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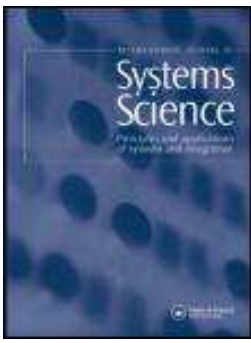
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Supervisory control based on minimal cuts and Petri net sub-controllers coordination

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ABSTRACT

This paper addresses the synthesis of Petri net (PN) controller for the forbidden state transition problem with a new utilisation of the theory of regions. Moreover, as any method of control synthesis based on a reachability graph, the theory of regions suffers from the combinatorial explosion problem. The proposed work minimises the number of equations in the linear system of theory of regions and therefore one can reduce the computation time. In this paper, two different approaches are proposed to select minimal cuts in the reachability graph in order to synthesise a PN controller. Thanks to a switch from one cut to another, one can activate and deactivate the corresponding PN controller. An application is implemented in a flexible manufacturing system to illustrate the present method. Finally, a comparison with previous works with experimental results in obtaining a maximally permissive controller is presented.

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1. Introduction

The previous research works in supervisory control led to the application of state feedback control to prevent the system to avoid some states in order to assure a desired behaviour. The first investigations developed for discrete event systems (DES) led Ramadge and Wonham (Ramadge & Wonham, 1987; Ramadge & Wonham, 1989) to the great results on the existence and synthesis of supervisors for DES. It was shown that if a set M of controllable legal markings, i.e. $\forall M \in M$ there is no reachable forbidden marking from a sequence containing uncontrollable transitions, and if this set is expressed by n_c linear inequalities constraints [(or generalised mutual exclusion constraints (GMEC))] then the Petri net (PN) has a solution and the behaviour is maximum permissive (Giua, DiCesare, & Silva, 1992).

The proposed method of Ghaffari, Rezg, and Xie (2002a) and Ghaffari, Rezg, and Xie (2002b) in a supervisory control problem is based on the theory of regions (Badouel & Darondeau, 1998). This theory is represented by a linear system (LS) composed of three kinds of conditions, its resolution leads to the synthesis of a PN controller. However, the discussed LS may contain state equations that form a convex combination of other states which make it impossible to resolve. Furthermore, the combinatorial explosion of states in the generated graph complicates the computation of PN supervisors. In this work, the application of the theory of regions is restricted in minimal cuts (MC) selected with specific criteria and not on the graph. Each monitor supervises one MC in the reachability graph. Therefore, there is a coordination between PN controllers, thanks to a Boolean control function that activates and deactivates the monitors.

Some research has been conducted to synthesise PN supervisors under some bounded assumptions to enforce disjunctive constraints called generalised linear constraints (Iordache & Antsaklis, 2007). Moreover, situations may exist where the

number of places of the least-restrictive controllers was exponentially related to the size of the specification. Iordache, Wu, Zhu and Antsaklis (2013) introduced an enhanced approach that generates PN monitors, in which the number of places is linearly related to the size of the specification. However, in these works, the PN controller is designed under full controllability. In addition, Haoxun (2000) described a method for constructing controller by forcing the net to obey the linear inequality constraints. Thus, the computation of its PN controller is based on the concept of minimal support S-decreases, but the monitor place is an extended PN. Furthermore, Ru, Cabasino, Giua, and Hadjicostis (2012) synthesised a PN supervisor under arbitrary forbidden state specifications. Indeed, the control policy is determined by checking certain controllable transitions. If it will lead to a subsequent reduced marking that belongs to the forbidden marking, the supervisor disables the corresponding controllable transition.

Our new control policy is focused on the research carried on the utilisation of the theory of regions that has been recognised as the powerful method of deadlock prevention policy for obtaining maximally permissive controllers. Many research works sought to lower the computational cost of the theory of regions; Li, Zhou, and Jeng (2008) utilised an approach called siphons control before using the theory of regions to calculate the PN controllers. Furthermore, Huang and Pan (2011) proposed another issue for deadlock prevention to decrease the number of reachability condition equations of the theory of regions in order to facilitate the computation of the supervisors. Their experimental results seem to be the most efficient method in the deadlock prevention policy compared with the works of Uzam and Zhou (2007), Uzam (2009), Park and Reveliotis (2000) and Qin et al. (2012).

In order to reduce the resolution complexity of the theory of regions, this work proposes a new method based on MCs (Rezig,

Achour, & Rezg, 2014; Rezig, Achour, Rezg, & Kammoun, 2014), which allows on the one hand to minimize the total number of equations in LS and the time calculation, and on the other hand to avoid the combinatory explosion of states.

The rest of the paper is structured as follows: Section 2 depicts the background of the theory of regions and PN tools. Section 3 introduces the concept of MCs in the reachability graph for synthesising supervisors. The switch between PN controllers is proposed in Section 4 due to the activation and deactivation of supervisors. A flexible manufacturing system (FMS) is presented in Section 5 in order to illustrate the contribution of the new method. Finally, Section 6 presents the comparison of our new method, which will be carried out with the classical approach of the theory of regions (Ghaffari, Njima, Rezg, & Xie, 2002) to the recent method of Huang and Pan (2011).

2. Preliminaries

A PN is a bipartite graph $N = (P, T, \text{Pre}, \text{Post})$ where P is a set of places, T is a set of transitions with $T = T_c \cup T_u$ the union of controllable and uncontrollable transitions (Murata, 1989). $\text{Pre} : P \times T \rightarrow \mathbb{N}$ is the pre-incidence function that specify weighted arcs from places to transitions, $\text{Post} : P \times T \rightarrow \mathbb{N}$ is the post-incidence function that specify weighted arcs from transitions to places, where \mathbb{N} is the set of nonnegative integers. Let $p^{(t)}$ (respectively ${}^{(t)}p$) be the set of output transitions (respectively input transitions) of the place p and $t^{(p)}$ (respectively ${}^{(p)}t$) the set of output places (respectively input places) of transition t . Let $G(N, M_0)$ be the reachability graph constructed from its initial marking M_0 . M denote by the set of markings $M \in G(N, M_0)$. A transition $t \in T$ is enabled from a marking M (denoted by $M[t >]$) if and only if $M \geq \text{Pre}(\cdot, t)$. An enabled transition t may fire yield a new marking M' , such that $M = M' + C(\cdot, t)$, this can be denoted by $M[t > M']$. A marking M' is reachable from a marking M if there exists a firing sequence $\sigma = t_1, t_2, \dots, t_n$ transforming M into M' . By firing a sequence σ , any marking M reachable from M_0 satisfies the following state equation of PN: $M = M_0 + C\vec{\sigma}$, where $\vec{\sigma} : T \rightarrow \mathbb{N}$ a vector of nonnegative integers is called the occurrence of transition t_i in σ and C is the incidence matrix of a marked PN. $C(p_i, t_j) = w(t_j, p_i)$ if $t_j \in {}^{(t)}P_i$, $C(p_i, t_j) = -w(p_i, t_j)$ if $t_j \in p_i^{(t)}$, else 0, where $w : F \rightarrow \mathbb{N}$ is a valuation function of arcs. ($F \subseteq (T * P) \cup (P * T)$: finite set of arcs). A PN is k -bounded if the number of tokens in each place does not exceed k . A PN is live if every transition is live. A PN is reversible if from any

reachable state M there is an enabled sequence to return to M_0 . Considering a set of legal markings, such admissible states correspond to a set of constraints $\text{GMEC}(\vec{w}, K)$ which defines the set of states (Giua et al., 1992): $M_{(\vec{w}, K)} = \{M \in M \mid \vec{w}^T \leq k\}$.

In certain cases, the control specification cannot be expressed by GMEC. Therefore, we developed in our previous work (Rezg, Rezig, Kammoun, & Achour, 2014) a new type of control specification constraint (\vec{v}, C) which defines a set of legal sequences of transitions of transitions called admissible paths constraints:

$$T_{(\vec{v}, C)} = \{\sigma \in \sigma^* \mid \vec{v}^T \leq C\}. \quad (2)$$

The application of the theory of regions allows the synthesis of the PN controller to add to the initial PN. The control place P_c with initial marking $M_0(P_c)$ and its incidence vectors $C(P_c, \cdot)$ has to satisfy the reachability conditions, i.e

$$M_0(P_c) + C(P_c, \cdot) \vec{\Gamma}_M \geq 0. \quad (3)$$

Furthermore, each P_c to add should solve at least one forbidden event (M, t) in the set Ω of prohibited state transitions:

$$M_0(P_c) + C(P_c, \cdot) \vec{\Gamma}_M + C(P_c, t) < 0. \quad (4)$$

Besides, P_c has to fulfil cycle equations:

$$\sum_{t \in T} C(P_c, t) \cdot \vec{\sigma}[t] = 0, \forall \sigma \in R. \quad (5)$$

$\vec{\sigma}[t]$ is the algebraic sum of occurrences of t in σ .

The (LS) composed of Equations (3)–(5) is used to determine P_c . Notably, many event separation instances may obtain the same solutions. As a result, the number of P_{c_i} will be much smaller than the set of a prohibited state transitions Ω .

Aside from the combinatorial explosion problem in the theory of regions, the convex combination of states equations can be found in LS which make its resolution impossible as in the following example (i.e. see Figure 1). By writing the following LS, the theory of regions is insoluble and the synthesis of the PN

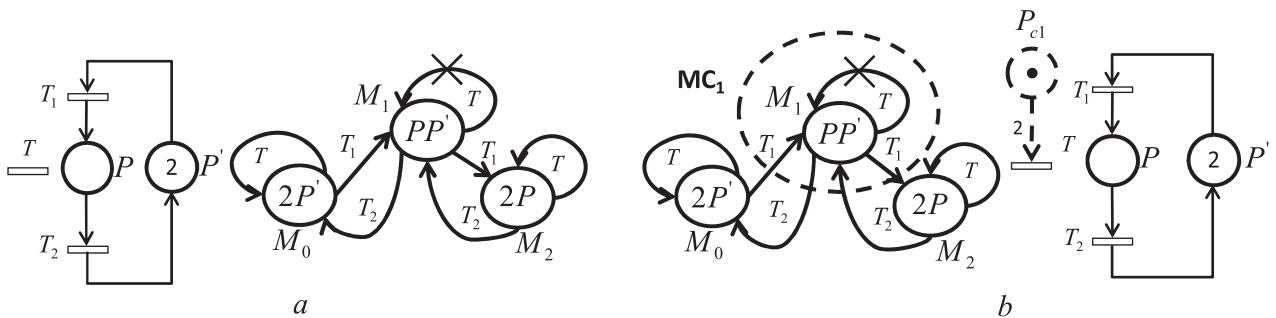


Figure 1. An insoluble problem with the classical approach of the theory of regions. a. Insoluble problem with the classical approach. b. Resolution of the insoluble problem.

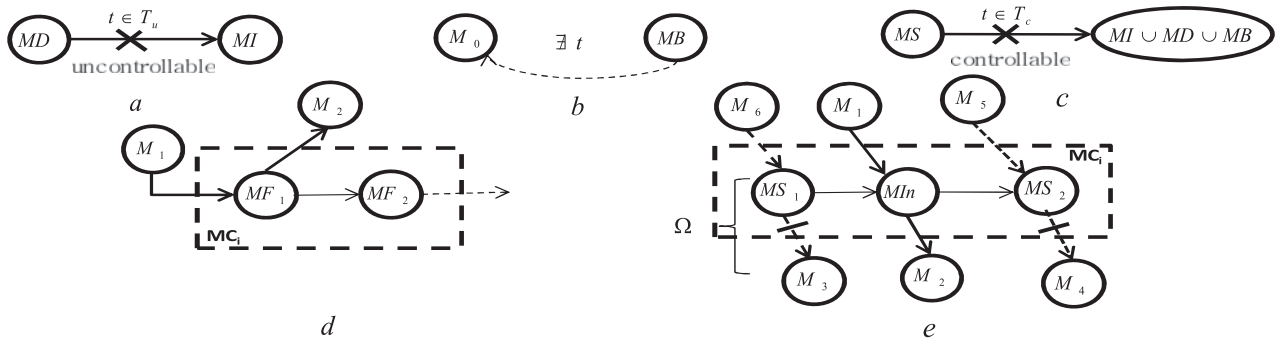


Figure 2. Six kinds of markings. a. Dangerous and forbidden state. b. Blocking state. c. Source state. d. Frontier state. e. Intermediate state.

controller is impossible:

$$LS : \begin{cases} M_0(P_c) = M_0(P_c) \geq 0 \\ M_1(P_c) = M_0(P_c) + C(P_c, T_1) \geq 0 \\ M_2(P_c) = M_0(P_c) + 2C(P_c, T_1) \geq 0 \\ M_0(P_c) + C(P_c, T_1) + C(P_c, T) < 0 \\ C(P_c, T) = 0 \\ C(P_c, T_1) + C(P_c, T_2) = 0 \end{cases}$$

Indeed, it is clear that $M_1(P_c)$ is a convex combination of $M_0(P_c)$ and $M_2(P_c)$; (i.e. $M_1(P_c) = \frac{1}{2} M_0(P_c) + \frac{1}{2} M_2(P_c)$). Therefore, the event (M_1, T) cannot be forbidden with the application of theory of regions.

In the following section, an efficient method to reduce the total number of equations in LS will be introduced. For convenience, our proposed new method will follow the above interpretation of the theory of regions.

3. Minimal cuts in a reachability graph

3.1 Problem setting

This work is focused on the reduction of the computational cost of the theory of regions by minimising the number of equations in its LS; hence, one can automatically save computing time of

the theory of regions. Therefore, a new concept of MC is introduced in the reachability graph to achieve these aims by two approaches.

Definition 1: A state is said forbidden state if it does not respect a given specification (\vec{w}, K) . (i.e. see Figure 2(a)). Let MI be the set of forbidden states: $MI = \{M \in M | M \neq M_{(w,k)}\}$.

Definition 2: The set of event separation instances that the supervisor has to disable is

$$\Omega = \{(M, t) | \exists M[t > M; ' \wedge M \in G(N, M_0) \wedge M' \in MI\}$$

(i.e. see Figure 3(c)).

Definition 3: A state is said a dangerous state if it leads to the forbidden state by firing uncontrollable transitions. Let MD be the set of dangerous states: $MD = \{M \in G(N, M_0) | \exists t \in T_u, \exists \sigma \in T_u^*, M[\sigma > M' \wedge M' [t > M'' \wedge (M', t) \in \Omega\}$ (Figure 2(a)).

Definition 4: A state is said a blocking state if it does not belong to a live sub-graph (LSG); LSG is a sub-graph of the reachability graph $G(N, M_0)$ containing all the states $M \in G(N, M_0)$ leading to M_0 . Let MB be the set of blocking states:

$$MB = \{M \in G(N, M_0) | \nexists \sigma \in T^*, M[\sigma > M_0\}$$
 (Figure 2(b)).

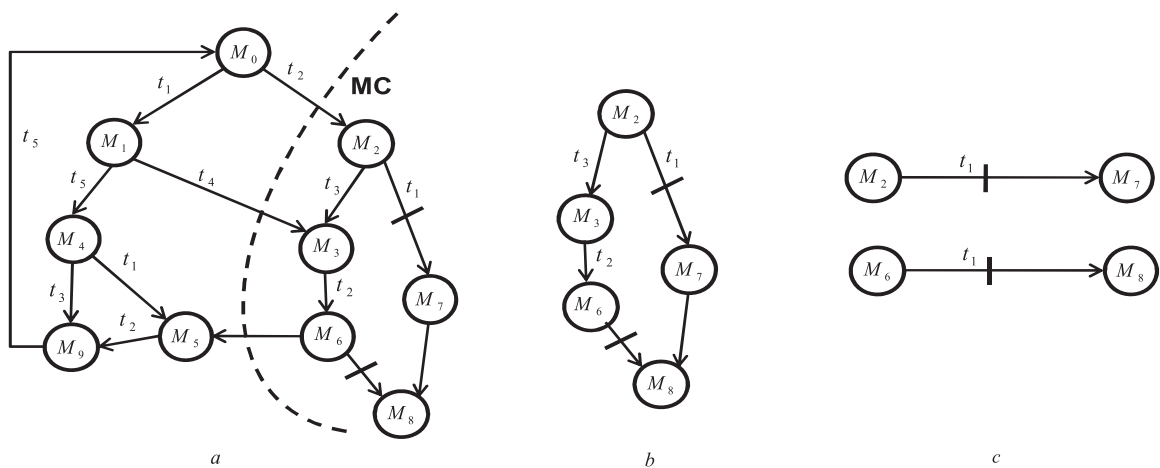


Figure 3. Example of minimal cut, MC. a. Reachability graph. b. MC. c. Correlated event separation instances with t_1 .

Definition 5: A state is said a source state if it is an admissible state in MC and from which we prohibit a forbidden state transition (M, t) of Ω . Let MS_i be the set of source states (Figure 2(c)): $MS_i = \{M \in G(N, M_0) | \exists (M[t > M'], M \in \Omega, M' \in (MI \cup MD \cup MB), t \in T_c)\}$.

Definition 6: A state is said frontier if it is a state from which we activate (deactivate) a MC. Let MF_i be the set of frontier states (Figure 2(d):

$$MF_i = \{M \in MC_i | \exists M_i [t > M], M_i \notin MC_i\}.$$

3.2 Approach 1: minimal cut with correlated event separation instances

The identification of a minimal cut MC_i in the reachability graph $G(N, M_0)$ is characterised by the correlation of event separation instances. Nevertheless, the application of the theory of regions in control synthesis will be restricted in MC_i , and will generate one control place P_{ci} for each MC_i .

Definition 7: A minimal cut MC_i is a zone of the reachability graph $G(N, M_0)$ with minimal number of nodes which is identified by one control place P_{ci} , ensures the reachability of all markings contained in MC_i and solves one or more event separation instances (i.e. see Figure 3(a) and 3(b)): $MC_i = MS_i \cup \{M[\sigma > M' | M, M' \in MS_i, \sigma \in T^*]\}$

Definition 8: Two event separation instances are correlated if their corresponding transition to prohibit is the same. Let $\Omega_{11} = (M, t_i)$, $\Omega_{12} = (M', t_j) \in \Omega$, if $t_i = t_j$ then Ω_{11} and Ω_{12} are correlated (i.e. see Figure 3(c)).

Definition 9: A state is said an intermediate state if it is a frontier state belonging MC_i and it is not a source state: $Min_i = \{M \in MC_i | M \in MF_i / MS_i\}$ (Figure 2(e)).

Proposition 1: An initial source state M_{0i} of a minimal cut MC_i belongs to the set of source states MS_i and is generated by the smallest transition vector x in the reachability graph $G(N, M_0)$ from its initial state M_0 .

Definition 10 (Darondeau & Xie, 2003): A transition vector x of a state $M \in G(N, M_0)$ is said the smallest transition vector if its corresponding state M is calculated with the smallest transition firing sequence σ from M_0 , (i.e. the smallest i , with $i = |\sigma|$: number of transitions in σ called its length).

Proof: Let x and x' be the respective transition vectors of M and M' ($M, M' \in G(N, M_0)$) and let C be the incidence matrix of a PN. $\vec{\sigma}$ and $\vec{\sigma}'$ are the associated transition firing vectors of M and M' , such that: $M = M_0 + C \cdot \vec{\sigma}$ and $M' = M_0 + C \cdot \vec{\sigma}'$. From Darondeau and Xie (2003), if $\vec{\sigma}' \geq \vec{\sigma}$ it exists a transition firing sequence: $\sigma'' = \sigma' - \sigma | M[\sigma'' > M']$ then x is smaller than x' .

The identification of MCs is described in the following algorithm 1.

Algorithm 1 : Identification of MC in $G(N, M_0)$.

Let Q be a given control specification and $MA = (MI \cup MD \cup MB)$ the set of markings to avoid.

(1) Determine sets of states and state transitions, respectively, MI and TF that does not respect Q .

(2) Specify the event separation instances:

$$T_f = \{(M[t > M'], M \in G(N, M_0), M' \in MI)\}; TF = TF \cup T_f.$$

(3) Identify the set of dangerous states MD in $G(N, M_0)$.

(4) Identify controllable transitions leading to MB .

(5) Determine the set of blocking states.

(6) Identify controllable transitions leading to MB

(7) Specify the event separation instances:

$$\Omega = \{(Mt > M'), M \in G(N, M_0), t \in T_c, M' \in MA\}.$$

(8) Define the set of source states:

$$MS = \{(M \in G(N, M_0) | \exists (M, t) \in \Omega, t \in T_c)\}.$$

(9) If $M \notin (MS \cap MB)$ then go to the next step. Else eliminate $M[t > \text{of } \Omega$ and go to step 5.

(10) Determine each set Ω_i of correlated event separation instances.

(11) Regroup source states MS_i from each set Ω_i and paths connecting them in MC_i .

(12) Define initial source state MS_i from MC_i .

END

In the first 9 steps, the avoided states and event separation instances are identified. Thus, one can determinate the set of event separation instances $\Omega = \{(M[t > M'], M \in G(N, M_0), t \in T_c, M' \in MA)\}$. In the following steps, MC are grouped according to the correlation of event separation instances in MC_i .

Remark 1: The application of the theory of regions in the method of Ghaffari et al. (2002a) in control synthesis considers the entire $G(N, M_0)$, however in this new method, the union of MC does not form the entire reachability graph: $\cup MC_i \subset G(N, M_0)$, which approves the reduction of computational cost of the theory of regions.

Theorem 1: If a marking enters a MC_i by any source state $M \in MS_i | M \neq M_{0i}$, the initial marking of P_{ci} does not need to be recalculated and the theory of regions remains valid.

Proof: (reduction ad absurdum): Let MC_i be a minimal cut in a reachability graph $G(N, M_0)$, correlated by transition t_1 (Figure 4) and the set of forbidden states, $MI_i = \{M', M''\}$. Hypothesis: If a marking enters a MC_i by $M \in MS_i | M \neq M_{0i}$, such that $(M_{0i} \rightarrow t_1 M')$; $(M \rightarrow t_1 M'')$, then with the theory of regions already calculated from M_{0i} , transition t_1 cannot be prohibited if a marking enters a MC_i by M and one can write: $M''(P_{ci}) = M(P_{ci}) + C(P_{ci}, t_1) \geq$

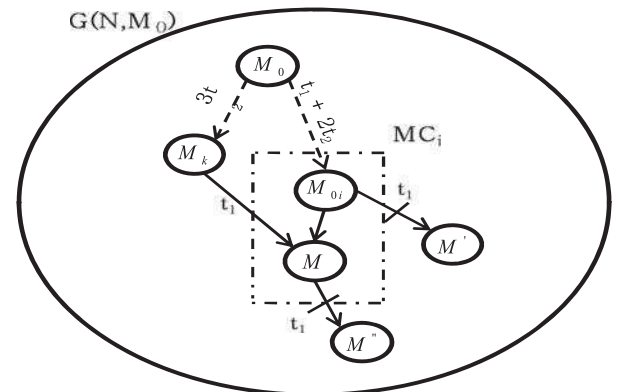


Figure 4. Example of minimal cut.

0 and the theory of regions is not valid. Suppose that a marking M_k enters a MC_i starting by t_1 , such that $M_k(P_{ci}) = M_0(P_{ci}) + 3C(P_{ci}, t_2)$, $M = M_k + C(P_{ci}, t_1) = M_0(P_{ci}) + 3C(P_{ci}, t_2) + C(P_{ci}, t_1)$. Since $M''(P_{ci}) = M(P_{ci}) + C(P_{ci}, t_1) = >M''(P_{ci}) = M_0(P_{ci}) + 2C(P_{ci}, t_1) + 3C(P_{ci}, t_2) \geq 0$ or M_{0i} is the state calculated with the smallest transition vector, so M'' can be expressed by M_{0i} of MC_i . From Figure 4, $M_{0i}(P_{ci}) = M_0(P_{ci}) + C(P_{ci}, t_1) + 2C(P_{ci}, t_2)$; $M''(P_{ci}) = M_{0i}(P_{ci}) + C(P_{ci}, t_1) + C(P_{ci}, t_2) \geq 0$. This is absurd; indeed, the application of the theory of regions from initial state M_{0i} of MC_i gives $M''(P_{ci}) = M_{0i}(P_{ci}) + C(P_{ci}, t_1) + C(P_{ci}, t_2) < 0$, whereas if a marking enters a MC_i by any state $M \in MS_i$, initial marking of control place $M_{0i}(P_{ci})$ does not need to be recalculated and the theory of regions remains valid.

Once MC_i are determined, relations 3, 4 and 5 make it possible to find P_{ci} , the following algorithm summarises the calculation of the PN supervisor.

Algorithm 2 : Synthesis of P_{ci} for each MC_i

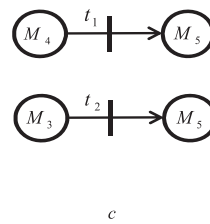
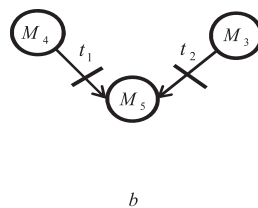
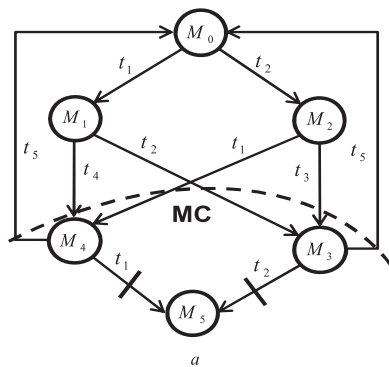
Let M_{0i} be the initial state of MC_i , and $n_i = \text{card}(\Omega_i)$.

- (1) Initialization, $i := 1$.
- (2) If $n_i = 1$.
 - (2.1) Write the reachability equation of M_{0i} .
 - (2.2) Write the event separation instance equation of $(M_{0i}[t > M'] \in \Omega_i)$.
 - (2.3) Solve the linear system LS_i and let $(M_{0i}(P_{ci}), C(P_{ci}, \cdot))$ be the solution of MC_i .
- (3) Else if $n_i > 1$:
 - (3.1) Identify independent equations of basic cycles using the Gauss method.
 - (3.2) Write the reachability equations of $M \in MC_i$ from M_{0i} .
 - (3.3) Write the event separation equations of the set Ω_i from M_{0i} .
 - (3.4) Solve the LS composed of equations of steps 3.1, 3.2 and 3.3. Let $(M_{0i}(P_{ci}), C(P_{ci}, \cdot))$ be the solution of MC_i .
 - (4) Eliminate redundant control places.
 - (5) While $i < n_i$, $i := i + 1$. Return to step 2.

END.

3.3 Approach 2: minimal cut for any event separation instances

In this approach, the design of MC_i is only interesting in forbidden state transitions, that does not respect a control specification Q .



$$MF_1 = \{M_3, M_4\}$$

$$\bar{\sigma}_A(M_3) = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \bar{\sigma}_A(M_4) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{\sigma}_D(M_3) = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \bar{\sigma}_D(M_4) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Definition 11: MC_i is a zone of $G(N, M_0)$ with minimal number of nodes which identify a PN supervisor, and is only interesting in event separation instances. $MC_i = \{M[t > M'] | M \in MS, M' \in MA\}$. (i.e. see Figure 5).

Next, the design of MC_i with any event separation instances is described in Algorithm 3. Indeed, steps 1–9 which identify the forbidden state and the forbidden state transition are the same as in Algorithm 1; the difference is the way to construct MC_i defined in steps 10–12.

Algorithm 3 : Identification of MC_i in $G(N, M_0)$

Let $T_a(M)$ be a set of arcs of a source state $M \in MS$.

(10) Determine for each $M \in MS$, arcs:

$$T_a(M) = \{t \in T | (M[t > M'], M \in MS, M' \notin MA)\}$$

(11) Associate each $M \in MS$ and its event separation instances $(M[t > M'] | M \in MS | M' \in MA)$ a MC_i .

(12) If $(M' \in MA | M[t > M'], M \in MS)$ is the common state between minimal cut MC_i ,

and $t \in T | (M[t > M']) \notin T_a(M)$ then merge

Minimal cuts MC_i .

END.

The most attractive advantage of the proposed approach is the design for each source states with its event separation instances a MC_i without basic cycles. However, the fusion of MC will reduce the number of MC_i , therefore the number of PN controllers is reduced.

The control synthesis is described in the following Algorithm 4.

Algorithm 4 : Synthesis of PN controller for MC_i

Let MC be the set of minimal cuts, M_{0i} the initial state of each MC_i and $n_i = \text{card}(\Omega_i)$.

(1) Initialization, $i := 1$

(2) If $n_i = 1$:

(2.1) Write the reachability equation of initial state M_{0i} of MC_i .

(2.2) Write the event separation instance equation of $M_{0i}[t > M', M' \in MA]$.

(2.3) Solve LS_i and let $(M_{0i}(P_{ci}), C(P_{ci}, \cdot))$ be the solution of MC_i .

(3) Else if $n_i > 1$:

(3.1) Write the reachability equation of initial state M_{0i} of MC_i .

(3.2) Write the event separation instance equation of $M_{0i}[t > M', M' \in MA]$.

(3.3) Solve the linear system LS_i and let $(M_{0i}(P_{ci}), C(P_{ci}, \cdot))$ be the solution of MC_i .

(3.4) Eliminate redundant control places.

(3.5) While $i < n_i$, $i := i + 1$. Return to step 2.

END.

Figure 5. Example of minimal cut. a. Reachability graph. b. Minimal cut, MC. c. Event separation instances. d. Activation/deactivation vectors.

(3.2) Write the event separation instance equations from M_{0i} .

(3.3) Solve LS_i and let $(M_{0i}(P_{ci}), C(P_{ci}, \cdot))$ Be the solution of MC_i

(4) Eliminate redundant control places.

(5) While $i < n_i, i = +1$;

Return to step 2.

END.

Using the same example of Figure 1, one can apply our policy to write the following LS: $M_{01}(P_{c1}) \geq 0$; $M_{01}(P_{c1}) + C(P_{c1}, T) < 0$. The resolution of LS gives $M_{01}(P_{c1}) = 1$; $C(P_{c1}, T) = -2$. The controlled PN is given in Figure 1(b).

Despite the reduction of the computation time and the equations number, our proposed method can solve insoluble problems with the original theory of regions. It is also required to calculate the activation/deactivation vectors of $MF_1 = \{M_1\}$ to enable/disable P_{c1} .

After synthesising the PN controllers for each MC_i , one can determine the Boolean control function F_{Bc} that allows the activation and the deactivation of supervisors. Thus, the role of each monitor P_{ci} is to control its zone (MC_i) in the reachability graph. The activation/deactivation vectors and the control Boolean function are described in the following Section 4.

4. The switch and coordination between Petri net controllers

In this section, an alternation policy of PN controllers is developed through a switch between them in the reachability graph by activating and deactivating the supervisor P_{ci} of each MC_i . Additionally, a Boolean control function will be introduced.

4.1 Activation and deactivation vectors

After the identification of MC_i and P_{ci} in the reachability graph, we proceed to the step of activation/deactivation of supervisors. Once a marking enters MC_i , each P_{ci} has the task to control its zone in the reachability graph. The PN controller is initially inactive (active) until a marking enters (exits) a MC_i by its frontier states. Here, one can formally define the activation/deactivation vector.

Definition 12: We associate for each $M \in MF_i$ an activation (deactivation) vector, which is a path between M_0 and $M \in MF_i$ by eliminating eventual cycles, which can reactivate (re-deactivate) P_{ci} .

Activation vector: $\vec{\sigma}_A(M) = \sigma \in T^* | \exists M_0[\sigma > M, M \in MF]$.

Deactivation vector: $\vec{\sigma}_D(M) = \sigma \in T^* | \exists M_0[\sigma > M \wedge M[t >, M \in MF, t \notin MC_i]$.

Theorem 2: Once a marking enters MC_i by intermediate states $M \in MIn_i$, the initial marking of P_{ci} changes as follows: $M_{0i}(P_{ci}) = M_{0i}(P_{ci}) + C(P_{ci}, \cdot)[\vec{\sigma}_A(M) - \vec{\sigma}_A(M_{0i})]$ (Figure 6).

Proof: The firing sequences $\sigma_A(M_{0i})$ and $\sigma_A(M)$ are enabled from M_0 and lead to markings M_{0i} and $M \in MIn_i$ of MC_i . Considering the proposition 1 ($\sigma_A(M_{0i}) < \sigma_A(M)$) and from Darondeau and Xie (2003), $\exists \sigma'' | M_{0i}[\sigma'' >, \sigma'' = \sigma_A(M) - \sigma_A(M_{0i})]$. The obtained marking after firing the sequence σ'' from M_{0i} is $M_{0i} + C[\vec{\sigma}_A(M) - \vec{\sigma}_A(M_{0i})] = [M_0 + C\vec{\sigma}_A(M_{0i})] + C[\vec{\sigma}_A(M) - \vec{\sigma}_A(M_{0i})] = M_0 + C\sigma_A(M) = M, M \in MIn_i$. One can deduce

that $M_0[\sigma_A(M_{0i})\sigma'' > M, M \in MIn_i$ and $\sigma'' = \sigma_A(M) - \sigma_A(M_{0i})$, (i.e. see in Figure 6).

4.2 Boolean control function

One can realise that all supervisors can control the system either simultaneously or alternately. More precisely, depending on the process state, the necessary PN controller is activated/deactivated. Here, one can formally define the Boolean control function F_{Bc} as follows: $F_{Bc} = F(P_{c1}, P_{c2}, \dots, P_{cm}) = P_{c1} + P_{c2} + \dots + P_{cm} = \sum_{i=1}^m P_{ci}$

This OR logic function characterises the union of supervisors and takes the value 1 if the system is controlled and 0 otherwise. A simple PN example that models two machines sharing one resource is employed in Figure 7(a). The following GMEC can be satisfied in the reachability graph (Figure 7(b)) to illustrate the present approach: $M(P_2) + M(P_3) + M(P_5) + M(P_6) \leq 1$. In this example, the supervisors will intervene alternatively: the Boolean control function is $F_{Bc} = P_{c1}P_{c2}$.

5. Example

A physical system composed of two stations in FMS in Ecole Nationale d'Ingenieurs de Metz (ENIM) whose assembly engraved glass with stands, and its PN model are presented in Figure 8(a) and Figure 8(b). The following scenario is applied: the first station modelled by P_2 represents our self-production, while the second station modelled by P_3 specifies a subcontractor C_M and C_S , respectively, indicate the capacity of three pieces of the machine and the subcontractor. Place P_1 represents our stock production, (t_1, t_2) and (t_4, t_5) model, respectively, the events of entry and exit of the pallets in stations through their annexes conveyors, otherwise the pallet can continue its way into the main conveyor by t_3 . Note that only (t_1, t_2) are controllable. The constraint to enforce is to keep the marking of the place P_2 plus the doubled marking of P_3 less than three tokens: $M(P_2) + 2M(P_3) \leq 3$. This GMEC expresses the following control specification (Q): the production is entrusted to a subcontractor within the context of sub-trade work. $G(N, M_0)$ contains 13 states (Figure 9).

5.1 Application of the first approach

This work proposes an efficient approach to determine MC with minimal number of nodes in which the control synthesis is applied. According to Definition 8, the set Ω contains five event separation instances correlated by t_2 or by t_1 :

$$\Omega_1 : \left\{ (M_2[t_2 > M_4]); (M_5[t_2 > M_8]); (M_{10}[t_2 > M_{13}]) \right\}$$

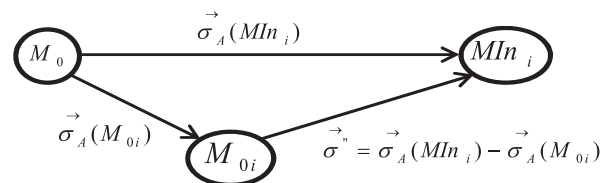


Figure 6. Relation between M_{0i} and MIn_i .

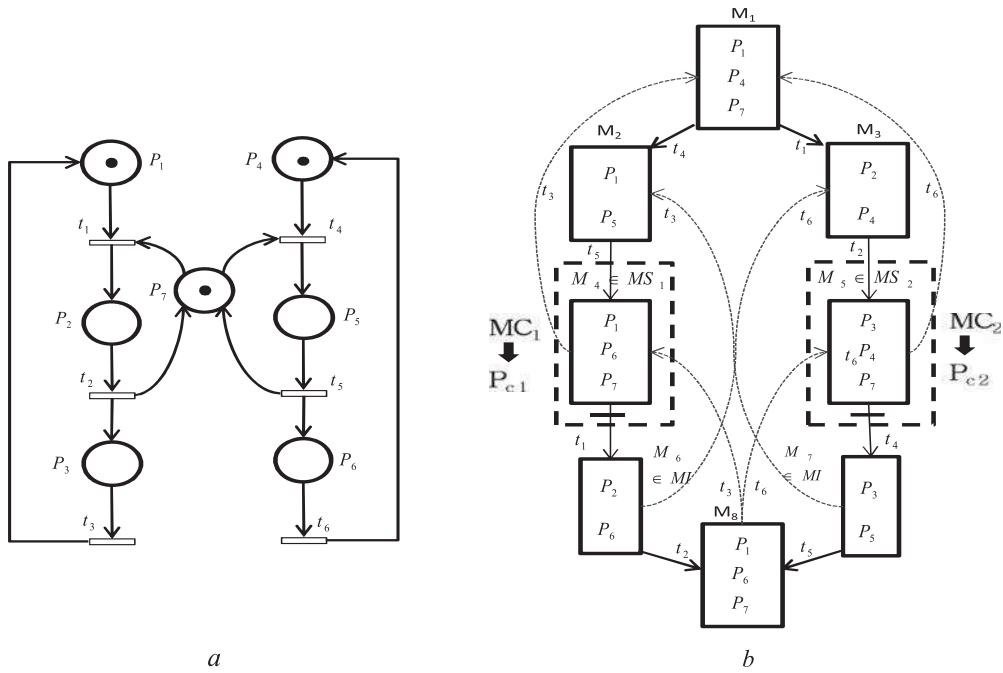


Figure 7. A simple PN model. a. PN model. b. Reachability graph.

$$\Omega_2 : \{M_5[t_1 > M_9]\}$$

$$\Rightarrow MC_1 : \left\{ \begin{array}{l} (M_2 [t_2 >]); (M_2 [t_1 >]); (M_3 [t_1 >]); (M_5 [t_2 >]); \\ (M_5 [t_5 >]); \\ (M_6 [t_2 >]); ([t_1 >]); (M_{10}[t_2 >]) \end{array} \right\}$$

$$MC_2 : \{(M_5[t_1 >])\}.$$

The detailed information of markings of MC is listed in Table 1. Owing to the limitation of paper space; the equations of the theory of regions are not listed.

Let $M_{01} = M_2$ and $M_{02} = M_5$ be the initial states of MC_1 and MC_2 . Additionally, by applying Algorithm 2, two control places P_{c1} and P_{c2} are obtained and shown in Figure 10 when equations (3), (4) and (5) for each MC_i are solved. $P_{c1} : M_2(P_{c1}) = M_{01} = 1; C(P_{c1}, \cdot) = (-1, -2, 0, 1, 2)$. $P_{c2} : M_5(P_{c2}) = M_{02} = 1; C(P_{c2}, \cdot) = (-2, 0, 0, 0, 0)$.

According to our new policy, the switch between PN controllers P_{c1} and P_{c2} is done by frontier states: $MF_1 = \{M_2, M_3\}$, $MF_2 = \{M_5\}$. One can infer that it is only in the state M_5 that the supervisors are involved simultaneously

otherwise they act alternatively. The Boolean control function is $F_{Bc} = P_{c1} + P_{c2}$.

If marking M_1 enters MC_1 by intermediate state M_3 , the initial marking of P_{c1} changes as follows: $M_{01}(P_{c1}) = M_{01}(P_{c1}) + C(P_{c1}, \cdot) [\sigma_A(M_3) - \sigma_A(M_2)] = 2$. To switch between P_{c1} and P_{c2} , detailed information of activation and deactivation vectors for P_{c1} and P_{c2} is listed in Table 2.

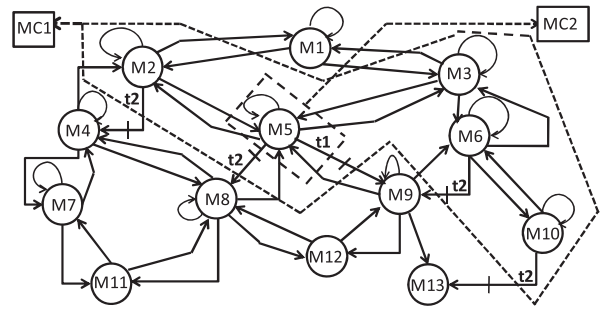


Figure 9. Reachability graph 1 with minimal cuts.

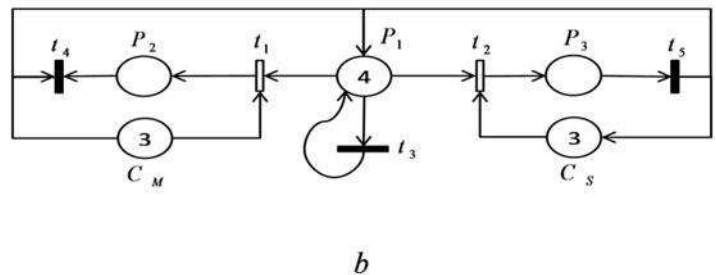
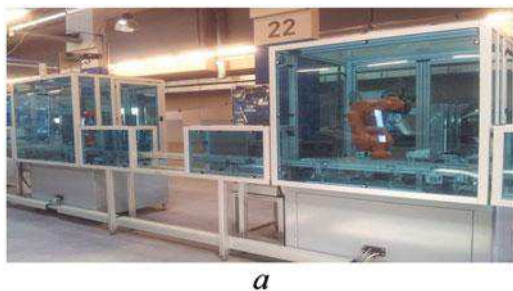


Figure 8. ENIM FMS. a. Experimental environment. b. PN model.

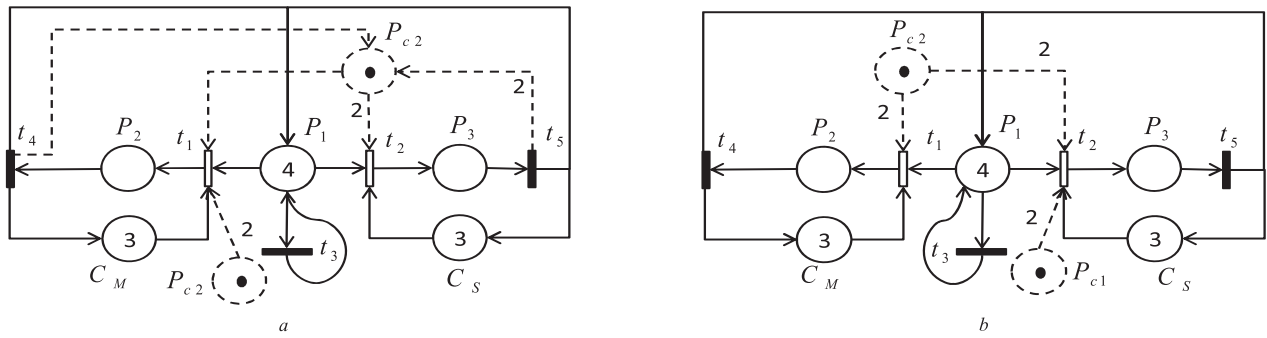


Figure 10. The controlled PN model of FMS. a Controlled PN model 1. b Controlled PN model 2.

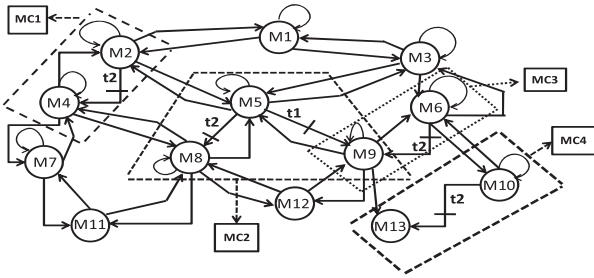


Figure 11. Reachability graph 2 with minimal cuts.

5.2 Application of the second approach

The design of MC in this work differs from the last approach and it is only interesting in event separation instances. Next, by using algorithm 3, one can generate the sources states: $MS : \{M_2; M_5; M_6; M_{10}\}$ and arcs $T_a(M_i)$ showed in Figure 11.

$$T_a(M_2) : \{(M_2 [t_1 >]); (M_2 [t_3 >]); (M_2 [t_5])\},$$

$$T_a(M_5) : \{(M_5 [t_3 >]); (M_5 [t_4 >]); (M_5 [t_5])\},$$

$$T_a(M_6) : \{(M_6 [t_1 >]); (M_6 [t_3 >]); (M_6 [t_4 >])\},$$

$$T_a(M_{10}) : \{(M_{10} [t_3 >]); (M_{10} [t_4 >])\}.$$

The set of outgoing arcs of source states $M_i \in MS$ which do not belong to the set of arcs $T_a(M_i)$ constitute initially the MC_i . $MC_1 : \{(M_2 [t_2 > M_4])\}$; $MC_2 : \{(M_5 [t_2 > M_8]), (M_5 [t_1 > M_9])\}$; $MC_3 : \{(M_6 [t_2 > M_9])\}$; $MC_4 = \{(M_{10} [t_2 > M_{13}])\}$.

Note that source states M_5 and M_6 share a common forbidden state M_9 and the fusion of MC_2 and MC_3 has not happened. Indeed, the transition t_1 from M_5 to M_9 belongs to $T_a(M_6)$. Next, Algorithm 4 generates the following PN controllers:

$$P_{c1} : M_{01}(P_{c1}) = 1; C(P_{c1}, \cdot) = (0, -2, 0, 0, 0);$$

$$P_{c2} : M_{02}(P_{c2}) = 1; C(P_{c2}, \cdot) = (-2, -2, 0, 0, 0)$$

$$P_{c3} : M_{03}(P_{c3}) = 1; C(P_{c3}, \cdot) = (0, -2, 0, 0, 0); P_{c4} : M_{04}(P_{c4}) = 1; C(P_{c4}, \cdot) = (0, -2, 0, 0, 0).$$

Notably, P_{c1} , P_{c3} and P_{c4} are equal. Therefore, the redundant control places P_{c3} and P_{c4} can be removed, and the Boolean control function is $F_{Bc} = P_{c1}P_{c2}$. The controlled PN model is shown in Figure 10.

Table 1. Information of MC markings.

Minimal cut no.	Marking no.	Classification	Marking $[p_1, p_2, p_3]^T$
MC ₁	M_2	MS_1	$[3, 0, 1]^T$
	M_4	MI_1	$[2, 0, 2]^T$
	M_5	MS_1	$[2, 1, 1]^T$
	M_8	MI_1	$[1, 1, 2]^T$
	M_6	MS_1	$[2, 2, 0]^T$
	M_9	MI_1	$[1, 2, 1]^T$
	M_{10}	MS_1	$[1, 3, 0]^T$
	M_{13}	MI_1	$[0, 3, 1]^T$
	M_3	Min_1	$[3, 1, 0]^T$
	MC ₂	M_5	MS_2
M_9		MI_2	$[1, 2, 1]^T$

Table 2. Activation and deactivation vectors.

PN controller	Marking no.	Activation vectors	Deactivation vectors
P_{c1}	M_2	$[01000]^T$	$[01001]^T$
	M_3	$[10000]^T$	$[10010]^T$
P_{c2}	M_5	$[11000]^T$	$[00011]^T$

6. Comparison with previous works

This section presents three tables with regard to compare our new method with previous works. First, we compare this work with the classical approach of the theory of regions developed by Ghaffari et al. (2002a) in Example 1.

Moreover, this method is confronted in Example 2 with the previous deadlock prevention policies and especially with the recent method of Huang and Pan (2011) that has many advantages and solves problem of forbidden state transition. Indeed, its experimental results with comparisons infer that its policy seems to be the most efficient approach among existing methods such as the research works of Li et al. (2008); Park and Reveliotis (2000); Uzam and Zhou (2007) and Piroddi, Cordone, and Fumagalli (2008). Although, this methods do not consider uncontrollable transitions in PN models used in experimentation.

Finally, it is shown in Example 3 that the resolution of the problem when the PN controller does not exist without resorting to partial PN controllers approach (Ghaffari et al., 2002). For convenience, the two approaches related to the two approaches

Table 3. Comparison results of Example 1.

Approach	No. of monitors	Reachable markings	No. of event separation instances Ω	No. of equations (L) in all LS_i	Computation time (ms)	Method
S_1	2	6	5	15	219	Minimal cuts
S_2	2	6	5	13	190	Minimal cuts
G_1	2	6	5	40	593	Reachability graph based

Table 4. Additional control places.

Approach	Additional control places	$M_{0i}(P_{ci})$	$C(P_{ci}, \cdot)$
S_1	P_{c1}	1	(-1,-2,0,1,2)
	P_{c2}	1	(-2,0,0,0,0)
S_2	P_{c1}	1	(0,-2,0,0,0)
	P_{c2}	1	(-2,-2,0,0,0)
G_1	P_{c1}	1	(0,-1,0,0,1)
	P_{c2}	3	(-1,-2,0,1,2)

are called S_1 and S_2 . Moreover, the related research works (Ghaffari, Rezg, & Xie, 2002a), (Ghaffari et al., 2002), (Huang & Pan, 2011) are called G_1 , G_2 and H , respectively.

6.1 Example 1 (ENIM FMS)

Table 3 lists the experimental results with the work of G_1 to synthesise PN controllers for the FMS example. One can observe that for the same number of reachable markings and event separation instances, one can synthesise the same number of maximally permissive supervisors with reduced number of equations (L) reflecting the theory of regions. Indeed, two control places founded with the classical approach of G_1 are obtained by solving 40 equations with computation time of 593 ms, whereas with MC approaches L is divided by 3. The reduction technology is involved in S_1 and S_2 by reducing systematically the computation time, which decreases from 593 to 219 and 190 ms for both approaches S_1 and S_2 . The controlled system net with the three approaches is obtained and shown in Table 4.

As a result, one can infer that our new method translated by S_1 and S_2 can drastically reduce the encountered difficulties in solving all equations of the theory of regions.

6.2 Example 2 (H)

Example 2 is taken from Huang and Pan (2011), based on Table 5, 4 event separation instances and 46 equations of the theory of regions are necessary to be processed if H is used. On the other side, our new method needs 10 event separations and respectively 22 and 18 equations to solve for approaches S_1 and S_2 . It is obvious that three control places are used to control this example with the H method against three and

Table 6. Additional control places.

(A)	Additional control places	$M_{0i}(P_{ci})$	$C(P_{ci}, \cdot)$
S_1	P_{c1}	0	(-2,0,0,0,0,0,0,0,0)
	P_{c2}	0	(0,-2,0,0,0,0,0,0,0)
S_2	P_{c3}	0	(0,0,0,0,-2,0,0,0,0)
	P_{c1}	1	(0,0,0,0,0,0,0,-2,0)
	P_{c2}	1	(0,0,0,-2,0,0,0,0,0)
	P_{c3}	1	(0,0,0,0,0,0,0,-2,-2,0)
H	P_{c4}	1	(0,0,0,-2,-2,0,0,0,0,0)
	P_{c1}	2	(0,0,0,0,1,-1,0,1,-1,0)
	P_{c2}	3	(0,0,0,1,-1,0,0,1,-1,0)
	P_{c3}	2	(0,0,0,1,-1,0,0,0,1,-1)

four control places for S_1 and S_2 . The controlled system net is obtained with the three approaches and shown in Table 6. The main advantage of the algorithms of S_1 and S_2 is the gain of computation time which is reduced from 489 to 234 and 191 ms. Indeed, for the same number of reachable markings of 20 states, one can solve only 22 and 18 equations of the theory of regions to synthesise the PN supervisors comparing to 46 equations for the H method. Another very important factor for the comparison between the three methods is not listed in Table 5. In fact, the H method does not consider uncontrollable transitions in the PN models, which is a weakness in this work. As a result, one can infer that our new method is the most efficient among H methods.

6.3 Example 3 (G_1 , G_2)

Example 3 is taken from Ghaffari et al. (2002) which present the forbidden state transitions problem insoluble with pure control places. Indeed, this method solves the problem when a single control place does not exist, and the main idea is to synthesise several control places to prevent a forbidden transition. The set of this control places is called a partial controller. Based on the experiment results in Table 7, 236 equations of the theory of regions to solve for 8 event separation instances are needed if one uses the G_2 method in this example. Although the reduced technology is involved in S_1 and S_2 , both the approaches need to handle 16 equations that shows the big reduction of time-consuming decreasing from 1540 to 110 and 115 ms. Moreover,

Table 5. Comparison results of Example 2.

Approach	No. of monitors	Reachable markings	No. of event separation instances Ω	No. of equations (L) in all LS_i	Computation time (ms)	Method
S_1	3	20	10	22	234	Minimal cuts
S_2	4	20	10	18	191	Minimal cuts
H	3	20	4	46	489	Reachability graph based

Table 7. Comparison results of Example 3.

Approach	No. of monitors	Reachable markings	No. of event separation instances Ω	No. of equations (L) in all LS_i	Computation time (ms)	Method
S_1	2	30	8	16	110	Minimal cuts
S_2	2	30	8	16	113	Minimal cuts
G_2	5	30	8	236	1540	Partial controllers

Table 8. Additional control places.

Approach (A)	Additional control places	$M_{0i}(P_{ci})$	$C(P_{ci}, \cdot)$
S_1	P_{c1}	0	(-2,0,0,0)
	P_{c2}	0	(0,-2,0,0)
S_2	P_{c1}	1	(0,-2,0,0)
	P_{c2}	1	(-2,0,0,0)
G_2	P_{c1}	4	(-1,-1,0,2)
	P_{c2}	7	(-2,-1,-2,5)
	P_{c3}	0	(1,-3,0,2)
	P_{c4}	9	(-3,-2,2,3)
	P_{c5}	0	(1,-3,2,0)

with the G_2 method, five monitors have been obtained what defines three partial controllers against two PN supervisors with approaches S_1 and S_2 . The controlled PN model that obtains maximally permissive supervisors with 30 reachable markings is shown in Table 8. Therefore, not only the resolution complexity of the theory of regions is reduced in this method, but the problem of partial controllers defined in Ghaffari et al. (2002) is also solved with a reduced approach. As a result, one can infer that the new approaches S_1 and S_2 are the most efficient among the G_2 method.

7. Conclusion and future work

The proposed policy based on MC approaches is implemented in a FMS. The results analysis shows that our control policy is more efficient than the previous work using the theory of regions for control synthesis. Indeed, for the same number of monitors, we can synthesise the PN controllers under fewer equations and computation time. The application of the theory of regions is confined to the MC and not to all the reachability graph. In fact, it reflects the great reduction of the resolution complexity of this theory. Moreover, our method solves insoluble problems of forbidden state transition problem under the combinatorial explosion of states and the convexity of state equations that may exist in the LS of the theory of regions.

Based on experimental results, it is obvious to notice the considerable reduction of the computational cost of the theory of regions. Our new policy provides an efficient method to obtain the maximally permissive supervisor.

Hence, in the near future, we intend to find other methodologies in order to reduce the resolution complexity of the theory of regions, where it is not necessary to generate the reachability graph to synthesise the PN controller which is an important issue in control synthesis.

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