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# An analytical thermomechanical modelling of peripheral milling process using a predictive machining theory

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## Abstract

In this work, a predictive machining theory, based on an analytical thermomechanical approach of oblique cutting [17,18], has been applied to the peripheral milling process. That leads to a three dimensional cutting force model for end milling operations which is an alternative approach in comparison with the mechanistic one. In this model, the material characteristics such as strain rate sensitivity, strain hardening and thermal softening are considered and thermomechanical coupling and inertia effects are accounted for. Calculated and experimental results are compared for up-milling.

## INTRODUCTION

The end milling is extensively used in aerospace industry for a large variety of roughing and finishing operations, and it is also largely employed in dies and moulds manufacturing. To optimize these machining operations, it is necessary to reduce the cutting forces and the cutting temperatures in order to achieve the surface quality requirements with an acceptable tool life. A predictive model may be very useful in this optimisation process. In the literature, several works are based on the mechanistic approach [1-11]. The cutting force coefficients or the specific pressures, depend on the cutting conditions, cutter geometry and tool and workpiece material properties. The determination of these coefficients requires many experimental milling tests for a given tool-workpiece material and cutter geometry. The functional relationships between the cutting conditions and the cutting coefficients vary from one model to another. In order to simplify the experimental identification of these coefficients and to avoid that the model becomes complex and unwieldy, the cutting force coefficients are taken to be function only of the uncut chip thickness. The mechanistic method has been used to predict the cutting forces variations during a cutter revolution, the tool vibration and the surface geometrical errors. However for each milling cutter geometry, the predictive capability of this semi-empirical approach depends on the experimental calibration.

In order to improve the mechanistic approach, the milling force coefficients have been determined in [7] from orthogonal cutting data for specified tool-workpiece materials and cutting

conditions. The approach developed in [7] can be applied for any cutter geometrical design by using an oblique cutting model which presents several drawbacks: (i) in the primary shear zone, the shear stress is supposed to be constant, (ii) the thermomechanical effects are not considered and (iii) the effect of cutting speed is not taken into account. In [12-16], a predictive machining theory based on Oxley's model of orthogonal cutting has been used to model milling processes. In Oxley's work, the mean friction coefficient at the tool-chip interface is calculated by the model and is independent on the tool material. In addition, in thermal study, namely heating in the primary and the secondary shear zones, there is many empirical equations where different coefficients have to be calibrated.

In the present work, an analytical thermomechanical approach of oblique cutting, developed and used in previous studies [17-18], is employed to determine the cutting forces distribution along the engaged cutting edges and the chip flow for helical end mills. That leads to a three dimensional cutting force model for end milling operations which is an alternative approach in comparison with the mechanistic one. In this model, the material characteristics such as strain rate sensitivity, strain hardening and thermal softening are considered and thermomechanical coupling and inertia effects are accounted for. Calculated and experimental results are compared for various cutting conditions and different tool geometries. In order to simplify the model presentation, tool eccentricity, spindle tilt, tooth runout and flexibility of the

machine-tool-workpiece are not considered in the present study.

## A THERMOMECHANICAL MODELLING OF MILLING PROCESS

To study the milling process, let us introduce the frame  $(\tilde{x}, \tilde{y}, \tilde{z})$  as shown in Fig. 1. For a point of cutting edge at position  $z = -\tilde{z}$ , the elementary chip, corresponding to an infinitesimal cutting edge element, is obtained from an oblique cutting process which is modelled by the thermomechanical approach developed in [17-18]. Although this predictive machining theory was established for stationary conditions, it can be applied to milling processes by using the assumption that at each instant of time the chip formation can be modeled by an equivalent stationary process, [19].

For a point of cutting edge, at position  $z = -\tilde{z}$ , of the  $i^{th}$  cutting tooth, the angular position  $\varphi_i(z)$  from the  $\tilde{x}$ -axis is reported in Fig. 1 and determined by:

$$\varphi_i(z) = \omega t - \frac{z}{R} \tan \alpha_h + (i-1) \frac{2\pi}{N_t} \quad (1)$$

where  $N_t$ ,  $\omega(\text{rad}/s)$ ,  $\alpha_h$  and  $R$  represent respectively the number of teeth, the angular velocity of the cutter, the helix angle and the tool radius.

According to the kinematics of milling, the  $i^{th}$  flute goes into cut if

$$\begin{cases} \varphi_{in} < \varphi_i(z=0) < \varphi_{out} \\ \text{or} \\ \varphi_{out} < \varphi_i(z=0) \text{ and} \\ \left( \varphi_i(z=0) - \psi_{out} \right) R < d_a \tan \alpha_h \end{cases} \quad (2)$$

where  $d_a$  is the axial depth of cut, Fig. 1. The entrance angle  $\varphi_{in}$  and exit angle  $\varphi_{out}$  are as follows:

for up-milling

$$\varphi_{in} = 0 \quad \varphi_{out} = \cos^{-1}(1 - d_r/R) \quad (3.1)$$

for down-milling

$$\varphi_{in} = \pi/2 + \sin^{-1}(1 - d_r/R) \quad \varphi_{out} = \pi \quad (3.2)$$

with  $d_r$  is the radial depth of cut, Fig. 1.

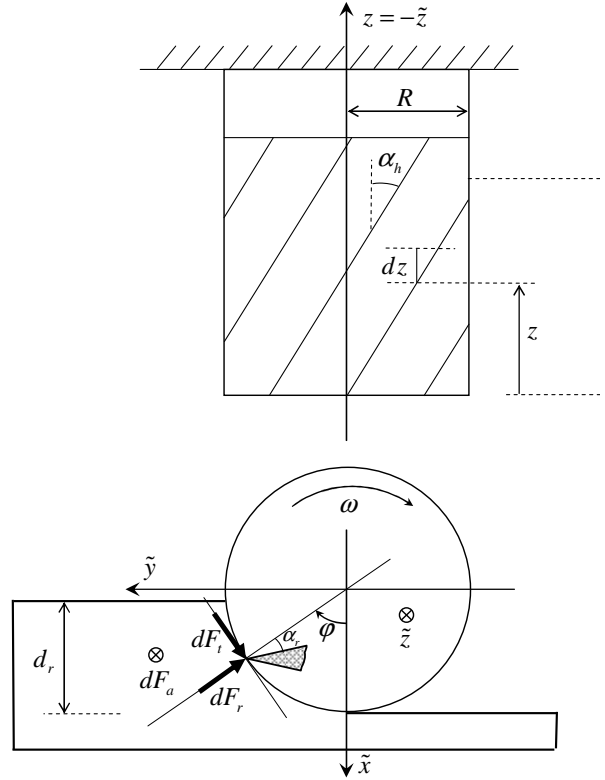


Figure 1: Peripheral milling process.

The elementary chip formation under oblique cutting process is modelled by a thermoviscoplastic approach. The thermomechanical properties and the inertia effects are accounted for to describe the material flow in the primary shear zone. On the tool rake face, the chip flow angle is calculated by assuming that the friction force is collinear to the chip flow direction. The oblique cutting conditions corresponding to a cutting edge element of the  $i^{th}$  cutting tooth are:

$$\begin{aligned} \lambda_s &= \alpha_h, \quad t_1 = f_t \sin(\varphi_i) \\ \alpha_n &= \tan^{-1}(\tan(\alpha_r) \cos \alpha_h), \quad V = \omega R \end{aligned} \quad (4)$$

where  $\lambda_s$ ,  $f_t$ ,  $V$ ,  $\alpha_n$  and  $\alpha_r$  correspond respectively to the inclination angle, the feed per tooth, the cutting speed, the normal rake angle and the rake angle measured in the velocity plane.

The primary shear zone is modelled as a shear band of constant thickness  $h$ . The deformation in the chip is supposed to be limited to this band which is characterized by the normal shear angle  $\phi_n$  measured in the plane perpendicular to the cutting edge. The secondary shear zone at the tool-chip interface and the complex material flow near the tool edge are not considered. This analysis is restricted to

stationary flow. Therefore, all the variables in the band depend solely on the coordinate along the normal to the band (one dimensional formulation). In addition, the shearing in the band is supposed to be adiabatic. This assumption is reasonable when the cutting speed is large enough (*High Speed Machining*).

The thermomechanical behaviour of the workpiece material has to be identified at conditions close to those of machining. In the proposed work, the workpiece material is supposed to be isotropic, rigid and viscoplastic, and described by a Johnson-Cook law:

$$\tau = \frac{1}{\sqrt{3}} \left[ A + B \left( \frac{\gamma}{\sqrt{3}} \right)^n \right] \left[ 1 + m \ln \left( \frac{\dot{\gamma}}{\dot{\gamma}_0} \right) \right] \left[ 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^\nu \right] \quad (5)$$

The variables  $\tau$ ,  $\gamma$ ,  $\dot{\gamma}$ , and  $T$  represent respectively the shear stress, the shear strain, the shear strain rate and the absolute temperature. The material characteristics are defined by the strain hardening exponent  $n$ , the strain rate sensitivity  $m$ , the thermal softening coefficient  $\nu$ , the constants  $A$ ,  $B$ ,  $\dot{\gamma}_0$ , the reference temperatures  $T_r$ , and the melting temperature  $T_m$ .

In primary shear zone, the distributions of the shear stress  $\tau$  and of the absolute temperature  $\theta$  are obtained respectively from the integration of the equations of motion and of heat (assuming adiabatic conditions) with respect to the coordinate along the normal to the shear band. The shear strain rate  $\dot{\gamma}$  is determined from the constitutive law (5). We get:

$$\tau = \rho (V \cos \lambda_s \sin \phi_n)^2 \gamma + \tau_0 \quad (6)$$

$$\theta = \theta_w + \frac{\beta}{\rho c} \left( \rho (V \cos \lambda_s \sin \phi_n)^2 \frac{\gamma^2}{2} + \tau_0 \gamma \right) \quad (7)$$

$$\left\{ \begin{array}{l} \dot{\gamma} = \dot{\gamma}(\gamma, \tau_0) = \dot{\gamma}_0 \exp \left( \frac{\tau \sqrt{3}}{m g_1(\gamma) g_2(\theta)} - \frac{1}{m} \right) \\ \text{with } g_1(\gamma) = \left[ A + B \left( \gamma / \sqrt{3} \right)^n \right] \\ g_2(\theta) = \left[ 1 - \left( \frac{\theta - T_r}{T_f - T_r} \right)^\nu \right] \end{array} \right. \quad (8)$$

where  $\beta$ ,  $c$ ,  $\rho$  and  $\tau_0$  represent the Taylor-Quinney coefficient, the heat capacity, the material density and the shear stress at the entry of the primary shear zone respectively.

The integration of the differential equation, corresponding to the material derivative of the shear strain  $\gamma$ , over the band width gives:

$$\int_0^{\gamma_h} \frac{V \cos \lambda_s \sin \phi_n}{\dot{\gamma}(\gamma, \tau_0)} d\gamma - h = 0 \quad (9)$$

This non linear integral equation is used to determine the shear stress  $\tau_0$ . It must be noted that due to the fact that the deformation in the chip is supposed to be limited to this band, the boundary condition giving the shear strain  $\gamma_h$  at the exit of the band is as follows:

$$\gamma_h = \frac{\cos \alpha_n}{\sin \phi_n \cos \eta_{sh} \cos(\phi_n - \alpha_n)} \quad (10)$$

where the angle  $\eta_s$  characterizes the shear direction in the primary shear plane:

$$\tan \eta_s = \frac{\tan \eta_c \sin \phi_n - \tan \lambda_s \cos(\phi_n - \alpha_n)}{\cos \alpha_n} \quad (11)$$

The shear force exerted on the primary shear band outflow is collinear to the shear velocity. In addition, on the tool rake face, the frictional force and the chip flow directions are supposed to be collinear. Therefore, the chip flow angle  $\eta_c$  can be calculated from this implicit equation:

$$\cos(\phi_n - \alpha_n) \sin \phi_n \sin \eta_c - \tan \lambda_s \cos^2(\phi_n - \alpha_n) \cos \eta_c + (\cos \alpha_n - \sin(\phi_n - \alpha_n) \sin \phi_n) \tan \lambda \sin \eta_c \cos \eta_c + \tan \lambda \tan \lambda_s \sin(\phi_n - \alpha_n) \cos(\phi_n - \alpha_n) \cos^2 \eta_c = 0 \quad (12)$$

The normal shear angle  $\phi_n$  is assumed to be independent on the inclination angle  $\lambda_s$ , see [20], and given by the Zvorykin law [21]:

$$\phi_n = A_1 + A_2 (\alpha_n - \lambda) \quad (13)$$

where  $\alpha_n$  is the normal rake angle and  $\lambda$  the mean friction angle at the tool-chip interface. The constants  $A_1$  and  $A_2$  depend on the workpiece-tool materials.

For a cutting edge element of the  $i^{\text{th}}$  flute, the specific cutting pressures  $K_{tc}$ ,  $K_{rc}$  and  $K_{ac}$  (the subscript c represents the cutting force coefficients due to the chip process formation) corresponding respectively to the tangential, radial, and axial directions are obtained from the elementary chip equilibrium:

$$\begin{cases} K_{tc} = G \{ \cos \alpha_n \cos \lambda_s + \\ \tan \lambda (\sin \eta_c \sin \lambda_s + \cos \eta_c \sin \alpha_n \cos \lambda_s) \} \\ K_{rc} = G (-\sin \alpha_n + \tan \lambda \cos \eta_c \cos \alpha_n) \\ K_{ac} = G \{ \cos \alpha_n \sin \lambda_s + \\ \tan \lambda (-\sin \eta_c \cos \lambda_s + \cos \eta_c \sin \alpha_n \sin \lambda_s) \} \end{cases} \quad (14)$$

with

$$G = \frac{\cos \lambda \cos \eta_s \tau_h}{\cos \lambda_s \sin \phi_n \cos \lambda (\cos(\phi_n - \alpha_n) - \tan \lambda \cos \eta_c \sin(\phi_n - \alpha_n))} \quad (15)$$

The shear stress  $\tau_h$  at the outflow of the primary shear band is given by (6):

$$\tau_h = \rho (V \cos \lambda_s \sin \phi_n)^2 \gamma_h + \tau_0 \quad (16)$$

Since in milling operation the undeformed chip thickness  $t_1$  varies strongly during cutting and becomes very small, the effect of the cutting edge has to be taken into account. Thus, the edge force coefficients  $K_{te}$ ,  $K_{re}$  and  $K_{ae}$  due to the rubbing forces are introduced as in [7]. The value of  $K_{ae}$  in the axial direction is often small and taken as zero [7].

The elementary force components  $dF_t$ ,  $dF_r$  and  $dF_a$ , reported in Fig. 1, correspond respectively to the tangential, radial and axial directions and are determined as follows

$$\begin{cases} dF_t = (K_{te} + K_{tc} t_1) dz \\ dF_r = (K_{re} + K_{rc} t_1) dz \\ dF_a = (K_{ae} + K_{ac} t_1) dz \end{cases}$$

where  $K_{tc}$ ,  $K_{rc}$  and  $K_{ac}$  are obtained from (14) and  $dz$  is an infinitesimal element thickness, Fig. 1.

The total forces exerted on the tool are obtained by integrating the elementary force components along the axial direction  $z$ . The interval of integration is determined by analysing where the mill enters and leaves the work material.

## EXAMPLE RESULTS

The present model is validated with respect to the milling tests for a titanium alloy Ti6Al4V with a carbide tool. The experimental data correspond to the tests performed in [22,23]. The material parameters of the Johnson-Cook law (5) have been deduced from [24]:

$$\begin{aligned} &: \\ A &= 862 \text{ MPa} \quad B = 331 \text{ MPa} \quad \dot{\gamma}_0 = 1 \text{ s}^{-1} \\ n &= 0.34 \quad m = 0.012 \quad \nu = 0.80 \end{aligned}$$

$$T_r = 296 \text{ }^\circ\text{K} \quad T_m = 1950 \text{ }^\circ\text{K}$$

The material density  $\rho$ , the heat capacity  $c$  and the thermal conductivity  $k$  are given by:

$$\rho = 4430 \text{ kg/m}^3 \quad c = 526 \text{ J/(kg }^\circ\text{K)}$$

$$k = 7 \text{ W/m }^\circ\text{K}$$

The edge force coefficients  $K_{te}$  and  $K_{re}$  have been determined by Budak et al. [7] from orthogonal cutting tests with different rake angle, at different cutting velocities and feedrates:

$$K_{te} = 24 \text{ N/mm}, \quad K_{re} = 43 \text{ N/mm}$$

In the figure 2, the present model is compared with the experimental results of [22,23] for a half-immersion up-milling test with the following cutting conditions:

$$\alpha_h = 30^\circ, \quad R = 9.525 \text{ mm}, \quad V = 30 \text{ m/min}$$

$$d_a = 5 \text{ mm}, \quad N_t = 4, \quad d_r = R$$

$$f_t = 0.05 \text{ mm/tooth}, \quad \alpha_r = 12^\circ$$

At the tool-chip interface, the mean friction angle  $\lambda$  is taken constant  $\lambda = 26^\circ$  according to the experimental values of [22,23].

As it can be seen from the figure 2, the predictions of the present model are in good agreement with the measured forces. It can be also noted that the shear stress in the primary shear zone estimated from cutting forces of the orthogonal cutting tests of Budak et al. [7] is very close to the predicted value given by the model. In [23], it has been show that the measured value of the chip flow angle  $\eta_c = 15^\circ$  is lower than the helix angle which is  $30^\circ$  and the Stabler rule is not valid. The value obtained from the present model is  $\eta_c = 16,71^\circ$ .

In the thermomechanical analytical model of oblique cutting, the material flow in the primary shear zone depends on two parameters: the normal shear angle  $\phi_n$  and the thickness  $h$  of the shear band. In this paper, the model has been used with the following values  $h = 0.010 \text{ mm}$  and  $\phi_n = \phi_{Merchant}$ . It can be noted that the effects of these parameters on the cutting forces are not significant. This can be explained as following: (i) considering the fact that the material strain rate sensitivity  $m$  is weak and the flow within the band is supposed to be adiabatic (which is acceptable for high cutting

speed), the precise value of  $h$  has no important effect on the results predicted by the model, see [19], and (ii) the value considered for  $\phi_n$  corresponds to minimum value of cutting forces, see [19].

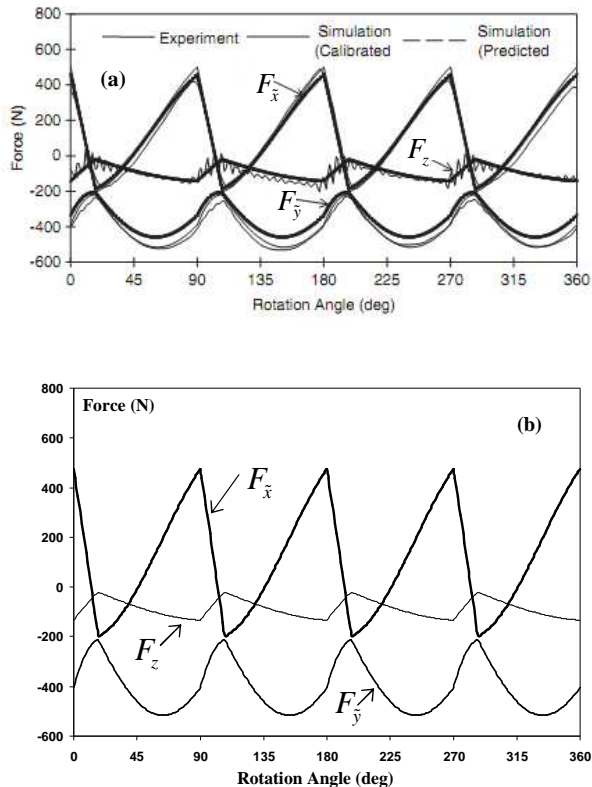


Figure 2: Comparison between measured milling forces (a) of [22,23] and the predicted values of the proposed model (b).

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