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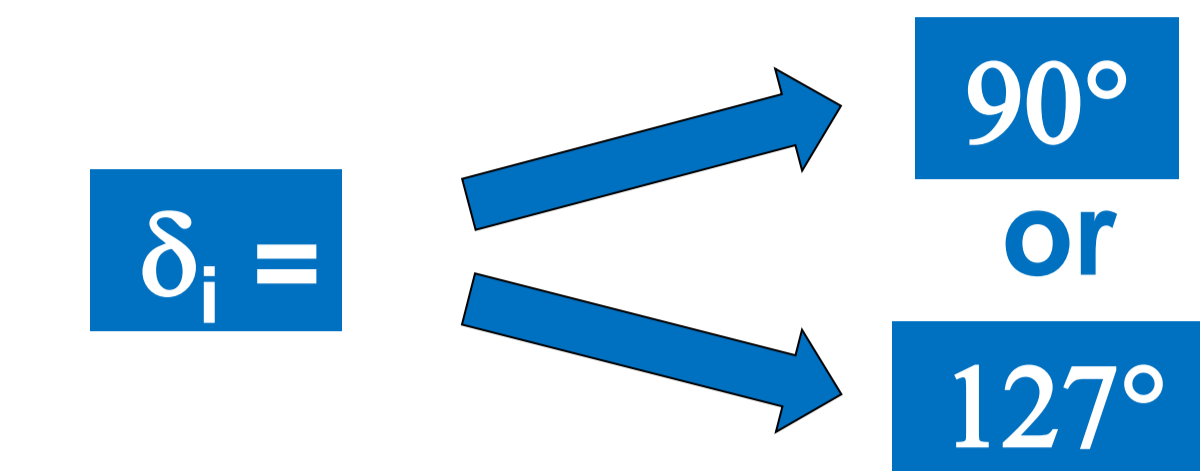
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# Analysis of systematic errors in Mueller matrix ellipsometry as a function of the retardance of the dual rotating compensators

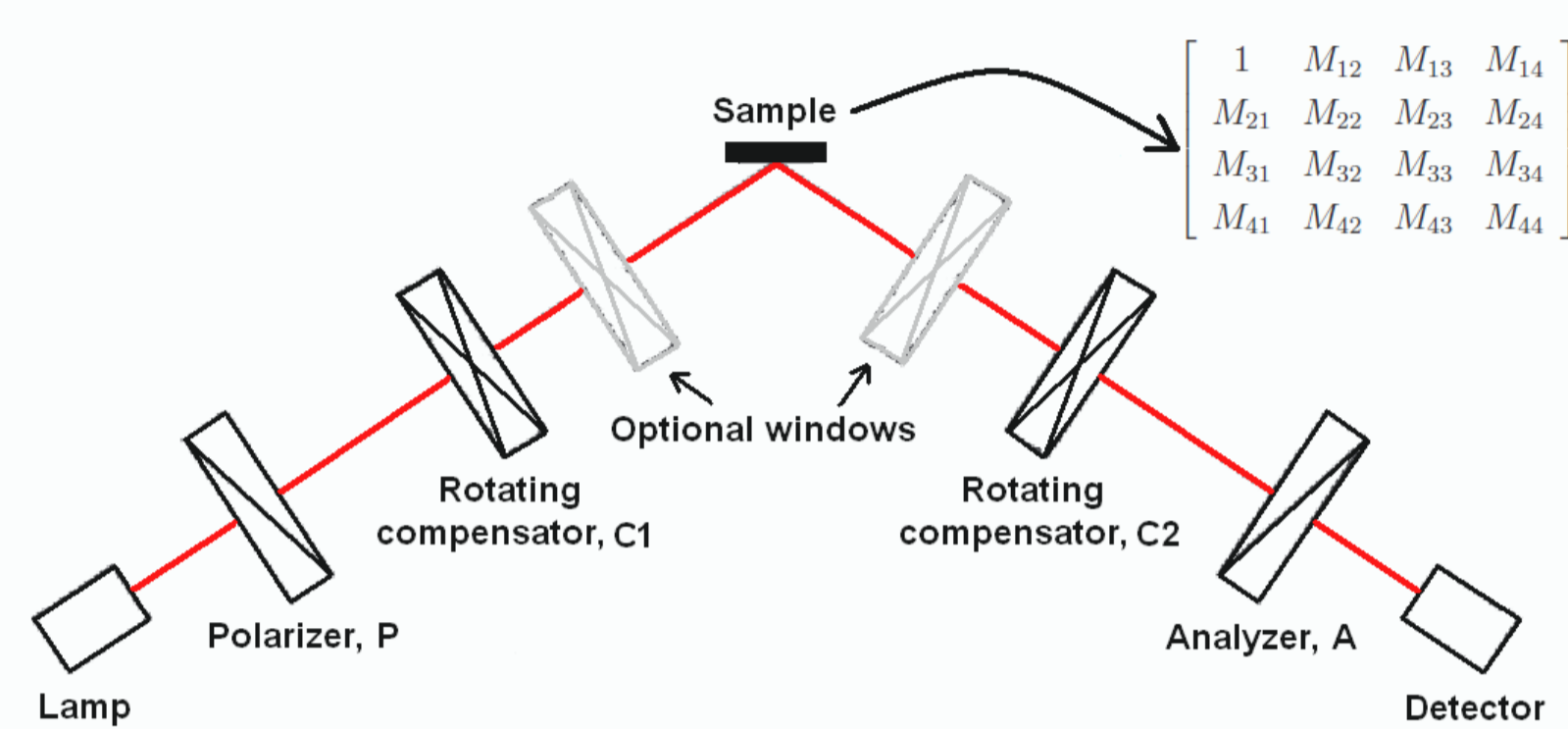
A dual rotating compensators ellipsometer has been developed. It is based on the optical PC<sub>1</sub>SC<sub>2</sub>A configuration, where C1 and C2 are the synchronized continuously rotating compensators [1]. The systematic errors if the compensators are quarter-wave plates have been already studied [2]. Smith [3] has demonstrated that the optimum retardance of a dual-rotating-retarder (DRR) instrument must be equal to 127° compared to the quarter-wave (90°) retarders generally used.

In these conditions random errors are optimized. ↔ What about systematic errors ?

What is the best configuration for the compensators ?



## The PC<sub>1</sub>SC<sub>2</sub>A ellipsometer



$$\begin{bmatrix} 1 & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$$

The detected light has the following form :

$$I = I_0 \left[ a_0 + \sum_n (a_{2n} \cos 2nC + b_{2n} \sin 2nC) \right]$$

- $C = \omega t$  is related to C1 and C2 by the relations:  $C_1 = m_1(C - C_{S1})\omega t$  and  $C_2 = m_2(C - C_{S2})\omega t$  respectively.
- $C_1$  and  $C_2$  are the angles of the fast axes of the first and second compensators at time t.
- $C_{S1}$  and  $C_{S2}$  are imposed by the mechanical assembly and correspond to the azimuth of the fast axis of the compensators with respect to the plane of incidence at the beginning of the acquisition.
- The frequency of the two compensators are synchronized and related to the base mechanical frequency  $\omega$  by the two integers  $m_1$  and  $m_2$ . In our configuration,  $m_1=3$  and  $m_2=5$ .
- 24 non-zero Fourier coefficients allow to deduce all normalized coefficients of the matrix.

## The systematic errors

The measured Mueller matrix,  $M_{measured}$ , is related to the Mueller matrix of the sample,  $M_{sample}$ , by :

$$M_{measured} = M_{sample} + \delta M = M_{sample} + \frac{\partial M}{\partial x} \delta x + \frac{1}{2} \frac{\partial^2 M}{\partial x^2} \delta x^2 + \dots$$

where  $x$  is the perturbation and  $\delta M$  is the systematic error matrix due to the perturbation. For this study, the effect of each error are studied individually and the remaining optical elements of the ellipsometer are assumed to be ideal.



### The azimuthal errors are :

- the analyzer,  $\delta A$
- the polarizer,  $\delta P$
- the compensators,  $\delta C_{S1}$  and  $\delta C_{S2}$

### The birefringence errors are :

- the (optional) windows,  $\delta W_1$  and  $\delta W_2$
- the compensators,  $\delta \delta_1$  and  $\delta \delta_2$

Some errors can be removed if a 4-zone averaging measurement method is performed. It is defined as:

$$(M_{ij})_{sample} = \frac{1}{4} \sum_{A; A+\frac{\pi}{2}} \sum_{P; P+\frac{\pi}{2}} (M_{ij})_{measured}$$

Systematic errors (1<sup>st</sup> and 2<sup>nd</sup> order) obtained with  $\delta_1 = \delta_2 = 90^\circ$  and  $\delta_1 = \delta_2 = 127^\circ$  for single measurement (Table 1) and 4-zone averaging measurement (Table 2).

For  $\delta C_{S1}$ ,  $\delta C_{S2}$ ,  $\delta C_{S1}^2$  and  $\delta C_{S2}^2$ , the errors are defined in the simple case,  $C_{S1} = C_{S2} = 0^\circ$ .

Table 1. Single zone measurement,  $A = P = 0^\circ$

	first order				second order							
	$\delta A$	$\delta P$	$\delta C_{S1}$	$\delta C_{S2}$	$\delta \delta_1$	$\delta \delta_2$	$\delta A^2$	$\delta P^2$	$\delta C_{S1}^2$	$\delta C_{S2}^2$	$\delta \delta_1^2$	$\delta \delta_2^2$
$\delta M_{12}$	127°	≡	≡	127°	127°	127°	0	≡	≡	127°	90°	90°
$\delta M_{13}$	127°	≡	≡	127°	127°	127°	0	≡	≡	127°	90°	90°
$\delta M_{14}$	127°	127°	127°	127°	90°	127°	0	≡	≡	127°	≡	90°
$\delta M_{21}$	≡	127°	127°	≡	127°	127°	≡	0	127°	≡	90°	90°
$\delta M_{22}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{23}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{24}$	≡	≡	≡	≡	90°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{31}$	≡	127°	127°	≡	127°	127°	≡	0	127°	≡	90°	90°
$\delta M_{32}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{33}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{34}$	≡	≡	≡	≡	90°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{41}$	127°	127°	127°	127°	127°	90°	≡	0	127°	≡	90°	≡
$\delta M_{42}$	≡	≡	≡	≡	127°	90°	≡	≡	≡	≡	90°	≡
$\delta M_{43}$	≡	≡	≡	≡	127°	90°	≡	≡	≡	≡	90°	≡
$\delta M_{44}$	≡	≡	≡	≡	90°	90°	≡	≡	≡	≡	≡	≡

Table 2. 4-zone averaging measurement,  $A = \pm 45^\circ$  and  $P = \pm 45^\circ$

	first order				second order							
	$\delta A$	$\delta P$	$\delta C_{S1}$	$\delta C_{S2}$	$\delta \delta_1$	$\delta \delta_2$	$\delta A^2$	$\delta P^2$	$\delta C_{S1}^2$	$\delta C_{S2}^2$	$\delta \delta_1^2$	$\delta \delta_2^2$
$\delta M_{12}$	0	≡	≡	0	127°	0	0	≡	≡	0	90°	0
$\delta M_{13}$	0	≡	≡	0	127°	0	0	≡	≡	0	90°	0
$\delta M_{14}$	0	0	0	0	90°	0	0	≡	≡	0	≡	0
$\delta M_{21}$	≡	0	0	≡	0	127°	≡	0	0	≡	0	90°
$\delta M_{22}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{23}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{24}$	≡	≡	≡	≡	90°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{31}$	≡	0	0	≡	0	127°	≡	0	0	≡	0	90°
$\delta M_{32}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{33}$	≡	≡	≡	≡	127°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{34}$	≡	≡	≡	≡	90°	127°	≡	≡	≡	≡	90°	90°
$\delta M_{41}$	0	0	0	0	0	90°	≡	0	0	≡	0	≡
$\delta M_{42}$	≡	≡	≡	≡	127°	90°	≡	≡	≡	≡	90°	≡
$\delta M_{43}$	≡	≡	≡	≡	127°	90°	≡	≡	≡	≡	90°	≡
$\delta M_{44}$	≡	≡	≡	≡	90°	90°	≡	≡	≡	≡	≡	≡

errors are smaller if  $\delta_1 = \delta_2 = 127^\circ$

errors are smaller if  $\delta_1 = \delta_2 = 90^\circ$

≡ : errors are identical ( $\delta_1 = \delta_2 = 127^\circ$  or  $90^\circ$ )

0 : errors are null ( $\delta_1 = \delta_2 = 127^\circ$  or  $90^\circ$ )

## Conclusions

### Single zone measurement (Table 1)

- $\delta_1 = \delta_2 = 127^\circ$  are well suited for errors due to  $\delta A$ ,  $\delta P$ ,  $\delta C_{S1}$ ,  $\delta C_{S2}$  (first and second order).
- For  $\delta \delta_1$  and  $\delta \delta_2$  (first order), 11 coefficients are in the best condition with  $\delta_1 = \delta_2 = 127^\circ$  and 4 coefficients are in the best condition with  $\delta_1 = \delta_2 = 90^\circ$ .
- $\delta_1 = \delta_2 = 90^\circ$  are well suited for  $\delta \delta_1^2$  and  $\delta \delta_2^2$  (second order).

### 4-zone measurement (Table 2)

- There is **no difference** between  $\delta_1 = \delta_2 = 127^\circ$  or  $90^\circ$  for errors due to  $\delta A$ ,  $\delta P$ ,  $\delta C_{S1}$  and  $\delta C_{S2}$  (the first and the second order).
- For  $\delta \delta_1$  and  $\delta \delta_2$  (first order), 8 coefficients are in the best condition with  $\delta_1 = \delta_2 = 127^\circ$  and 4 coefficients are in the best condition with  $\delta_1 = \delta_2 = 90^\circ$ .
- $\delta_1 = \delta_2 = 90^\circ$  are well suited for  $\delta \delta_1^2$  and  $\delta \delta_2^2$  (second order).

### References

- [1] R.W. Collins, in: Handbook of ellipsometry, edited by Harland G. Tompkins and Eugene A. Irene, (William Andrew Publishing & Springer-Verlag, 2005), Chap. 7.3.3, p 546-566.
- [2] L. Broch, A. En Naciri, L. Johann, Optics Express 16, 12, 8814-8824 (2008).
- [3] M.H. Smith, Appl. Opt., 41, 2488-2493 (2002).