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# Interval Branch and Bound Algorithms dealing with Black-Box Constraints

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**Abstract** Interval Branch and Bound Algorithms have shown their intrinsic interest to solve exactly some difficult mixed non-convex and non-linear programs. A code named IBBA was developed to solve some design problems for electromechanical actuators. The idea of this paper is to extend the exact global optimization algorithm by inserting the direct use of an automatic numerical tool (based on a finite element method). This new methodology permit to solve more rationally design problems. Some numerical examples validate the interest of this new approach.

**Keywords:** Interval Arithmetic, Branch and Bound, Black-box constraint

## 1. Introduction

*IBBA* is a code that we developed in order to solve some particular design problems of electromechanical actuators (such as electrical machines), [4]. Because *IBBA* is based on interval arithmetic, it needs all the expressions of the equations of the optimization problem. That was exploited by using analytical models of actuators developed by researchers in electrical engineering since the 80th. Associating analytical models and *IBBA* code yields to an efficient and rational approach to solve these design problems which can be formulated as non-homogeneous mixed constrained global optimization ones:

$$\left\{ \begin{array}{l} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} f(x,y) \\ \text{u.c.} \quad \quad \quad g_i(x,y) \leq 0, \forall i \in P = \{1, \dots, p\} \\ \quad \quad \quad h_j(x,y) = 0, \forall j \in Q = \{1, \dots, q\} \end{array} \right. \quad (1)$$

where  $f$  is a real function,  $\mathbb{R}$  and  $\mathbb{N}$  are respectively the real and the positive integer sets. Normally the discrete variables can also be boolean or categorical ones [4]. However, it is sufficiently general for this paper to use only these classical mixed formulation.

A large part of these kind of design problems were solved efficiently by using *IBBA* code with analytical models [1]. Nevertheless, before making the prototype of the optimized actuator it is preferable to validate the optimal solution by using a finite element code (because for developing analytical models some assumptions have to be done and it is less the case for a numerical model). Thus, some differences generally occur between some important characteristic values yielding the designer to correct some parameters of the solution. In [2], we developed an extension of *IBBA*, named *IBBA+NUMT* to solve a problem of type (1) but where one constraint has to be satisfied via a finite element code *NUMT*. This was extended recently when more than one constraint have to be taken into account.

In this paper, we will present in a more general case this extension of *IBBA*, that we name *IBBA\_BBC* for *IBBA* with Black-Box Constraints (constraints which do not have an explicit form but which has to be computed via an algorithm).

## 2. Formulations of the problem

The problem that we are interested with is the following:

$$\left\{ \begin{array}{l} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} f(x,y) \\ \text{u.c.} \quad \begin{array}{l} g_i(x,y) \leq 0, \forall i \in I \subseteq P \\ h_j(x,y) = 0, \forall j \in J \subseteq Q \\ g_i^A(x,y) \leq 0, \forall i \in P \setminus I \\ h_j^A(x,y) = 0, \forall j \in Q \setminus J \end{array} \end{array} \right. \quad (2)$$

where  $A$  on the constraints indicates that they are computed using an algorithm.

Unfortunately, this problem is actually impossible to be solved by a code based on *IBBA*; because for the black-box constraints, it is not possible to compute bounds. Hence, we introduce a new more constrained program but related to Program (2):

$$\left\{ \begin{array}{l} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} f(x,y) \\ \text{u.c.} \quad \begin{array}{l} g_i(x,y) \leq 0, \forall i \in I \subseteq P \\ h_j(x,y) = 0, \forall j \in J \subseteq Q \\ g_i^A(x,y) \leq 0, \forall i \in P \setminus I \\ h_j^A(x,y) = 0, \forall j \in Q \setminus J \\ \underline{r}_j \leq h_j(x,y) \leq \bar{r}_j, \forall j \in Q \setminus J \\ g_i(x,y) \leq \bar{s}_i, \forall i \in P \setminus I \end{array} \end{array} \right. \quad (3)$$

where  $\bar{s}_i, \underline{r}_j$  and  $\bar{r}_j$  associated with the analytical equations define a *zone* where the global solution has to be searched. Of course, analytical constraints  $g_i, \forall i \in P \setminus I$  and  $h_j, \forall j \in Q \setminus J$  as to be related to the numerical ones, respectively  $g_i^A, \forall i \in P \setminus I$  and  $h_j^A, \forall j \in Q \setminus J$ .

## 3. IBBA\_BBC Algorithm

The following corresponds to the iterations of *IBBA\_BBC* Algorithm:

1. Set  $(X, Y) :=$  the initial hypercube.
2. Set  $\tilde{f} := +\infty$  and set  $\mathcal{L} := (+\infty, (X, Y))$ .
3. Extract from  $\mathcal{L}$  the lowest lower bound.
4. **Bisect the considered box** chosen by its midpoint:  $V_1, V_2$ .
5. For  $j := 1$  to 2 do
  - (a)  $V_j :=$  **Propagation of the analytical constraints on  $V_j$** , [3].
  - (b) if  $V_j \neq \emptyset$  then
    - i. **Compute**  $v_j := lb(f, V_j)$ .
    - ii. **Compute all the lower and upper bounds** of all the analytical constraints on  $V_j$ .
    - iii. if  $\tilde{f} \geq v_j$  and no analytical constraint is unsatisfied then
      - insert  $(v_j, V_j)$  in  $\mathcal{L}$ .
      - set  $m$  the midpoint of  $V_j$
      - if  $m$  satisfies all the analytical constraints and then if the black-box constraints are also satisfied then  $\tilde{f} := \min(\tilde{f}, f(m))$ .
      - if  $\tilde{f}$  is changed then remove from  $\mathcal{L}$  all  $(z, Z)$  where  $z > \tilde{f}$  and set  $\tilde{y} := m$ .
6. If  $\tilde{f} - \min_{(z,Z) \in \mathcal{L}} z < \epsilon$  (where  $z = lb(f, Z)$ ) then STOP.  
Else GoTo Step 4.

In this algorithm, one has:  $\tilde{f}$  which represents the current best known solution of program (3);  $\mathcal{L}$  is the list which contain all the boxes which can possibly have the global solution at each stage of the iterations;  $-lb$  represents a technique using interval arithmetic which allows

to compute lower bounds of a function (explicitly known) over a box; -the constraint propagation code which is used is detailed in [3] and it is based on the calculus trees of the constraints (it is only for constraints which are explicitly known). Remark that there are a few differences between *IBBA* and *IBBA\_BBC*; the differences are underlined in the above algorithm.

Now, we want to prove that this algorithm *IBBA\_BBC* finds the global solution of Program (3). In order to prove that point, we introduce the following relaxed program:

$$\left\{ \begin{array}{l} \min_{(x,y) \in (X,Y) \subseteq \mathbb{R}^n \times \mathbb{N}^m} f(x, y) \\ \text{u.c.} \quad \begin{array}{l} g_i(x, y) \leq 0, \forall i \in I \subseteq P \\ h_j(x, y) = 0, \forall j \in J \subseteq Q \\ g_i(x, y) \leq \underline{s}_i, \forall i \in P \setminus I \\ \underline{r}_j \leq h_j(x, y) \leq \overline{r}_j, \forall j \in Q \setminus J \end{array} \end{array} \right. \quad (4)$$

where the black-box constraints are eliminated from Program (3). In fact, the main principle of *IBBA\_BCC* is to solve the relaxed program (4) and just to consider the Program (3) for improving the current solutions  $\tilde{f}$ . Such a Branch and Bound algorithm proceeds by exclusions of boxes when it is proven that the global solution cannot be inside it. Thus, suppose that a box  $Z$  which contains the global minimum of Program (3) is discarded by *IBBA\_BBC*.  $Z$  has only 3 distinct possibilities to be eliminated: (i)  $Z = \emptyset$  after the propagation step. This implies that at least one constraint of the relaxed problem (4) is unsatisfiable, see [3]. Hence, no point in  $Z$  can satisfy all the constraints of problem (4) and therefore of problem (3). Thus, this possibility cannot occur; (ii) using interval arithmetic, an analytical constraint is proved to be unsatisfiable for all points in  $Z$ . It is also an impossible case such as in (i); (iii)  $\tilde{f} \geq z := lb(f, Z)$ . This implies that no point in  $Z$  can have a value lower than  $\tilde{f}$ , therefore  $Z$  does not contain a global minimum. Therefore, we show that these 3 possibilities cannot occur and then that such a box  $Z$  cannot exist. Hence, *IBBA\_BBC* provides a global minimum of Program (3).

## 4. Numerical experiments

To illustrate our methodology, we consider the design of an electromechanical machine with permanent magnets. See [1] for details about its mixed non-convex analytical formulation. For the computations of the electromagnetic torque of the machine, we use *NUMT* which is an efficient finite element numerical algorithm that we developed, [2]. Thus, here we have just one black-box constraint.

In Table 1, parameters of the machine are:  $-D$  its bore diameter,  $-L$  its length,  $-l_a$  the thickness of the magnets,  $-E$  the winding thickness,  $-C$  the thickness of yoke,  $-\beta$  the polar arc factor,  $-k_d$  the filling factor,  $-p$  the number of poles pairs,  $-m$  the number of slots per pole and phase,  $-b_r$  the rotor configuration,  $-b_f$  the kind of electromotive force waveform,  $-\sigma_m$  the type of permanent magnet,  $-\sigma_{mt}$  the type of magnetic conductor. On this example, we minimize the mass (see [2] for complementary results), the main constraint is about the torque of the machine which must be equal to 10 N·m (with a tolerance about  $\pm 0.2$  N·m). In the column named *IBBA*, we solve program (1) with only the analytical model and just using *IBBA*. Remark that the numerical value for the torque (computed using *NUMT*) is equal to 9.12 which is quite far from imposed value 10. In column *IBBA\_BBC*, we solve Program (3) with a zone of 10% around 10 for the torque and using *NUMT*. Then, we obtain 9.83 N·m for the numerical torque; notice that the analytical value is 10.21. We emphasize in bold all the parameters which have changed between these two resolutions (nine parameters have changed including the integer ones).

In Table 2, we represent the solutions found when the zone is progressively increased: 2%, 5% and so on. We note that a zone about 5% is sufficient to solve efficiently this design problem.

Table 1. Results for the minimization of the Mass.

Parameter	Bounds	Unit	IBBA	IBBA_BBC
$D$	[0.01, 0.3]	m	<b>0.1430</b>	<b>0.1320</b>
$L$	[0.01, 0.3]	m	<b>0.0491</b>	<b>0.0519</b>
$l_a$	[0.003, 0.01]	m	<b>0.0034</b>	<b>0.0047</b>
$E$	[0.005, 0.03]	m	<b>0.0075</b>	<b>0.0082</b>
$C$	[0.003, 0.02]	m	<b>0.0040</b>	<b>0.0039</b>
$\beta$	[0.7, 0.9]		<b>0.73</b>	<b>0.79</b>
$k_d$	[0.4, 0.6]		<b>0.5020</b>	<b>0.5024</b>
$p$	[[3, 10]]		<b>8</b>	<b>9</b>
$m$	{1, 2}		1	1
$b_r$	{0, 1}		<b>0</b>	<b>1</b>
$b_f$	{0, 1}		1	1
$\sigma_m$	{1, 2}		2	2
$\sigma_{mt}$	{1, 2}		2	2
Mass		kg	<b>2.92</b>	<b>3.35</b>
Analytical Torque		N·m	<b>9.81</b>	10.21
Numerical Torque		N·m	9.12	<b>9.83</b>
CPU - Time		min	0'51	34'44
Iterations			152,126	216,623
Iterations of NUMT			-	2,898

Table 2. Results when the zone varies.

zone: $pc =$	Mass	$\Gamma_{em}$	NUMT	Time	Its	Its of NUMT
IBBA	2.92	9.81	9.12	0'51	152,126	-
2%	3.44	10.18	9.92	7'59	223,769	585
<b>5%</b>	<b>3.35</b>	<b>10.21</b>	<b>9.83</b>	<b>17'08</b>	<b>213,094</b>	<b>1,404</b>
10%	=	=	=	34'44	216,623	2,898
20%	=	=	=	79'29	223,118	7,162
30%	=	=	=	159'25	228,324	14,751
40%	=	=	=	281'09	231,513	25,004

## 5. Conclusion

We show in this paper that it is possible to extend IBBA code in order to take into account global optimization problems with black-box constraints. We prove that our new algorithm provides the global minimum of such a problem. Our new method is validated by solving a problem of design of an electromechanical machine with permanent magnets.

## References

- [1] E. Fitan, F. Messine, and B. Nogarede, *The Electromagnetic Actuator Design Problem : A General and Rational Approach*, IEEE Transactions on Magnetics, Vol. 40, N. 3, pp. 1579–1590, May 2004.
- [2] J. Fontchastagner, F. Messine, Y. Lefèvre, *Design of Electrical Rotating Machines by Associating Deterministic Global Optimization Algorithm With Combinatorial Analytical and Numerical Models*, IEEE Transactions on Magnetics, Vol. 43, N. 8, pp. 3411–3419, 2007.
- [3] F. Messine, *Deterministic Global Optimization using Interval Constraint Propagation Techniques*, RAIRO Operations Research, Vol. 38, N. 4, 2004.
- [4] F. Messine, *A Deterministic Global Optimization Algorithm for Design Problems*, chapter in Essays and Surveys in Global Optimization, Editors Charles Audet, Pierre Hansen et Gilles Savard, pp. 267–294, 2005.