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**Mesoscopic strain field analysis in a woven composite using a
spectral solver and 3D-DIC measurements**

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ABSTRACT:

Spectral solvers are mesh-free, fast and very efficient simulation tools which emerged in the last two decades for solving continuum mechanics problems for heterogenous materials. Applied exclusively on particle-type composites, this paper demonstrates that they can model woven laminated composites with very valuable issues as they are able to capture with high spatial resolution the details of the stress and strain fields in the fabric periodic cell. Assessment of the modelling tool is given within in-plane shear mechanical testing: (i) at the mesoscale, through the comparison of local full-field strain maps obtained experimentally with Digital Image Correlation; (ii) at the macroscale, through the estimation of macroscopic properties from direct computation and comparison to their measured values. As shown in the last part of the paper, this tool provides in-depth knowledge of the strain-stress tensor in the microstructure as it offers an opportunity for precise local 3-D investigation. It should be a matter of high concern in the future as it allows the rapid design of new composite structures.

Keywords: Woven composite materials, Spectral solver, Digital Image Correlation, Textured strain distributions, CraFT

Introduction

Industry's widespread use of composite materials remains of paramount importance especially in the field of transport for evident reasons linked to structure alleviation, fuel economy and environmental concerns. Nowadays composite materials are not just used for the fabrication of accessories (appearance parts) for vehicles but enter into the design and production of structural parts (car industry) and high performance parts (aeronautics). This shift brings additional constraints which requires continued research efforts in order to produce better predictive calculations. These efforts are also enforced by the multiple possibilities of selecting the basic materials for the building blocks of a composite material:

- Different reinforcement materials - either technical or bio-sourced,
- Different armor architectures (e.g. types of fabrics, laminated or 3D textile structural composites...),
- Wide range of possible materials for the matrix, especially in the present context of replacing thermoset materials by thermoplastics for recycling purposes,
- Different techniques to modify the reinforcement material-matrix adhesion which is a key point for the global mechanical performance.

To theoretically analyse the influence of the internal structure of the material on the overall mechanical properties, it is essential to take into consideration both

averaged and full local fields. For instance, damage and fracturing are heavily dependent on the local details of stress or strain fields. For such analysis, it is essential to possess reliable and efficient tools which are also able to produce calculations in a systematic way, moving easily from one type of architecture to another while correctly taking the structure at different scales into account. Modelling issues in this field are therefore increasingly important. In this paper we used a spectral method to model the mechanical behaviour of a 2D textile composite at the mesoscopic scale. We show that this type of approach, which had never been used previously for such an application, allows the above mentioned requirements to be met.

Widely used 2D textile composites are periodically structured at the mesoscopic (fabric) scale. Accordingly, analytical homogenization approaches over the periodic fabric RVE (Representative Volume Element) were initially favoured and further developed to predict macroscopic equivalent material properties [1-3]. The joint development of powerful computers and softwares made it possible to develop numerical simulations allowing the computation of stress and strain distributions over the fabric RVE. The overwhelming majority of these studies were performed through discretization methods of the Finite Element (FE) type [4-12]. Without doubt, FE modelling of the local stress fields in the periodic RVE of textile composites has allowed significant advances in our understanding of the mechanical behaviour of these materials at the mesoscopic scale: localization of damage [13-14], influence of the

nesting effect on the mechanical properties in the case of a multi-layered composite [15], etc. FE modelling remains however a challenging task given that a correct meshing for the textile RVE is difficult to achieve. The problem is related to the contact surface regions between the yarns, where to avoid the generation of spurious voids and interpenetrations between the yarn surface meshes, the location of the nodes associated to the meshes have to coincide. For a particularly clear illustration, the reader is referred to figure 1 of the article written by Grail et al. [16]. A lot of strategies can be considered to circumvent properly this drawback of FE approaches [8, 15, 17, 18] which unfortunately bring additional troubles [16, 19]. Related to this mesh step, obtaining highly resolved calculations through FE based techniques requires very thin meshing in multiple areas, which results in very high computational costs.

It seems then desirable to assess the potential of mesh-free methods for studying the mesoscopic-scale mechanical properties of textile composites. To our knowledge, such an analysis has only been carried out by the Wen and Aliabady group [20, 21]. In these studies, a radial basis function coupled with moving interpolations were used for the shape function constructions, and the formulation of the discretized system was obtained through the well-known Galerkin scheme. However, the interpolation was performed using a relatively small number of nodes which makes it impossible to obtain highly-resolved stress or strain fields.

The so called “spectral solvers” are alternative meshless methods that permit computation of the local mechanical response of heterogeneous materials. These codes rely on the use of 3D Fourier Transforms and are well-suited to the case of materials with periodic microstructure. They give results with very good spatial resolution, since the fields of interest (stress or strain) are calculated at every pixel of the microstructure image. The use of spectral methods is growing markedly and is gaining increasing interest especially with the recent application of phase field approaches to metallurgical transformations [22].

In this study, we show for the first time that a spectral solver can successfully be used to model the mesoscopic scale mechanical behaviour of a textile composite. The outcomes of the simulation in terms of strain fields are validated at both the macroscopic and mesoscopic scales by measurements obtained with 3D Digital Image Correlation (DIC). The pixel size with which the RVE is digitalized for the simulation corresponds roughly to the fibre diameter size. This means that the stress and strain fields are provided with the best possible spatial resolution in the framework of a simulation performed at the mesoscopic scale.

The paper is structured as follows. In section 1, in relation with the considered application, the characteristics of the spectral solvers will be briefly summarized. In Section 2 we present the idealized representation of the plain-weave composite under study in terms of geometry and mechanical material properties. Section 3 is mainly

devoted to the experimental validation of the results obtained by simulation: the composite effective mechanical properties and the strain fields obtained at the mesoscopic scale are compared to their experimental counterparts. We also show that the use of a spectral solver allows the details of the strain field in very specific regions of the composite microstructure to be obtained, in particular in the core of the material where it is difficult to envisage direct measurements to be performed. Finally, we summarize our findings in the conclusion section.

1 Main characteristics of spectral solvers

The details of the mathematical framework on which the spectral solvers are based is well-documented in the context of solid mechanics and can be found in one of our previous studies [23]. In the following description, we will merely recapitulate the main characteristics of these approaches, bearing in mind the specific objectives of the present study. Namely, we will describe modelling of the mechanical behaviour of a textile composite at the mesoscopic scale. The use of a spectral method for solving the partial differential equation system that results from the strain compatibility conditions and from the equilibrium equations was introduced by Moulinec and Suquet two decades ago [24]. The efficiency of this approach is based on the Fourier series development of strain and stress fields when considering periodic RVE with strong heterogeneities in their material properties. Thanks to the repetitive use of Fast Fourier Transforms (FFT), the iterative solution algorithm operates alternatively in the Fourier

and in the real spaces (see the algorithm in Moulinec and Suquet [24]). In particular, the local equilibrium that involves partial derivatives in real space is expressed in a very efficient manner in terms of computational costs in the Fourier space using the Green operator. In this way, except for calculating the FFT, a given iteration of the algorithm only involves very fast elementary operations limited to each individual pixel of the real or Fourier space. For a given iteration, the limiting step is the FFT computation, which is nowadays obtained with highly optimized algorithms. The crucial point is that the spectral solvers directly use the microstructure numerical images. No meshing or coarser additional image discretization is needed. The stress and strain fields are obtained on a grid that simply corresponds to each pixel of the initial image, which corresponds to the highest possible resolution for a given digitalized image. In the case of comparisons performed with the same spatial resolution for the stress and strain fields, the spectral solvers are proved to be much faster than the FE codes in the linear elastic domain [25], as well as in plastic or viscoplastic ranges [26]. The method was first applied to the framework of elasticity and later extended to the case of materials with arbitrary phase contrast, elasto-plastic natures, and linear hardening behaviour [27]. It remains nevertheless restricted to small transformations (kinematic framework of infinitesimal strains).

There are currently two spectral solvers in free access dedicated to the modelling of the mechanical behaviour of solid materials: DAMASK (Düsseldorf Advanced

MAterial Simulation Kit) developed by Roters et al. [28] and CraFT (Composite response and Fourier Transforms) developed by Moulinec and Suquet [24, 29]. Both were used in a benchmark test case [23] to secure our results, those being presented in this paper having been obtained by the latter one.

2 Modelling information

The composite material under study is modelled at the mesoscopic scale, i.e. the scale corresponding to the structure of the fabric. In all following figures, the (XY) or (12) planes correspond to the plane of the plain weave lamina (Fig. 1). Thus the through-thickness (or stacking) directions Z or 3 are the same in both frameworks. The (12) directions refer to the Cartesian framework defined by the averaged warp (1) and weft (2) tow directions. The (XY) directions refer to the testing machines system of axis (X=tensile direction).

2.1 Microstructure generation

The plain weave microstructure was ideally reproduced using Solidworks® with the general characteristics of the real material (Fig. 1) as provided by the weaving company or as measured. The 2D plane view of the millimetric periodic RVE is a square of size $T = 7.94 \text{ mm}$ and consists of four intertwined identical tows surrounded by the matrix. Optical microscopy was used to characterize tows that have a cross-section with an elliptic shape of $3.26 \text{ mm} \times 0.22 \text{ mm}$. It also leads to an estimation of the fibre

diameter in the range $15\mu\text{m}-20\mu\text{m}$ and a fibre volume fraction in the tow of $V_f^{\text{tow}}=0.7$.

The overall fibre volume fraction is $V_f=0.47$ measured by the resin burning-off method.

It is then easy to check that the volume fraction of tows in the composite is 0.67. It should be noted that the plain weave balanced fabric creates a periodic distribution of full resin pockets (denoted R in Fig. 1) of approximately square shape that fill the whole ply thickness. The larger fibre-rich region, with also approximately square shape and where warp and weft tows are superimposed is denoted F in Fig 1.

The 3D modelling of the RVE in a ply based on Solidworks® is shown in Fig. 1. It clearly shows the distribution of materials within sections at different depths of the fabric lamina following the tow undulations. The tows are shown here with the same 'color' but in the material description under CraFT, two sets of properties are given with same mechanical properties but in different orders, so as to take into account their different orientations. The real laminate composite is simulated as a lay-up of 9 plies of thickness 0.44 mm corresponding to a total composite thickness of 4 mm . Two additional layers (one-pixel size) on the bottom and up surfaces were considered as having null mechanical properties. These "air" layers were added to impose the stress free boundary conditions on these two surfaces of the periodic RVE. Each ply was divided into 9 pixels along direction 3 which gives 83 pixels for the whole discretization along this direction. In the (12) plane, a RVE was made up of 443×443 pixels which conserves the pattern's real aspect ratio and gives an in-plane resolution of

about: $7.94/443=18\mu\text{m}/px$. It can be noted that it roughly corresponds to the fibre diameter size, which means that the resolution is the best possible in the framework of a study carried out at the mesoscopic scale.

Once this geometrical description of the RVE has been made, the second step of spectral modelling just requires assigning mechanical properties to each "phase" of the material.

2.2 The mechanical properties at the meso-scale.

Most of the experimental features and results on this material (including fatigue analysis and mechanical spectroscopy) have been explained and reported elsewhere [30] and are only used here as inputs or validation outputs for the numerical computation. Only the elastic behaviour is considered for each phase and Table 1 lists the values of all relevant moduli. The acrylic resin was a newly referenced thermoplastic (Elium® - Arkema) considered to be homogeneous and isotropic and whose properties were characterized experimentally (Dynamic Mechanical Analysis-DMA at 1 HZ was used in the three points bending mode for the elastic modulus determination) or from a well-established data base (Poisson coefficient for acrylic material). E-Glass fibres with well-known mechanical properties were used to perform the tows. Each tow was modelled as a unidirectional composite whose mechanical behaviour can therefore be described with 5 independent elastic constants: E_{11} , E_{22} , ν_{12} , G_{12} and G_{23} (1 is the average fibre direction in the tow). Many micromechanical models

are available to calculate these homogenized elastic constants, for instance the Chamis model [31], the Hopkins and Chamis model (HC) [32] and the Composite Cylinder Assemblage model (CCA) [33]. CraFT code has also been used at this stage to obtain the elastic constants of the tows (see Supplementary Information File for additional information on the procedure) and compare the results with those of the above-mentioned micromechanical model. The RVE is taken within a tow cross section at the micrometric scale. An average stress state is applied to the RVE as an input for CraFT simulation. The averaging of the corresponding output full field strains allows calculating the five independent elastic constants of the tow. Moreover, the determined elastic constants are the mean value resulting from repeated simulations over 10 different random fibre configurations of the tow RVE. The final results are given in the last line of table 1 with the resulting standard deviations indicated in brackets. The standard deviations obtained are smaller than 5%, which shows that the RVE we considered with 64 fibres is large enough to describe the statistics of the fibre and matrix arrangement.

For all the micro-mechanic models and for CraFT simulation, E_{11} -and to a lesser extent ν_{12} - are nearly the same and are very close to the well-known mixture rule (see table 1). However for E_{22} and G_{12} , the discrepancies between the different models are of the order of 25%. It should be noted that, among the micromechanics models, only the CCA model provides an analytical expression for the G_{23} value (see table 1). For the

simulations shown in this paper we used the Chamis model for E_{11} , E_{22} , ν_{12} and G_{12} , and the CCA model for G_{23} . However, we checked that the main conclusions obtained during our study remain qualitatively and quantitatively true even if other micromechanical models were used to evaluate the elastic constants in the tows. The variations of elastic constants expressed in the (1, 2, 3) coordinate system due to the fibre undulation effect were not taken into account. To check the validity of this approximation, we modelled the tow undulation using a sinusoidal function and found the variations of the local value (along the tow) of E_{11} to be less than 5%.

Table 1: Mechanical properties of the different phases of the studied composite

(1 = average tow direction).

		E (GPa)		ν		
Elium® resin		3.6		0.37		
E-Glass fibre		73		0.22		
Tow $V_f^{tow} = 0.7$	Model	E_{11} (GPa)	E_{22} (GPa)	ν_{12}	G_{12} (GPa)	G_{23} (GPa)
	Chamis	52.2	17.6	0.265	6.57	Not provided
	HC	52.2	15.3	0.265	5.71	Not provided
	CCA	52.2	19.1	0.259	6.01	7.17
	CraFT (Mean and Standard deviations over the 10 fibre configurations)	52.2 (5 10 ⁻³ %)	19.7 (3%)	0.254 (0.8%)	7.45 (4%)	7.19 (2.5%)

2.3 In-plane shear loading.

On woven composites showing weak undulation, it is well-known that tensile tests performed with either the warp or weft directions aligned along the tensile axis can be correctly modelled considering an equivalent cross-ply laminate of UniDirectional (UD) composites [34]. This means that in that case, the mesoscopic scale organization has no strong incidence on the general behaviour. In order to obtain results at a macroscopic scale that depends strongly on the material organization at the mesoscopic scale, the tensile tests considered in this analysis were made for a force applied at $\pm 45^\circ$ from the warp (1) and weft (2) fabric directions (in-plane shear loading, ASTM D3518 or NF ISO 14129).

In-plane shear loading conditions introduce very harsh conditions being applied to the fibre/matrix interface and one interesting perspective of this work rests upon the analysis of the mesoscopic stress fields calculated under these loading conditions. In the (XY) plane, a tensile load was applied in the X-direction, at $\pm 45^\circ$ from the warp (1) and weft (2) fabric directions. The following plane stress tensor was therefore applied:

$$\sigma = \begin{bmatrix} \sigma_L & 0 \\ 0 & 0 \end{bmatrix}_{(X,Y)} = \begin{bmatrix} \sigma_L/2 & -\sigma_L/2 \\ -\sigma_L/2 & \sigma_L/2 \end{bmatrix}_{(1,2)} \quad (1)$$

where σ_L was considered to be the average loading stress imposed on the whole RVE in CraFT software. The average stress applied over the whole thickness along the X-direction, is 50 MPa . Due to the presence of the two one pixel air layer, the average stress actually applied to the composite material is therefore: $\sigma_L = 83/81 \times 50 = 51.2 \text{ MPa}$. We will prove in section 3.2.2 that the stress field computed at the mesoscale are perfectly in line with this uniform constant stress at the macroscale.

Typically, the loading stress is imposed with a ramp of 20 steps. Each step requires less than 300s to perform stress and strain computations over a 3D grid of 16.8 Millions of Voxels (standard PC, Intel® Xeon CPU W3520 @ 2.67GHz with 4 threads).

3 Results and discussion

In this section, all quantities with upperscripts M and m will denote values corresponding respectively to the Macroscale and the meso-scale. This section is organized into two parts. First we validate CraFT modelling, mainly by comparing the simulation results to surface measurements obtained by 3D DIC on the main and lateral specimen faces. In the second part, we used our CraFT simulations to analyse certain characteristics of the stress and strain fields in the core of the laminated composite. A modelling approach is inevitable in that case, because in the composite core region, far from the specimen-free surfaces, no simple way exists to perform strain field measurements. In particular, we studied the variations of the transverse normal strain in the resin region that extends throughout the thickness of the composite (region R in Fig. 1). Taking into account the small size of these resin pockets, such a study is only possible using highly resolved strain fields such as those obtained with a spectral solver like CraFT.

3.1 *Experimental validation of the numerical model*

Experimental data of whole-field strain measurements were obtained using the 3D DIC Aramis® device provided by the GOM-Instruments® company. These measurements were performed on the so-called main and lateral specimen faces. A random pattern was applied to the specimen with an air brush. We used 30x30 pixels

facets (or subsets) and a 7 pixels step-size (distance between the centres of two adjacent facets). On the surface studied, one pixel represents 16 μm in size. The spatial measurement resolution is thus $\text{DIC_SR}=44\text{pixels}\approx 700\ \mu\text{m}$ [30, 35]. The two cameras are placed so that both the main and lateral specimen faces are visible in the recorded images. On the main face (XY plane, Fig. 2b), the measurement of the mesoscopic strains ε_{XX}^m and ε_{YY}^m are obtained. Simultaneously, on the lateral face (XZ plane, Fig. 3b), the mesoscopic strains ε_{XX}^m and ε_{ZZ}^m are measured.

From this mesoscopic scale experimental information, the validation of our spectral computations will be firstly assessed through recalculated properties at the macroscale, and secondly by a close comparison of local strain fields.

3.1.1 Macroscopic scale: Composite effective elastic moduli

The strain fields ε_{XX}^m at the mesoscopic scale obtained by CraFT simulation are shown in Fig. 2a: $\varepsilon_{XX}^m(X,Y)$ on the top of the RVE (specimen main face), and in Fig. 3a: $\varepsilon_{XX}^m(X,Z)$ along the cross-section indicated in Fig 2a. In Figures 2b and 3b we show the strain fields $\varepsilon_{XX}^m(X,Y)$ and $\varepsilon_{XX}^m(X,Z)$ measured by 3D DIC on the specimen main face and edge, respectively. The variation of the strain field over the RVE will be analysed in the following section and here we will only focus on the composite macroscopic elastic moduli that can be obtained from the average strain distributions over the RVE

simulation in response to the macroscopic applied stress: $\sigma_{XX}^M = \sigma_L = 51.2 \text{ MPa}$. For the average strain on the RVE, we found: $\langle \epsilon_{XX}^m \rangle_{RVE} = 0.0038$, $\langle \epsilon_{YY}^m \rangle_{RVE} = -0.0017$ and $\langle \epsilon_{ZZ}^m \rangle_{RVE} = -0.00076$. As expected, $\langle \epsilon_{XY}^m \rangle_{RVE}$, $\langle \epsilon_{XZ}^m \rangle_{RVE}$ and $\langle \epsilon_{YZ}^m \rangle_{RVE}$ are very close to 0.

Three macroscopic elastic constants of the composite can be evaluated:

- the longitudinal modulus: $E_{XX}^M = \sigma_{XX}^M / \langle \epsilon_{XX}^m \rangle_{RVE}$,
- the in-plane effective Poisson's ratio: $\nu_{XY}^M = -\langle \epsilon_{YY}^m \rangle_{RVE} / \langle \epsilon_{XX}^m \rangle_{RVE}$
- the transverse effective Poisson's ratio: $\nu_{XZ}^M = -\langle \epsilon_{ZZ}^m \rangle_{RVE} / \langle \epsilon_{XX}^m \rangle_{RVE}$.

The results obtained through CraFT simulation are summarized in Table 2. The experimental values of ν_{XY}^M and ν_{XZ}^M are obtained in the same way but using averaged mesoscopic strain fields measured by 3D DIC on the specimen main face : $\langle \epsilon_{XX}^m \rangle_{\text{MainFace}}$ and $\langle \epsilon_{YY}^m \rangle_{\text{MainFace}}$ as well as on the specimen edge: $\langle \epsilon_{XX}^m \rangle_{\text{Edge}}$ and $\langle \epsilon_{ZZ}^m \rangle_{\text{Edge}}$. Taking into account the measurement noise, the two experimental Poisson ratios ($-\langle \epsilon_{YY}^m \rangle_{\text{MainFace}} / \langle \epsilon_{XX}^m \rangle_{\text{MainFace}}$ and $-\langle \epsilon_{ZZ}^m \rangle_{\text{Edge}} / \langle \epsilon_{XX}^m \rangle_{\text{Edge}}$) are found to remain constant in the strain range $\langle \epsilon_{XX}^m \rangle \in [2 \cdot 10^{-4} \quad 10^{-3}]$. The averaged values obtained in this strain range are indicated in table 2. For evaluating the longitudinal modulus E_{XX}^M , it was found that the

stress-strain curve obtained experimentally during $\pm 45^\circ$ tensile testing with strains measured using 3D-DIC was not linear [30], even in a very small strain range as that in which the moduli needed to be evaluated according to the ASTM D3518 standard. This suggests that the resin has a noticeable visco-elastic effect in this kind of stress state. As a result, the elastic behaviour needs to be looked for at very early times of the tensile test which is unfortunately when the machine response can be smoothed by the feedback loop resulting in a biased estimation of the longitudinal Young Modulus. This fact was identified when measuring the instantaneous modulus of semi-crystalline polymers - a visco-elastic physical model needs to be applied to the whole tensile curve to obtain a good estimation of the modulus [36]. Therefore, to avoid this drawback, the experimental measurement of E_{xx}^M provided in Table 2 was obtained through DMA performed on a Bose 3300N fatigue machine in the same way as the Young modulus of neat PMMA resin was evaluated. DMA enables the measurement of the “true” elastic modulus of a visco-elastic material (real part of the complex modulus). The experimental value for E_{xx}^M is shown in table 2.

In table 2, the agreement between the experimental values for E_{xx}^M , ν_{xy}^M and ν_{xz}^M compared to those obtained through simulation for the moduli is fairly good. Many factors can explain the few % of discrepancies between the experimental results and those obtained through simulation (see table 2). The shape of tows may be more

complex than that used for CraFT simulation. Measurement errors can be made when evaluating the elastic constants by 3D DIC and the fibre volume fractions V_f^{tow} and V_f . The volume fraction V_f^{tow} may also not be constant within a tow. The choice of the micro-mechanical model for E_{22} and G_{12} (see table 1) can also affect the results given by the CraFT simulation for the composite effective moduli. A parametric study has shown that the most influential tow elastic constant affecting the results for E_{XX}^M is G_{12} . In table 1, it can be seen that the scattering of the G_{12} values given by the different micro-mechanics models is of order of 25%. Increasing the micro-mechanical parameter G_{12} by 10% results in a 6% increase of E_{XX}^M , which is larger than the discrepancy between the experimental value and that obtained through modelling (see Table 2).

Table 2 Composite effective moduli obtained by experience and modelling

	E_{XX}^M (MPa): DMA	ν_{XY}^M : DIC (measurement on the main Face)	ν_{XZ}^M : DIC (measurement on the specimen edge)
Experimental	14.0	0.49	0.206
CraFT Simulation	13.5	0.45	0.198
Discrepancy	4 %	8.5%	4%

In what follows, we will not focus anymore on the RVE averaged strain, but we will compare local strain variations over the RVE as provided by CraFT simulation to those measured by 3D DIC.

3.1.2 Mesoscopic scale analysis of the strain fields

Qualitative analysis

Qualitatively, on the main face, it is possible to compare the mesoscopic strain field ϵ_{xx}^m obtained by CraFT simulation (Fig. 2a) and by experimental DIC measurements (Fig. 2b), see also Fig. 4a. It is clear that the exact same pattern is obtained for strain distribution which displays a strong correlation with the fabric geometry. In short, the squared areas of high strains were found to be located in the small regions containing only resin throughout the thickness of a layer (region R in Fig. 1), those of small strains, in the warp and weft superimposed regions (regions F in Fig. 1). The areas with intermediate strains (bands surrounding the interlaced regions and forming a cross pattern) correspond to the undulation nodes of the tows where the resin forms strong junctions between the laminates. On this qualitative comparison, we can also highlight the significant difference in spatial strain resolution between the simulation (highly resolved spatial resolution set as $1/N_p \times \text{RVE size} = 18 \mu\text{m}$ where $N_p = 443$ is the number of pixels chosen for the RVE size) and the DIC measurements. As previously mentioned, the best resolution achieved for the latter was about $700 \mu\text{m}$ or about one twelfth the RVE size. The ratio of simulation versus experimental resolution was found to be within a factor of 40 (see Fig. 4a for a clear illustration of the resolution difference). In brief, to compare experimental and simulated data, one should bear in mind that the experimental strain distribution is averaged by the measurement process

over zones of non-negligible size and that the strain measurements are affected by significant noise.

On the lateral face, for the numerical (Fig. 3a) and experimental (Fig. 3b) results, it can be seen that the high strain regions (R) form approximately a continuous band that is oriented nearly normally to the specimen main surface. For the simulated case with “in-phase” ply configuration, it can be checked that the distance between two consecutive bands is $T/\sqrt{2}$. In the experimental case (Fig. 3b), the coincidence of regions R throughout consecutive layers can be visually appreciated as well as the estimated resulting distance between bands of strong strain values which is also roughly equal to $T/\sqrt{2}$. It shows that the fabric of the different layers was placed approximately in an “in-phase configuration” and explains why this case was chosen for CraFT simulation (Fig. 3a).

Quantitative analysis

In Fig. 4b we compare on the same plot the ε_{xx}^m strain profile (solid line) theoretically obtained along the X direction in the RVE to 8 experimental curves (dashed lines) corresponding to the same quantity measured by 3D-DIC in 8 different RVE taken on the specimen surface (see Fig. 2b). However, to allow the reader to visualize easily the aspect of a given individual experimental profile, one of these curves was drawn in blue solid line in Fig. 4b. As expected, the experimental curves

appear very noisy but it is possible to check that the measurement is fairly reproducible: same curve shape with roughly identical minimum and maximum values at same locations. Because of the difference in resolution between the simulation and the measurement, it was impossible to capture experimentally the strong strain variations shown by the simulations in the frontier region between the rigid tows and the softer resin material (see the blue curve in Fig 4b). As a result, the highly deformed zones associated with the resin pockets appeared to be magnified to some extent. A strong quantitative validation of both results can be obtained nevertheless by looking at the profiles of Fig. 4b. The 8 experimental strain profiles were found to have similar features as the theoretical one. In particular, the ratio between the strains in the fibre rich and resin rich regions (respectively regions F and R in Fig. 1) was found to be approximately 3 both experimentally and by simulation (Fig. 4a). However, the theoretical profiles exhibit two secondary peaks at the interface between tow and resin (see Fig. 4b), which were not present on the experimental curve. Below, we will show that because of the measurement spatial filtering, these narrow secondary peaks were blurred as far as not appearing on experimental profiles. The size of the strain concentration regions associated with the secondary peaks is of the order of 100 or 200 μm . This justifies the need to obtain results with high spatial resolution ($\approx 18 \mu\text{m}/\text{pixel}$), to capture these details of the strain field. In an attempt to reproduce the experimental results more closely, we considered the highly resolved calculated strain patterns to be

real objective measurements and tried to reproduce the spatial filtering operated when using DIC. In short, the highly resolved strain field was integrated to obtain the displacement field (in the small displacement-gradient theory framework). The detail of the procedure can be found in Boufaïda et al. [23]. For this, the heterogeneous component of the strain (considered as periodic over the RVE) was transformed in the Fourier domain to produce the Fourier transform of the displacement which was back-transformed in the real domain. From this highly-resolved displacement field, we implemented an averaging procedure identical to that performed by the Aramis® software to obtain the transformation gradient tensor at a smaller resolution corresponding to that of the real measurement. Fig. 5 shows the new strain pattern obtained numerically after lowering the resolution and the corresponding numerical profile along direction X. On the same plot we also show one of the strain profiles measured by 3D-DIC (blue curve in Fig. 4b). The secondary peaks found at the resin-tow interfaces can be seen to have disappeared while the strain profiles now match the experimental ones much better (apparent enlargement of the peak bottom).

3.2 Analysis of strain and stress fields obtained using CraFT

As was previously mentioned, with a spectral solver the strain (or stress) fields are obtained on the same grid as the microstructure, which allows for a highly detailed analysis of the local stress or strain variations caused by material heterogeneity. To

illustrate this point with a concrete example, we studied the spatial variations of ε_{33}^m in the very small zones corresponding to the neat resin regions that cover the whole composite thickness (region R in Fig. 1). In particular, we highlight the complex influence of the surrounding tows on the strain state in these zones. In a second part, we show and briefly analyse the stress fields obtained through CraFT simulation in the specimen core region.

3.2.1 Variations of the transverse normal strain in the resin pockets

The transversal normal strain $\varepsilon_{33}^m(x_1, x_2, x_3 = 38 \text{ pixels})$ map obtained with the CraFT simulation on the top of the central ply (ply 5) of the woven laminate is shown in Fig. 6a. To analyse the spatial variations of this strain component in the resin pockets (region R in Fig. 1), we plotted $\varepsilon_{33}^m(x_3)$ along 4 cross-section profiles (parallel to the x_3 -axis) taken at points A through D i.e. that are situated strictly in the resin region (Fig. 6a). In Fig. 6b, the corresponding profiles are denoted A,B,C, and D accordingly. Point A ($x_1=333$ pixels, $x_2=109$ pixels, $x_3=38$ pixels) is taken at the centre of the resin pockets and, using the digitalized microstructure image, it was checked that the closest point belonging to a tow is situated at 18 pixels from Line A. Points B (326 pixels, 116 pixels, 38 pixels), C (320 pixels, 122 pixels, 38 pixels), and D (316 pixels, 126 pixels, 38 pixels) are taken along the Y-axis (bisector to the x_1 and x_2 axes, see Fig. 6a). The closest points belonging to a tow are respectively situated at 11 pixels, 5 pixels and 1 pixel

from lines B, C and D. On the profiles of Fig. 6b, it can be seen that the dependence of ε_{33}^m on x_3 shows a strong quasi-periodic modulation resulting from the ply alternation with maxima located in the frontier regions between the layers and minima in the middle of the plies. The quasi-periodic variation of the $\varepsilon_{33}^m(x_3)$ curves becomes much more significant when moving away from the resin pocket centre (Profile A) to the resin pocket extremity (Profile D). In profile D, it can be seen that in the composite material core region, the strain component ε_{33}^m , although transverse to the tensile axis, gets positive values in small zones situated at the ply frontiers. As will be seen in the following subsection, this is due to the complex tri-axial stress state to which the material is subjected. For the sake of clarity we have only shown four curves in Fig. 6b but, thanks to the resolution of the strain fields obtained through CraFT simulation, 17 curves could have been plotted to show the progressive evolution from profile A to profile D.

3.2.2 Stress fields in RVE

In Fig. 7, we show the maps associated with the 6 stress tensor components: $\sigma_{11}^m(x_1, x_2)$, $\sigma_{22}^m(x_1, x_2)$, $\sigma_{33}^m(x_1, x_2)$, $\sigma_{12}^m(x_1, x_2)$, $\sigma_{13}^m(x_1, x_2)$ and $\sigma_{23}^m(x_1, x_2)$ obtained upon CraFT simulation on the top of the central ply ($x_3 = 38 \text{ pixels}$). As expected, the average values for σ_{33}^m , σ_{13}^m and σ_{23}^m are zero over this (x_1, x_2) central plane. Similarly, we have checked that the average values for σ_{11}^m and σ_{12}^m over the (x_2, x_3) planes as well as the average

values for σ_{22}^m and σ_{12}^m over the (x_1, x_3) planes correspond well to the applied macroscopic stress: $\sigma_L/2 = 25.6 \text{ MPa}$ with an accuracy of the order of $3 \times 10^{-3} \%$. However, in the case of the $\sigma_{33}^m(x_1, x_2)$ map of Fig. 7, taken in the specimen core, it is worth noting that locally σ_{33}^m can reach significant values. It is positive in the neat resin pockets (region R in Fig. 1): the material there is under traction along the 3-axis ($\sigma_{33}^m > 0$). In the corner of these resin pockets, the normal transverse stress reaches its highest values ($\sigma_{33}^m \approx 20 \text{ MPa}$) that cannot be neglected compared to the in-plane stresses ($\sigma_{11}^m(x_1, x_2)$, $\sigma_{22}^m(x_1, x_2)$, and $\sigma_{12}^m(x_1, x_2)$) and should be taken into account to study damage initiation in textile composites. This explains the positive values found for ε_{33}^m in the composite core at the ply frontiers (see Profile D in Fig. 6b). On the other hand, in the tows (region F in Fig. 1), it can be checked that the material is under compression along the 3-axis.

Conclusion

The mechanical behaviour at the mesoscopic scale of a woven composite was modelled for the first time with a spectral solver, a mesh-free method particularly adapted to materials with periodic microstructure. We show that the stress and strain details over RVE can be obtained in very short computation time, with excellent spatial resolution. All the data necessary for simulations were provided by independent characterizations

performed by the authors and introduced, in some cases, into micromechanical models to determine properties at the mesoscopic scale. Despite the complex geometry and strong heterogeneous properties of RVE (acrylic resin, Glass fibres arranged in warp and weft tows at 90°), the results serve here as an explanation for in-depth experimental observation in terms of strains (direct comparison with full-field DIC measurements). The high potential of this tool has also been illustrated in view of well-controlled industrial developments of new composites taking into consideration the complex stress distribution at the mesoscale.

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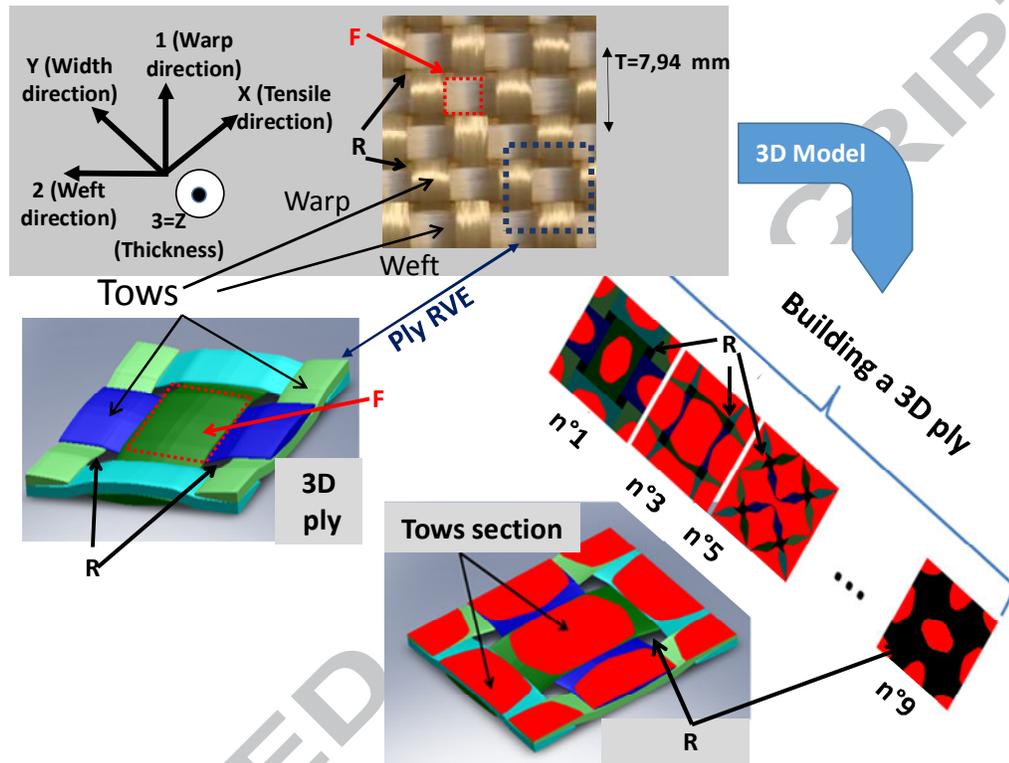


Figure 1. 3D numerical RVE in a ply corresponding to the real woven and material distribution for different slices

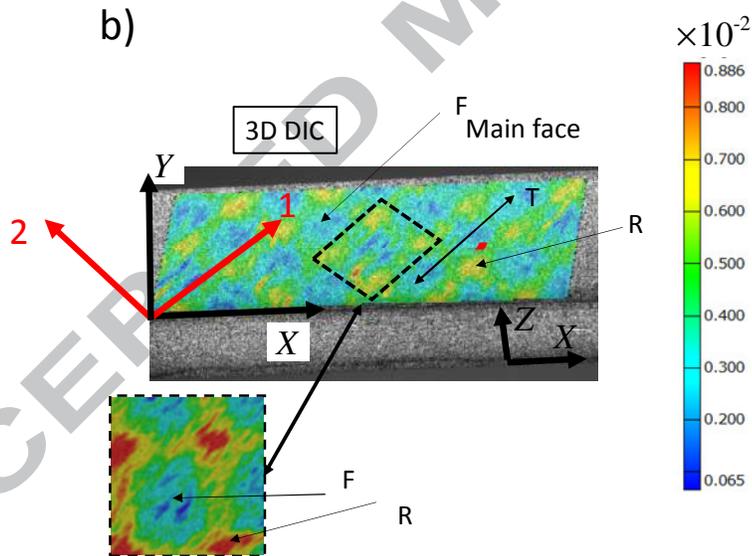
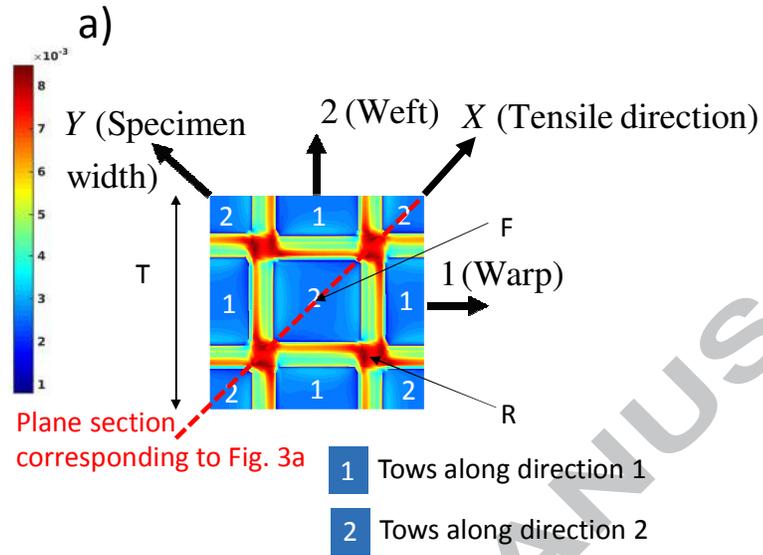


Figure 2. ε_{xx}^m strain field in the (X, Y) plane: a) Numerical result on the top of the RVE; b) Experimental result on the main face

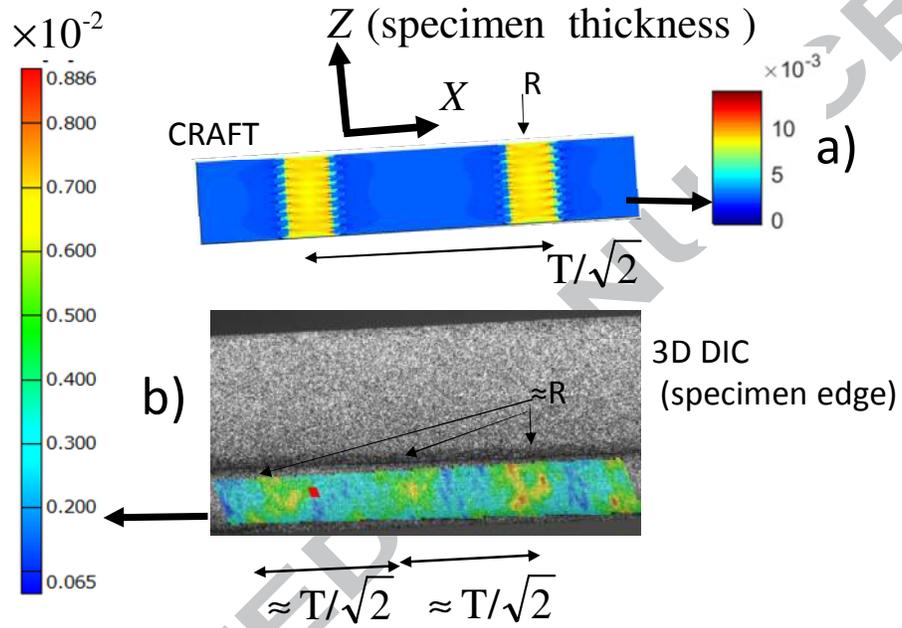


Figure 3. ε_{xx}^m strain field in the (X, Z) plane: a) Numerical result for the plane section indicated in Fig. 2a; b) Experimental result on the lateral face

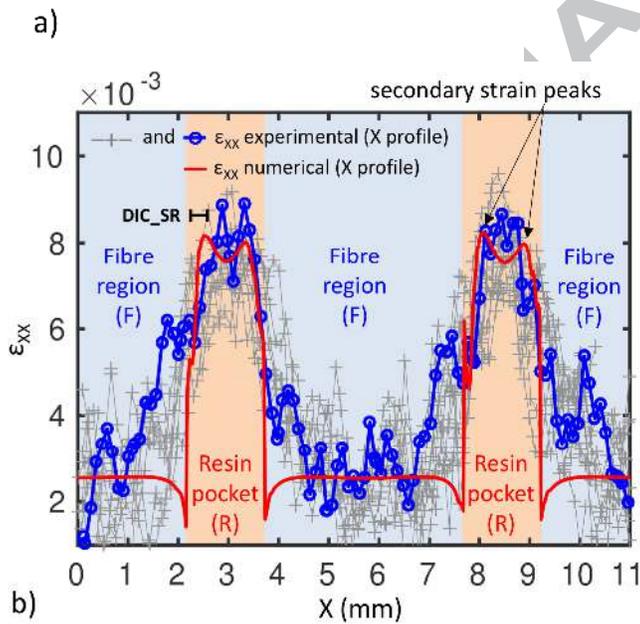
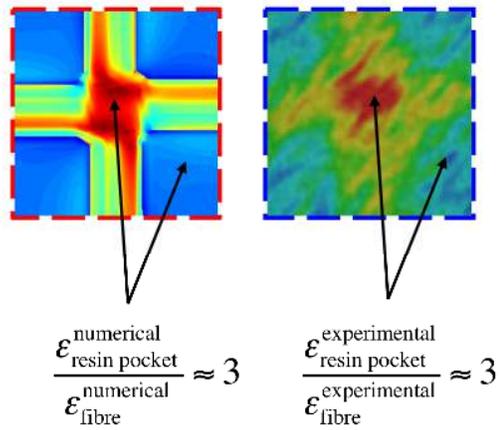


Figure 4. a) Numerical and experimental strain patterns of ε_{xx}^m around a resin pocket; b) Numerical and experimental profiles of ε_{xx}^m along the X-axis.

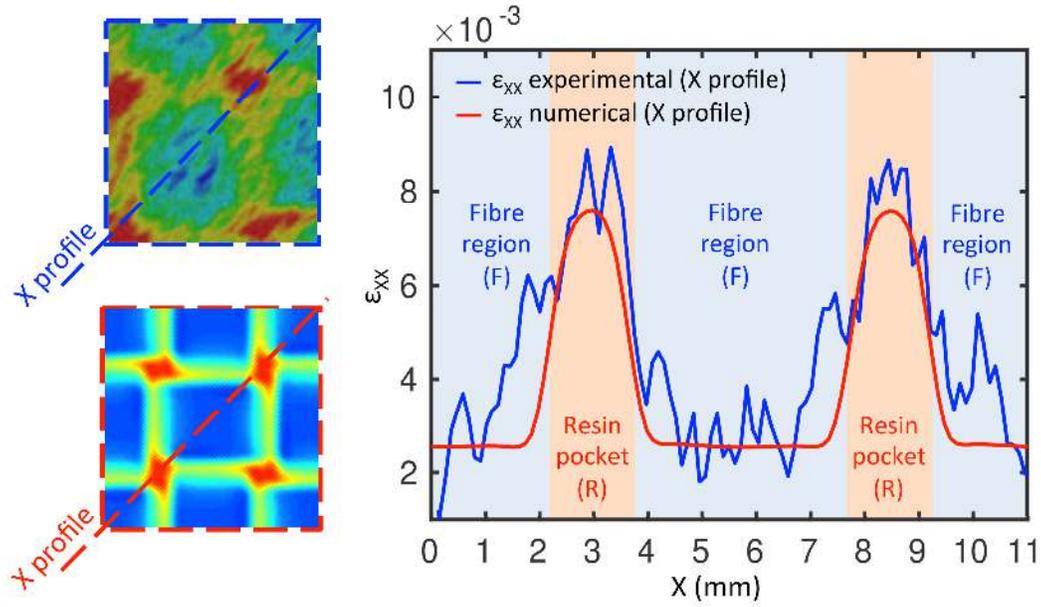


Figure 5. Numerical and experimental profiles of the ε_{xx}^m strain along the X-axis after applying a smoothing procedure to the CraFT results.

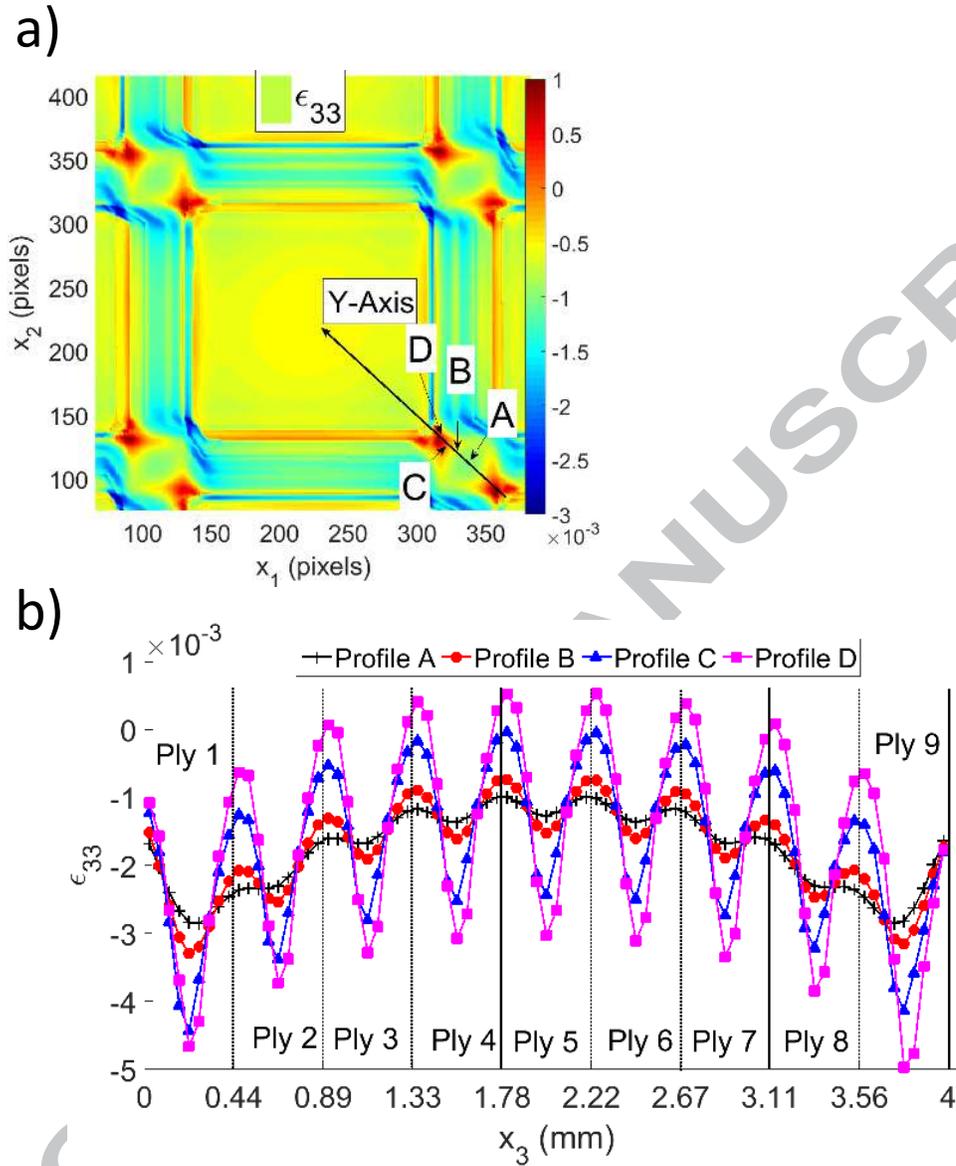


Figure 6. a) $\epsilon_{33}^m(x_1, x_2)$ on the top of ply 5 ($x_3 = Z = 38$ pixels); b) Through the thickness dependence of the transverse normal strain $\epsilon_{33}^m(x_3)$ in the resin pockets (region R in Fig. 1)

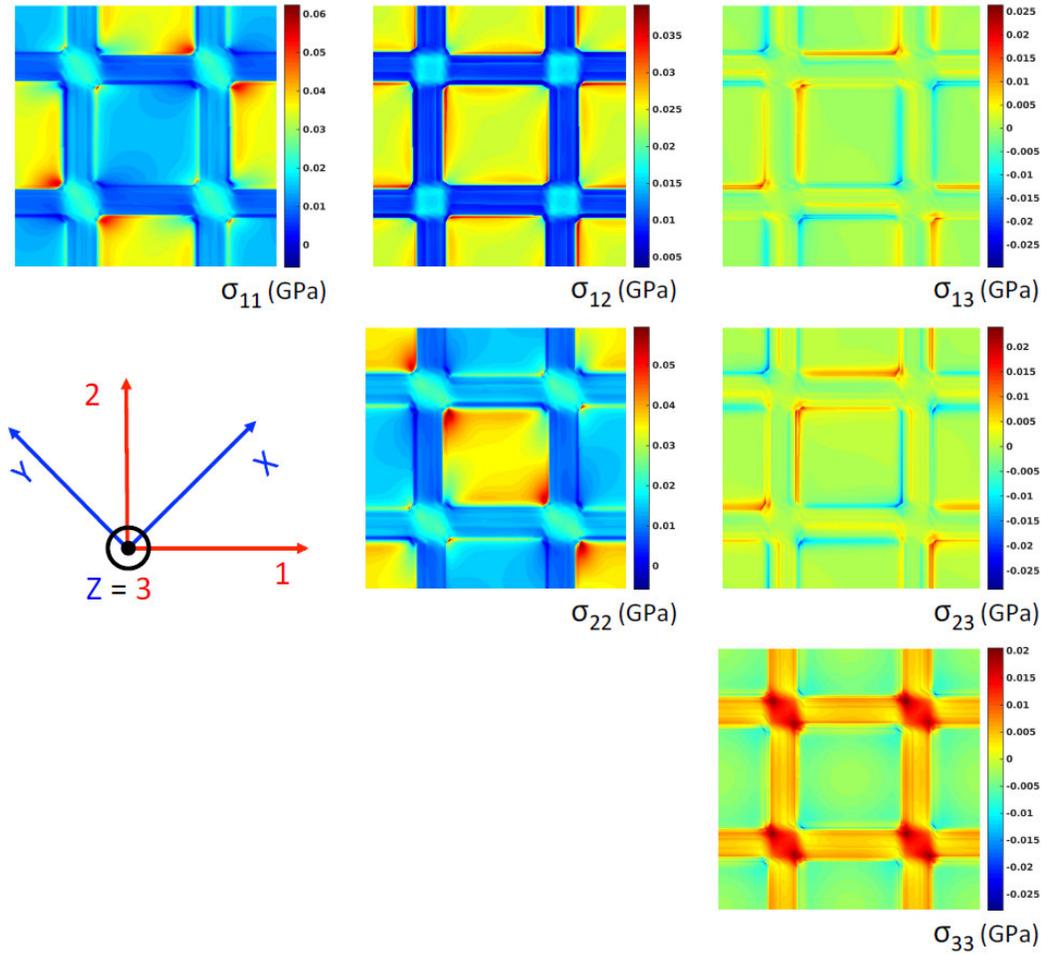


Figure 7. Maps of the stress tensor components on the top of ply 5 ($x_3 = 38$ pixels)

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