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# GEOSTATISTICS ON STRATIGRAPHIC GRIDS

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## ABSTRACT

*For stratigraphic grids, the traditional method used to model properties does not consider the volume of the cells and ignores the volumetric distortion between physical and depositional space. Therefore, it introduces some biases. A sampling based method allows to solve this problem for variogram-based geostatistics.*

## INTRODUCTION

The fundamental idea underlying the field of geostatistics is the concept of random function, whereby the set of unknown values is considered as a set of spatially dependent random variables. This variable is defined on a specific support, the term support reflecting the geometric characteristics (size, shape and spatial orientation) associated with the measurement. Changing of support will create a new variable. Though related to the original one, this variable has different statistical and spatial characteristics. While this issue has been well addressed in the mining industry, solving this problem remains a real challenge in cases where the underlying grid has cells with changing support. In that case, the size and shape of each grid cell is controlled by stratigraphy (layering and faulting). The grid no longer provides a constant support for property modeling. Some analytical methods, designed for mining applications, exist but applied to stratigraphic modeling, become computationally prohibitive. In this paper, we develop a new method to account for this change of support when modeling on stratigraphic grids.

## GEOSTATISTICS AND GRID GEOMETRY REPRESENTATION

### Grid definition

A critical step in many spatial modeling methods is to represent accurately the geometry of the domain being modeled, as well as the properties modeled within that domain. The volume of the domain is discretized by a set of grid cells. Different topologies can be used such as Cartesian grid, a stratigraphic grid, or any unstructured grid. One type of grid commonly employed is the stratigraphic grid (Caumon, 2006). This is an irregular, structured grid, locally unstructured

when faulted and exclusively composed of hexahedral cells, distorted and indexed with three axis (stratigraphic coordinate): U and V parallel to the layering, and W perpendicular to U and V and representing the time of the deposition.

### **Advantages of a stratigraphic grid**

Geostatistical algorithms use intensively distance calculations between the data location and the block where the property is interpolated. This raises the problem of how these distances should be computed. If the domain is neither faulted nor folded then using a Euclidian distance is relevant. This is, however, rarely the case. Hence, a curvilinear coordinates system U V W is more appropriate (Mallet, 2004). Such curvilinear coordinates account for the shape of the horizons (which themselves control the geological continuity). It defines implicitly a Geo-Chronological (depositional space) model of the subsurface structures. A gridding following these structures (stratigraphic grid) defines then the stratigraphic heterogeneities of the reservoir and simplifies many problems: realistic and efficient spatial modeling of properties within the grid, (Mallet (2004) and Caumon (2006)). This approach could be considered as a specific numerical representation of "Time-Stratigraphy" concept introduced by Wheeler (Wheeler, 1958), the Geochron model providing just a numerical generalization of this concept.

### **Volume distortion**

In the chronological space, the vertical axis corresponds to the age of deposition (time axis). From a stratigraphic grid, the corresponding grid in the depositional space has constant block size, the boundary between each layer corresponding to an iso-time line. The thickness of the stratigraphic grid cells are not controlled only by the duration of the sedimentation event, but also by its nature, the presence of erosion. As a consequence, the volumes are not conserved between the physical and the chronostratigraphic spaces (Fig.1).

### **Link between the physical space and depositional space**

As previously explained, two grids are used, one in the physical space, where all the structural and geological features are incorporated (stratigraphic grid) and one in the depositional space (Cartesian grid) where geostatistical algorithms are applied. The volumes are not conserved between the two spaces. Therefore, linking these two spaces is not straightforward. More precisely, it raises three main concerns: the first one is the ability to transfer consistently (hard and soft) data from the real space to the depositional space. The second one is to respect some target statistics defined in the physical space while the geostatistical modeling is executed in the depositional space. The last problem requires accounting for the volumetric distortion in the Cartesian space, even if this is not explicitly represented (there is no information in the depositional space about the geometry of the model).

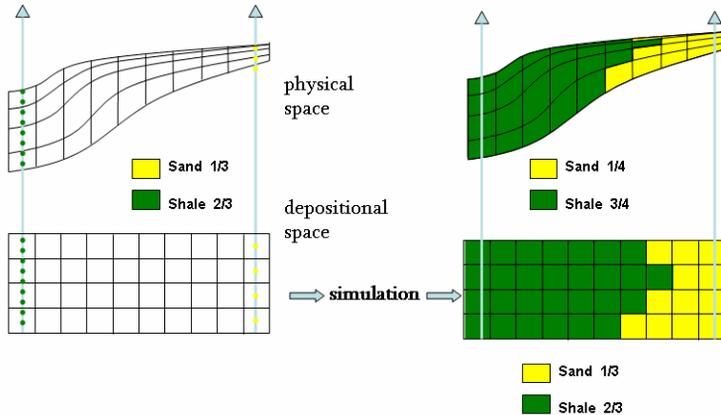


Figure 1: Geostatistical simulations performed in the depositional space. The data comes from the physical space. Assuming that the histograms are the same in the two spaces will not guarantee the respect of the target histogram in the real space

## LIMITATION OF STATE-OF-THE ART APPROACHES

### Direct mapping and point-support simulation

The traditional method consists of a direct mapping between the two spaces, which is fast and easy to complete. A one-to-one relationship is established between each grid cell of the stratigraphic (physical space) and Cartesian grid (depositional space). The simulation, done on the Cartesian grid, assumes each block to be a point. This approach ignores the volume of the cells and distortion between the two spaces. Therefore, it introduces biases.

### Problem of a direct mapping between the two spaces

#### *Biasing of the statistics*

The data are defined in the physical space. The histogram is valid only in this space. On the contrary, the algorithms are performed in the depositional space, so the target histogram must be representative for this space. The transformation of the distribution is tedious. We can keep the same histogram (assuming that in both spaces, the histogram is the same (Fig.1) or modify the histogram in accordance with the volume distortion at the well location (Fig.2). In both cases, the volume distortions are not accounted for in the entire grid. Hence a bias is introduced when the properties are mapped back into physical space.

#### *Bias in the dispersion variance*

The variance decreases when the volume of the block increases: the small scale heterogeneities are not anymore represented (Journel and Huijberts, 1978). In the

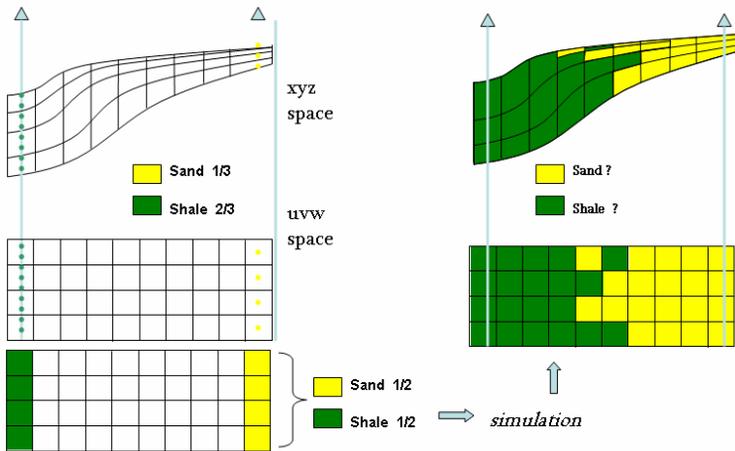


Figure 2: Correcting the histogram in accordance to the volume distortion at the wells locations will bias the results because it ignores the global distortion (it is just calculated from a few points).

current approaches, this is not taken into account. The variance used by default is the quasi point support one, which is very high compared to the blocks theoretical variance.

***Categorical variables***

When dealing with categorical variables, such as for example a rock-type, then this definition is usually made for a specific volume of support. Hence defining categorical variables at different sizes of support does not make much sense. We propose therefore to work on the proportion of each category within a volume, which seems to be more appropriate.

**BLOCK KRIGING**

The practical problems motivating the initial development of geostatistics were those encountered in the mining industry. One of the critical issue was the prediction of the average grade of a mining block from drill core samples. Thus, most change of support methods were focus on "upscaling", the prediction of a variable whose support is larger than that of the observed data. The change of support methods are generally based on kriging and co-kriging equations (block kriging, nonlinear geostatistical methods and isofactorial models, Chiles and Delfiner (1999)). These methods are computationally intensive and limited to variogram-based geostatistical methods. Also, in traditional block-kriging, the calculation of block-to-point covariances becomes difficult when the block sizes vary considerably.

## PROPOSED APPROACH

For stratigraphic modeling, the change of support issue can be seen as a problem of mapping between two spaces with non-conservation of the volume. The idea is, then, to transfer in the depositional space information about the physical volumes present in the physical space. One way to accomplish this is by sampling the stratigraphic grid with points, which are not affected by the volume deformation, and map those sample points into the depositional space. Hence, in depositional space, these points reflect the structure and volume distortion in the physical space. Performing geostatistical simulations on these points allows considering at the same time the change of support and the volume distortion. The main problem is to do this within acceptable computation time.

### Algorithm Steps

#### Sampling

The main purpose of the sampling is to handle the geometric variability of the grid cells by points. In general, the main variation in the cells shape and volume is in the vertical direction. Then, it makes sense to sample only in this direction (Fig. 3).

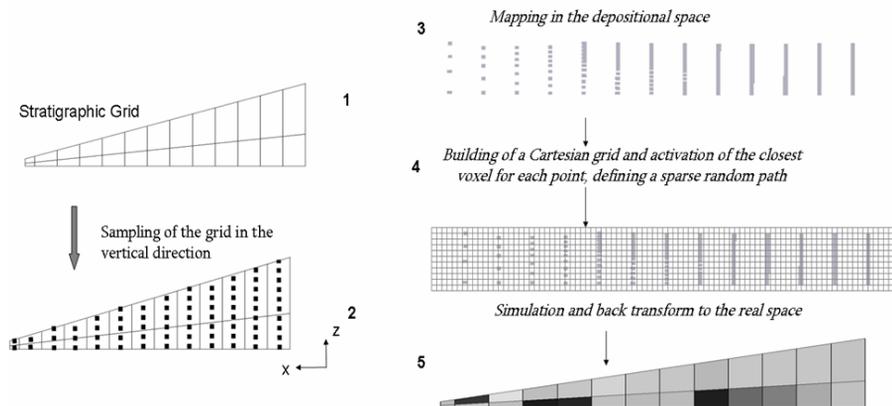


Figure 3: The sampling is done only in the vertical direction, in the direction where the distortion is the most important. For tartan grids, the sampling could be made in all three directions, incurring higher computational time. Once mapped, the points are inside a parallelepiped. In the depositional space, the information about the geometry of the stratigraphic grid is represented by the arrangement and the spatial density of the points.

### ***Mapping of the points***

The sampled points are mapped in the depositional space. The parametric function used here comes from Mallet (2002). It consists of the calculation, from the vertical coordinate of each point, of the corresponding relative time of deposition. The spread of the point cloud and their density in the depositional space are related to the size and the shape of the cells in the physical space (Fig. 3).

### ***Simulations***

A very fine Cartesian grid is built to the background and each sampled point is associated to its closest node on the grid, defining then a sparse random path. Simulations are directly performed on that grid, which is less CPU demanding than simulating directly on the sampled points.

### ***Back transform***

Each cell of the structural grid contains a corresponding set of points in the depositional space. Once the simulation is completed, the point values are averaged into a block value.

## **RESULTS**

This method has been implemented for Sequential Gaussian Simulation (SGS, Goovaerts (1997)) and for Sequential Indicator Simulation (SIS, Goovaerts (1997)). The upscaling method is a simple arithmetic averaging.

### **Decrease of the dispersion variance**

An increase in the cells size leads to a smoothing of the property (decrease of the dispersion variance). The averaging performed inside each block tends to dampen the fluctuations between extremes values. The intensity of this smoothing effect is directly correlated to the number of sample points inside the block (i.e. the volume). Two tests, using SGS, have been performed on a specifically designed grid, where the cells become increasingly thinner toward the pinch-out. The first one, a simulation with a pure nugget effect, shows a fast decrease of the dispersion variance as the cells size increases (Fig. 4). The visual effect is a strong smoothing of the property. For the second example (Fig. 5), the variables are now correlated through an exponential variogram. The smoothing effect is still clearly visible, but less marked. This is induced by the presence of the variogram. Indeed, when the range of the variogram increases, the relative size of the support (compared to the scale of the geological phenomena) decreases. Inside each block, the sample points values are then, more correlated and the values are closer from one to each other. The smoothing effect of the averaging is reduced, the impact of the change of support becomes less significant.

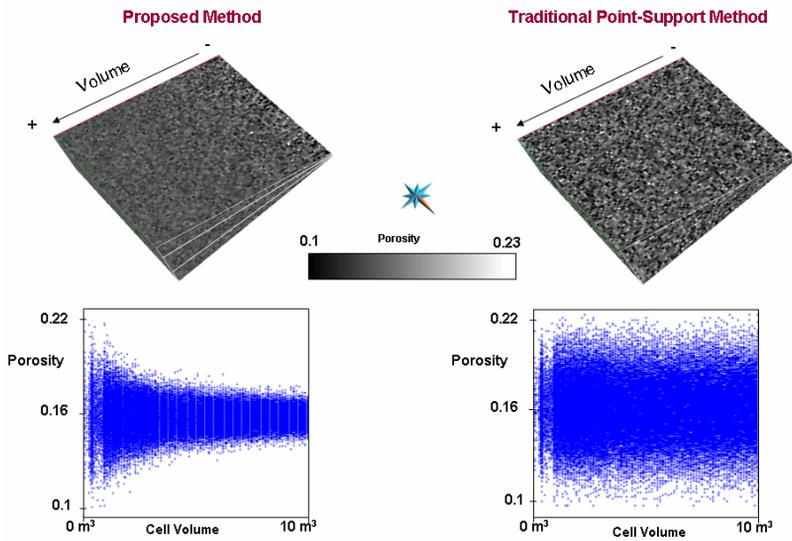


Figure 4: The fluctuations of the block values decrease as the block size increases. The visual result is a smoothing of the property (in accordance with the block size). The point-support method (SGS) does not reproduce this tendency.

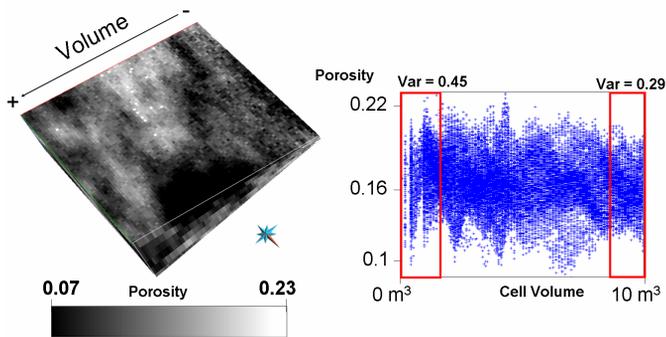


Figure 5: For an exponential variogram, the smoothing effect due to the increase of the block size is clearly visible. The point support variance is 1, the final block support variance is 0.4.

### Change in the histogram shape

In theory, the shape of the distribution associated to each block is defined partially by its volume. If a large number of geostatistical simulations (100 simulations in this case) are performed and a histogram of the cell values is computed, we see clearly that the histogram tends to be more symmetric for high volume blocks (Fig. 6). This is due to the decrease of the dispersion variance.

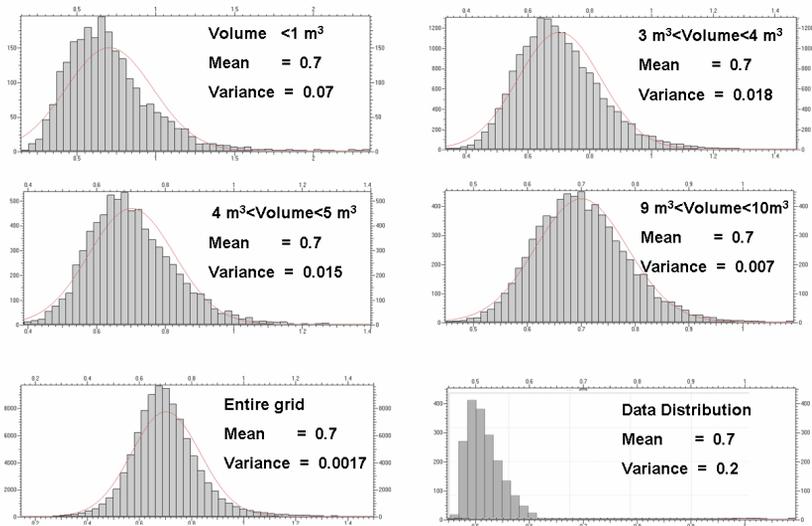


Figure 6: Change of the histogram shape. When the volume of the block increases, the associated distribution becomes more symmetric. The target and final distribution are different, because the variable is not represented at the same scale (quasi point support for the target distribution, block support for the final distribution)

**Reproduction of the statistics**

We construct another example where the same grid is modeled with a binary variable indicating for example sand versus shale (using SIS). In this case, due to the configuration of the wells, the cells filled by the sand are globally thinner than the cells filled by the shale (Fig.7). The point support algorithm does not account for this trend and then, induces a systematic bias in the final proportion of sand. Indeed, the back-transform (from the depositional space to the real one) tends to preferentially decrease the volume of the cells filled by the sand (Fig.1 and Fig.2). The target proportion, honored in the depositional space, is no longer reproduced in the real space. With the sampling method the statistics are reproduced at the point support level. Because the arrangement of the points is representative of the geometry of the grid, the statistics are respected according to the volume of the cells (Table 1). More over, the facies are now defined at the point-support scale. The block property value is an average of the corresponding sample-point values. Hence, in this case, the sampling method provides in each block a proportion of sand (continuous variable).

**CPU costs**

This method is more CPU demanding than when simulation is directly performed on the grid. For example, the number of cells in the grid is 33750 and the simulation time is less than 5 seconds. The number of points used to sample the grid is 427500

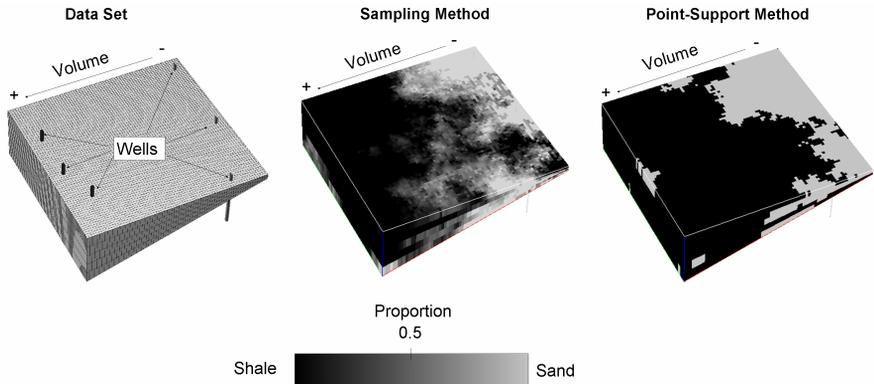


Figure 7: The volumes of the cells filled by the shale are larger than the ones filled by the sand. During the property modeling, considering or not this trend will modify the final result (Table 1).

Table 1: Comparison of the sand proportion obtained with the sampling and point-support method. In this case, the cells filled with shale are preferentially thicker (Fig 7). The sampling method honors the target statistics, which is not the case of the commonly used point-support method. It induces indeed a systematic bias, controlled by the global geometric deformation between the stratigraphic grid and the corresponding Cartesian grid in the depositional space.

<b>Sampling Method</b>	<b>real 1</b> 0.248	<b>real 2</b> 0.251	<b>real 3</b> 0.249	<b>real 4</b> 0.247	<b>real 5</b> 0.250	<b>real 6</b> 0.249
Target Sand Proportion = 0.250	<b>real 7</b> 0.248	<b>real 8</b> 0.252	<b>real 9</b> 0.261	<b>real 10</b> 0.255	<b>real 11</b> 0.260	<b>real 12</b> 0.255
<b>Point Support Method</b>	<b>real 1</b> 0.132	<b>real 2</b> 0.128	<b>real 3</b> 0.121	<b>real 4</b> 0.127	<b>real 5</b> 0.129	<b>real 6</b> 0.135
Target Sand Proportion = 0.250	<b>real 7</b> 0.122	<b>real 8</b> 0.113	<b>real 9</b> 0.124	<b>real 10</b> 0.131	<b>real 11</b> 0.141	<b>real 12</b> 0.124

and it increases the simulation time to 79 seconds. The simulation time is directly correlated to the number of points. This number is defined by the size and the geometric complexity of the grid. However, the method is likely to be much more efficient than any (even efficient) block kriging approach. Moreover, it also work for categorical variables and multiple-point techniques.

### Limitations of the method

These results emphasize the importance of considering the volume distortion during the property modeling: ignoring these problems leads to a bias of the property variance and mean, which are key parameters in a reservoir uncertainty assessment. However, because the simulation is performed in the depositional space, the properties simulated depend on the interpreted depositional process. Any

properties related to a post-sedimentation event can not be represented directly by this method.

## CONCLUSION

We have addressed the problem of support for non-Cartesian grid. More specifically, these grids are stratigraphic, specifically designed to represent complex geometry. In these types of grid, traditional geostatistical approach introduce bias. This sampling method allows to solve this problem.

## ACKNOWLEDGMENT

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