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Prediction of elastic and acoustic behaviors of calcarenite used for construction of historical monuments of Rabat, Morocco

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abstract

Natural materials (e.g. rocks and soils) are porous media, whose microstructures present a wide diversity. They generally consist of a heterogeneous solid phase and a porous phase which may be fully or partially saturated with one or more fluids. The prediction of elastic and acoustic properties of porous materials is very important in many fields, such as physics of rocks, reservoir geophysics, civil engineering, construction field and study of the behavior of historical monuments. The aim of this work is to predict the elastic and acoustic behaviors of isotropic porous materials of a solid matrix containing dry, saturated and partially saturated spherical pores. For this, a homogenization technique based on the MorieTanaka model is presented to connect the elastic and acoustic properties to porosity and degree of water saturation. Non-destructive ultrasonic technique is used to determine the elastic properties from measurements of P-wave velocities. The results obtained show the influence of porosity and degree of water saturation on the effective properties. The various predictions of MorieTanaka model are then compared with experimental results for the elastic and acoustic properties of calcarenite.

1. Introduction

The study of the role of porosity, geometry of the pore space and fluid flow in the elastic and acoustic behaviors of porous media is fundamental to understanding phenomena in different fields (e.g. geophysical subsurface and reservoirs, seismology, engineering, monuments, and construction). Indeed, presence of porosity and fluid results in a modification to elastic and acoustic properties which can be modeled by homogenization techniques.

The state of saturation of the natural rock influences the evolution of its elastic and acoustic properties and has been the subject of numerous works, many of which are related to prediction of elastic and acoustic properties of a multiphase porous rock and generally deal with a fully saturated state in comparison to a dry state. The propagation of an elastic wave in a biphasic medium (solideliquid) was described by Biot (1956), who introduced inertial coupling concepts to the fluid-solid and relative movements of these two phases.

The relationship between the partial saturation and the acoustic properties of rock is more complex. Its analysis requires following experiment very rigorously and involves, from a theoretical point of view, the mechanisms at pore scale and fluid distribution concepts in porous networks. The acoustics of porous and heterogeneous media aims to characterize the seismic waves by synthesizing between the rigor of the laws of mechanics and the natural disorder of porous media (Pham, 2003; Wang and Li, 2007).

Many fields of geophysical and geological investigations are almost invariably faced with the need to determine the properties such as porosity, degree of cracking or fracturing, degree of water saturation, permeability, thermal conductivity, temperature and pressure of fluid in different parts of the crust. Studies have focused on the influence of rock properties and fluids on geophysical, petrophysical and thermal measurements (Gregory, 1976; Han et al., 1986; Klimentos, 1991; Popov et al., 2003; Samaouali et al., 2010; Rahmouni et al., 2013, 2014a, b).

The first models devoted to calculating the effective elastic properties of composite materials were proposed by Voigt (1889) and Reuss (1929). The Voigt approximation provides an upper bound, while the Reuss approximation is a lower bound for the effective moduli. Hashin and Shtrickman (1963) developed a variational approach that can identify, for isotropic media, a narrower range for the effective moduli than the average of Voigt (1889) and Reuss (1929) approximations. These models lead to a broad framework of elastic moduli of

case, to determine the effective elastic properties of a rock whose pore space is either empty or unsaturated or saturated with fluid.

The models such as Eshelby, dilute, MorieTanaka, self-consistent, Ponte Castaneda and Willis are characterized by taking into account the geometry of the constituent phases of porous media. Some authors have applied these models to describing the behavior of porous media. For example, Guéguen et al. (1997) used the differential self-consistent model to determine the elastic moduli of porous media. Xu (2004) applied changing scale methods to modeling the behavior of unsaturated porous media whose solid phase consists of a linearly elastic material. The prediction of the hydromechanical behavior of clays compacted by homogenization models was studied by Barboura (2007). Sarout (2006) applied the MorieTanaka model to describing the propagation of elastic waves in dry argillaceous rocks. Miled et al. (2011) studied the prediction of elastic moduli of isotropic porous materials constituting a solid matrix based on well-known mean-field Eshelby-based homogenization schemes.

Many studies have been devoted to the modeling of effective elastic and acoustic properties of partially saturated porous media (Gregory, 1976; Mavko and Hoeksema, 1994; Cadoret, 1995; Le Ravalec et al., 1996; Dvorkin et al., 1999; King et al., 2000; Li et al., 2001; Toms et al., 2006). Gassmann (1951)'s equations provide the effective elastic moduli of saturated rock for low frequency, provided that the effective elastic modulus of dry rock is known in advance. The MorieTanaka model has the advantage of allowing the determination of effective elastic properties even for a rock whose pore space is empty. We can then obtain the effective elastic modulus of dry or saturated rock.

The approach developed by Domenico (1976) allows to take into account the phenomena of partial saturation. This author suggested replacing the compression modulus of the fluid by an equivalent modulus. Two approaches can be envisaged classically: equivalent modulus is assessed either from the average of Voigt approximation (uniform deformation hypothesis), or from the average of Reuss approximation (uniform stress hypothesis). The second proposal is necessary here since the objective is to calculate the effective elastic and acoustic properties for partially saturated rock at low frequency: the fluid pressure is uniform. The bulk modulus of fluid k_f is then written as

$$\frac{1}{k_f} = \frac{S}{k_w} + \frac{1-S}{k_g}$$

where k_w and k_g are the bulk moduli for the liquid and gas, respectively; and S is the degree of water saturation. Eq. (1) has a very contrasting behavior because k_g

composite media, mainly when phases presented have very different elastic properties. This remark is particularly appropriate that we seek, in the present

$\ll k_w$. If $S \ll 1$, $k_r \ll k_w$. Instead, when a small amount of gas is present in pore space, k_r tends to be k_g .

In addition, the pore fluid increases the effective density of the rock, ρ , i.e.

$$\rho = \rho_m(1 - \phi) + \rho_w \phi S + \rho_g \phi(1 - S) \quad (2)$$

where ρ_m , ρ_w and ρ_g are the densities of solid matrix, water and gas phases, respectively; and ϕ is the porosity.

In a homogeneous isotropic elastic medium, the velocities of P and S-waves can be respectively given by (Bourbie et al., 1986):

$$V_P = \sqrt{\frac{k + 4\mu/3}{\rho}} \quad (3)$$

$$V_S = \sqrt{\mu/\rho} \quad (4)$$

where k and μ are the bulk and shear moduli of the medium, respectively.

According to Eqs. (3) and (4), the P-wave velocity (V_P) depends on the bulk and shear moduli and density of the rock, while the Swave velocity (V_S) depends only on the density (ρ) of the rock and the shear modulus (μ).

The objective of this paper is to predict the elastic and acoustic behaviors

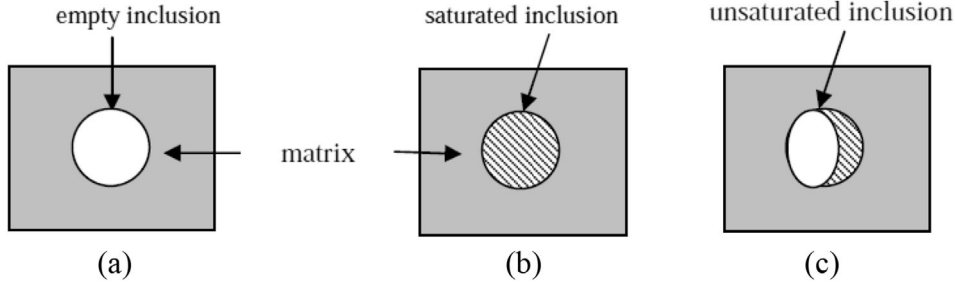


Fig. 1. Scheme of homogenizing porous media: (a) dry, (b) saturated and (c) unsaturated.

of isotropic porous materials of a solid matrix containing dry, saturated and partially saturated spherical pores, using a homogenization technique based on the Mori-Tanaka model. The elastic properties of calcarenite are determined from P-wave velocity measurements. The comparisons between the predictions of Mori-Tanaka model and experimental results are analyzed.

2. Theoretical background

The homogenization method is usually used to replace a real heterogeneous medium by a fictitious homogeneous medium equivalent (Xu, 2004). We consider that a linearly elastic isotropic two-phase medium constitutes a matrix containing inclusions (Fig. 1). The elastic tensor, bulk and shear moduli of the phase i (i = matrix or inclusion) are respectively denoted by C_i , k_i and μ_i , then the macroscopic behavior is linearly elastic and isotropic. Macroscopic stress (Σ) and strain (E) tensors are connected by

$$\Sigma = C^{\text{hom}} : E \quad (5)$$

where

$$C^{\text{hom}} = 3k^{\text{hom}}J + 2\mu^{\text{hom}}K \quad (6)$$

where k^{hom} and μ^{hom} denote the bulk and shear moduli homogenized, respectively; J and K represent the spherical and deviatoric

tensors, $J = \frac{1}{3} \delta_{ijkl} \delta_{ijkl}$ and $K = \frac{1}{2} (d_{ij} d_{ij} - \frac{1}{3} \delta_{ij} \delta_{ij})$, and $J_{ijkl} = \frac{1}{24} (\delta_{ij} \delta_{kl} + \delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{kj} + \delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl})$, $d_{ij} = \delta_{ij} - \frac{1}{3} \delta_{ij} \delta_{ij}$.

(Giraud et al., 2008), where J_{ijkl} is the fourth-order symmetric identity tensor, and d_{ij} is the Kronecker symbol ($d_{ij} = 1$ if $i = j$, $d_{ij} = 0$ otherwise).

The elastic tensors of the matrix and inclusion can be respectively written by

$$C_m = 3k_m J + 2\mu_m K \quad (7)$$

$$C_{\text{inc}} = 3k_{\text{inc}} J + 2\mu_{\text{inc}} K \quad (8)$$

where subscripts 'm' and 'inc' represent matrix and inclusion, respectively.

At the microscopic scale, the law of behavior at the point r is $\sigma(r) = C(r) : \epsilon(r)$

$$\sigma(\vec{r}) = C(\vec{r}) : \epsilon(\vec{r}) \quad (9)$$

where $\sigma(\vec{r})$ and $\epsilon(\vec{r})$ are the microscopic stress and strain tensors, $C(\vec{r})$ is the microscopic elastic tensor.

The averages of the macroscopic stress and strain are given by

$$\left. \begin{aligned} \Sigma &= \langle \sigma(\vec{r}) \rangle \\ E &= \langle \epsilon(\vec{r}) \rangle \end{aligned} \right\} \quad (10)$$

Relation of averages in Eq. (10), representing the homogenization of heterogeneous material behavior, must consider the relation between local strain and stress that links microscopic fields $\epsilon(\vec{r})$ and $\sigma(\vec{r})$ to macroscopic fields E and Σ . The local tensor $A(\vec{r})$ of fourth order can be written by

$$\epsilon(\vec{r}) = A(\vec{r}) : E \quad (11)$$

By using Eqs. (5), (10) and (11), the macroscopic elastic tensor can be written in the following form:

$$C^{\text{hom}} = \langle C(\vec{r}) : A(\vec{r}) \rangle \quad (12)$$

The resolution of the homogenization problem is to determine the local tensor \mathbf{A}^{MT} . In our case, we will use the expression of local tensor of Mori-Tanaka model.

2.1. Mori-Tanaka model

The Mori-Tanaka model (Mori and Tanaka, 1973) is effective for sedimentary rocks (Huynh, 2006) and composite materials (Benveniste, 1987; Reiter et al., 1997; Li and Wang, 2005; Pichler et al., 2007). This model takes into account the geometry of the constituent phases of porous media, and also the interaction between the inclusions (Xu, 2004; Huynh, 2006).

The local tensors of the matrix and inclusion can be respectively written as

$$\mathbf{A}_m = \left[\phi_m \mathbf{I} + (\mathbf{C}_{\text{inc}} - \mathbf{C}_m)^{-1} : \langle \mathbf{T} \rangle \right]^{-1} \quad (13)$$

$$\mathbf{A}_{\text{inc}} = [\mathbf{I} + \mathbf{P}_{\text{inc}} : (\mathbf{C}_{\text{inc}} - \mathbf{C}_m)]^{-1} : \mathbf{A}_m \quad (14)$$

where ϕ_m is the volume fraction of matrix; \mathbf{P} is the Hill tensor, and $\mathbf{P}^{\text{SE}} = \frac{1}{15} (5\mathbf{I} + 3\mathbf{S}^E)$ where \mathbf{S}^E is the Eshelby tensor; and

$\mathbf{T} = \frac{1}{2} \mathbf{S}^E$

Based on Eqs. (12)(14), the Mori-Tanaka macroscopic elastic tensor can be written as

$$\mathbf{C}^{\text{MT}} = \mathbf{C}_m + \phi_{\text{inc}} \left[\phi_m \mathbf{S}^E : \mathbf{C}_m^{-1} + (\mathbf{C}_{\text{inc}} - \mathbf{C}_m)^{-1} \right]^{-1} \quad (15)$$

For a medium consisting of an isotropic matrix and spherical inclusions, the behavior of the macroscopic material is still isotropic. The effective bulk and shear moduli (k^{MT} and μ^{MT}) are therefore respectively given by (Xu, 2004):

$$k^{\text{MT}} = k_m \left[1 + \phi_{\text{inc}} \frac{k_{\text{inc}} - k_m}{k_m + (1 - \phi_{\text{inc}}) \alpha (k_{\text{inc}} - k_m)} \right] \quad (16)$$

$$\mu^{\text{MT}} = \mu_m \left[1 + \phi_{\text{inc}} \frac{\mu_{\text{inc}} - \mu_m}{\mu_m + (1 - \phi_{\text{inc}}) \beta (\mu_{\text{inc}} - \mu_m)} \right] \quad (17)$$

2.2. Effective elastic properties of dry porous media

By putting $k_{\text{inc}} \rightarrow 0$ and $\mu_{\text{inc}} \rightarrow 0$ in Eqs. (16) and (17), the elastic properties including bulk and shear moduli and Young's modulus

(k_{dry} , μ_{dry} , E_{dry}) for porous media containing dry spherical pores are obtained:

$$k_{\text{dry}} = k_m (1 - \phi) (1 + a\phi)^{-1} \quad (18)$$

$$\mu_{\text{dry}} = \mu_m (1 - \phi) (1 + b\phi)^{-1} \quad (19)$$

$$E_{\text{dry}} = E_0 (1 - \phi) (1 + c\phi)^{-1} \quad (20)$$

where

$$a = \frac{3k_m}{4\mu_m}, \quad b = \frac{6k_m + 1}{9k_m + 1}, \quad c = \frac{3k_m}{4\mu_m}, \quad b = \frac{6k_m + 1}{9k_m + 1}$$

$$E_0 = \frac{9k_m \mu_m}{3k_m + \mu_m}$$

where E_0 is the Young's modulus of the solid matrix.

By replacing Eqs. (18) and (19) in Eqs. (3) and (4), the effective acoustic properties (V_P and V_S) are obtained for dry porous media containing spherical pores:

$$V_{P \text{ dry}}^2 = V_{Pm}^2 \frac{1 + d\phi}{(1 + a\phi)(1 + b\phi)} \quad (21)$$

$$V_{S \text{ dry}}^2 = V_{Sm}^2 \frac{1}{1 + b\phi} \quad \text{where} \quad (22)$$

$$V_{Pm} = \sqrt{\frac{k_m + 4\mu_m/3}{\rho_m}}, \quad V_{Sm} = \sqrt{\frac{k_m + 4\mu_m/3}{\rho_m}},$$

$$d = \frac{3k_m b + 4\mu_m a}{3k_m + 4\mu_m}, \quad d = \frac{3k_m b + 4\mu_m a}{3k_m + 4\mu_m}$$

where V_{Pm} and V_{Sm} are the velocities of P- and S-waves in the matrix, respectively.

2.3. Effective elastic properties of saturated porous media

For saturated porous media by a fluid, the effective elastic tensor corresponding to the Mori-Tanaka scheme is

$$\mathbf{C}_{\text{sat}}^{\text{MT}} = \mathbf{C}_f - (1 - \phi)(\mathbf{C}_f - \mathbf{C}_m) \left\{ (1 - \phi) \mathbf{I} + \phi \left[\mathbf{I} + \mathbf{S}_E : \mathbf{C}_m^{-1} : (\mathbf{C}_f - \mathbf{C}_m) \right]^{-1} \right\}^{-1} \quad (23)$$

This elastic tensor depends on the mechanical properties of the liquid phase ($C_f \neq 3k_f$, $m_f \neq 0$) in the pores and the solid matrix.

The effective elastic properties (k_{sat} , m_{sat} , E_{sat}) of saturated porous media containing spherical pores are obtained:

$$k_{\text{sat}} = k_m(1 - e\phi)(1 + h\phi)^{-1} \quad (24)$$

$$\mu_{\text{sat}} = \mu_{\text{dry}} = \mu_m(1 - \phi)(1 + b\phi)^{-1} \quad (25)$$

$$E_{\text{sat}} = E_0 \frac{(1 - \phi)(1 - e\phi)}{1 + f\phi - g\phi^2} \quad (26)$$

where

$$e = \frac{4\mu_m(k_m - k_f)}{k_m(3k_f + 4\mu_m)}, \quad f = \frac{3k_m(b - e) + \mu_m(h - 1)}{3k_m + 4\mu_m},$$

$$g = \frac{3k_me b + \mu_m h}{3k_m + 4\mu_m}, \quad h = \frac{3(k_m - k_f)}{3k_f + 4\mu_m}$$

The shear modulus of saturated porous media is the same as that for dry porous media, because the fluid does not support shear stress ($m_f \neq 0$).

By replacing Eqs. (24) and (25) in Eqs. (3) and (4), we obtain the acoustic properties (V_P and V_S) of saturated porous media containing spherical pores:

$$V_{P \text{ sat}}^2 = V_{P \text{ m}}^2 \frac{1 + f\phi - g\phi^2}{(1 + h\phi)(1 + b\phi)(1 - \phi)} \quad (27)$$

$$V_{S \text{ sat}}^2 = \frac{\mu_m(1 - \phi)}{[\rho_m(1 - \phi) + \rho_f\phi](1 + b\phi)} \quad (28)$$

2.4. Degree of water saturation

The degree of water saturation represents the proportion of the volume of water V_w relative to the void volume V_p . It is possible to calculate, at a given instant, the saturation of the rock which can be expressed as

$$S_t = \frac{\overline{V_w}}{\overline{V_p}} = \frac{\overline{W_{2t}} - \overline{W_{se}}}{\overline{W_2} - \overline{W_s}} \quad (29)$$

where S_t is the degree of water saturation at time t , W_{2t} is the weight increase at time t , W_{se} is the dry weight at the beginning of capillary experience, W_2 is the weight soaked and weighed in air, and W_s is the dry weight.

The saturation of the rock can be divided into three regions: dry region, unsaturated region and saturated region. The water and gas in pores of unsaturated rock are shown in Fig. 2.

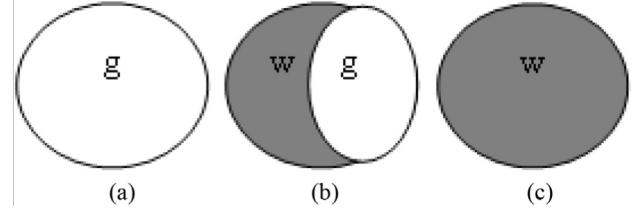


Fig. 2. Water and gas in pores: (a) dry pore, (b) unsaturated pore and (c) saturated pore.

2.5. Effective elastic properties of partially saturated porous media

From Eqs. (1), (18), (20), (24) and (26), we derive the effective elastic moduli (k_{unsat} and E_{unsat}) depending on the degree of water saturation S for porous media constituting a solid matrix containing spherical pores:

$$k_{\text{unsat}} = \frac{1 + jk_f}{k_{\text{dry}} + lk_f} \quad (30)$$

$$E_{\text{unsat}} = \frac{E_{\text{dry}}}{1 - m \frac{k_f}{1 + jk_f}} \quad (31)$$

where

$$j = \frac{3k_m + 4\mu_m\phi}{4k_m\mu_m(1 - \phi)}, \quad l = \frac{3(1 - \phi)}{4\mu_m + 3k_m\phi},$$

$$m = \left(\frac{3k_m + 4\mu_m}{4k_m\mu_m} \right)^2 \frac{E_0\phi}{(1 - \phi)(1 + n\phi)},$$

$$n = \frac{3k_m(33k_m + 56\mu_m)}{(12k_m + 4\mu_m)(9k_m + 8\mu_m)}$$

The relationship between the bulk modulus and density of the unsaturated rock which describes the variation of P-wave velocity of a porous medium according to the degree of water saturation S is given by

$$V_P^{\text{unsat}} = \sqrt{\frac{k_{\text{unsat}} + \frac{4}{3}\mu}{\rho_{\text{unsat}}}} \quad (32)$$

3. Experimental procedure

3.1. Materials

The calcarenite is a kind of sedimentary rock and widely used in many fields, such as civil construction, and renovation of historical monuments, stones and sculptures. This sedimentary rock in yellow ochre color is found mainly in the Rabat-Salé region, and it was used for the construction and restoration of various historical monuments of Rabat (e.g. Kasbah Oudayas, Chellah Necropolis, Hassan Tower, and Bab El Had). It is characterized by high porosity (18e47%), elevated permeability (Azouaoui et al., 2000; Zaouia et al., 2005; Boulanour et al., 2012; Rahmouni et al., 2014a, b), and good

load-bearing capacity (stresses and shocks) (Ben Boughaba, 2001). The calcarenite is very rich in calcium carbonates and its rough surface allows a high receptivity to the atmospheric gaseous pollutants (Samaouali et al., 2010).

Six samples of calcarenite with dimensions of 7 cm 7 cm 7 cm were used in this study. All samples were dried at 105 C for 24 h to remove the moisture.

3.2. Porosity

The porosity of rock sample is determined using the method recommended by AFPC-AFREM (1997), which provides that the samples are soaked in the absence of air. All samples are dried in an oven at 105 C to constant mass.

The porosity f is then obtained by the method of triple weighing. Knowing the dry weight (W_{dry}) and the weight soaked (W_{sat}) of the sample, we obtain the total pore volume occupied by water ($W_{sat} - W_{dry}$)/ r_{water} , then from a hydrostatic weighing (W), the total volume of the sample is calculated by $V_t = (W_{sat} - W) / r_{water}$. The porosity of the sample can be given by

$$\phi = \frac{W_{sat} - W_{dry}}{W_{sat} - W} \quad (33)$$

3.3. P-wave velocity

Ultrasonic velocity measurement is a non-destructive method based on measuring the velocity of P-waves. The velocities of these waves traveling in a solid material depend on the density and elastic properties of the material (Yasar and Erdogan, 2004; Kahraman, 2007).

The measuring principle of the P-wave velocity is based on the production of a vibration pulse train by means of a transmitter (transducer) applied on one surface of the sample. After crossing a path length (L) in the rock, the vibration pulse is converted into an electrical signal using a receiver. The propagation time of the ultrasonic waves (t) is displayed on the meter as soon as the measured value remains stable for 3 s. The P-wave velocity V_P can be given by

$$V_P = L/t \quad (34)$$

To have a perfect contact between the surfaces of rocks and the transducers, it is necessary to use a paste of coupling kaolin and glycerol at both sides of rocks and taking care to ensure that the transducers are pressed against the surface to be tested. The standard 54 kHz transducers are calibrated on the display unit which must be controlled by a calibration test (20.6 mm). All measurements were conducted at room temperature and atmospheric pressure according to ISRM (1978) suggested method.

In a homogeneous isotropic elastic medium, the P-wave velocity V_P is related to Young's modulus E by the following relationship (Lafhaj and Goueygou, 2009):

$$V_P = \sqrt{\frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}} \quad (35)$$

where ν is the Poisson's ratio.

4. Results and discussions

Measurement of different samples extracted from the same block (same mineralogy and petrography) shows that these samples have a difference in

porosity values. The porosity of all samples varies between 25.69% and 35.83%.

The measurement results of P-wave velocities of dry samples range from 3.56 km/s to 3.8 km/s and those of water saturated samples vary between 3.59 km/s and 3.9 km/s. The measurement results of P-wave velocity and porosity are given in Table 1.

Before comparing the predictions of the MorieTanaka model with experimental results, a brief description of the data on the elastic constants of calcarenite is listed in Table 2.

Table 1
P-wave velocity and porosity of calcarenite samples.

| Samples | P-wave velocity V_P (km/s) | | Porosity, f (%) |
|---------|------------------------------|-----------|-------------------|
| | Dry | Saturated | |
| 1 | 3.8 | 3.9 | 25.69 |
| 2 | 3.7 | 3.77 | 29.82 |
| 3 | 3.62 | 3.74 | 31.07 |
| 4 | 3.64 | 3.7 | 33.5 |
| 5 | 3.61 | 3.65 | 35.07 |
| 6 | 3.56 | 3.59 | 35.83 |

Table 2
Properties of calcarenite in three phases.

| Phase | Bulk modulus, k (GPa) | Shear modulus, m (GPa) | Density, ρ (g/cm ³) |
|---------------------------|-------------------------|--------------------------|--------------------------------------|
| Solid matrix (Lion, 2004) | 63.3 | 17.1 | 2.7 |
| Water | 2.3 | e | 1 |
| Gas | 0.013 | e | 0.0012 |

Fig. 3 shows the results obtained by considering a solid matrix containing voids or saturated spherical pores. The illustrated curves show that the effective elastic properties decrease with increasing porosity, which is in the same sense as the work of Gregory (1976), Le Ravalec et al. (1996), Guéguen et al. (1997), Xu (2004) and Huynh (2006).

The elastic properties (k and E) in the saturated porous media are more important than those in dry porous media (Figs. 4 and 5). The comparisons between predictions of the MorieTanaka model and experimental results of Young's modulus E are illustrated in Fig. 5. It is shown that the lowest values are recorded for highporosity rocks (Gregory, 1976). From Fig. 5, we can see that this theoretical model fits well with the experiments data in dry and saturated calcarenite containing spherical pores.

The numerical results of modeling dry and saturated porous media containing spherical pores using the MorieTanaka model show that the effective acoustic properties decrease with porosity (Fig. 6), which is in the same sense as the results obtained by Bourbie et al. (1986) on Fontainebleau sandstone. The experimental results obtained on porous calcarenite are consistent with theoretical results, as shown in Fig. 7.

From Eqs. (22) and (28), the S-wave velocity V_S in dry media is greater than that in saturated media. The explanation for this is that since the fluid does not affect the S-wave velocity, and the shear modulus m is constant ($m_{sat} = m_{dry}$ in Eq. (25)), the factor affecting the S-wave velocity is only the density.

The bulk modulus k is greater in saturated porous media, the shear modulus m remains unchanged, and P-wave velocity V_P is more important in dry porous media (Fig. 6). This result is interpreted by the fact that the density is higher in saturated porous media.

The comparison between Eqs. (21) and (27) and experimental results is plotted in Fig. 7. This figure shows a good agreement between MorieTanaka model and experimental results for dry and saturated calcarenite, which indicates that the MorieTanaka model can be used to predict the acoustic behavior of calcarenite.

Fig. 8 shows the variations of the effective modulus (k and E) as a function of degree of water saturation S for partially saturated porous media containing spherical pores using the MorieTanaka model. It is shown that the bulk modulus k and Young's modulus E increase with the degree of water saturation S . From the curves in Fig. 8, we can see a slow increase of elastic moduli up to $S \approx 80\%$.

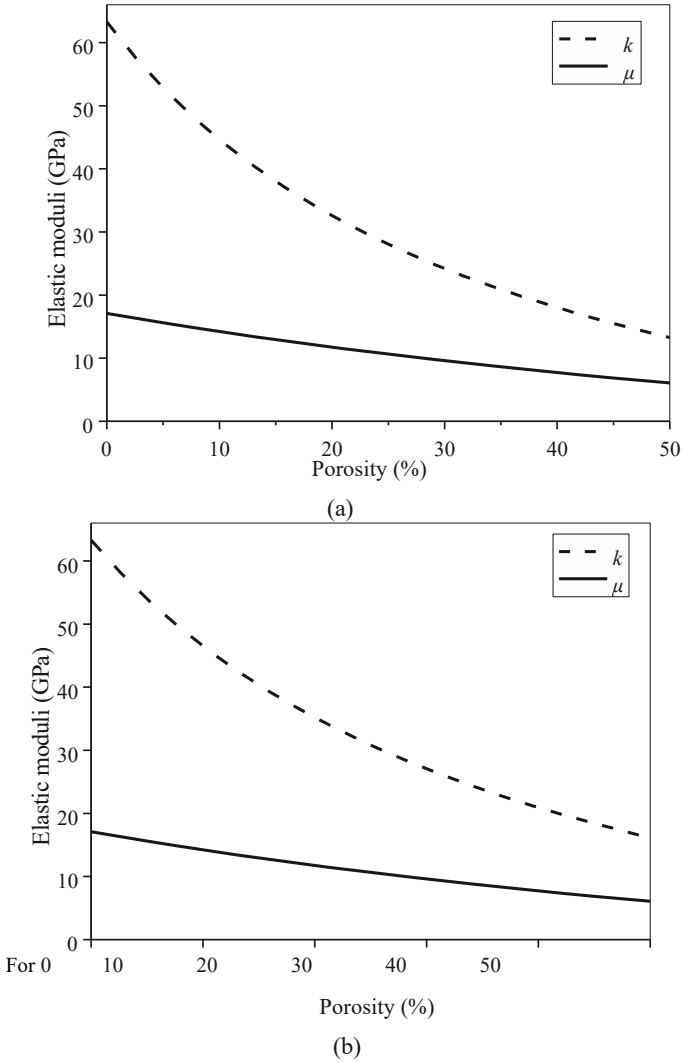


Fig. 3. Effective elastic moduli (μ and k) as a function of porosity for porous media containing (a) voids and (b) saturated spherical pores.

strong saturation, the bulk modulus k and Young's modulus E are increasing rapidly, which is in the same sense as the works of Gregory (1976), Beck (2006), Makarynska et al. (2008) and Adam (2009).

The comparison between Eqs. (30) and (31) and experimental results is also illustrated in Fig. 8. For degrees of water saturation lower than 80%, the theoretical model does not give good agreements with the experimental results of partially saturated calcarenite. But for degrees of water saturation greater than 80%, the experimental results are consistent with the theoretical model.

For a partially saturated rock, the variation of P-wave velocity with degree of water saturation is relatively complex, as several effects are superimposed, and it depends on the porosity of the rock (Gregory, 1976; Bourbie and Zinszner, 1985; Damaj, 2006; Ezzdine, 2009).

Theoretically, using the MorieTanaka model for partially saturated porous media containing spherical pores, and when the

porosity is important, the effect of density is dominant under low or medium saturation: the P-wave velocity decreases as the density

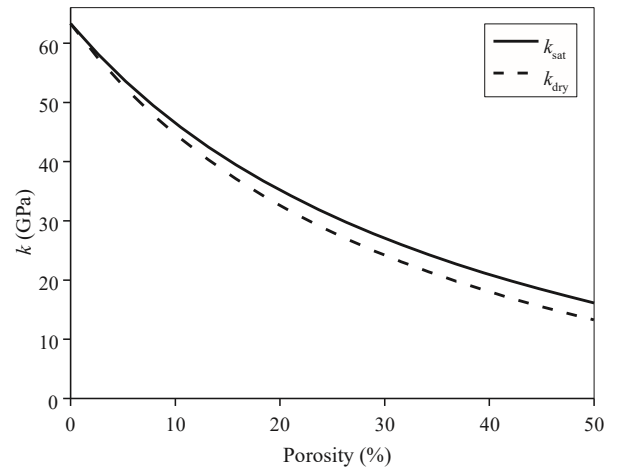
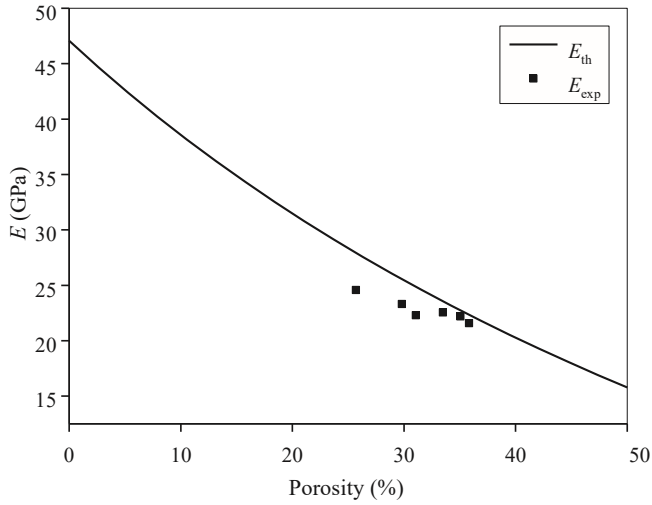
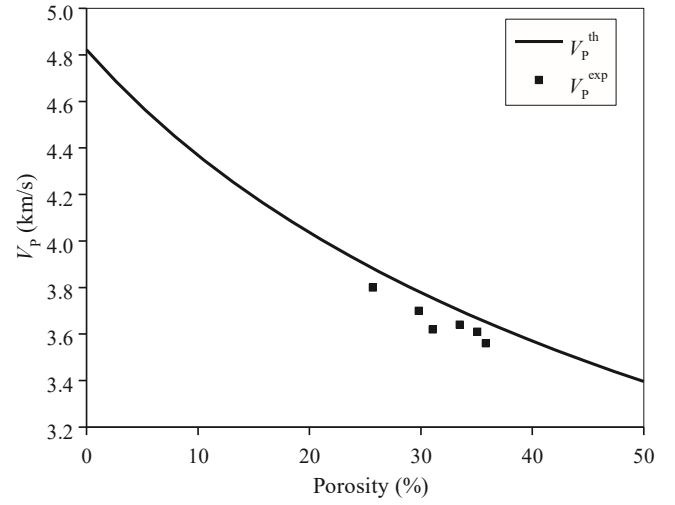


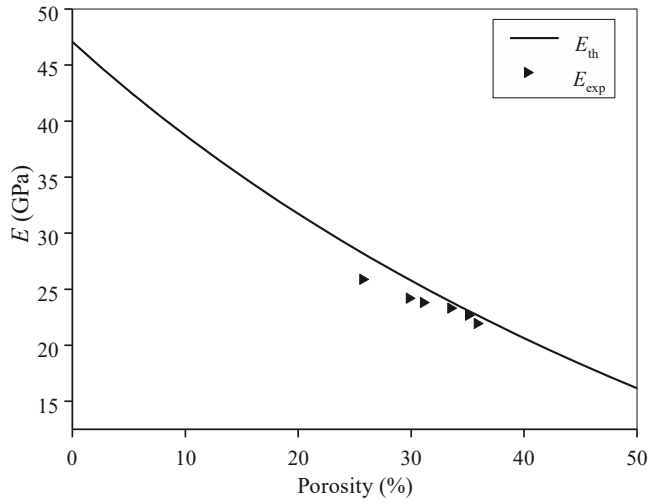
Fig. 4. Effective bulk modulus k in dry and saturated states as a function of porosity.



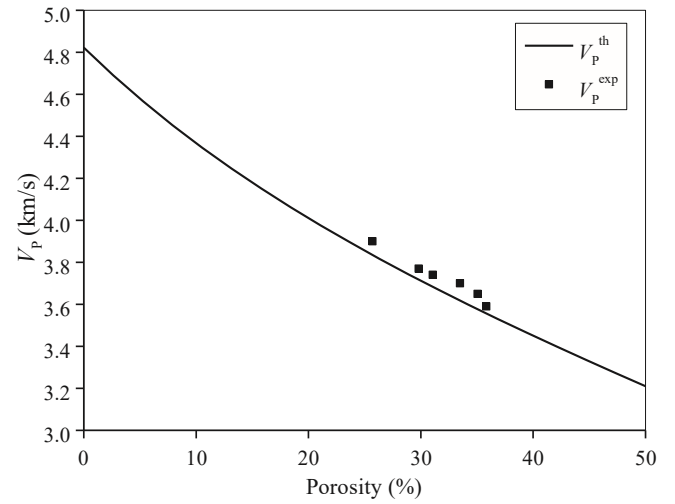
(a)



(a)



(b)



(b)

Fig. 5. Comparisons of measurements of Young's modulus E as a function of porosity for (a) dry and (b) saturated calcarenite with predictions of MorieTanaka model. Subscripts 'th' and 'exp' represent the theoretical and experimental results, respectively.

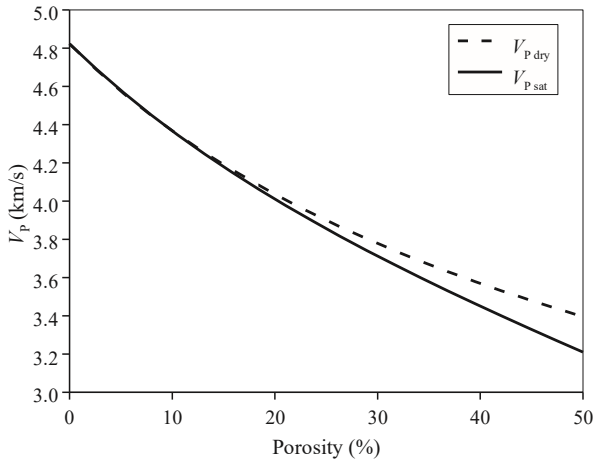
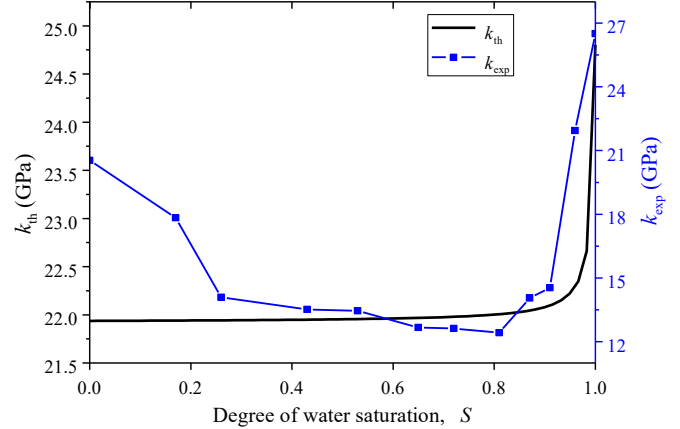


Fig. 6. P-wave velocity V_p in dry and saturated states as a function of porosity for porous media containing spherical pores.

Fig. 7. Comparisons of P-wave velocity as a function of porosity for (a) dry and (b) saturated calcarenite with predictions of MorieTanaka model.



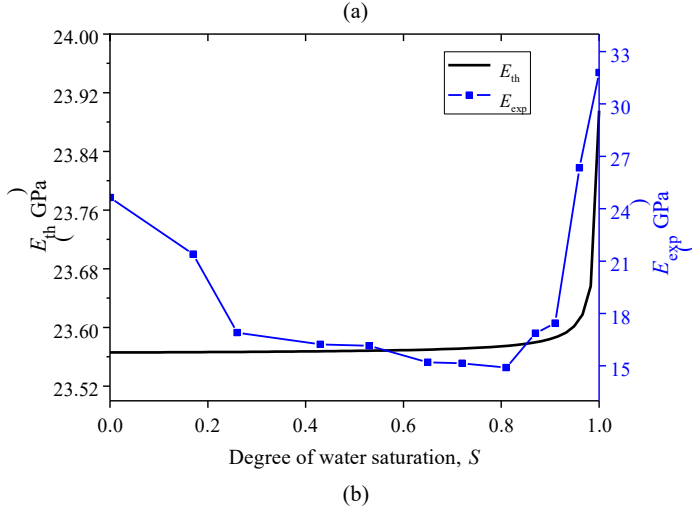


Fig. 8. (a) Bulk modulus k and (b) Young's modulus E versus degree of water saturation S for a calcarenite sample with porosity of 33.5%.

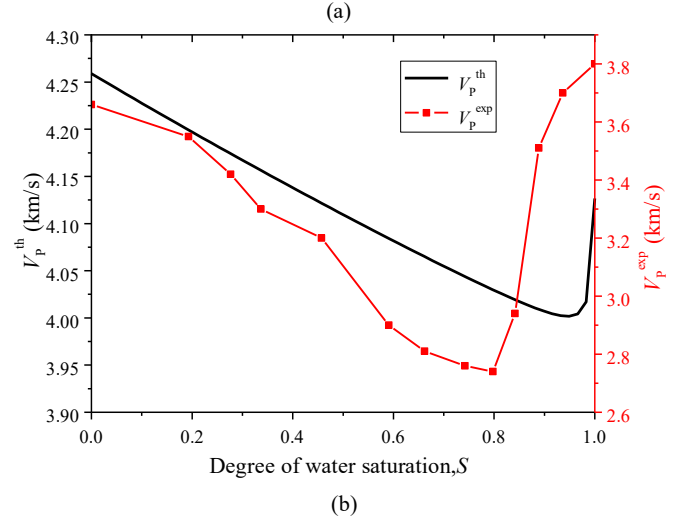


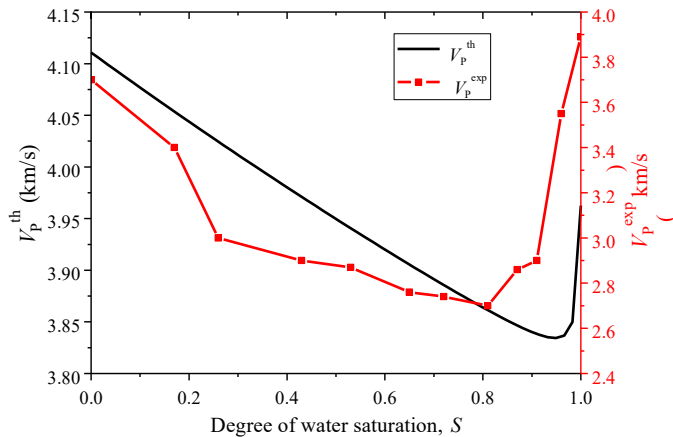
Fig. 9. P-wave velocity versus degree of water saturation S for a calcarenite sample with porosities of (a) 33.5% and (b) 29.82%.

increases. By approaching a strong saturation, the effect of bulk modulus becomes preponderant: P-wave velocity increases with increase in density. There is a minimum value on the curve of variation of P-wave velocity with the degree of saturation S (Fig. 9). This minimum value often corresponds to a fairly high degree of water saturation, greater than 80%.

Many studies have interpreted the variation of the P-wave velocity with the degree of saturation at the pore scale (Tallon, 1976; Beck, 2006). Beck (2006) explained the presence of the minimum value of the V_p - S curve by the fact that, starting from a dry state to a saturated state, the system passes through two successive steps.

The first step corresponds to the period from a dry state to a degree of saturation of about 80%, during which the P-wave velocity drops very sharply and quickly arrives at its lowest value. In this domain of saturation, the system passes a two-phase medium (solid $\&$ air) to a three-phase medium (solid $\&$ air $\&$ water) with the emergence of air bubbles that gradually fill the pore space. Therefore, there is an increase in the number of interfaces (water/air and solid/water). In the dry state, the wave propagates more easily in the presence of solid/air interfaces, which gives a greater value of the P-wave velocity in the dry state.

In the presence of water and at the critical value of the water content, when a compressive wave arrives, the water can flow inwards pore since the latter is not filled with water. The stiffness of the material is almost identical to that of dry material, and this will increase the density of media ($\rho_{\text{media}} \approx \rho_{\text{solid}} \& \rho_{\text{water}}$). This increase



in density is then reflected by a decrease in P-wave velocity to a minimum value, which corresponds to the critical degree of water saturation S_{cr} , since the bulk modulus hardly varies (Fig. 8a). In general, this value of the degree of water saturation is 80%.

The second step corresponds to the period from the critical degree of water saturation S_{cr} to a state of complete saturation. When $S \geq S_{cr}$, the pore walls are all saturated and beyond this value, the water begins to fill the pores. The number of interfaces (water/ air) is stable, and the thickness of the various layers of water lining the walls of the solid skeleton increases due to the increasing volume of water absorbed by capillarity. The water confined by the walls of the pores, which have already been saturated, can no longer flow which contributes to stiffening the material. The rigidity of the material increases, and the effect of bulk modulus becomes dominant and outweighs the effect of density. This then results in an increase in P-wave velocity up to a maximum value which corresponds to a complete state of saturation.

We carried out measurements of P-wave velocity on two samples with different degrees of water saturation and porosities. Fig. 9 shows the variation of P-wave velocity measured with the degree of water saturation for two calcarenite samples. We find the general shape of the curve obtained in the literature (Bourbie et al., 1986; Le Ravalec et al., 1996; Beck, 2006) and from the theoretical model, with minimum corresponding to degrees of water saturation between 80% and 90%.

At the scale of the material and from Eq. (32), we can translate the acoustic behavior of calcarenite as follows:

- (1) If $S < S_{cr}$, the bulk modulus of the rock is almost unchanged (Fig. 8a). It can be considered that the bulk modulus of the unsaturated rock (with a degree of water saturation S) is equivalent to that of the dry rock ($k_{\text{unsat}} \approx k_{\text{dry}}$). The shear modulus does not vary with the degree of water saturation. One can express the velocity V_p in the rock containing water as a function of the velocity in the dry rock (V_p^{dry}), the bulk densities of the solid matrix (ρ_m) and water (ρ_w), and porosity (ϕ):

$$V_p^2 = V_p^{\text{dry}2} \frac{\rho_m(1-\phi)}{\rho_m(1-\phi) + \rho_w\phi S} \quad (36)$$

(2) If $S \leq S_{cr}$, the velocity of the rock can be expressed as

$$V_P^2 = V_{P\text{ dry}}^2 \frac{\rho_m(1-\phi)}{\rho_m(1-\phi) + \rho_w\phi S_{cr}} \quad (37)$$

(3) If $S > S_{cr}$, the velocity of the rock can be expressed as

$$V_P^2 = V_{P\text{ dry}}^2 \frac{\rho_m(1-\phi) + V_w^2 \rho_w \phi (S - S_{cr})}{\rho_m(1-\phi) + \rho_w \phi S} \quad (38)$$

Conflict of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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5. Conclusions

This work is to predict the elastic and acoustic behaviors of calcarenite used in the construction of historical monuments of Rabat, Morocco. We have studied porous media containing dry, saturated and partially saturated spherical pores using a technique of homogenization based on the MorieTanaka model.

The results show that the elastic and acoustic properties decrease with increasing porosity. The MorieTanaka model has firstly the advantage of determining the elastic moduli for porous media whose pore space is empty, and this model offers the advantage of taking into account the effects of interaction between the pores, which allows to calculate the effective elastic and acoustic properties up to high values of porosity.

The elastic properties in saturated state are more important than those in dry state. A good agreement is shown between the MorieTanaka model and experimental results for dry and saturated calcarenite, indicating that the MorieTanaka model can be used to predict the elastic behavior of calcarenite.

Furthermore, we have established two new equations that describe the variation of bulk modulus k and Young's modulus E as a function of degree of water saturation by using the MorieTanaka model.

This study also shows that the elastic and acoustic behaviors of calcarenite are also highly dependent on the degree of water saturation. Indeed, a strong increase of the elastic properties is observed from a critical degree of water saturation. From dry state, the bulk modulus k and Young's modulus E evolve slowly until the rock reaches a critical degree of water saturation. From this value, specific to each rock, the elastic moduli are evolving rapidly to the maximum values. This critical degree of water saturation, which is unique to each rock, is dependent on structure of porous media. For degrees of water saturation greater than 80%, the theoretical model gives a good agreement with the experimental results of partially saturated calcarenite.

The effect of bulk modulus outweighs in the case of P-wave and V_p increases. For S-wave, the density is the only factor influencing the S-wave velocity and V_s decreases. Increasing P-wave velocity depends on the structure of porous network and value of porosity.

The predictions of MorieTanaka model fit well with the experimental results obtained on calcarenite. The theoretical model proposed in this paper is valid for predicting the elastic and acoustic properties of dry and saturated calcarenite.

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