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Integrated production, maintenance and control chart of supply chain management under quality constraint

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Abstract - This paper develops a joint control of production, maintenance, and quality for a supply chain management system. The manufacturing system is subjected to a random failure that directly affects the quality of the products. The degradation of the manufacturing system is impacted by the variation of the production and its use. The control of the process and the quality of the products is carried out using statistical process control (SPC) and its tool "control chart of average". The problem consists of several warehouses allow satisfying random demands during a finite horizon, under quality level. The goal of this study is to establish an optimal production planning and maintenance strategy, taking into account the influence of the production rate on the system degradation and according to the average of the measures of the quality indicator x compared to the control limits, which minimize the total cost.

Keywords - Failure rate, random demand, multi-warehouse, process quality, statistical process control,

I. INTRODUCTION

Due to the changing faster of the global & economic conditions for the markets, the high-quality product at competitive prices with product variety and reduced lead-time has become the requirement of the customers in terms of consciousness and demand. The quality of a product or service is only reduced when it affects either the process which creates it by increasing costs, time and constraints, or by affecting the final product or service used by a customer. This implies that quality does not simply achieve perfection of a product or service, but rather that high quality achieves perfection in the performance of a product or service, or that it is as close as possible.

To guarantee the conformity of the quality as well as the satisfaction of the customer several methods of continuous improvement which can be applied in each phase at the level of a manufacturing process such as Six Sigma Methods, Measurement system analysis, Analysis of the causes and consequences. In this study, we focus on the Six sigma method which uses in the production companies or service industries. Six Sigma uses the base tools to improve the quality of products and processes by reducing the process variation and the associated high defect rate such as the Control chart which is based on the statistical process control (SPC) approach.

On the other hand, the optimal maintenance and production plans which minimize the total cost including production, inventory, and maintenance are one of the first actions of a hierarchical decision-making process. The failure or dysfunction of the production system is considered among the causes of the production of non-conforming items. Hence, the integration of statistical process control, maintenance, and production is considered the solution to reduce the defective of production. The integration of the control chart "x-bar" and the periodic preventive maintenance strategy is dealt with by several researchers to stabilize the process and avoid the production of non-conformity ([1], [2], [3],[4])

The optimal production plan and maintenance strategy required by the company to manufacture their products which satisfy a random demand over future periods. In this context, Ref. [5] studied the problem of integrated maintenance to production strategies by studying the role of buffer stocks in increasing system productivity. More recently, Ref. [6] determined simultaneously the optimal plans of production and maintenance as well as the optimal delivery plan took into account the delivery time, machine failures, random demand, and withdrawal right. Indeed, delivery time and transported quantity are important characteristics of manufacturing systems. Thus many manufacturers are working to reduce transportation delays such as the delivery time, which is the period that the part takes between a manufacturing store and a purchase warehouse (customer), and which usually has a great impact on performance measures. Ref. [7] developed a model for supporting investment strategies about inventory and preventive maintenance in an imperfect production system take into account the delivery time to the customer.

In this paper, we dealt with a system that consists of several warehouses allow us to satisfy random demands with quality constraint. The originality of this work is that determined the optimal production, maintenance, and delivery quantities plans based on the result of the control chart and the correlation between production and maintenance. Indeed, to respect the service level, each warehouse should contain enough parts for satisfying customer demands. Thus, optimal planning of the transported quantity between the manufacturing store and purchase warehouses should be determined based on

the relationship with the production/maintenance planning and the service level

II. PRODUCTION AND MAINTENANCE PROBLEM

A. Notations

The following parameters are used in the mathematical formulation of the model:

- τ_i : delivery time for warehouse w_i
- L : Number of warehouses
- Δt : length of a production period
- H : number of production periods in the planning horizon
- $H.\Delta t$: length of the finite planning horizon
- $u(k)$: production rate of machine M during period k ($k=0, 1, \dots, H-1$)
- $U=\{u(0), u(1), \dots, u(H-1)\}$
- $Q_i(k)$: delivery rate during period k ($k=0, 1, \dots, H-1$) for each warehouse
- $Q_i=\{Q_i(0), Q_i(1), \dots, Q_i(H-1)\}$
- $\hat{d}_i(k)$: average demand during period k ($k=0, 1, \dots, H$) for each customer
- $V_{di(k)}$: variance of demand during period k ($k=0, 1, \dots, H$) for each customer
- $S(k)$: inventory level of S at the end of period k ($k=0, 1, \dots, H$)
- $w_i(k)$: inventory level of S_i ($i:0 \dots L$) at the end of period k ($k=0, 1, \dots, H$) for each warehouse
- C_p : unit production cost of machine M
- C_h : inventory holding cost of one product unit during one period at the first store S .
- C_{hi} : inventory holding cost of one product unit during one period at the ware-house S_i ($i:0 \dots L$)
- C_d : shortage cost one product unit during one period
- C_M : total maintenance cost
- C_{pm} : preventive maintenance action cost
- C_{cm} : corrective maintenance action cost
- mu : monetary unit
- U_{max} : maximal production rate of machine M
- U_{min} : minimal production rate of machine M
- θ_i : probability index related to each customer i satisfaction and expressing the service level.
- J : average number of samples to detect the 'out of control' state.
- δ_i : The magnitude of the shift to the 'out of control' state compared to the centerline
- A : Number of standard deviations between the center line of the control chart and the control limits
- C_i : Unit cost of inspection
- C_r : Unit cost of one defective unit

B. Problem Description

In this study, the joint optimization of production, maintenance planning of the quality control problem is

presented. We consider a supply chain composed of a manufacturing system with a single machine that produces one type of product, a principal manufacturing store S and multi-purchases warehouse (W_0, W_1, \dots, W_L) (where the customer receives his demand (products)). Each warehouse aims to satisfy several random demands under a given quality specification limit over a finite horizon H and with a delivery time t_i ($i=0, \dots, L$) for each demand (see figure1). The machine M is subject to random failure. The degradation degree of the machine is influenced by the production rates, consequently, the failure rate $\lambda(t)$ increases with time and production rate $u(k)$ and affects the reliability of the production process and generates the production of non-conforming units. We assumed that the machine is controlled at every h time units during production period Δt by a quantitative quality statistic X with n measurements for the sample t . It is assumed that the individual measures must all be between USL upper specification limit and LSL lower specification Limit. The measurement results are saved in a measurement card with a risk of the first species. It's assumed that the production process is stable if the law of X_t is Gaussian constant with mean m_0 and standard deviation s which are known or well estimated. The process is under control when all statistics are within between control limits: lower control limit LCL and upper control limit UCL.

To reduce the degradation, as well as the failure rate that depends on the variation of the production rate of the machine, a maintenance strategy (PMMR: perfect preventive maintenance actions with minimal repair), is performed during the finite horizon of production. Perfect Preventive maintenance actions negligible durations are periodically planned $T = a$. $\Delta t = \alpha.\beta.h$ (α and β are integer numbers) and a minimal repair for each failure between two successive preventive maintenances during the production horizon when the production process is "under control" state between two preventive maintenance actions (Scenario 1). in the case where the production process is "out of control" (Scenario 2) between the j^{th} and $(j+1)^{\text{th}}$ sampling due to occurrence of an assignable cause, and consequently the sample average of the quality indicator measurements is located beyond the control

limits ($LCL \leq \bar{X}_i \leq UCL$) with $LCL = \mu_0 - \frac{A}{\sqrt{n}} \times \sigma_0$ and

$UCL = \mu_0 + \frac{A}{\sqrt{n}} \times \sigma_0$ or the mean value of the quality

characteristic changes for from and. Hence, production is stopped by performing a perfect maintenance action restored the machine to as good as a new state with a duration until the end of the production period and the classical periodically planning of the preventive maintenance according to cumulative failure rate is starts again from the end of this period (Fig.1).

Our objective lies in establishing the best production, delivery, and maintenance strategy. To do this conjugated production, delivery, and maintenance optimization, we minimize the sum of the inventory costs at the different stores, the manufacturing and delivery costs along with the costs associated with the maintenance strategy.

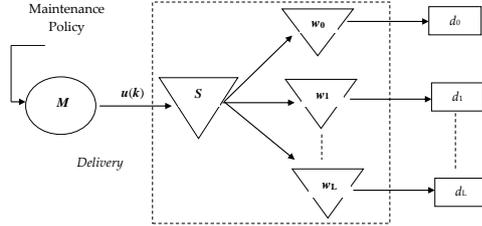


Fig. 1. Supply chain management

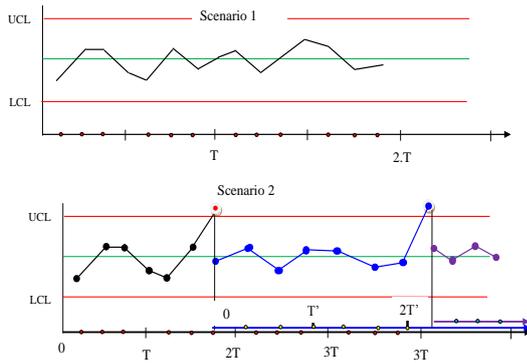


Fig. 2. Integrated quality control and maintenance strategy

III. PROBLEM FORMULATION

The objective of this study is to determine firstly the optimal plan of production to satisfy the random demand of customers under a given service and quality level. Secondly, according to the economic plan of production, the optimal parameters of the control chart which are: the sample size n , the control interval h , and the control limits coefficient as well as the optimal maintenance strategy characterized by the optimal number of preventive maintenance actions N . the objective is to minimize the expected costs related to production, inventory, maintenance and quality including sampling and rejection costs during the finite time horizon. It's assumed that the horizon is portioned equally into H periods with a length equal to Dt . Moreover, we assume that the fluctuation of the demands is a normal process with mean and variance are given respectively by μ and σ^2 and the demands are satisfied at the end of each period.

The effectiveness of a control card can be measured by using the probability of not detecting an adjustment when taking a sample of size n . The effectiveness of the card is all the greater when this probability is low. When the process does not work properly, the average can vary and it takes as value μ . We note c the expression of the adjustment of the mean in number of standard deviations:

$$\delta_j = \frac{\mu - \mu_0}{\sigma_0} \quad (1)$$

Let $P_j(c)$ be the probability of not detecting an adjustment of c standard deviations when taking a sample of n pieces.

$$P_j(\delta_j) = F(-\delta_j \cdot \sqrt{n} + A) - F(-\delta_j \cdot \sqrt{n} - A) \quad j=\{1,2,\dots\} \quad (2)$$

With F : distribution function of the reduced centered normal law.

Hence, the average operational period (AOP) characterized the average number of successive samples leading to the first point out of range, for a given adjustment.

$$AOP_j = \frac{1}{1 - P_j(\delta_j)} = \frac{1}{1 - F(-\delta_j \cdot \sqrt{n} + A) + F(-\delta_j \cdot \sqrt{n} - A)} \quad (3)$$

- Holding cost

$$HC_S = \sum_{k=1}^H C_h \times Z_s(k) \quad (4)$$

$Z_s(k)$ is the area generated by the inventory level evolution during the period k ($k=1, \dots, H$)

The progress of the inventory is characterized by the inventory balance equation level of the principal store S is given by the following equation for ($k=1, \dots, H$):

$$S(k) = S(k-1) + u(k) \cdot \Delta t - \sum_{i=1}^L Q_i(k) \quad (5)$$

The generated area during a period k is given as follows:

$$Z_s(k) = \max(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \quad (6)$$

The inventory holding cost for the all warehouses is given by the following expression

$$HC_w = \sum_{k=1}^H C_h \times Z_w(k) \quad (7)$$

$Z_w(k)$ is the area generated by the inventory level evolution during the period k ($k=1, \dots, H$)

The inventory level of each warehouse inventory w_i at the period k is given by the following equation:

$$w_i(k) = w_i(k-1) + Q_i(k - \tau_i) - d_i(k) \quad (8)$$

The inventory level of each warehouse w_i at the period k equals to the inventory level of w_i at period $k-1$ plus the quantity of products that arrives to w_i (i.e. $Q_i(k - \tau_i)$) minus the customer demand d_i at period k .

The generated area of the inventory level evolution for each warehouse during a period is given as follows:

$$Z_{w,i}(k) = \max(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot Q_i(k - \tau_i) \cdot \Delta t^2 \quad (9)$$

Consequently, the holding cost is expressed as follows:

$$HC = C_h \times \sum_{k=1}^H \left(\max(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 + \max(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot Q_i(k - \tau_i) \cdot \Delta t^2 \right) \quad (10)$$

The service level requirement constraint for each warehouse at each period k is expressed by the following constraint. For $(k=1, \dots, H-1)$ and $(i=1, \dots, L)$

$$\text{Prob}[w_i(k) \geq 0] \geq \theta_i \quad (11)$$

The following constraint defines an upper and lower bounds on the production level during each period k .

$$U_{\min} \leq u(k) \leq U_{\max} \quad (12)$$

- Production Cost

the production cost for period k

$$PC = C_p \times \sum_{k=1}^H u(k) \cdot \Delta t \quad (13)$$

- Delay Penalties

The delay penalties are characterized by the consequence of a delay to satisfy all the demands. If a delay situation occurred at the end of period k caused a shortage recovered during the next period $(k+1)$:

The penalties are determined as a function of the required duration $dw(\cdot)$ to produce the missed quantity at the end of each period, given by the following expression:

$$PC = C_d \times \left(\sum_{k=1}^H \left(\sum_{i=1}^L dw_i \right) \right), \quad dw_i = \frac{\min(w_i(k), 0)}{Q(k+1 - \tau_i)} \quad (14)$$

- Maintenance Policy

Maintenance cost consists of the false alarm cost, preventive and corrective maintenance costs, and also depends on the scenario that happens “scenario 1: in-control” and “scenario 2: out-control”. the resolution of maintenance planning problem consists of minimizing costs related to preventive and corrective maintenance. The maintenance strategy considered in this work is preventive maintenance with minimal repair in the ‘in-control’ state of the system. Preventive actions should be scheduled over the finite horizon H which is divided equally into N parts of duration T . We suppose that performing a preventive action corresponds to times $i.T$ ($i=1, 2, \dots, N$) consists to restore the machine to an as good as new condition. However, between successive preventive interventions, breakdowns may happen, the minimal repair is performed. It is assumed that the repair and overhaul durations are negligible.

So, it is necessary to keep in mind and to consider that the status of the machine depends on their production and transported plan variation.

The maintenance strategy is characterized by the optimal

number N^* of preventive maintenance actions and the most adequate time between them noted T^* .

$$C_M = P(S_1) \times C_{M1} + P(S_2) \times C_{M2} \quad (15)$$

$$P(S_1) = \text{Prob}(LSL \leq \bar{X} \leq USL) = F\left(\mu_0 + A \times \frac{\sigma_0}{\sqrt{n}}\right) - F\left(\mu_0 - A \times \frac{\sigma_0}{\sqrt{n}}\right)$$

$$P(S_2) = \text{Prob}(\bar{X} \leq LSL) + \text{Prob}(USL \leq \bar{X}) = F\left(\mu_0 - A \times \frac{\sigma_0}{\sqrt{n}}\right) + 1 - F\left(\mu_0 + A \times \frac{\sigma_0}{\sqrt{n}}\right)$$

$$C_{M1} = C_{pm} \times \left[\frac{H}{T} \right] + C_{cm} \times \varphi_M(U, N) \quad (16)$$

Where $\varphi(U, N)$ is the average number of failures as a function of the production plan defined by the vector U and the number of preventive maintenance actions N ?

$$\varphi_M(U, T) = \sum_{j=0}^{N-1} \left[\begin{aligned} & \int_{i=\ln\left(\frac{j}{\Delta t}\right)+1}^{\ln\left(\frac{j+1}{\Delta t}\right)} \lambda_i(t) dt + \int_0^{(j+1) \times T - \ln\left(\frac{j+1}{\Delta t}\right) \times \Delta t} \lambda_{\ln\left(\frac{j+1}{\Delta t}\right)+1}(t) dt \\ & + \int_{(j+1) \times T}^{\dots} \frac{\left(\ln\left(\frac{j+1}{\Delta t}\right) + 1 \right)}{U_{\max}} \times \lambda_n(t) dt \end{aligned} \right] \quad (17)$$

With $\lambda(t)$ represents the linear failure rate function at production period k is expressed as follows:

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{\max}} \cdot \lambda_n(t) \quad \forall t \in [0, \Delta t] \quad (18)$$

$\lambda_n(t)$: failure rate for nominal conditions (maximal production during the all horizon $H \cdot \Delta t$).

$$C_{M2} = \sum_{j=0}^M \left(C_{pm} \times \left[\frac{AOP_{j+1} - AOP_j}{T_j} \right] + C_{cm} \times \varphi_{M_j}(U, N_j) \right) + C_{pm} \times \left[\frac{H - AOP_M}{T_M} \right] + C_{cm} \times \varphi_M(U, N_M) \quad (19)$$

M : number of detection of the out of control

$$\varphi_{M_j}(U, N_j) = \sum_{z=0}^{N_j} \left[\frac{AOP_z}{T_j} \right] \sum_{i=\left\lfloor \frac{AOP_z}{T_j} \right\rfloor + 1}^{\left\lfloor \frac{(z+1) \times T_j}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \quad N_j = \left[\frac{AOP_{j+1} - AOP_j}{T_j} \right]$$

And

$$\varphi_M(U, N_M) = \sum_{z=0}^{N_M} \left[\frac{AOP_z}{T_j} \right] \sum_{i=\left\lfloor \frac{AOP_z}{T_j} \right\rfloor + 1}^{\left\lfloor \frac{(z+1) \times T_M}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt, \quad N_M = \left[\frac{H - AOP_M}{T_M} \right]$$

- The average total quality cost

The average total cost of quality corresponds to the sum of the cost of sampling and the cost of rejection of non-conforming items.

The average total cost of sampling is expressed by:

$$SC = \sum_{j=0}^M C_i \times n \times (AOP_{j+1} - AOP_j) \quad (20)$$

The average cost of non-conforming (NC) items is corresponding to the production quantity at “out control” period expressed by:

$$CNC = \sum_{j=0}^M C_r \times u \left(\left[\frac{AOP_j}{\Delta t} \right] \right) \times \left(\frac{AOP_j}{\Delta t} - h \right) \quad (21)$$

Therefore, the average cost of quality is given by

$$ACQ = \sum_{j=0}^M C_i \times n \times (AOP_{j+1} - AOP_j) + C_r \times u \left(\left[\frac{AOP_j}{\Delta t} \right] \right) \quad (22)$$

$$AOP_0 = 0$$

IV. NUMERICAL EXAMPLE

A supply chain is composed of a production system that produces one type of product to meet the delivery for $L=2$ warehouses that will satisfy random demands below over a finite planning horizon: $H=12$ months each of period length $\Delta t=1$ month. Assumed that the standard deviation of each demand of a product is the same for all periods and each demand $\sigma_{d(i:1,2)}=20$ and the initial inventory level we assume that $S(0)=0$. The average demand for customer of warehouse 1 and warehouse 2: $d_1(k)=d_2(k)=150 \{k:0, \dots, H-1\}$.

- Lower and upper boundaries of production capacities: $U_{min}=0$ and $U_{max}=500$ and $C_p = 2 \text{ mu}$, $C_h=0.2 \text{ mu/k}$, $C_s=0.2 \text{ mu/k} \{i:1, \dots, L=2\}$, $S_i(0)=0$ with $\{i:1, 2\}$

- $\mu_0 = 5$, $\sigma_0 = 1.5$

- $C_i = 15\$/\text{product}$, $C_r = 70\$/\text{defective item}$, $\delta = 0.8$

- customer satisfaction degree, is equal to 90% ($\theta=0.9$ ($i=1, 2$)).

- The degradation law of the production system characterized by a Weibull distribution with scale and shape parameters are respectively $\beta_l=100$ and $\alpha_l=2$.

- The costs of corrective and preventive maintenance actions are respectively $C_{cm} = 3000 \text{ mu}$ $C_{pm}=500 \text{ mu}$ (monetary unit).

The economical production and delivery plans for each warehouse 1 and 2 are given in Table I and II:

TABLE I. OPTIMAL PRODUCTION PLAN

$u^*(1)$	$u^*(2)$	$u^*(3)$	$u^*(4)$	$u^*(5)$	$u^*(6)$
440	290	250	320	320	420
$u^*(7)$	$u^*(8)$	$u^*(9)$	$u^*(10)$	$u^*(11)$	$u^*(12)$
410	420	360	230	140	440

TABLE II. OPTIMAL DELIVERY PLAN FOR WAREHOUSE 1 ($\tau^*_1=3$)

$Q_1^*(1)$	$Q_1^*(2)$	$Q_1^*(3)$	$Q_1^*(4)$	$Q_1^*(5)$	$Q_1^*(6)$
250	250	250	250	220	90
$Q_1^*(7)$	$Q_1^*(8)$	$Q_1^*(9)$	-	-	-
160	210	70	-	-	-

TABLE III. OPTIMAL DELIVERY PLAN FOR WAREHOUSE 2 ($\tau^*_1=3$)

$Q_2^*(1)$	$Q_2^*(2)$	$Q_2^*(3)$	$Q_2^*(4)$	$Q_2^*(5)$	$Q_2^*(6)$
160	120	150	160	250	170
$Q_2^*(7)$	$Q_2^*(8)$	$Q_2^*(9)$	-	-	-
100	250	70	-	-	-

Hence, the obtained best strategy consists of taking one sample of size 60 (n^*) every 5 days (h^*). Furthermore, according to the control chart, the optimal number of standard deviations between the center line and the control limits is 3.5 (A^*). Before the first shift to the ‘out of control’ state that occurs on average after $AOP_1=6 \cdot \Delta t=36$ samples, two periodically preventive maintenance actions MP are carried out every $T=3$. $\Delta t=18.h$ and $2T=6 \cdot \Delta t=36.h$. After this first ‘out of control’ and to the end of horizon $H \cdot \Delta t$, an average of 2 PM actions would be carried out.

V. CONCLUSION

This paper dealt with a joint approach of production, maintenance, delivery, and quality problem. The best combination between the degradation of the machine depends on production rates and the control chart parameters are developed to reduce the non-conforming items as well as to guarantee the reliability of the production system. Basing on the proposed production policy and maintenance strategy (PMMR), a mathematical model is developed to minimize the total cost of production, maintenance, inventory, quality by obtaining the different decision variables characterized by the sample size, the sampling interval, the control limits coefficients and the PM period.

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