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► **To cite this version:**

Julien Fontchastagner, Frédéric Messine. Interval Branch and Bound Algorithms dealing with Black-Box Constraints. TOULOUSE GLOBAL OPTIMIZATION WORKSHOP - TOGO 2010, Aug 2010, Toulouse, France. hal-02944330

HAL Id: hal-02944330

<https://hal.univ-lorraine.fr/hal-02944330>

Submitted on 21 Sep 2020

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Interval Branch and Bound Algorithms dealing with Black-Box Constraints

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Abstract Interval Branch and Bound Algorithms have shown their intrinsic interest to solve exactly some difficult mixed non-convex and non-linear programs. A code named IBBA was developed to solve some design problems for electromechanical actuators. The idea of this paper is to extend the exact global optimization algorithm by inserting the direct use of an automatic numerical tool (based on a finite element method). This new methodology permit to solve more rationally design problems. Some numerical examples validate the interest of this new approach.

Keywords: Interval Arithmetic, Branch and Bound, Black-box constraint

1. Introduction

IBBA is a code that we developed in order to solve some particular design problems of electromechanical actuators (such as electrical machines), [4]. Because *IBBA* is based on interval arithmetic, it needs all the expressions of the equations of the optimization problem. That was exploited by using analytical models of actuators developed by researchers in electrical engineering since the 80th. Associating analytical models and *IBBA* code yields to an efficient and rational approach to solve these design problems which can be formulated as non-homogeneous mixed constrained global optimization ones:

$$\begin{cases} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} & f(x, y) \\ \text{u.c.} & g_i(x, y) \leq 0, \forall i \in P = \{1, \dots, p\} \\ & h_j(x, y) = 0, \forall j \in Q = \{1, \dots, q\} \end{cases} \quad (1)$$

where f is a real function, \mathbb{R} and \mathbb{N} are respectively the real and the positive integer sets. Normally the discrete variables can also be boolean or categorical ones [4]. However, it is sufficiently general for this paper to use only these classical mixed formulation.

A large part of these kind of design problems were solved efficiently by using *IBBA* code with analytical models [1]. Nevertheless, before making the prototype of the optimized actuator it is preferable to validate the optimal solution by using a finite element code (because for developing analytical models some assumptions have to be done and it is less the case for a numerical model). Thus, some differences generally occur between some important characteristic values yielding the designer to correct some parameters of the solution. In [2], we developed an extension of *IBBA*, named *IBBA+NUMT* to solve a problem of type (1) but where one constraint has to be satisfied via a finite element code *NUMT*. This was extended recently when more than one constraint have to be taken into account.

In this paper, we will present in a more general case this extension of *IBBA*, that we name *IBBA_BBC* for *IBBA* with Black-Box Constraints (constraints which do not have an explicit form but which has to be computed via an algorithm).

2. Formulations of the problem

The problem that we are interested with is the following:

$$\left\{ \begin{array}{l} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} f(x,y) \\ \text{u.c.} \quad \begin{array}{l} g_i(x,y) \leq 0, \forall i \in I \subseteq P \\ h_j(x,y) = 0, \forall j \in J \subseteq Q \\ g_i^A(x,y) \leq 0, \forall i \in P \setminus I \\ h_j^A(x,y) = 0, \forall j \in Q \setminus J \end{array} \end{array} \right. \quad (2)$$

where A on the constraints indicates that they are computed using an algorithm.

Unfortunately, this problem is actually impossible to be solved by a code based on *IBBA*; because for the black-box constraints, it is not possible to compute bounds. Hence, we introduce a new more constrained program but related to Program (2):

$$\left\{ \begin{array}{l} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} f(x,y) \\ \text{u.c.} \quad \begin{array}{l} g_i(x,y) \leq 0, \forall i \in I \subseteq P \\ h_j(x,y) = 0, \forall j \in J \subseteq Q \\ g_i^A(x,y) \leq 0, \forall i \in P \setminus I \\ h_j^A(x,y) = 0, \forall j \in Q \setminus J \\ \underline{r}_j \leq h_j(x,y) \leq \bar{r}_j, \forall j \in Q \setminus J \end{array} \end{array} \right. \quad (3)$$

where $\bar{s}_i, \underline{r}_j$ and \bar{r}_j associated with the analytical equations define a *zone* where the global solution has to be searched. Of course, analytical constraints $g_i, \forall i \in P \setminus I$ and $h_j, \forall j \in Q \setminus J$ as to be related to the numerical ones, respectively $g_i^A, \forall i \in P \setminus I$ and $h_j^A, \forall j \in Q \setminus J$.

3. IBBA_BBC Algorithm

The following corresponds to the iterations of *IBBA_BBC* Algorithm:

1. Set $(X, Y) :=$ the initial hypercube.
2. Set $\tilde{f} := +\infty$ and set $\mathcal{L} := (+\infty, (X, Y))$.
3. Extract from \mathcal{L} the lowest lower bound.
4. **Bisect the considered box** chosen by its midpoint: V_1, V_2 .
5. For $j := 1$ to 2 do
 - (a) $V_j :=$ **Propagation of the analytical constraints on V_j** , [3].
 - (b) if $V_j \neq \emptyset$ then
 - i. **Compute** $v_j := lb(f, V_j)$.
 - ii. **Compute all the lower and upper bounds** of all the analytical constraints on V_j .
 - iii. if $\tilde{f} \geq v_j$ and no analytical constraint is unsatisfied then
 - insert (v_j, V_j) in \mathcal{L} .
 - set m the midpoint of V_j
 - if m satisfies all the analytical constraints and then if the black-box constraints are also satisfied then $\tilde{f} := \min(\tilde{f}, f(m))$.
 - if \tilde{f} is changed then remove from \mathcal{L} all (z, Z) where $z > \tilde{f}$ and set $\tilde{y} := m$.
6. If $\tilde{f} - \min_{(z,Z) \in \mathcal{L}} z < \epsilon$ (where $z = lb(f, Z)$) then STOP.
Else GoTo Step 4.

In this algorithm, one has: \tilde{f} which represents the current best known solution of program (3); $-\mathcal{L}$ is the list which contain all the boxes which can possibly have the global solution at each stage of the iterations; $-lb$ represents a technique using interval arithmetic which allows

Table 1. Results for the minimization of the Mass.

Parameter	Bounds	Unit	IBBA	IBBA_BBC
D	[0.01, 0.3]	m	0.1430	0.1320
L	[0.01, 0.3]	m	0.0491	0.0519
l_a	[0.003, 0.01]	m	0.0034	0.0047
E	[0.005, 0.03]	m	0.0075	0.0082
C	[0.003, 0.02]	m	0.0040	0.0039
β	[0.7, 0.9]		0.73	0.79
k_d	[0.4, 0.6]		0.5020	0.5024
p	[[3, 10]]		8	9
m	{1, 2}		1	1
b_r	{0, 1}		0	1
b_f	{0, 1}		1	1
σ_m	{1, 2}		2	2
σ_{mt}	{1, 2}		2	2
Mass		kg	2.92	3.35
Analytical Torque		N·m	9.81	10.21
Numerical Torque		N·m	9.12	9.83
CPU - Time		min	0'51	34'44
Iterations			152,126	216,623
Iterations of NUMT			-	2,898

Table 2. Results when the zone varies.

zone: $pc =$	Mass	Γ_{em}	NUMT	Time	Its	Its of NUMT
IBBA	2.92	9.81	9.12	0'51	152,126	-
2%	3.44	10.18	9.92	7'59	223,769	585
5%	3.35	10.21	9.83	17'08	213,094	1,404
10%	=	=	=	34'44	216,623	2,898
20%	=	=	=	79'29	223,118	7,162
30%	=	=	=	159'25	228,324	14,751
40%	=	=	=	281'09	231,513	25,004

5. Conclusion

We show in this paper that it is possible to extend IBBA code in order to take into account global optimization problems with black-box constraints. We prove that our new algorithm provides the global minimum of such a problem. Our new method is validated by solving a problem of design of an electromechanical machine with permanent magnets.

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