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Laminar flow friction factor in highly curved helical pipes: numerical investigation, predictive correlation and experimental validation using a 3D-printed model

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Abstract

Highly curved helical pipes offer attracting potentialities for intensified mass/heat transfer performances as they generate intense Dean-type vortices. The evaluation of friction factor in such geometries is necessary for assessing the trade-offs between the increase of transfer efficiency and the associated specific energy requirement. Unfortunately, such data are lacking for highly curved helices, probably due to the difficulty to manufacture these geometries through traditional manufacturing techniques.

In this paper, CFD simulations are carried out for determining the laminar flow friction factor in helical pipes, particularly highly curved ones. For an experimental validation of the numerical results, a highly curved helix was built by 3D-printing. Existing correlations are shown to fail for the accurate prediction of the friction factor in highly curved helices. A new correlation is thus proposed. An excellent agreement is obtained between the experimental pressure drop measurements and the proposed correlation predictions.

Keywords: 3D printing; CFD; Correlation; Dean vortices; Friction factor; Helical pipe

1. Introduction

Hydrodynamics in curved pipes is characterized by the occurrence of secondary flows, i.e. recirculations of relatively low magnitude in the cross-stream direction, perpendicularly to the primary flow. In toroidal pipes and bends, they consist of a symmetric pair of counter-rotating vortices known as Dean cells, Fig. 1a, after Dean (1927, 1928). In these pioneering papers, Dean derived an analytical solution for the laminar flow hydrodynamics in toroidal pipes which curvature radius is much larger than the pipe diameter. The so-called Dean number, De , is generally used to quantify the intensity of these secondary flows:

$$De = Re \sqrt{\frac{d}{2R}} = \frac{\rho d U}{\mu} \sqrt{\frac{d}{2R}} \quad \text{Eq. 1}$$

where Re represents the Reynolds number, ρ the fluid density, μ the fluid dynamic viscosity, U the mean velocity of the primary flow, d the internal diameter of the pipe and R the radius of the torus/bend. This dimensionless number is often used as the sole representative parameter in correlations concerning the flow characteristics and the heat and mass transfer efficiencies in curved pipes; indeed, the magnitude of these transport phenomena is strongly correlated to the intensity of Dean vortices.

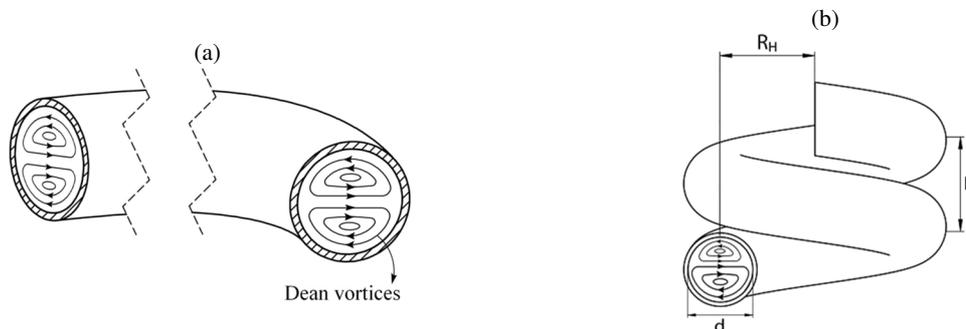


Figure 1: Schematic representation of Dean cells: (a) in a bend or a toroidal pipe. (b) in a helical tube.

In the case of helical pipes, Dean-type cells still develop, however, their symmetry is broken because of torsion effects (Figure 1b). This geometry is widely used in industrial processes and applications where the presence of secondary flows allows reaching much better performance than straight pipes, e.g.: (1) Heat transfer

enhancement. Helically coiled heat exchangers are commonly encountered in anaerobic digesters (Liu et al., 2019), refrigeration systems (Gill and Singh, 2018), power plants and nuclear reactors (Pioro, 2016) among many other applications. (2) Mass transfer enhancement in catalytic reactors (Abdel-Aziz et al., 2010), dense (Mendez et al., 2017) and porous (Ghidossi et al., 2006) membrane contactors, etc. (3) Fouling and clogging reduction in filtration membranes (Moll et al., 2007). (4) Improvement of mixing efficiency and homogenization (Mansour et al., 2017).

Many studies focusing on flow and/or transport phenomena in helical pipes relied upon the Dean number as the sole dimensionless parameter accounting for both flow and geometry effects (see Table 1 for example). Most of these authors extended the Dean number definition to helical geometries simply by replacing the pipe curvature radius with the helix radius, R_H (Figure 1b), in the original form (Eq. 1):

$$De_{R_H} = \frac{\rho d U}{\mu} \sqrt{\frac{d}{2R_H}} = Re \sqrt{\frac{d}{2R_H}} \quad \text{Eq. 2}$$

However, doing so, the effects of the helix pitch are completely ignored, which may lead to important errors in several situations. Effectively, as discussed later in this paper, the flow characteristics may substantially vary with the helix pitch, especially in the case of pipes with a small helix radius.

Table 1: Set of commonly used correlations for predicting the friction factor of fully developed laminar flow in helical pipes. $C_{f,H}^\infty$ and $C_{f,S}^\infty$ correspond to the asymptotic friction factor in helical and straight pipes respectively. $C_{f,S}^\infty$ is equal to $64/Re$ following the Darcy-Weisbach definition which is used in this paper.

Reference	Correlation	Correlation's validity range according to its authors
White (1929)	$C_{f,H}^\infty = C_{f,S}^\infty \left\{ 1 - \left[1 - \left(\frac{11.6}{De_{R_H}} \right)^{0.457} \right]^{2.2} \right\}^{-1}$	$11.6 \leq De_{R_H} \leq 2\,000$
Mori and Nakayama (1965)	$C_{f,H}^\infty = C_{f,S}^\infty \frac{0.108 De_{R_H}^{0.5}}{1 - 3.253 De_{R_H}^{-0.5}}$	Entire laminar regime when $R_H^* \gg 1$
Schmidt (1967)	$C_{f,H}^\infty = C_{f,S}^\infty \left\{ 1 + 0.14 \left(\frac{1}{2R_H^*} \right)^{0.97} Re^{1-0.644(1/2R_H^*)^{0.312}} \right\}$	Laminar regime with $Re \geq 100$
Srinivasan et al. (1968) (cited by Ali (2001))	$C_{f,H}^\infty = \begin{cases} 128/Re & \text{for } De_{R_H} \leq 30 \\ 20.9 De_{R_H}^{-0.6} & \text{for } 30 \leq De_{R_H} \leq 300 \\ 7.2 De_{R_H}^{-0.5} & \text{for } De_{R_H} \geq 300 \end{cases}$	Entire laminar regime
Mishra and Gupta (1979)	$C_{f,H}^\infty = C_{f,S}^\infty \left[1 + 0.033 (\log_{10} De_\gamma)^4 \right]$	$1 \leq De_\gamma \leq 3\,000$
Hart et al. (1988)	$C_{f,H}^\infty = C_{f,S}^\infty \left[1 + \frac{0.09 De_{R_H}^{1.5}}{70 + De_{R_H}} \right]$	Entire Laminar regime

Geometry of the pipe(s)
with which experiments
were conducted

$7.6 \leq R_H^* \leq 1.024$
The range of p^* was not
reported.

$R_H^* = 80$. The value of p^*
was not reported.

$2.5 \leq R_H^* \leq 42$
 $3 \leq p^* \leq 137$

$3.7 \leq R_H^* \leq 50.5$
The range of p^* is not
reported in [Ali \(2001\)](#).

$13.8 \leq p^* \leq 172$
 $3.33 \leq R_H^* \leq 167$

Not clearly mentioned but
seemingly: $p^* = 5.8$, $R_H^* =$
 14.4

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On the other hand, some authors adopted a different expression for the Dean number in helical pipes as follows:

$$De_\gamma = \frac{\rho d U}{\mu} \sqrt{\frac{d}{2\gamma}} = Re \sqrt{\frac{d \kappa}{2}} = Re \sqrt{\frac{d}{2R_H \left[1 + \left(\frac{p}{2\pi R_H} \right)^2 \right]}} \quad \text{Eq. 3}$$

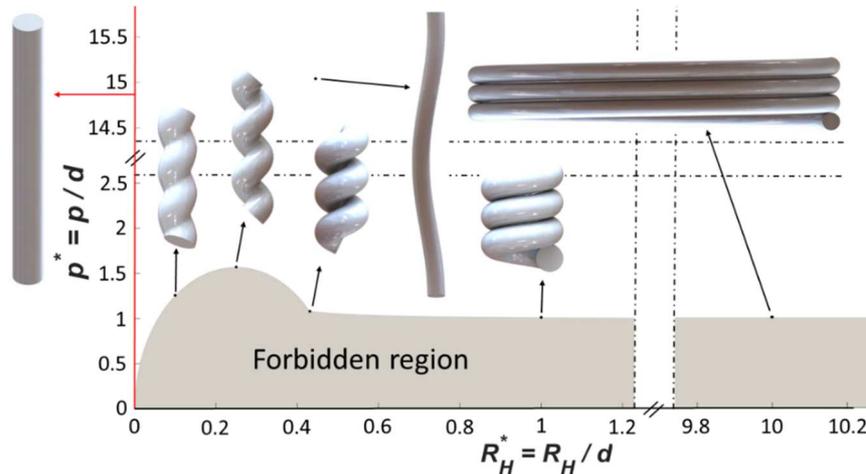
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where γ , κ and p are respectively the radius of curvature, the curvature (i.e. the inverse of γ) and the pitch of the helix. Unlike [Eq.2](#), this expression for the Dean number is consistent with the original definition, [Eq. 1](#). Indeed, in the case of a torus/bend, the radius, R , and the radius of curvature, γ , are identical. Comparing [Equations 2](#) and [3](#), it can be noticed that they lead to significantly different results in the case of geometries with a high pitch to helix radius ratio, as γ becomes much higher than R_H . To the authors' best knowledge, among the studies dealing with friction factor in helical pipes ([Table 1](#)), only [Mishra and Gupta \(1979\)](#) have based their correlation on the Dean number expression given in [Equation 3](#).

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In order to clarify the reasons why the Dean number definition given in [Eq. 3](#) is much more convenient than that of [Eq. 2](#), [Figures 2](#) and [3](#) should be considered. [Figure 2](#) presents the limits of the so-called forbidden region, i.e. the zone in which it is not possible to design helical shapes because the consecutive turns of the helix would intersect/overlap one with/on another. The equation of this frontier has been determined by [Przybył and Pierański \(2001\)](#). It represents the set of geometric parameters describing closely packed helices, i.e. which pitch cannot be further decreased. Helix designs at some points are also represented at [Figure 2](#). They illustrate the fact that the helical pipe geometry tends toward that of a straight tube at three asymptotic limits: (1) when the dimensionless pitch $p^* = p/d$ tends to infinity. (2) when the dimensionless helix radius $R_H^* = R_H/d$ tends to infinity. (3) when the dimensionless helix radius R_H^* tends to zero.

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[Figure 2](#): Limit of the forbidden region in the (R_H^*, p^*) space (adapted from [Przybył and Pierański \(2001\)](#)) and some representative helix geometries.

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[Figure 3](#) shows the contour plot of the dimensionless helix curvature, $\kappa^* = \kappa \cdot d$, in the (R_H^*, p^*) space. It illustrates the fact that κ^* tends to zero at the abovementioned asymptotic limits, as the helix turns into a straight pipe. Thus, the Dean number as defined by [Eq. 3](#) vanishes at these limits, which is consistent with the absence of Dean vortices and centrifugal effects in straight pipe flows. On the contrary, the Dean number as defined by [Eq. 2](#) does not vanish at infinite helix pitches since it does not depend on the pitch value. More importantly, it tends

99 to infinity when the helix radius approaches zero, which corresponds to Dean cells of infinite intensity.
 100 Therefore, as proved later in this paper, correlations based on this latter definition lead to systematic errors at
 101 low helix radiuses or relatively high helical pitches.
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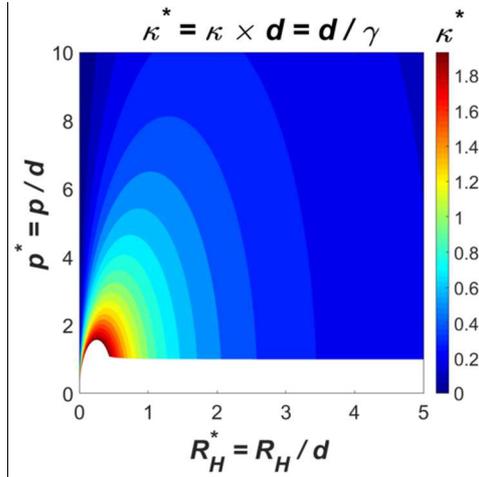


Figure 3: Contour plot of the dimensionless helix curvature in the (R_H^*, p^*) space.

103 Moreover, Figure 3 reveals that the highest curvatures are associated to helices of low pitch and relatively
 104 low helical radius. These geometries will be referred to as ‘highly curved helical pipes’ further on in this paper.
 105 They are expected to exhibit the highest centrifugal effects, i.e. the most intense Dean vortices. Indeed, a CFD
 106 (computational fluid dynamics) study has confirmed that the mass and/or heat transfer efficiencies in highly
 107 curved helical pipes can be an order of magnitude higher than in straight pipes, which makes these designs
 108 potentially interesting for heat exchangers and hollow fiber membranes applications (Abushammala et al., 2019).
 109 Moreover, their elongated shape allows them to be densely packed. Thus, highly curved helices enable a great
 110 improvement of the overall transfer rates, and thus, allow a great unit volume reduction of heat/mass transfer
 111 equipment (Abushammala et al., 2019). It is worthy to underline that classic ‘non-highly’ curved helices may
 112 lead to lower volumetric transfer rates than straight tubes. Indeed, the transfer efficiency improvement that they
 113 provide can be counterbalanced by their low packing density (Kaufhold et al. 2012).
 114

115 Nevertheless, to the authors’ knowledge, no other study has investigated yet the hydrodynamics or transport
 116 phenomena in highly curved helices. As can be noticed from Table 1 for example, literature has focused on
 117 helices with a high R_H to d ratio (i.e. R_H^*), probably because these geometries are easier to manufacture using
 118 conventional manufacturing techniques. However, nowadays, with the development of additive manufacturing,
 119 the elaboration of highly curved helical geometries has become easily achievable. It is noteworthy that 3D
 120 printing offers a great potential for improving heat exchangers and membranes designs (Cardone and Gargiulo,
 121 2018; Low et al., 2017).
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123 The current study focuses on laminar flow friction factor in highly curved helical pipes: laminar flow is
 124 encountered in many applications of practical interest, in particular small-scale devices, e.g. microfluidics,
 125 micro-structured heat exchangers and hollow fiber membranes. Although intense Dean vortices in highly curved
 126 helices lead to improved heat and mass transfer efficiencies compared to straight pipes, they also induce higher
 127 pressure drops and hence greater pumping costs. Therefore, friction factor data in highly curved helices are
 128 required so as to be able to compare the provided benefits with the additional operating costs and therefore to
 129 assess the usefulness of such designs.
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131 In this paper, a series of CFD simulations are carried out to evaluate the friction factor in helical pipes,
 132 particularly highly curved ones. Different operating conditions, described by the Reynolds number, and various
 133 helix designs are examined. It is shown that available correlations fail in estimating the friction factor in highly
 134 curved helices with a sufficient accuracy. Therefore, a new correlation predicting the friction factor over a wide
 135 range of helical pipe geometries and operating conditions is proposed. In order to validate the numerical results,
 136 a highly curved helical pipe was 3D-printed and pressure drops measurements are performed over it. The
 137 experimental data were in very good agreement with the proposed correlation results. Additionally, the
 138 correlation is shown to be robust, efficient and predictive since it correctly estimates experimental data from
 139 literature which were obtained under geometric and operating conditions beyond those investigated in the
 140 present paper (and upon which the correlation was built).
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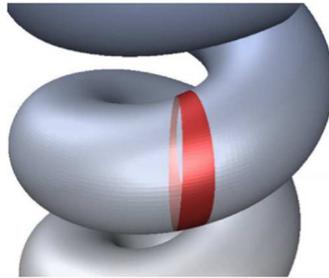
142 **2. Materials and methods**

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144 *2.1. Friction factor and dimensional analysis*

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146 In helical and curved pipes, the wall shear stress and so the local friction factor are not uniform over the pipe
147 wall. In most engineering applications, only the friction coefficient averaged over the pipe circumference is of
148 practical interest (it will be simply referred to as friction factor in the remaining part of this paper):
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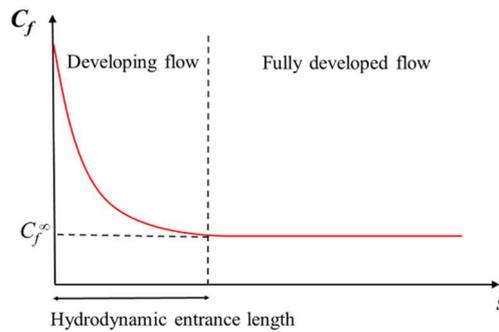
$$C_f = \lim_{s \rightarrow 0} \left[\frac{8}{\rho U^2} \frac{1}{S} \iint \tau_w dS \right] \quad \text{Eq. 4}$$

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151 where C_f is the averaged local friction factor (defined after Darcy-Weisbach) at a given longitudinal position, τ_w
152 the averaged local wall shear stress and S the surface of a wall annular element sliced over the helix surface
153 (Figure 4), and over which τ_w and C_f are averaged.
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156 **Figure 4:** Shape of a wall element (in red) over which C_f is averaged.
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158 **Figure 5** illustrates a qualitative variation of C_f along a helical or a straight tube, the abscissa axis being the -
159 curvilinear - position along the pipe centerline. The friction factor is the highest at the pipe entrance and
160 decreases over a relatively short distance called ‘hydrodynamic entrance length’ before reaching an asymptotic
161 value, C_f^∞ . Even though C_f^∞ is lower than the friction factor at the pipe entrance, whenever the pipe is
162 sufficiently long, it gives the largest contribution to the mechanical energy dissipation, the so-called ‘major head
163 losses’. The additional head losses due to the higher friction coefficient at the entrance length are called ‘minor
164 losses’. The effects of minor losses and the hydrodynamic entrance length are generally negligible in the case of
165 pipes of small diameter and/or sufficient length, which is generally the case in the applications targeted in this
166 study, namely heat exchangers and hollow fiber membranes. Therefore, only major losses are addressed in this
167 paper.
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170 **Figure 5:** Typical variation of the friction factor from the entrance of a straight or a helical pipe.
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172 Due to the occurrence of secondary flows, the asymptotic friction factor in helical pipes, $C_{f,H}^\infty$, is higher than
173 that in straight ones, $C_{f,S}^\infty$ (which equals $64/Re$ in the case of a laminar flow), under similar operating conditions.
174 Assuming an incompressible flow under laminar conditions, $C_{f,H}^\infty$ value depends on the pipe internal diameter, d ,
175 the helical radius of the helix, R_H , its pitch, p , the fluid density, ρ , its dynamic viscosity, μ , and the mean velocity
176 of the primary flow, U . Therefore, according to the Buckingham π theorem, the following three independent
177 dimensionless parameters are required to correlate $C_{f,H}^\infty$ data: (1) The dimensionless helix pitch, $p^* = p/d$. (2)
178 The dimensionless helix radius, $R_H^* = R_H/d$. These first two parameters characterize the helix shape. (3) The
179 Reynolds number, $Re = \rho U d / \mu$, which accounts for the operating conditions.
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181 *2.2. CFD modeling and simulation*

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Flow simulations were conducted for various helical pipe designs and operating conditions. The geometries were drawn using Autodesk Inventor Professional 2018 software, considering a sufficient tube length to attain the fully developed flow region. The 3D numerical domain was meshed using ANSYS Meshing. The grid consisted of hexahedral cells only, with a boundary layer mesh in the near-wall zone for a more accurate calculation of the steep gradients prevailing in this zone. Based on preliminary simulations, the cells' size and density were chosen so as to ensure a mesh-independent solution.

CFD simulations were conducted using the commercial code ANSYS Fluent 16. The flow field in the pipe was determined by solving the continuity and Navier-Stokes equations in the case of a Newtonian, incompressible, isothermal and steady flow:

$$\begin{aligned} \text{div}(\vec{v}) &= 0 \\ \rho \overrightarrow{\text{div}}(\vec{v} \otimes \vec{v}) &= -\vec{\nabla}p + \mu \overrightarrow{\Delta}\vec{v} \end{aligned} \tag{Eqs. 5}$$

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where \vec{v} is the velocity vector. As boundary conditions, a uniform velocity profile was set at the pipe inlet: as the flow rate is imposed, the gravity force has no effect on the velocity field and therefore this body force term was not included in the Navier-Stokes equations. It is noteworthy that the velocity profile at the inlet only affects the flow in the entrance length region and does not have any effect on the value of $C_{f,H}^\infty$. At the pipe outlet, a uniform pressure condition was used and the no-slip condition was set at the pipe walls. The advective terms in Eqs. 5 were discretized using the QUICK scheme while the diffusive ones were central-differenced. Pressure interpolation was carried out using a second order scheme. The equations were solved using a coupled iterative solver until convergence.

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More than 150 simulations were carried out in the following range of operating conditions and aspect ratios: (a) $1.25 \leq p^* \leq 25$. (b) $0.05 \leq R_H^* \leq 10$. (c) $10 \leq Re \leq 2\,000$. In each simulation, the local friction factor was calculated at different longitudinal positions across the pipe length using Eq. 4 (considering a sufficiently small wall element as that depicted on Figure 4), and its asymptotic value, $C_{f,H}^\infty$, was determined. These CFD results were used to correlate $C_{f,H}^\infty$ as a function of the helix design and flow conditions. They are provided in an Excel sheet as supplementary material to this paper.

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2.3. 3D-printed helical pipe

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In order to validate the numerical results, a highly curved helical pipe was built with nylon material using a fused-filament 3D printer (Stream 30 Dual MK2). The geometric parameters of the helix that were targeted are summarized in the first column of Table 2.

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Table 2: Targeted and obtained geometric parameters of the 3D-printed helical pipe.

	Targeted dimensions	Obtained dimensions
Pipe inner diameter, d (mm)	1.6	1.25 ± 0.1
Helix radius, R_H (mm)	0.88	0.80 ± 0.1
Helix pitch, p (mm)	6	5.80 ± 0.1

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A first helical pipe having a wall thickness around 0.8 mm and a total height of 156 mm was 3D-printed (Figure 6a). Printing was carried out in the vertical direction, z . However, once the pipe has reached a certain height, the back and forth movement of the printing platform (in the x -direction) causes the pipe free end to vibrate: hence, the printing quality decreases and the pipe cross-section becomes no longer properly circular (Figure 6a).

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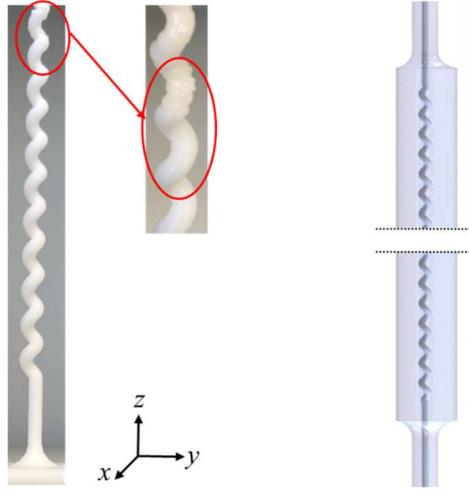


Figure 6: 3D-printed helical tubes: (a) Photo of the failed model, (b) CAD design of the successful model.

Therefore, a new design was tested (Figure 6b). It consists in surrounding the helical tube with a cylindrical block having an external diameter of 15 mm. Indeed, weighting down the structure enhances its stability and reduces its oscillations during the printing process. This design is 200 mm high, which is the maximum height authorized by the 3D printer. It consists of a 156 mm high helical pipe connected at its ends to two straight pipes of 22 mm length each. The straight tubes are required to connect the pipe ends to a pressure transducer for pressure drop measurements.

The model represented in Figure 6b could be successfully 3D-printed without any detectable vibration to the naked eye. In order to check the printing quality, the pipe interior has been visualized using magnetic resonance imaging (MRI), a non-intrusive imaging technique commonly used in medical investigations. The pipe was filled with water and plugged at both ends and then introduced into the scanner. The model was visualized at different sections with different scanning angles. MRI images showed that the helical tube cross-section was properly circular and uniform all over the pipe length. However, it revealed that the tube internal diameter is slightly lower than the one initially targeted (Table 2). This difference is mainly due to the limited precision of the 3D-printer (especially in the x and y directions) and to the relatively large diameter of the nozzle used, which is about 0.4 mm.

The geometric parameters of the obtained helical pipe are summarized in the second column of Table 2: they correspond to a R_H^* of about 0.64 and a p^* around 4.64. The total curvilinear length of the tube is about 206 mm, and is calculated as follows:

$$L = n [(2\pi R_H)^2 + p^2] \quad \text{Eq. 6}$$

where n is the number of helix turns which is equal to the total height of the helical pipe divided by its pitch.

2.4. Experimental setup for pressure drop measurements

The experimental setup for pressure drop measurements is represented in Figure 7. The 3D-printed model was placed horizontally and fed with an aqueous glycerol solution using a high precision micro annular gear pump (MZR-7205, HNP mikrosysteme, Schwerin, Germany. Operating flow rate range: 0.048-288 ml/min). A differential pressure sensor (Kobold PAD-d, Kobold instruments, Pittsburgh, USA. Operating range from 19 mbar to 1.9 bar, with a precision of about 2 mbar) connected to both ends of the pipe (Figure 6b) allows measuring the total pressure drop across the system. Three aqueous solutions with different glycerol concentrations were used as working fluids. Their dynamic viscosity was measured and their Newtonian behavior checked using a strain-imposed rheometer (AREF, TA Instrument, New Castle, USA) equipped with a parallel plates geometry of 50 mm diameter. To ensure reliable viscosity measurements, only solutions of viscosity higher than 20 times the viscosity of water were used.

Steady-state pressure drop measurements were conducted for twelve different Reynolds numbers ranging from 10 to 65. Three replicates were performed for each measurement. The investigated operating conditions were chosen so as to respect the precision limits and the optimal working ranges of the pump, pressure transducer and rheometer. They are summarized in an Excel file provided as supplementary material to this paper. To the authors' best knowledge, these data constitute the first pressure drop measurements in highly curved helical pipes.

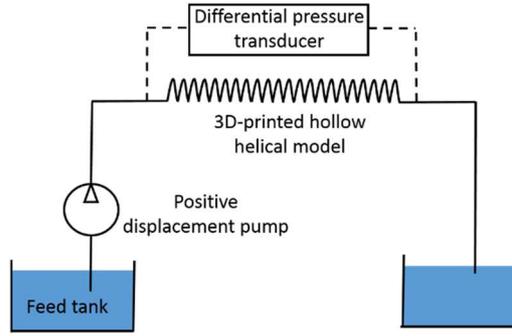


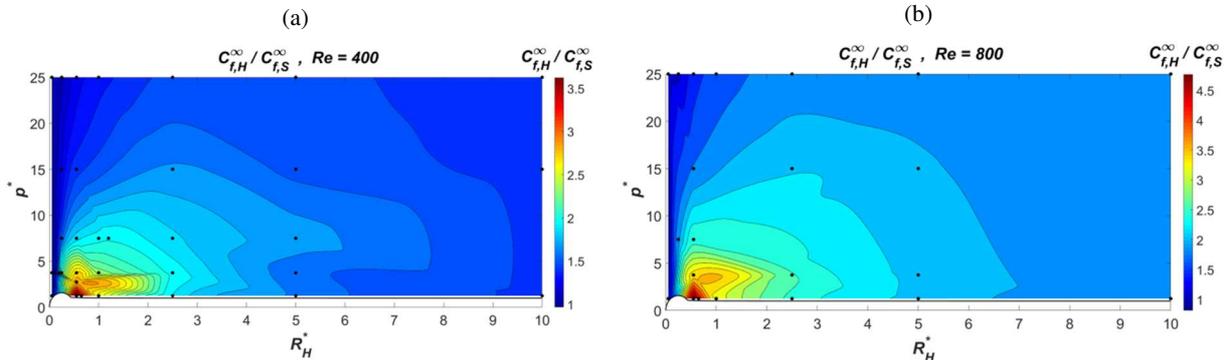
Figure 7: Schematic layout of experimental setup.

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3. Results and discussion

3.1. CFD results

Figure 8 shows contour plots of the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio that were obtained using a triangulation-based cubic interpolation of the CFD results. The geometric parameters for which simulations were performed are represented by black dots. It is worthy to note that, at these points, the values of $C_{f,H}^\infty$ (divided by $C_{f,S}^\infty$) that are displayed on the contour plots are exactly the same than those provided by CFD. On the other hand, the results predicted between these points are generated by interpolation. Therefore, they may be expected not to be very accurate since they are quite sensitive to the interpolation scheme used. Indeed, as can be noticed from Figure 8, the contour plots exhibit some relatively irregular variations, and at some positions, the predicted $C_{f,H}^\infty$ to $C_{f,S}^\infty$ values are even slightly lower than unity.



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Figure 8: Contour plots of the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio at a Reynolds number of (a) 400 and (b) 800 respectively. The black dots represent the geometric conditions for which CFD results are acquired ($1.25 \leq p^* \leq 25$ and $0.05 \leq R_H^* \leq 10$). The contours are derived by interpolating these data.

Nonetheless, these contour plots provide valuable information:

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- (1) Comparison to Figure 3 confirms that the more curved is the pipe, the higher the magnitude of $C_{f,H}^\infty$. Indeed, highly curved helices engender the most intense Dean-type vortices and hence the highest friction factors.
- (2) For a given dimensionless pitch, p^* , there exists a dimensionless helix radius, R_H^* , at which the friction coefficient is maximal. The $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio decreases for higher or lower helical radiuses and tends toward unity when R_H^* tends to zero or to infinity as the helix geometry approaches that of a straight pipe.
- (3) For a given R_H^* , the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio decreases when p^* increases (although interpolated data indicate the opposite at some positions). Indeed, as can be noticed from Figure 3 and Eq. 3, the pipe curvature (and hence centrifugal effects) diminishes when p^* is increased. In fact, the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio should tend towards unity at infinite p^* as the helix design approaches a straight pipe.
- (4) Although $C_{f,H}^\infty$ decreases when p^* is increased, as can be seen from Figure 8, this effect is only significant in the case of highly curved helices. Indeed, for relatively high R_H^* , the curvature is not much sensitive to the value of p^* (Figure 3). As only helical pipes with rather large R_H^* were investigated in the literature, most authors concluded that the pitch value has no significant effect on $C_{f,H}^\infty$ (Table 1). Thus, it is obvious that their

302 correlations, which are based on De_{RH} (Eq. 2), cannot accurately predict the friction coefficient in highly curved
 303 helical pipes. Only the one derived by Mishra and Gupta (1979) may lead to acceptable results since it considers
 304 pitch effects through the use of De_γ (Eq.3).

305 (5) When Re is increased, centrifugal effects get more intense, thus the flow and so $C_{f,H}^\infty$ become more sensitive
 306 to the helix geometry. Hence, while the maximum value of the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio is about 3.5 under $Re = 400$, it
 307 exceeds 4.5 for $Re = 800$. However, the geometric parameters for which $C_{f,H}^\infty$ is maximal do not seem to be
 308 significantly affected by the Reynolds number value.

310 3.2. Correlation development

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 312 Although the $C_{f,H}^\infty$ field in the (R_H^*, p^*) space (Figure 8) presents a single maximum (no local optima) for a
 313 given Re , finding a mathematical model that correctly fits the CFD data was very challenging and tedious. In
 314 fact, the $C_{f,H}^\infty$ field exhibits steep variations in the highly curved helices region, with highly non-uniform and
 315 anisotropic gradients, which magnitudes are very sensitive to the helix geometry.

316 Accordingly, the development of the regression model for $C_{f,H}^\infty$ has been conducted by trial-and-error. The
 317 following expression has been found to provide the best fit of the CFD data:

318

$$\begin{aligned}
 C_{f,H}^\infty &= \frac{64}{Re} + A \cdot B \cdot \exp(-C) \\
 \text{where:} \quad A &= p_1 \cdot D \cdot \left(\frac{D}{Re}\right)^{p_2} \\
 B &= \left(R_H^* + \frac{1}{R_H^*}\right)^{p_3} \\
 C &= p_4 \cdot D \cdot p^* \cdot R_H^{*-p_5} \\
 D &= \left[R_H^{*p_6} \left(1 + \left(\frac{p^*}{2\pi R_H^*} \right)^2 \right) \right]^{-p_7}
 \end{aligned}
 \tag{Eqs. 7}$$

319
 320 The present expression includes 7 positive regression parameters denoted p_i (i being an integer ranging from
 321 1 to 7) which values are determined using an optimization procedure, as explained later, in order to minimize the
 322 differences between the correlation outputs and the CFD data.

323 The term $64/Re$ corresponds to the asymptotic friction factor in a straight tube, $C_{f,S}^\infty$, with the Darcy-
 324 Weisbach definition. A and B being positive terms, for any given Re , the present correlation guarantees that the
 325 predicted $C_{f,H}^\infty$ value remains greater (or equal) than $C_{f,S}^\infty$.

326 Coefficient D is analogous to κ^* , with the difference that p_6 and p_7 are treated as optimization variables
 327 instead of assigning their values to 1 and 2 respectively. Parameters p_6 and p_7 being strictly positive, term D
 328 vanishes when R_H^* tends to zero or infinity or when p^* tends to infinity, i.e. when the helical pipe geometry tends
 329 toward that of a straight one.

330 The structure of term A expresses the fact that the magnitude of $C_{f,H}^\infty$ increases in zones of high curvatures
 331 and decreases with increasing Re (as the optimal value of p_2 was found to be strictly positive). The term D
 332 appears with a strictly positive power, $1 + p_2$, in the expression of A . Accordingly, term A vanishes at the
 333 asymptotic limits for which the helical pipe reduces to a straight one. Thus, the structure of the developed
 334 correlation guarantees that the predicted $C_{f,H}^\infty$ becomes equal to $C_{f,S}^\infty$ at these limits.

335 Term B allows adjusting the model behavior at large and low R_H^* values, and its incorporation into the
 336 mathematical expression significantly improved the correlation accuracy.

337 Term C being positive, $\exp(-C)$ represents a decreasing exponential. Since D appears with a positive power in
 338 the expression of C , $\exp(-C)$ allows reproducing the steep variations of $C_{f,H}^\infty$ in the regions of high helix
 339 curvature, and the flattening of the $C_{f,H}^\infty$ contour plot in zones of low helix curvature, namely at high R_H^* and p^*
 340 values.

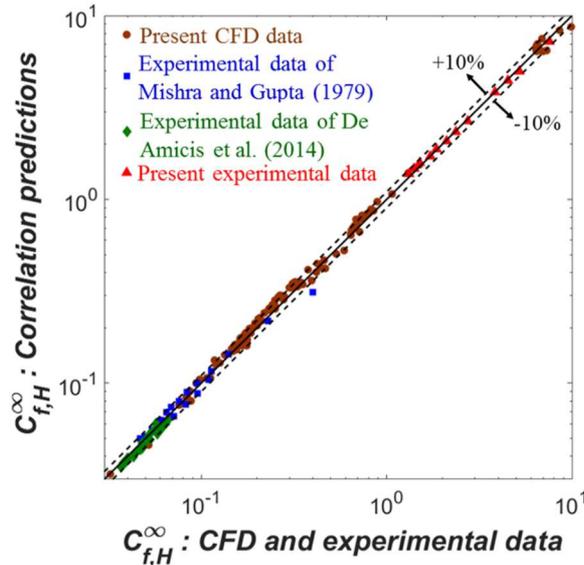
341 The mathematical expression presented in Equations 7 was used to correlate the CFD data. It should be
 342 recalled that, as mentioned in Section 2.2, these results were obtained in the following range of dimensionless
 343 parameters: $1.25 \leq p^* \leq 25$, $0.05 \leq R_H^* \leq 10$, $10 \leq Re \leq 2\,000$. For a better accuracy, two correlations were
 344 derived: the first one using the data where Re ranges from 10 to 400, and the second one for Re ranging from 400
 345 to 2 000. The optimization problem was formulated as a minimization of the maximum relative difference
 346 between the model predictions and the numerical data. The optimal set of the parameters p_i , which is reported in

347 Table 3, was determined using a genetic algorithm and the results were further refined using a local optimizer,
 348 GRG2.

349 Table 3: Optimal set for the parameters of the correlation expressed by Equations 7.

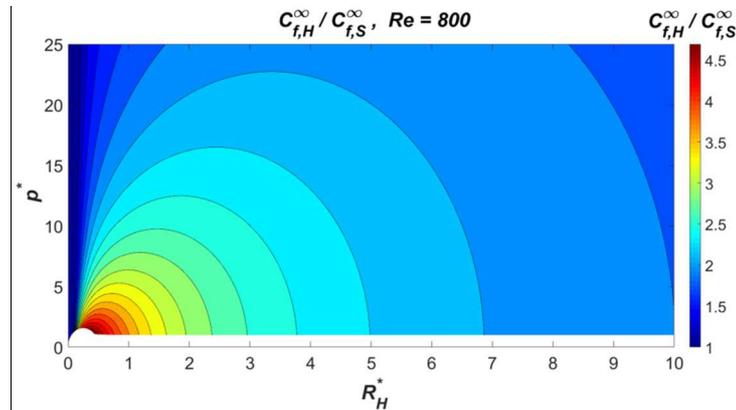
Validity range	p_1	p_2	p_3	p_4	p_5	p_6	p_7
$10 \leq Re \leq 400$	1.98	4.07×10^{-1}	8.49×10^{-1}	8.71×10^{-2}	8.91×10^{-1}	2.31	3.67×10^{-1}
$400 \leq Re \leq 2\,000$	2.88	3.82×10^{-1}	9.16×10^{-3}	2.48×10^{-3}	2.62	1.10	3.23×10^{-1}

352 Figure 9 compares the correlation predictions and CFD results (represented by brown disks). It shows that the
 353 proposed correlation correctly fits the $C_{f,H}^\infty$ data as most points are within an error margin of 10%. Indeed, the
 354 maximum relative difference between the numerical results and the correlation predictions is respectively 13.8%
 355 for Re below and 400 and 13.2% for Re above 400.
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 357



358 Figure 9: Parity diagram of $C_{f,H}^\infty$: CFD and experimental data versus correlation predictions.

359 Figure 10 shows the contour plot of the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio calculated using the present correlation for a Re of
 360 800. Comparison to Figure 8b shows a good agreement, with the difference that the model data are much
 361 smoother since they are not affected by interpolation approximations. As can be noticed from Figure 10, $C_{f,H}^\infty$
 362 exhibits the largest values as well as the highest gradients in the highly curved helixes region. The $C_{f,H}^\infty$ to $C_{f,S}^\infty$
 363 ratio tends toward unity at very low values of R_H^* . Indeed, the formulation of the mathematical expression (Eqs.
 364 7) ensures that $C_{f,H}^\infty$ becomes equal to $C_{f,S}^\infty$ at the three asymptotic limits at which the helix geometry tends toward
 365 that of a straight pipe.
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369 Figure 10: Contour plot of the $C_{f,H}^\infty$ to $C_{f,S}^\infty$ ratio at a Reynolds number of 800: present correlation data (Equations 7).

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 372 3.3. Comparison with literature correlations

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In order to assess the potentiality of the new correlation, its performance are compared to that of literature correlations in Figures 11, 12 and 13. Figure 11 depicts the variations of $C_{f,H}^\infty$ as a function of R_H^* for two values of p^* . Given that literature correlations were derived by regressing experimental measurements acquired on helixes with large R_H^* (Table 1), they are expected to be accurate for non-highly curved helixes. As can be noticed, at relatively large R_H^* values, the results provided by all of the plotted correlations are in good agreement with the current CFD data. This constitutes a first validation of the present CFD simulations.

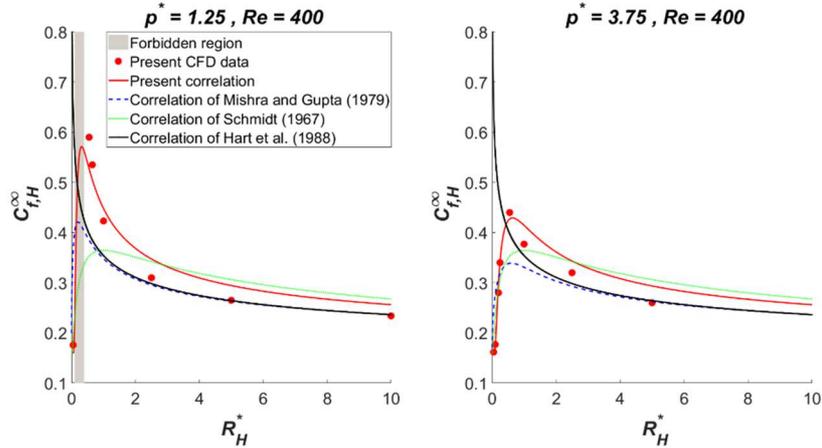


Figure 11: $C_{f,H}^\infty$ versus R_H^* at two different p^* : comparison of different correlations.

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However, Figure 11 also shows that literature correlations do not correctly estimate $C_{f,H}^\infty$ in the case of highly curved helixes. Indeed, the correlation of Hart et al. (1988) and all the correlations based on De_{RH} (Equation 2) (see Table 1) do predict an infinite friction factor when R_H^* tends towards zero.

On the other hand, the correlations of Schmidt (1967) and Mishra and Gupta (1979) reproduce correctly the shape of the curves showing $C_{f,H}^\infty$ versus R_H^* . However, the correlation of Schmidt (1967) does not account for any pitch effect. Therefore, it leads to systematic errors in the highly curved helixes zone where the friction factor is very sensitive to the pitch (Figures 8, 10, 11 and 12). The correlation of Mishra and Gupta (1979) proved to be the most accurate among literature models. In some cases as illustrated in Figure 11, it does even provide slightly better predictions than the current correlation. However, its major weak point is that it generally largely underestimates the friction factor in highly curved helixes as can be noticed from Figures 11, 12 and 13.

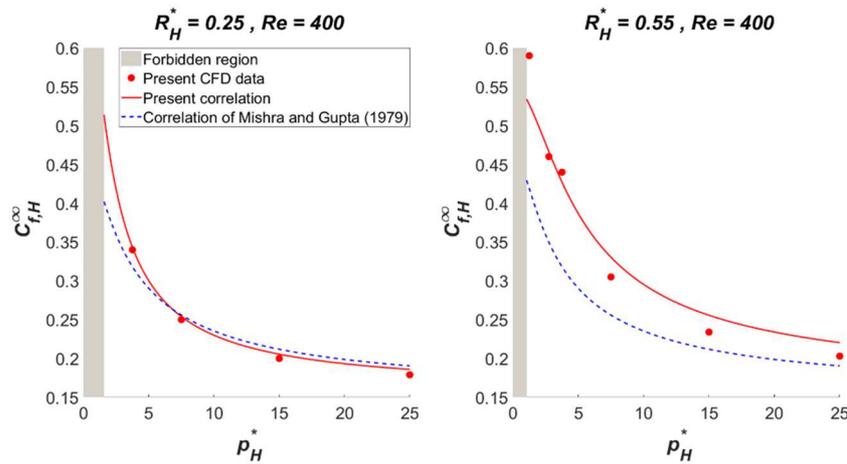


Figure 12: $C_{f,H}^\infty$ versus p_H^* at two different R_H^* : comparison between the present correlation and that of Mishra and Gupta (1979).

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Finally, the variations of $C_{f,H}^\infty$ with the Reynolds number are plotted on Figure 13 for four highly curved helixes. This figure illustrates the fact that the present model (Eqs. 7) correctly captures the Re effects, and is generally more accurate than that of Mishra and Gupta (1979). A discontinuity in the curves obtained with the present correlation can be noticed. It is due to the use of different parameter values depending on whether Re is below or above 400 (Table 3).

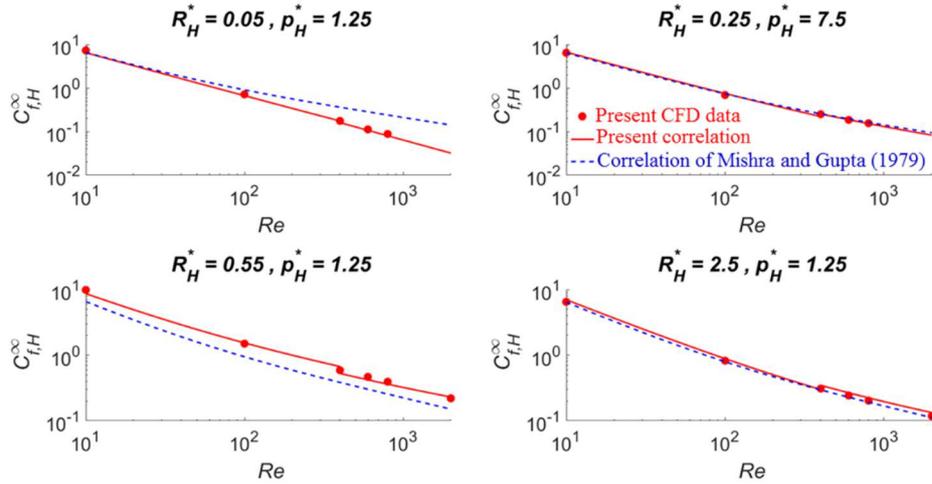


Figure 13: $C_{f,H}^{\infty}$ variation with Re for four different helices.

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3.4. Correlation validation using experimental data from literature

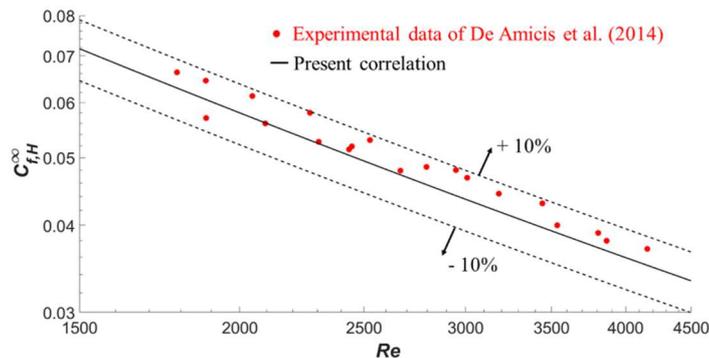
In addition to CFD data, for a more trustworthy validation, the correlation results were compared to the experimental data of Mishra and Gupta (1979) and De Amicis et al. (2014). Table 4 summarizes the geometric and operating conditions over which these measurements were obtained. It is noteworthy that, although most of these experiments were performed for a Re higher than 2 300, they do all correspond to a laminar flow regime. Indeed, in the case of helical pipes, centrifugal forces do generally have a stabilizing effect on the flow which delays the transition to turbulence. Hence, the flow can remain laminar for Reynolds values much higher than in straight pipes (De Amicis et al., 2014; Ghobadi and Muzychka 2016).

The last column in Table 4 recalls the range of geometric and operating conditions over which the current CFD data were performed, and hence, upon which the present correlation was built. It can be noticed that all of the literature experimental results were acquired for conditions beyond the validity range of the present correlation. Nevertheless, as shown in Figures 9 and 14, all of De Amicis et al. (2014) results are predicted within a margin error of 10%, and most of Mishra and Gupta (1979) data are predicted with a relative error less than 8%. Only a single point falls beyond this limit and is predicted with an error of about 28%. It corresponds to a measurement acquired by Mishra and Gupta (1979) under a Re of 266, which represents the lowest Re investigated in their study (Table 4). Such a low Re leads to a low pressure drop especially since the helix considered is not highly curved; hence, this measurement is expected to be much affected by experimental errors.

Table 4: Ranges of geometric and operating conditions over which the literature experimental data were acquired and the present correlation was built.

	Mishra and Gupta (1979): helical pipe 1	Mishra and Gupta (1979): helical pipe 2	De Amicis et al. (2014)	Range of parameters investigated using CFD and upon which the present correlation was built
R_H^*	3.4	20.2	39.9	[0.05 – 10]
p^*	27.5	18.9	63.8	[1.25 – 25]
Re	[266 – 5 500]	[1 100 – 2 550]	[1 790 – 4 160]	[10 – 2 000]

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433 Figure 14: Comparison between the present correlation predictions and the experimental data of De Amicis et al. (2014).

434
 435 The results presented in this section demonstrate the predictive capacities of the present correlation, as it
 436 successfully estimated experimental data which were acquired beyond its supposed range of validity. This
 437 advantageous feature is probably due to the structure of the present model (Eqs. 7): in particular, it guarantees
 438 correct results at the asymptotic limits where the helical pipe geometry tends toward that of a straight one, i.e. it
 439 ensures results to be correctly bounded. Therefore, the present correlation is believed to lead to realistic and
 440 accurate friction factor values even beyond the range of geometric and operating conditions investigated in this
 441 paper.

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 443 3.5. Correlation validation using data acquired on the 3D-printed highly curved helix

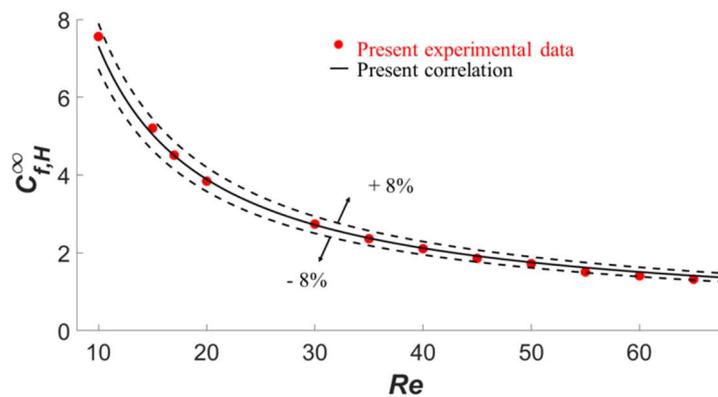
444
 445 The performed pressure measurements correspond to the total pressure drop across the 3D-printed model, i.e.
 446 through the helical pipe (which R_H^* is about 0.64 and p^* around 4.64, see Section 2.3) and the two straight pipes
 447 connections. Assuming that the friction factor in the straight tubes equals $64/Re$, the pressure drop across the
 448 helical pipe, Δp_H , could be calculated (calculation details are given in the Excel file provided as supplementary
 449 material to this paper). Given the important length and the small diameter of the helical pipe, minor head losses
 450 can be neglected. Thus, $C_{f,H}^\infty$ is calculated following Darcy-Weisbach:

$$C_{f,H}^\infty = \frac{\Delta p_H d}{\frac{1}{2} \rho L U^2} \quad \text{Eq. 8}$$

451
 452 where L is the total curvilinear length of the helical pipe.

453 Figures 9 and 15 compare the experimental friction factor to the correlation (Equations 7) predictions. All the
 454 experimental data are predicted within a relative error margin less than 8%, which demonstrates the accuracy of
 455 the proposed correlation in the case of highly curved helical pipes.

456
 457 This correlation is the first one to accurately predict the pressure drop in highly curved helical pipes.
 458 Moreover, as shown in this paper, it proved to be efficient over a broad range of geometric and operating
 459 conditions. The precise estimation of the specific energy requirement of a whole set of heat and mass exchangers
 460 designs, over a broad range of operating conditions (under the assumption of a laminar flow regime) is thus now
 461 achievable. This offers the possibility to systematically explore which heat or mass exchanger geometry offers
 462 the best combination of increased transfer performances together with a minimal specific energy requirement.
 463 Process intensification studies typically correspond to such an objective and the results reported in this paper
 464 should be helpful for optimal exchangers design studies.



466
 467 Figure 15: Comparison between the present correlation predictions and the present experimental data.

468
 469 4. Conclusion and perspectives

470
 471 This paper focused on the laminar flow friction factor in helical pipes, especially highly curved geometries.
 472 CFD simulations were conducted to determine the friction factor for various helix designs under different
 473 Reynolds numbers. On the basis of these numerical results, a general correlation for calculating the friction
 474 factor in helical pipes was developed. The correlation proved to be robust, efficient and predictive as its results
 475 were in excellent agreement with literature experimental data as well as with measurements performed on a 3D-
 476 printed highly curved helical tube.

477 Future studies will concentrate on developing correlations predicting the heat and mass transfer efficiencies
478 in helical pipes. These correlations, along with the one developed in this paper, will then be used in model-based
479 optimization of helically coiled heat exchangers and hollow fiber membrane contactors in order to determine the
480 optimal helical tubes design, i.e. the one leading to the most lucrative trade-off between the transfer efficiency
481 enhancement and the increase of pumping costs.

482 483 **Acknowledgment**

484
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