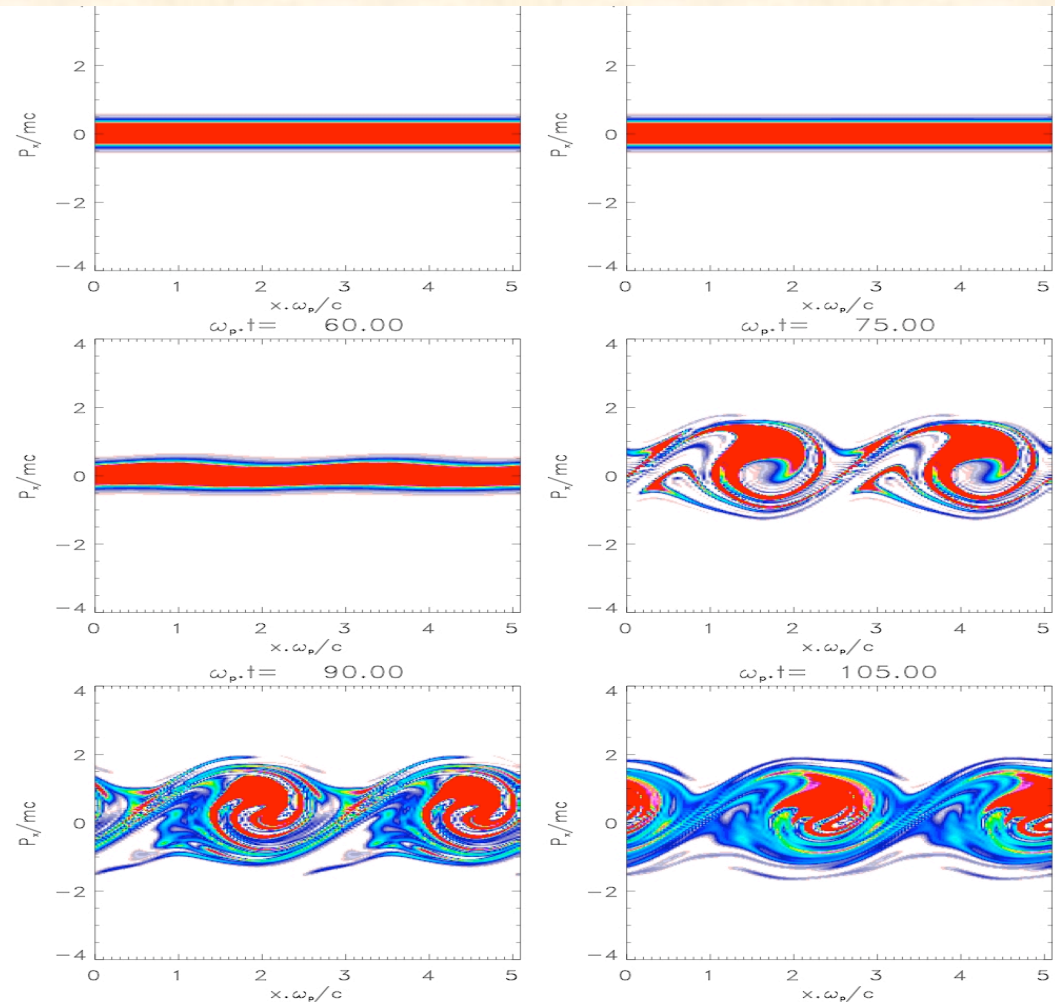
- Growth rate is now close to 0.56

Physical Parameters

$T_e = 15 \text{ keV}$

$a_0 = 1.732$

$n/n_c = 0.50$



The Relativistic Parametric Instability (4)

- Phase space behavior

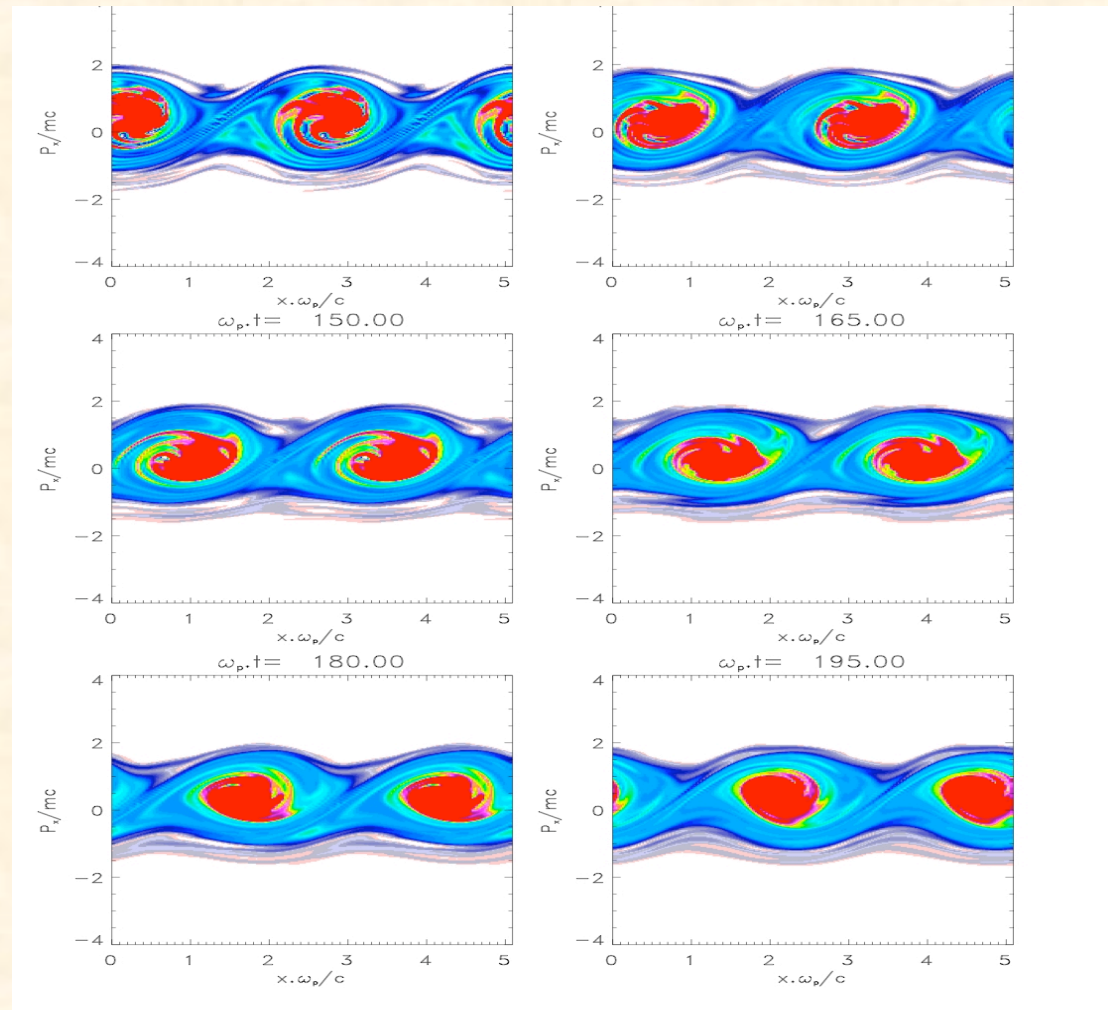
- Growth rate is now close to 0.56

Physical Parameters

$T_e = 15 \text{ keV}$

$a_0 = 1.732$

$n/n_c = 0.50$

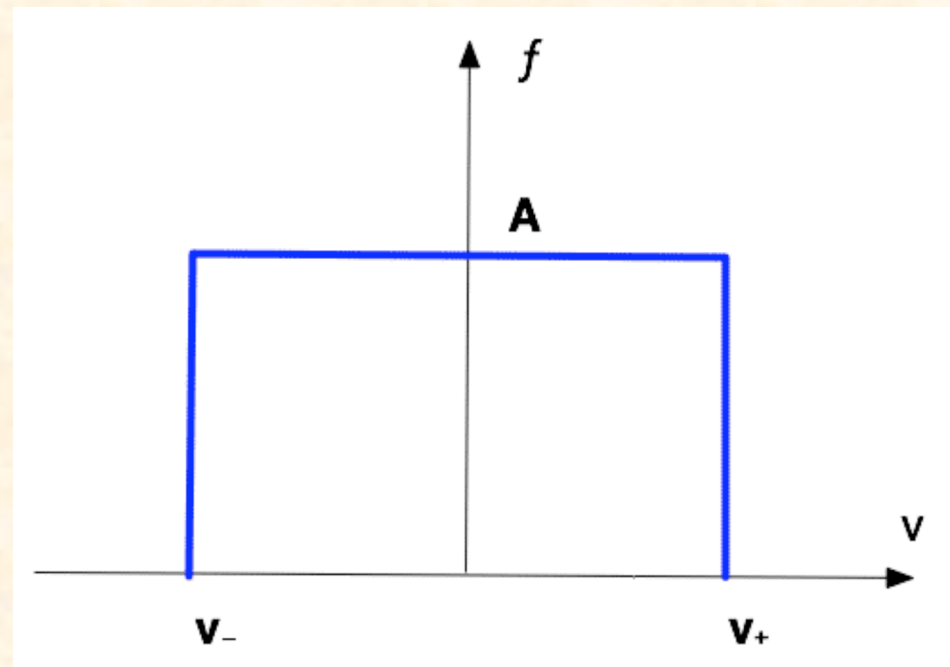


The Water-Bag approach: the case of a hot plasma

The distribution function may be described as:

$$f_0(p) = \frac{1}{2p_m} [H(p - p_m) - H(p + p_m)]$$

H is the Heaviside fstep function and $+p_m$ and $-p_m$ are the limit values in momentum



Water-Bag approach (2)

The new dispersion relation for a hot plasma is then:

$$D_+ D_- = \frac{\omega_p^2 a_0^2}{2\gamma_0^2 \gamma_m D_p} \left(k^2 c^2 + \frac{\omega_p^2}{\gamma_m} - \omega^2 \right) (D_+ + D_-)$$

$\gamma_m = \sqrt{1 + a_0^2 + \frac{p_m^2}{m^2 c^2}}$ is the new Lorentz factor

$$D_p = \omega^2 - \frac{\omega_p^2}{\gamma_m} - k^2 v_m^2$$

for an EPW in the hot case

$$D_{\pm} = (\omega \pm \omega_0)^2 - (k \pm k_0)^2 c^2 - \omega_p^2 \langle \gamma^{-1} \rangle$$

for an electromagnetic wave

with

$$\langle \gamma^{-1} \rangle = \int_{-\infty}^{+\infty} \frac{dp F_0(p)}{\gamma} = \frac{mc}{2p_m} \ln \frac{c + v_m}{c - v_m}$$

Water-Bag approach (3)

- Growth rate is now strongly reduced

Physical Parameters

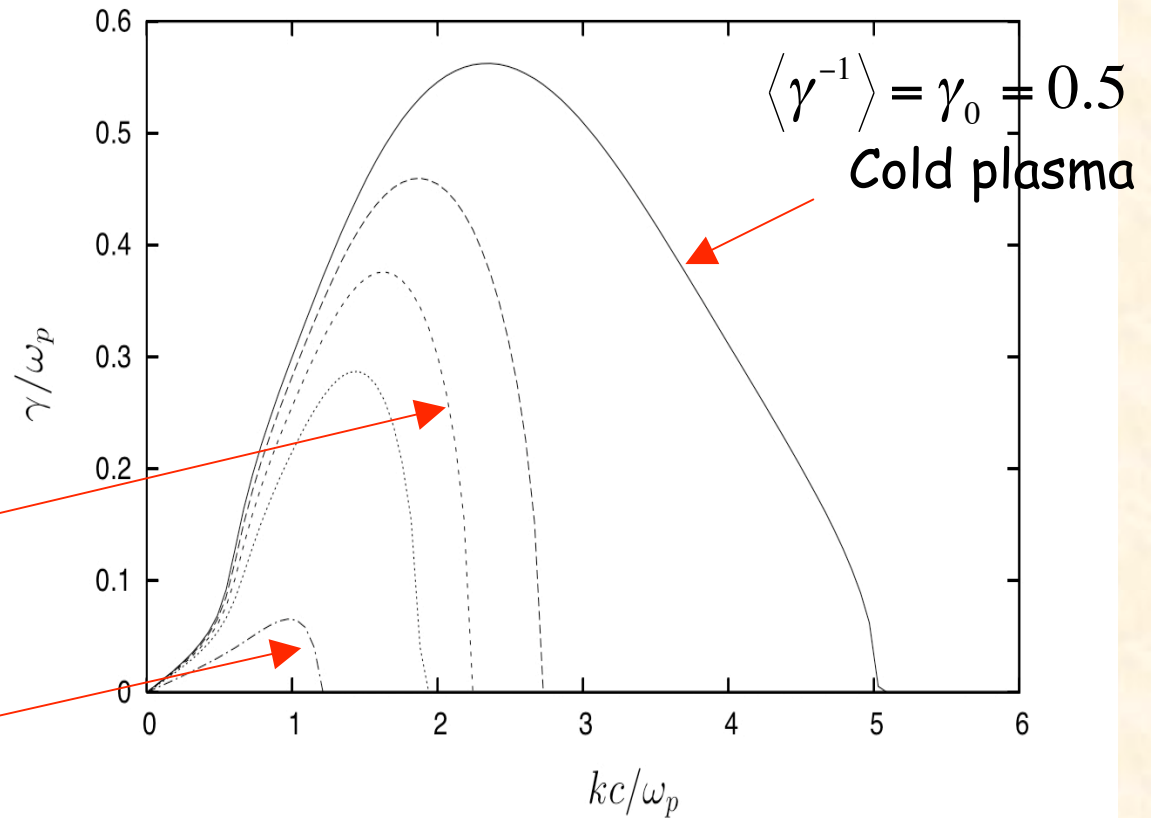
$T_e = 15 \text{ keV}$

$a_0 = 1.732$

$n/n_c = 0.50$

$$\langle \gamma^{-1} \rangle \approx 0.48$$

$$\langle \gamma^{-1} \rangle \approx 0.36$$



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Semi-lagrangian Vlasov simulations: Hole Boring scenario (1)

Situation is somewhat different in an overdense open plasma:

The laser pulse is capable to propagate through an over-dense due to relativistic effects.

Conventional Hole Boring scenario or forward motion of the critical surface caused by light pressure



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Conventional Hole Boring scenario (2)

- Wave front velocity

$$v_F \approx 0.055c$$

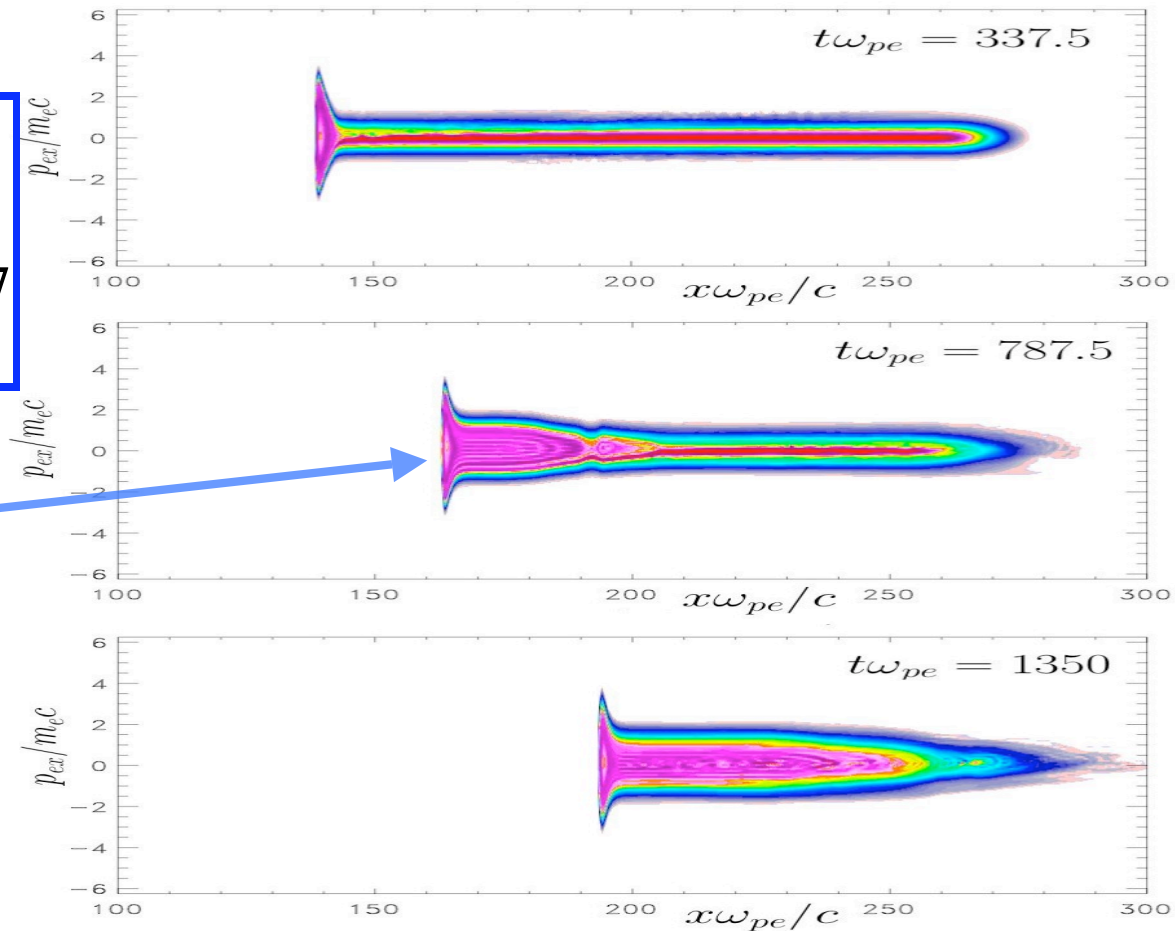
$$\frac{v_F}{c} = \left(\frac{n_c}{n_0} \frac{m_e}{m_i} a_0^2 \right)^{1/2} = 0.057$$

Motion of the
critical surface

$$a_0 = 5$$

$$n_0 = 4n_c$$

$$T_e = 100 \text{ keV}$$

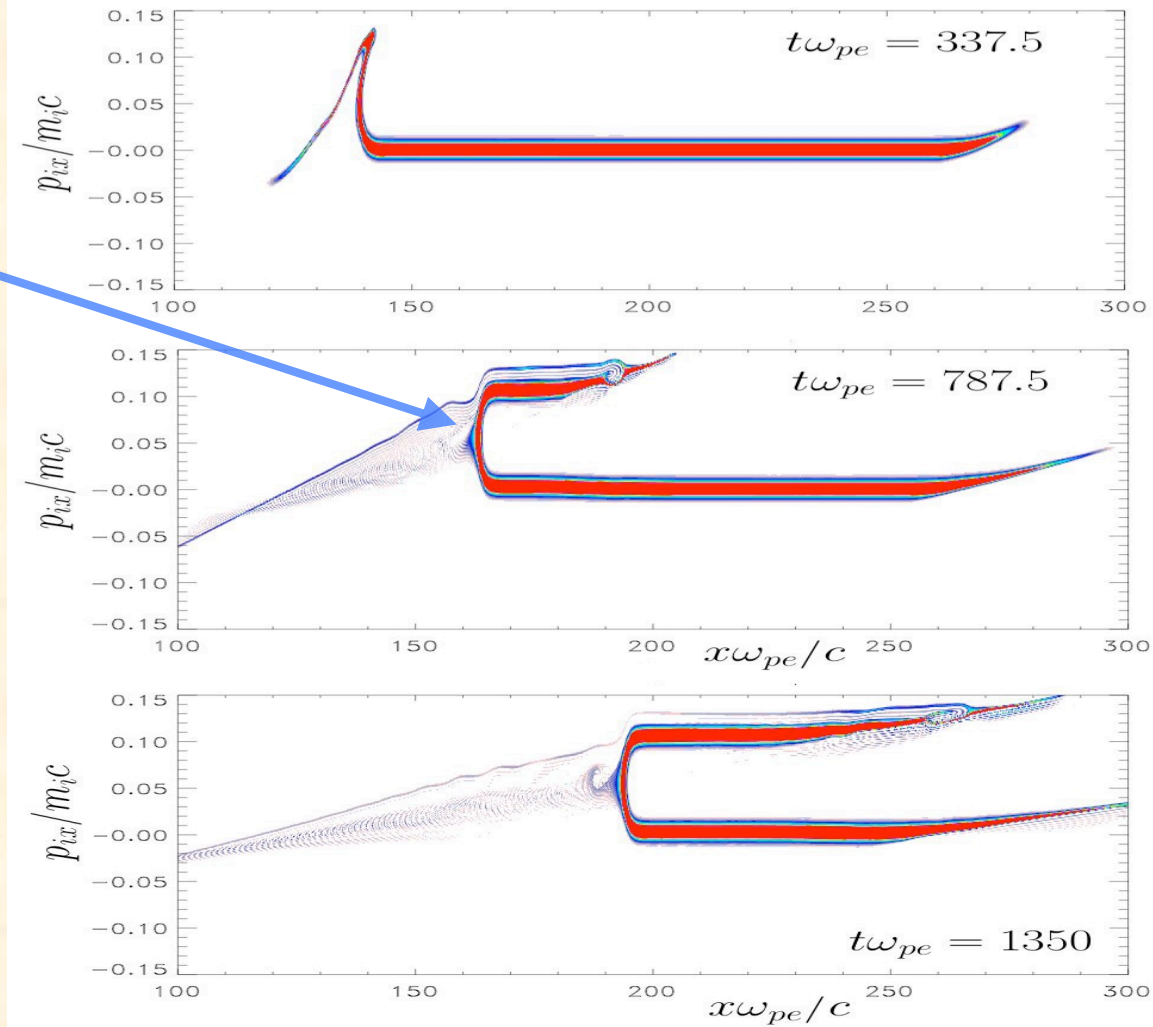


Conventional Hole Boring scenario (3)

- Electrostatic collisionless ion shock
- M Mach number

$$v_{shock} = 2v_F \approx 0.11c$$

$$v_{shock} = Mc_s$$



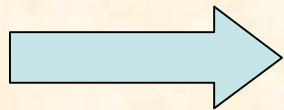
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Anomalous electron Heating: Self Induced Transparency regime

Relativistic Doppler shift takes place at moderate density
Due to the formation of a discontinuity in density

Beat-wave between the pump (ω_0, k_0) and its reflected (ω_r, k_r)
electromagnetic wave



(Doppler) Beat-Wave Heating -DBWH mechanism
responsible to anomalous electron heating

Self Induced Transparency Doppler Beat Wave Heating-DBWH

Beat-Wave between the the incoming pump wave and the reflected wave at the wave front.

Matching conditions required for the resonance are:

$$\omega_0 = \omega_r + \omega_{EAW} \quad \text{and} \quad k_0 = -k_r + k_{EAW}$$

Low-frequency Electron
Acoustic Wave (EAW) in a
NonLinear regime

$$\omega_{EAW} = k_{EAW} v_{front} \approx k_{EAW} v_{th}$$

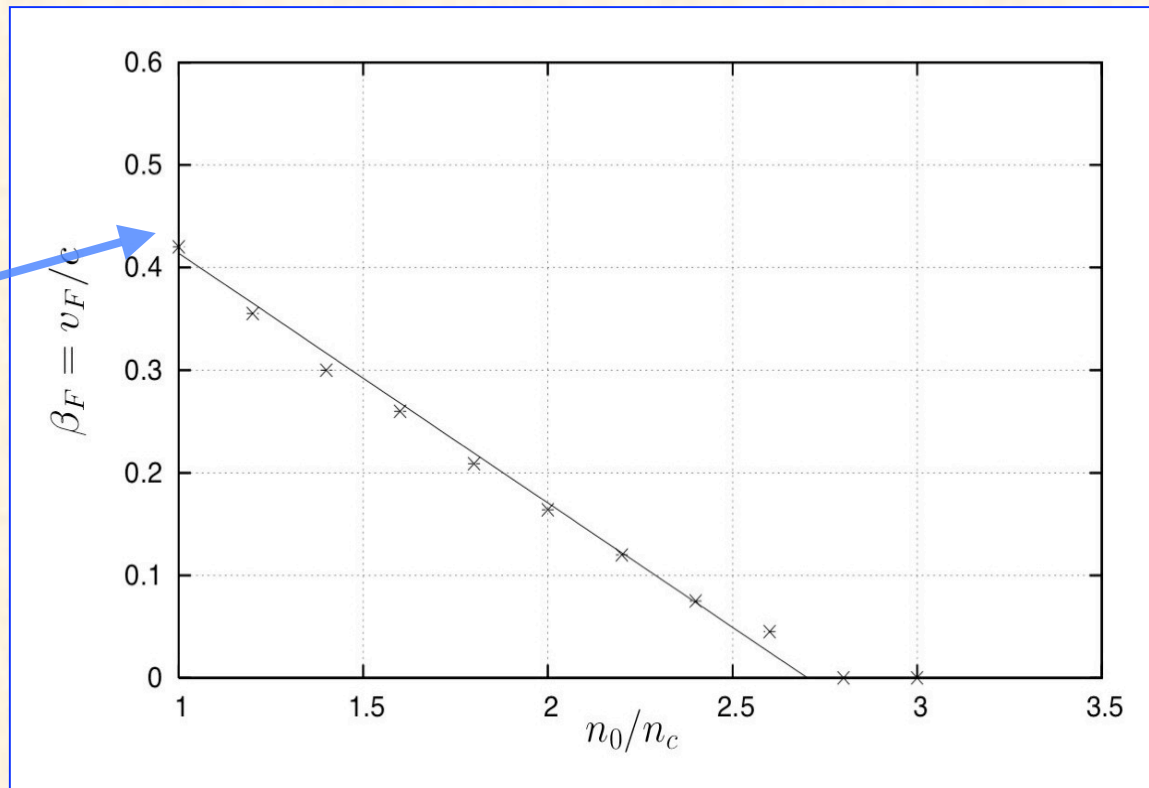
$$\omega_{0;r}^2 = \omega_{pe}^2 \langle \gamma^{-1} \rangle + k_{0;r}^2 c^2$$

$$\langle \gamma^{-1} \rangle = \frac{1}{n_0} \int_{system} dx \int_{-\infty}^{+\infty} dp_x \frac{f_e}{\gamma(x, p_x, t)}$$

Self Induced Transparency Doppler Beat Wave Heating (2)

Wave front velocity depends on $n_0/n_c; T_e$ and a_0

High value of the
wave-front
velocity



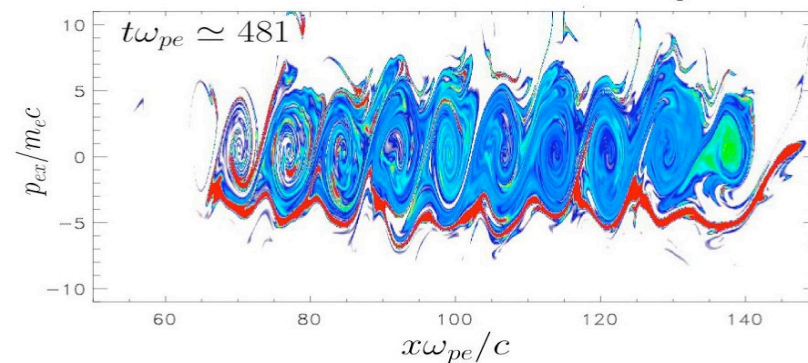
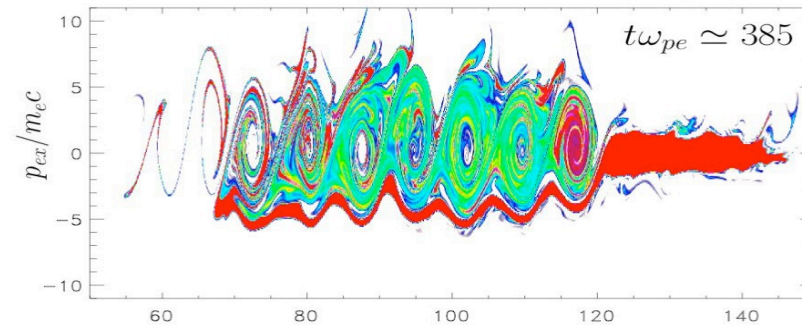
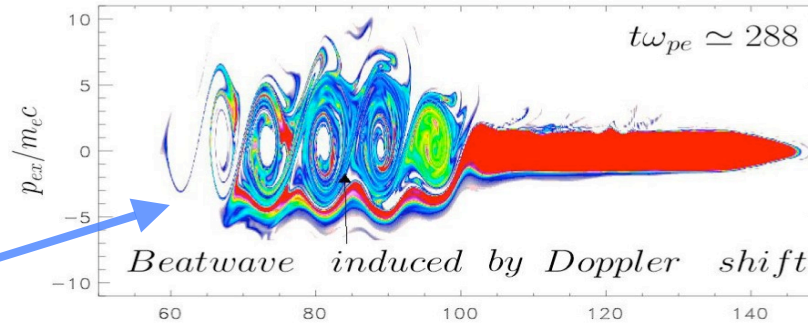
Doppler BeatWave Heating (3)

- Phase space « bumps » rather than holes
- EAWs are driven at a nonlinear level
- Ions do not play a role in the DBWH regime
- Wave front velocity is high close to $v_F \approx 0.22c$

$$a_0 = \sqrt{5}$$

$$n_0 = 2n_c$$

$$T_e = 400 \text{ keV}$$



Doppler BeatWave Heating (4)

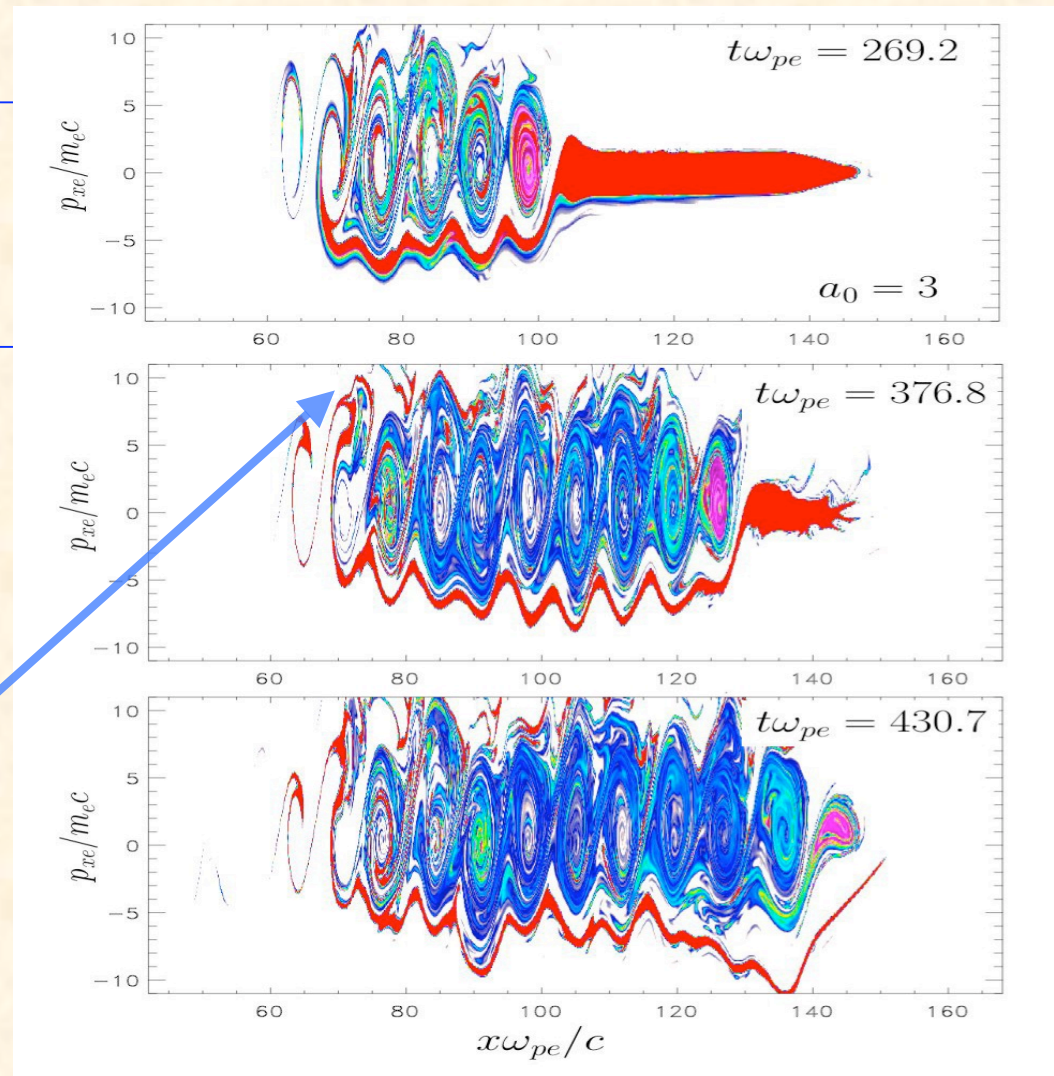
- We have increased the quiver momentum keeping other parameters identical

$$a_0 = 3$$

$$n_0 = 2n_c$$

$$T_e = 400\text{keV}$$

Size of the « trapping » structures are bigger



SIT- second step of the instability Stimulated KEEN Slab Scattering

- Once the plasma slab crossing by the wavefront is achieved, the plasma is subject to a new parametric SRS-like instability.
- The scattered wave is found to be always driven near critical:

$$\omega_0 = \omega_s + \omega_{KEEN} \quad \text{and} \quad \omega_s \approx \omega_p \sqrt{\langle \gamma^{-1} \rangle}$$
$$k_0 = \pm k_s + k_{KEEN} \approx k_{KEEN}$$

- Excitation of Kinetic Electrostatic Electron Nonlinear waves (the nonlinear « cousins » of EAWs characterized by trapping structures in phase space)
- Experimental observation of TEAWs by Montgomery et al from experiments on low-density plasmas (PRL **87**, 155001 (2001))



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Stimulated KEEN Slab Scattering (2)

- First observed in PIC simulations by Nikolic et al in PRE 66, 036404 (2002) and reinterpreted by Ghizzo et al in PRE 74, 046407 (2006)
- KEEN waves are quite nonresonant, being relatively easy to drive up over a significant range of frequencies for a given wavenumber
- The lack of low-level narrow band resonance in the electrostatic oscillation precludes the usual stimulated scattering scenario of three-wave resonance amplification from a low level seed.
- The plasma is in the form of a relatively sharp-edged slab and this supports an electromagnetic quasi-mode or *radiating plasma pseudocavity mode with*

$$k_s = \frac{n\pi}{L_{slab}} \ll 1$$

Stimulated KEEN Slab Scattering (3)

$$a_0 = \sqrt{5}$$

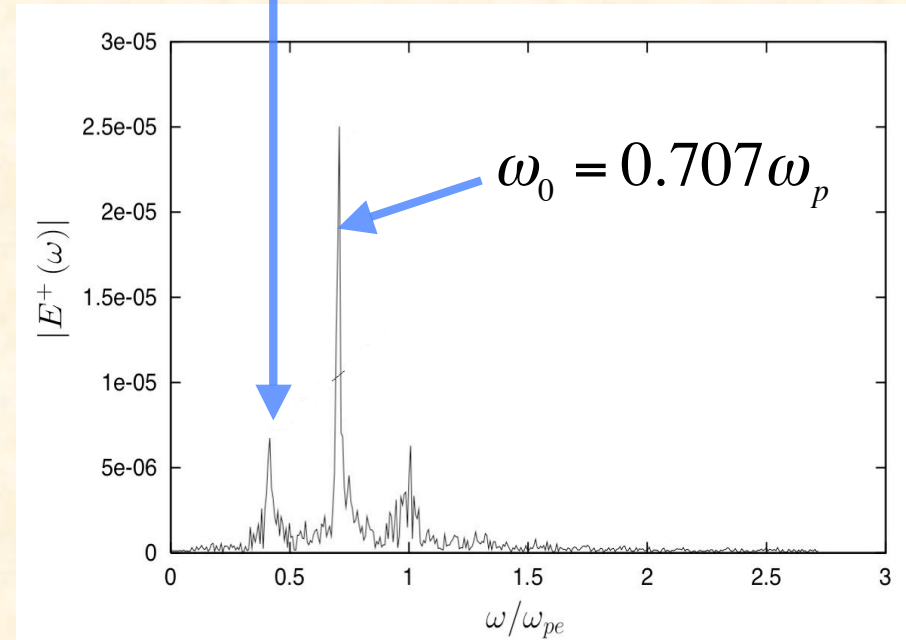
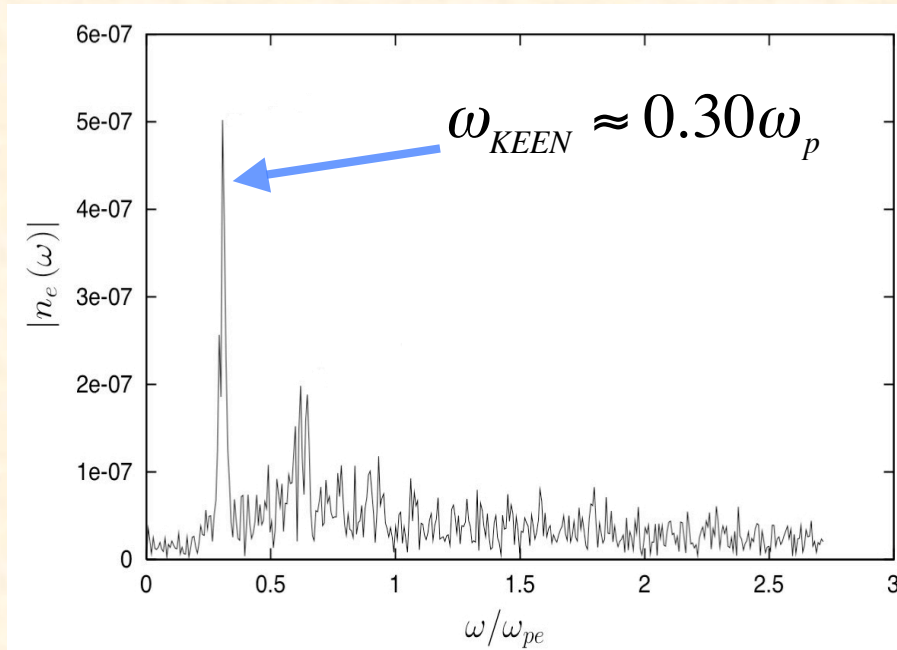
$$n_0 = 2n_c$$

$$T_e = 400keV$$

$$\omega_0 = \omega_s + \omega_{KEEN}$$

$$k_0 = \pm k_s + k_{KEEN} \approx k_{KEEN}$$

$$\omega_s = \omega_p \sqrt{\langle \gamma^{-1} \rangle} \approx 0.40\omega_p$$



Stimulated KEEN Slab Scattering (4)

- Trapping structures at a wavenumber k_{KEEN} close to 0.5 ie k_0 or k_{PUMP}

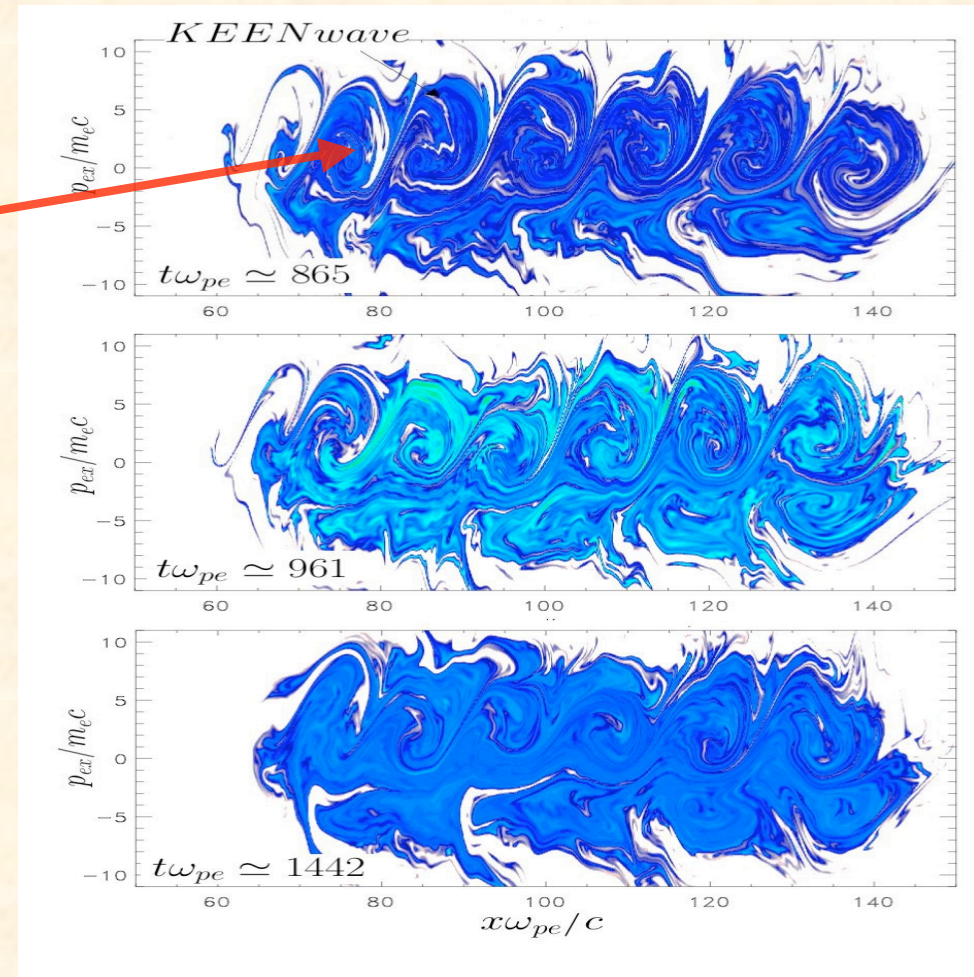
$$\omega_0 = \omega_s + \omega_{KEEN}$$

$$k_0 = \pm k_s + k_{KEEN} \approx k_{KEEN}$$

$$a_0 = \sqrt{5}$$

$$n_0 = 2n_c$$

$$T_e = 400keV$$



Transition regime (DBWH and Hole-Boring)

- Strong decrease of the wave-front velocity (which now obeys the Denavit's formula)

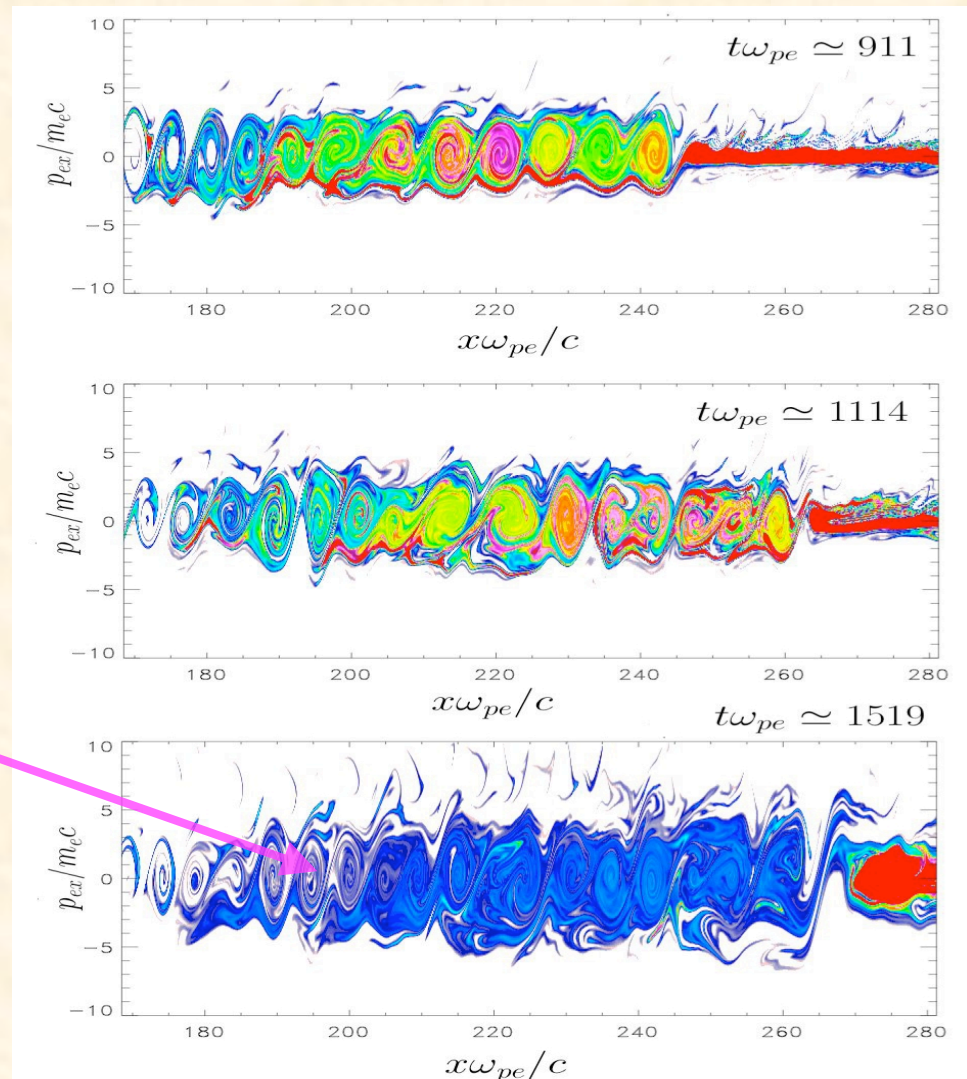
• BEAT-WAVE HEATING

- Initial phase of DBWH
- Strong heating

$$a_0 = \sqrt{2}$$

$$n_0 = 1.6n_c$$

$$m_i = 1836m_e$$



Transition regime (2)

- Strong decrease of the wavefront velocity (which now obeys the Denavit's formula)

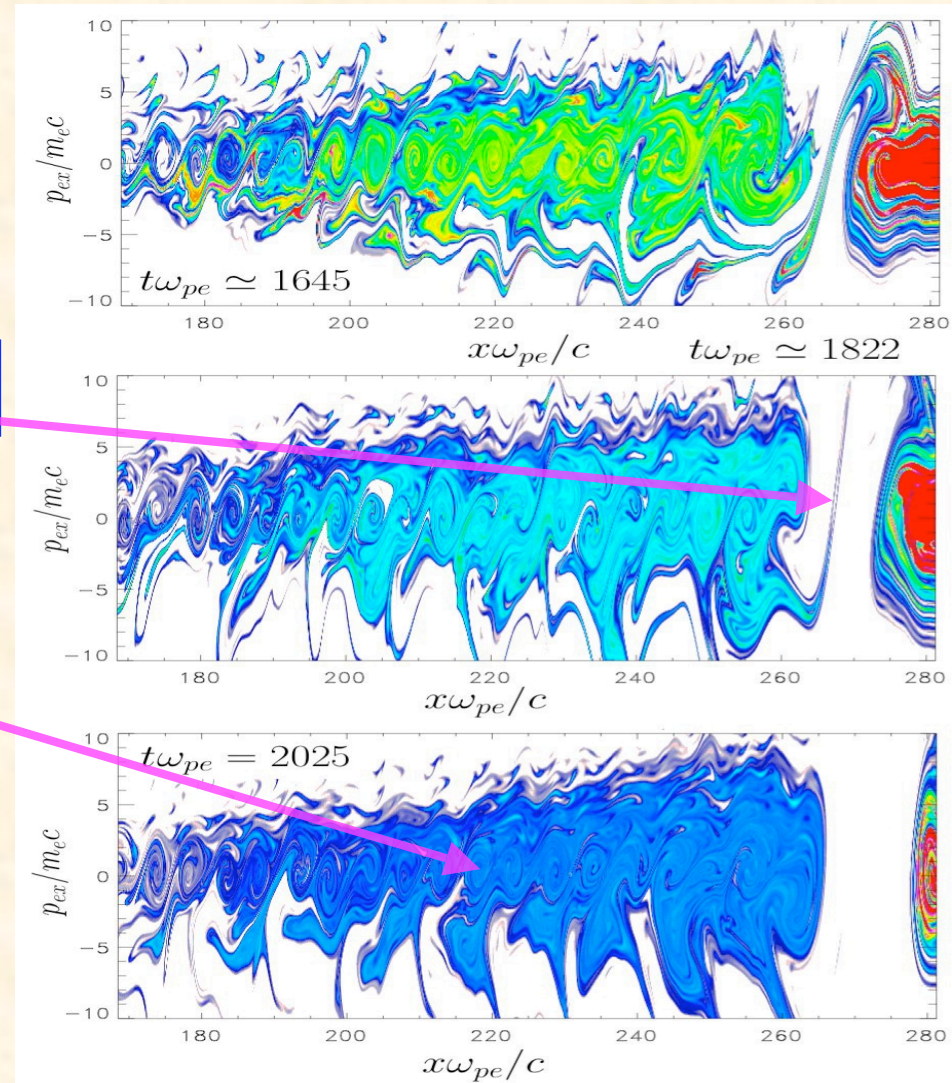
- Electron cavitation

- Initial phase of DBWH
- Strong heating

$$a_0 = \sqrt{2}$$

$$n_0 = 1.6n_c$$

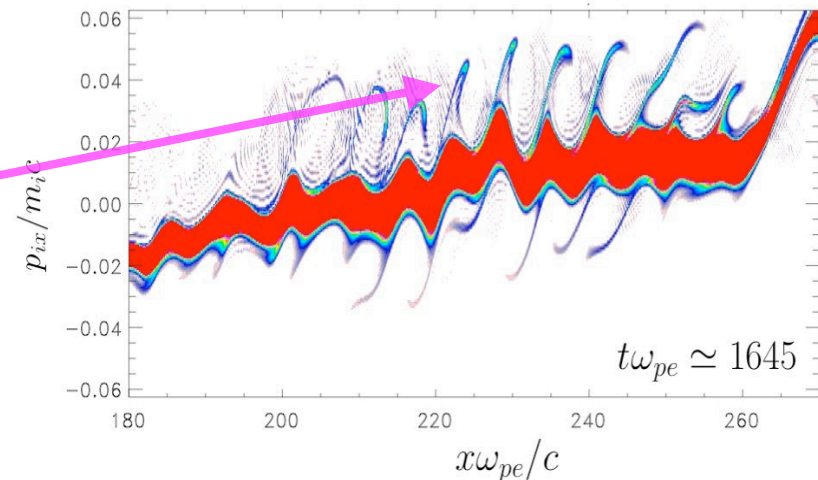
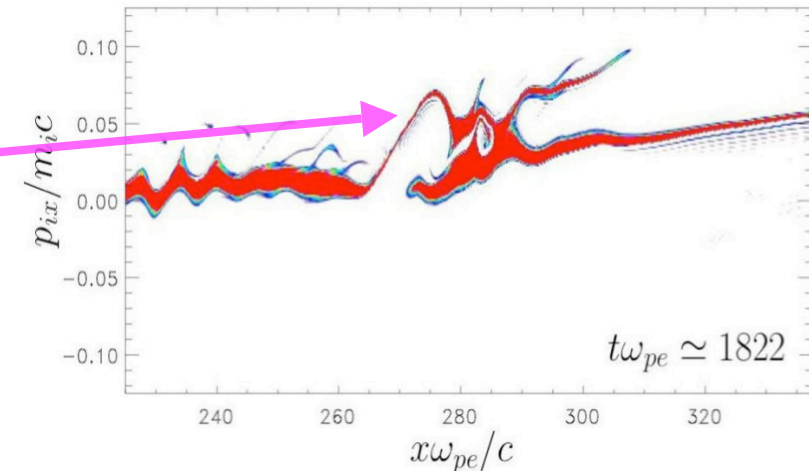
$$m_i = 1836m_e$$



Transition regime (3)

Collisionless Ion Shock observed
in Ion Phase Space plus ion
Wave-Breaking

- Modification to the original picture: now IAWs are driven and replace EAWs till Ion trapping



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Conclusion

- Semi-Lagrangian Vlasov models gives very well description of particle trapping and particle heating
- Canonical invariant in transverse directions (Weibel instability)
- Extension possible to Wavelet-Multi-Resolution Analysis - adaptative-SL method for the relativistic Vlasov-Maxwell system (for instance relativistic parametric instability)

