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Free Piston Stirling Engine as a new heat recovery option for an Internal Reforming Solid Oxide Fuel Cell

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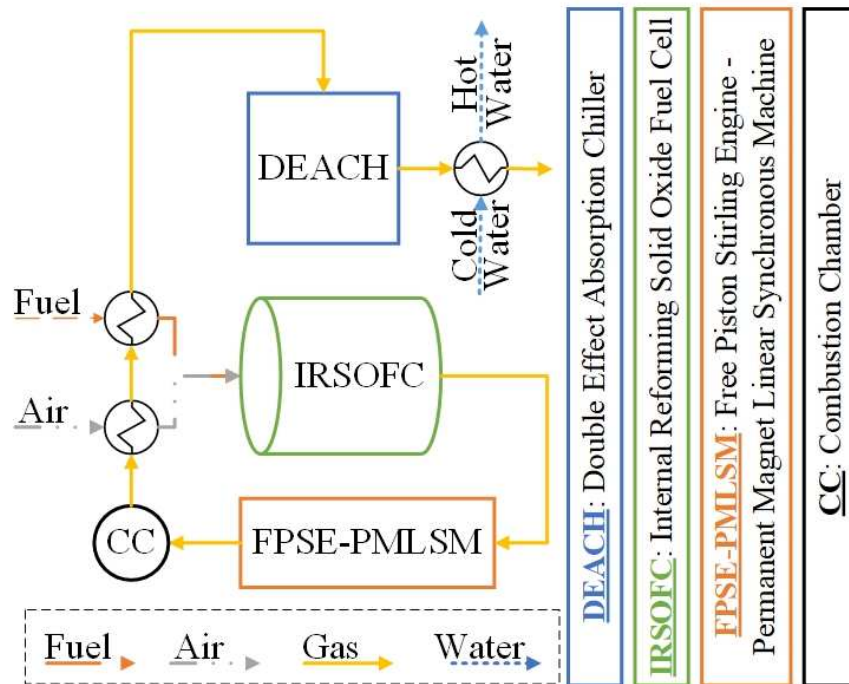
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1 **Abstract** – Energy demand increment besides the reduction of fossil fuel resources from one side, and
 2 climate issues concerning the conventional power production methods from another side lead humanity
 3 to more efficient and cleaner power production methods. One of these methods is polygeneration
 4 microgrids based on a renewable energy source. In the present study, a combined cycle of a Free Piston
 5 Stirling Engine with a Permanent Magnet Linear Synchronous Machine was proposed as a new heat
 6 recovery option for the high-quality thermal energy of an Internal Reforming Solid Oxide Fuel Cell as
 7 electricity. Then, a Double Effect Absorption Chiller and a heat exchanger were used to recover the rest
 8 of the heat as cooling and hot water, respectively. After modeling and validating all the systems, they
 9 were combined. The results of the combined system showed 31.7% (or 40% by including cooling
 10 production of Double Effect Absorption Chiller instead of its heat consumption for efficiency
 11 calculation) of total efficiency improvement compared to the standalone Internal Reforming Solid Oxide
 12 Fuel Cell. Furthermore, the electrical efficiency of the system was improved by 14.1%.

13 **Keywords:** Free Piston Stirling Engine (FPSE); Permanent Magnet Linear Synchronous Machine
 14 (PMLSM); Internal Reforming Solid Oxide Fuel Cell (IRSOFC); Double Effect Absorption Chiller
 15 (DEACH); Waste heat recovery; Microgrid

16 **Graphical Abstract**



17

18 **Nomenclature**

		<i>Greek symbols</i>	
A	Area, m^2	γ	Specific heat ratio
$a; b; c$	Reaction extents, mol/s	ΔG	Gibbs free energy changes, J/mol
$\underline{A}; \underline{B}; \underline{C}; \underline{D}; \underline{E}$	Constants	η	Efficiency
B_v	Friction coefficient	$\theta; \phi$	Angle
C	Clearance, m	Υ	Pre-exponential factor, A/m^2
C_f	Darcy friction factor	ρ	Density, kg/m^3
C_p	Heat transfer coefficient at constant pressure, $J/(kgK)$		

Cv	Heat transfer coefficient at constant volume, J/(kgK)	ψ_f	Flux linkage, Wb
d	Diameter, m	ω	Frequency, rad/s
E	Energy, J/kg	<hr/> <i>Index and exponent</i> <hr/>	
F	Force, N	0	Standard state
Fr	Faraday constant, 96487 C/mol	<i>Act</i>	Activation
h	Convective heat transfer coefficient, W/(m ² K)	<i>An</i>	Anode
H	Enthalpy (J/kg)	<i>b</i>	Buffer
i	Current, A; Current density, A/m ²	<i>c</i>	Compression
i_0	Exchange current density, A/m ²	<i>Ca</i>	Cathode
i_l	Limiting current density, A/m ²	<i>Con</i>	Concentration
K	Reaction equilibrium constant	<i>D</i>	Displacer
L	Inductance, mH, Length, m	<i>d; q</i>	Axis
LHV	Lower Heating Value, J/kg	<i>e</i>	Expansion
l	Length, m	<i>el</i>	Electric
$M; m$	Mass, kg	<i>em</i>	Electromagnetic
\dot{n}_j^i	Molar flow rate of component i in channel j , mol/s	<i>eq</i>	Equilibrium
n_e	Number of electrons	<i>gs</i>	Gas spring
P	Pressure, Pa	<i>h</i>	Heater
\dot{Q}	Heat transfer rate, W	<i>k</i>	Cooler
Q_{rxn}	Reaction heat, J	<i>l</i>	Limiting
R	Gas constant	<i>m</i>	Mover
r	Resistance, Ω	<i>Ohm</i>	Ohmic
Re	Recirculation ratio	<i>p</i>	Power piston
S	Entropy, J/K	<i>pr</i>	Product
T	Temperature, K	<i>R</i>	Regenerator
u	Velocity, m/s	<i>r</i>	Reforming
U_f	Fuel utilization factor	<i>re</i>	Reactant
U_{ox}	Air utilization factor	<i>rev</i>	Reversible
V	Volume, m ³	<i>s</i>	Shifting
v	Voltage, V	<i>w</i>	Wire
X^i	Molar concentration of component i	<hr/> <i>Abbreviation</i> <hr/>	
x	Displacement, m	FPSE	Free Piston Stirling Engine

\dot{x}	Velocity, m/s	PMLSM	Permanent Magnet Linear Synchronous Machine
\ddot{x}	Acceleration, m/s ²	IRSOFC	Internal Reforming Solid Oxide Fuel Cell
Xm	Mass concentration	DEACH	Double Effect Absorption Chiller

1

2 1. Introduction

3 Reduction of fossil fuel resources, on the one hand, and increasing the energy demand, on
4 the other hand, create a big concern about the future of the world energy supply. Adding climate
5 changes to these issues cause much attention to be drawn to an alternative clean and efficient
6 power production source. Nowadays, most of the required power is produced by centralized
7 power plants, which generally have a low efficiency at 30-40%. The results of such a low
8 efficient system are high running costs, high waste of energy, and also many CO₂ productions.
9 The solution to this problem is moving from centralized power production systems to the high
10 efficient and clean microgrids [1].

11 Recently, fuel cells attract much attention as the primary power source for microgrids [2].
12 These systems have higher efficiency compared to internal combustion engines and coal-fired
13 power plants. Also, since fuel cells do not have any moving parts, they have silent and reliable
14 operation. Furthermore, for combined heat and power systems, the exhaust heat of the high-
15 temperature fuel cells can be used. Also, virtually there is no SO_x, NO_x, and particulate
16 emissions [3,4].

17 Coupling the high-temperature fuel cells with heat recovery systems improves the
18 performance of the system. One of the interesting methods to recover this heat is using Stirling
19 engines due to their high efficiency and high potential to reduce overall costs [5]. Furthermore,
20 Stirling engines, because of their external combustions, can be coupled with different systems
21 like solar energy-based systems [6], gasification systems [7], external combustion chambers
22 [8], and high-temperature fuel cells [9]. Stirling engines, compared to conventional systems,
23 like gas turbines, that can be used beside the high-temperature fuel cells, have a silent operation.
24 Another advantage is that conventional heat recovery systems, like gas turbines, generally
25 require high-pressure fuel cells. However, by using Stirling engines, the fuel cell can work at
26 atmospheric pressure [10]. Furthermore, a Stirling engine combined with a fuel cell is more
27 adapted for variable energy demands than a gas turbine or an organic Rankine cycle [11]. These
28 features encouraged researchers to study the combination of the fuel cell with a Stirling engine.

29 Obara et al. [12] have studied the combination of a Woody Biomass Stirling Engine with a
30 Proton Exchange Membrane Fuel Cell. They have used the exhaust high-temperature gases of
31 the Stirling engine for the reforming of the fuel cell. Their goal of this combination has been to
32 improve the exergy efficiency of the Woody Biomass Stirling Engine. Sánchez et al. [10] have
33 used an externally fired - Stirling - piston engine instead of the conventional gas turbine system
34 combined with a fuel cell. They have compared the Stirling engine's combination with Molten
35 Carbonate Fuel Cell with two other commonly used systems for fuel cell heat recovery. They
36 have shown that the combined system's performance with the Stirling engine has been better
37 than other systems. They have confirmed that the benefit of using the Stirling engine has not
38 been just efficiency improvement. Using Stirling engines has made it possible to use fuel cells
39 in the coupled mode at atmospheric pressure and disconnect the Stirling engine from the fuel
40 cell system to operate in standalone mode. Rokni [13] has analyzed a combined cycle of a SOFC

1 with a Stirling engine. He has been able to improve the efficiency of the combined system
2 compared to standalone systems. The power generation of the combined system compared to
3 the SOFC standalone system has been increased by 10%. The efficiency of the combined
4 system has also been about 60%. He has continued his work on this topic with different system
5 combinations [9,14,15]. Escalona et al. [16] have studied the combination of a Molten
6 Carbonate Fuel Cell with two closed systems: a Stirling engine and a carbon dioxide turbine.
7 They have compared both systems' results and finally have concluded that these systems have
8 had better performance compared to the high-pressure fuel cells combined with a gas turbine.
9 Chen et al. [17] have also analyzed a combined system of a Molten Carbonate Fuel Cell with a
10 Stirling engine. They have shown that the performance of the Molten Carbonate Fuel Cell has
11 been greatly improved when it has been combined with the Stirling engine. They have also
12 studied the effect of multiple irreversible losses and different operating conditions like
13 operating temperature and pressure on the system. Mehrpooya et al. [18] have analyzed a
14 combined system of a Molten Carbonate Fuel Cell, a Stirling engine, and a double-effect
15 absorption chiller. They have done an energy and exergy analysis of the system. The overall
16 and electrical efficiency of the system have been 71.71% and 42.28%, respectively.
17 Hosseinpour et al. [19] have also studied a combined system of a SOFC with a Stirling engine.
18 They have studied the effect of current density, SOFC inlet temperature, compression ratio, and
19 regenerator effectiveness on system performance. The efficiency of the combined system has
20 been around 24% more than the SOFC standalone system. The results have also revealed that
21 the SOFC inlet temperature increment to a certain value has increased the combined system's
22 exergy efficiency. Ansarinasab et al. [20] have studied a combined system of a Molten
23 Carbonate Fuel Cell, a Stirling engine, and a gas turbine to produce heating, cooling, and
24 electricity. For cooling, a double-effect absorption chiller has been used. They have done an
25 exergy cost analysis on the system. Habibollahzade et al. [21] have compared three systems
26 from energy, exergy, exergoeconomic, and environmental perspectives. The first system has
27 been a SOFC. The second one has been a SOFC combined with a Stirling engine. The third
28 system has been a SOFC combined with a Stirling engine that the surplus power of the SOFC
29 has been used in a proton exchange membrane electrolyzer to produce hydrogen. From all
30 points of view, SOFC combined system with a Stirling engine has been the best system. Luo et
31 al. [11] have studied a combined cooling and power generation system based on four power
32 systems: SOFC, SOFC-gas turbine, SOFC-Stirling engine, and SOFC-organic Rankine cycle.
33 The effect of some parameters like fuel inlet temperature, SOFC current density, and gas turbine
34 pressure ratio on the system has been studied. For constant energy demand, the SOFC-gas
35 turbine system has been recommended, while for the variable energy demand of the building,
36 the SOFC-Stirling engine system has been suggested. Marefati et al. [22] have analyzed a
37 combined system of a SOFC, a Stirling engine, a steam turbine, a linear Fresnel solar field, and
38 a double effect absorption chiller to produce electricity, heating, and cooling. The electrical,
39 energy, and exergy efficiencies of the system have been 49.7%, 67.5%, and 55.6%,
40 respectively. Also, a sensitivity analysis of the system has been done. Salehi et al. [23] have
41 studied and compared three different systems coupled with a biomass gasifier: Molten
42 Carbonate Fuel Cell system, Molten Carbonate Fuel Cell with Stirling engine, and Molten
43 Carbonate Fuel Cell with Stirling engine and Rankine cycle. Energetic, exergetic, and
44 environmental analyses have been done on the systems. The maximum efficiency of the whole
45 combined system has been 28.4% more than the Molten Carbonate Fuel Cell standalone system.
46 Moradpoor et al. [24] have analyzed a combined system of a Molten Carbonate Fuel Cell,
47 Stirling engine, absorption chiller, and Kalina power generation cycle. They have also done a
48 parametric and sensitive analysis. The results have shown that this system has saved up to 45%
49 of fuel than a conventional system. The overall and electrical efficiencies of this system have
50 been 70% and 50%, respectively. Moradi et al. [25] have studied a combined cycle of a SOFC

1 with a multi-effect desalination system and a Stirling engine. They have shown that the
2 combined system has had 11.7% higher efficiency compared to the same system but in the
3 absence of the Stirling engine.

4 As can be seen, the Stirling engine has been used as a heat recovery option for different types
5 of fuel cells. For the present study, an IRSOFC is chosen as the main power production source.
6 SOFC, compared to other fuel cells, has the advantage that can work with a wide range of fuels
7 [26,27], including methane [28], ethanol [29], carbon [30], syngas, hydrogen [31], biogenous
8 fuels such as biogas [32], and so on. However, for some types of fuels, there might be some
9 constraints and considerations related to trace contaminants [33,34]. The ability to work with
10 different fuels makes it possible for SOFC to handle fluctuations of fuel compositions or fuel
11 type change [35]. Also, compared to most other types of fuel cells, SOFC produces a high
12 amount of exhaust heat, which is useful for combined systems, specially multigeneration ones
13 [4].

14 Stirling engines also can be classified into different categories. From the energy conversion
15 point of view, they can be classified into standard Stirling engines and Free Piston Stirling
16 Engines (FPSE). Standard Stirling engines convert the linear movement of the piston to a rotary
17 one through a crankshaft to produce electricity by a rotary generator. FPSEs directly use the
18 linear movement of the piston in a linear generator to produce electricity. FPSEs are more
19 compact and lightweight than standard Stirling engines, and also, due to the better sealing, they
20 can keep working gasses for more working cycles inside them [36,37]. Such aspects make FPSE
21 combined with a linear generator a perfect choice for the heat recovery of microgrids. Also, for
22 the linear generator, a three-phase Permanent Magnet Linear Synchronous Machine (PMLSM)
23 is used. A three-phase machine needs less wire compared to an equivalent one-phase one, and
24 also, a permanent magnet generator has higher efficiency compared to an induction one [38,39].

25 Thus, an FPSE-PMLSM system is used to recover the high-quality thermal energy of the
26 exhaust gasses of the IRSOFC. Then, a double-effect absorption chiller and a heat exchanger
27 are used to recover the rest of the thermal energy in the forms of cooling and heating,
28 respectively.

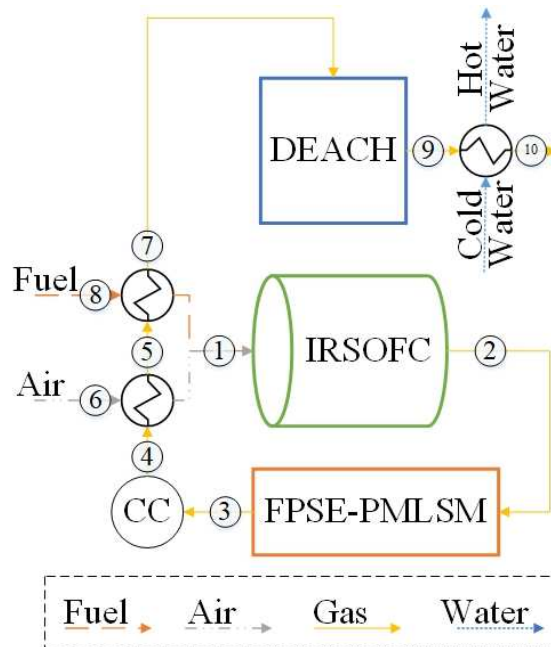
29 The most important point which makes the current study original, and has not been already
30 done, is using a Free Piston Stirling Engine combined with a Permanent Magnet Linear
31 Synchronous Machine (FPSE-PMLSM) as a waste heat recovery option for a Fuel Cell.
32 Considering the advantages of Solid Oxide Fuel Cell (SOFC) and FPSE-PMLSM (that were
33 already discussed in detail) makes them a great candidate for distributed power generation
34 systems. These interesting aspects of both systems encouraged the authors to study their
35 combination in such a newly developed trigeneration configuration. The detailed modeling of
36 the systems, especially the FPSE-PMLSM one that a precise thermo-dynamic and electric
37 model of it was developed, is another strength of the present study that has not been done before
38 in such a combination. Providing only a thermic model of the FPSE-PMLSM system and
39 ignoring the electric and thermodynamic model as done by many researchers and explained in
40 [40,41] may cause a loss of precision, leading to unrealistic performance estimation.

41 In the present study, at first, an IRSOFC was modeled and validated. In the next step, a
42 nonlinear thermodynamic model of an FPSE was presented. This model could predict the
43 behavior of the system in dynamic conditions. After FPSE validation, a PMLSM was modeled
44 as its generator. Then, FPSE was coupled with PMLSM, and the combined system was
45 controlled. After that, a Double Effect Absorption Chiller (DEACH) was modeled and
46 validated. Since there was no need to have a high-pressure fuel cell, the IRSOFC in the
47 combined system worked at atmospheric pressure. Since electricity production was the main
48 interest of the proposed combined system which is a part of a microgrid, to recover the high-

1 quality heat of the IRSOFC, the FPSE-PMLSM system was put before the IRSOFC gas/air
 2 preheaters. Finally, the results of the whole combined system were presented in the last section.

3 2. System Description

4 In Figure 1, the schematic of the combined system is shown. At first, air and fuel are fed to
 5 IRSOFC ① to produce electricity. The exhaust gas of the Internal Reforming Solid Oxide Fuel
 6 Cell (IRSOFC) ② has a high temperature that is used in a Free Piston Stirling Engine combined
 7 with a Permanent Magnet Linear Synchronous Machine (FPSE-PMLSM) to produce electricity.
 8 Then, the gas leaves the FPSE-PMLSM system ③ with a decreased temperature. Since this
 9 gas still contains burnable substances, a Combustion Chamber (CC) is used to burn them and
 10 increase the gas temperature ④. Then, this gas passes through a heat exchanger ⑤ to preheat
 11 the input air ⑥. After that, it passes through another heat exchanger ⑦ to preheat the input
 12 fuel ⑧. This gas still has enough energy to run a Double Effect Absorption Chiller (DEACH)
 13 ⑨. The final low-quality thermal energy heats the water ⑩ and provides hot water.



14
 15

Figure 1: Combined system schematic

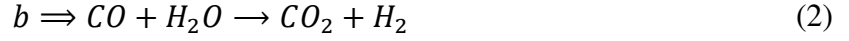
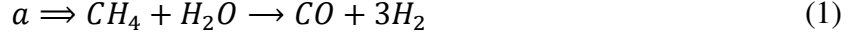
16 3. IRSOFC Analysis

17 Here a zero-dimensional model of IRSOFC is developed. The following assumptions are
 18 made to model the IRSOFC system:

- 19 - The IRSOFC operates at a steady-state
- 20 - The IRSOFC is isolated and has no heat transfer with the environment
- 21 - It is a tubular IRSOFC
- 22 - The radiation heat transfer between solid structure and gas channels is ignored
- 23 - The contact resistances are ignored

24 3.1. Internal Reformer

25 An internal reformer, compared to an external one, has higher efficiency and lower cost [42];
 26 thus, an internal reformer was chosen. The three main reactions that happen inside the reformer
 27 are:



1 In these equations, a , b , and c are extents of reforming, shifting, and electrochemical
2 reactions (mol/s), respectively. The equilibrium constants of reforming (r subscript-eq (1))
3 and shifting (s subscript- eq(2)) reactions can be calculated based on partial pressures of gasses
4 in the reactions which are related to their equilibrium molar concentrations:

$$K_r = (P^{CO}(P^{H_2})^3)/(P^{CH_4}P^{H_2O}) = (X_{eq}^{CO}(X_{eq}^{H_2})^3)/(X_{eq}^{CH_4}X_{eq}^{H_2O}) \times (P/P_0)^2 \quad (4)$$

$$K_s = (P^{CO_2}P^{H_2})/(P^{CO}P^{H_2O}) = (X_{eq}^{CO_2}X_{eq}^{H_2})/(X_{eq}^{CO}X_{eq}^{H_2O}) \quad (5)$$

5 These coefficients are also a function of temperature:

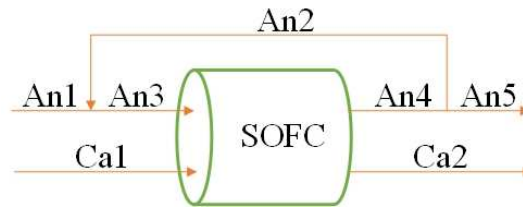
$$\ln K = \underline{A}T^4 + \underline{B}T^3 + \underline{C}T^2 + \underline{D}T + \underline{E} \quad (6)$$

6 \underline{A} , \underline{B} , \underline{C} , \underline{D} , and \underline{E} are constants, and their values are presented in Table 1.

7 *Table 1: Constants to calculate equilibrium constants of equations [43]*

	Reforming (for K_r)	Shifting (for K_s)
\underline{A}	-2.63121×10^{-11}	5.47301×10^{-12}
\underline{B}	1.24065×10^{-7}	-2.57479×10^{-8}
\underline{C}	-2.25232×10^{-4}	4.63742×10^{-5}
\underline{D}	1.95028×10^{-1}	-3.91500×10^{-2}
\underline{E}	-6.61395×10	1.32097×10

8 Since in equation (4) and (5), partial gas pressures are related to their equilibrium molar
9 concentrations, and these equations are rewritten based on equilibrium molar concentrations,
10 the first step to model the reformer is to calculate gas compositions at different channels of
11 Figure 2. Since the chemical reactions in the anode need water as a reactant, and the output gas
12 mixtures of the anode ($An4$) has high water content, the fuel ($An1$) before entering the anode
13 mixes with the recirculated gas mixture ($An2$). Also, the oxidant is fed to cathode ($Ca1$).



14
15 *Figure 2: SOFC flow channels*

16 Based on Figure 2, the molar gas flow rate for $An3$ and $An4$ for each gas (i) can be
17 calculated as [44]:

$$\dot{n}_{An3}^i = \dot{n}_{An1}^i + \dot{n}_{An2}^i = \dot{n}_{An1}^i + X_{eq}^i \dot{n}_{An2} = \dot{n}_{An1}^i + X_{eq}^i (Re \dot{n}_{An4}) \quad (7)$$

$$\dot{n}_{An4}^i = \dot{n}_{An3}^i + d^i \rightarrow \dot{n}_{An4} = \dot{n}_{An3} + 2a \quad (8)$$

Here \dot{n}_j^i is the molar flow rate of component i in channel j and \dot{n}_j is the total molar flow rate in channel j . The values of d^i based on a , b , and c are given in Table 2.

Table 2: d^i values to calculate An4 gas flow rates [44]

d^{CH_4}	$-a$	d^{CO_2}	b
d^{H_2O}	$-a - b + c$	d^{H_2}	$3a + b - c$
d^{CO}	$a - b$	d^{N_2}	0

Thus, the total gas flow rate in An3 is [44]:

$$\dot{n}_{An3} = (\dot{n}_{An1} + 2aRe)/(1 - Re) \quad (9)$$

Re is the recirculation ratio that here is assumed to be equal to 0.6. Combination of equations (7) - (8) allows calculating the equilibrium gas molar composition in An4 Channel [44]:

$$X_{eq}^i = (\dot{n}_{An4}^i)/(\dot{n}_{An4}) = (\dot{n}_{An1}^i + d^i)/(\dot{n}_{An1} + 2a) \quad (10)$$

For CH_4 , CO_2 , CO , and N_2 , the d^i values are based on Table 2, and for H_2O and H_2 the equilibrium gas molar compositions in An4 are as [44]:

$$X_{eq}^{H_2} = (\dot{n}_{An1}^{H_2} + 3a + b)/(\dot{n}_{An1} + 2a) \times ((1 - Re)(1 - U_f))/(1 - Re + ReU_f) \quad (11)$$

$$X_{eq}^{H_2O} = (\dot{n}_{An1}^{H_2O} + (-a - b + U_f(\dot{n}_{An1}^{H_2} + 3a + b)/(1 - Re + ReU_f)))/(\dot{n}_{An1} + 2a) \quad (12)$$

U_f in these equations is the fuel utilization factor, which is assumed to be equal to 0.85. For the cathode side (air channel), gases molar flow rates and equilibrium gases molar compositions are as [44]:

$$\dot{n}_{Ca1}^{O_2} = c/(2U_{ox}) \quad (13)$$

$$\dot{n}_{Ca1}^{N_2} = c/(2U_{ox}) \times 79/21 \quad (14)$$

$$\dot{n}_{Ca2}^{O_2} = c/(2U_{ox}) - c/2 \quad (15)$$

$$\dot{n}_{Ca2}^{N_2} = c/(2U_{ox}) \times 79/21 \quad (16)$$

Here U_{ox} is the air utilization factor, which is assumed to be 0.25. The unknown variables of these equations are a , b , and total mass flow rate in An1. These variables can be found by solving the equations (4), (5), and the following equation (17) at the same time [44]:

$$i_{cell}A_{cell} = 2cFr = 2 \left(\left((\dot{n}_{An1}^{H_2} + 3a + b)U_f \right) / (1 - Re + ReU_f) \right) Fr \quad (17)$$

Unknown parameters of these equations are a , b and fuel flow rate (\dot{m}_{fuel}) which is needed to calculate \dot{n}_{An1}^i . Since c can be calculated based on a and b , it is not an independent parameter. The flow chart of the internal reformer modeling procedure is shown in Figure 3.

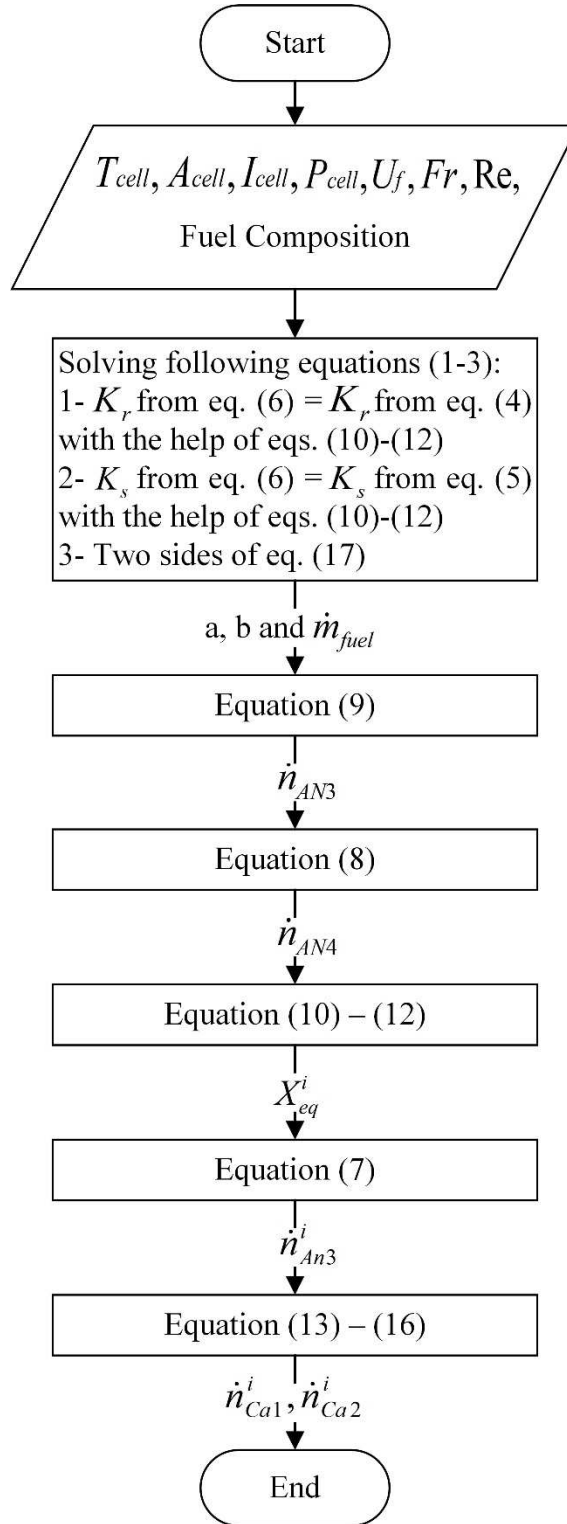


Figure 3: Internal reformer flow chart

1
2

3.2. SOFC Voltage

4 The reversible voltage of the cell can be calculated based on the Nernst equation [45]:

$$\begin{aligned}
 v_{rev} &= v_0 + \Delta v = -\Delta G_0 / (n_e Fr) - \Delta G / (n_e Fr) \\
 &= -\Delta G_0 / (2Fr) + (RT) / (2Fr) \ln(P^{H_2} (P^{O_2})^{0.5} / P^{H_2O})
 \end{aligned} \tag{18}$$

1 n_e is the number of electrons that here is equal to 2. Partial pressures of gasses are related to
 2 their molar concentrations in *An4* that were already calculated in section 3.1. Due to
 3 irreversibilities, the actual voltage is smaller than this value and is equal to [45]:

$$v = v_{rev} - (v_{Act} + v_{Ohm} + v_{Conc}) \quad (19)$$

4 As can be seen, there are three different polarization types: activation (*Act* subscription),
 5 ohmic (*Ohm* subscription), and concentration (*Conc* subscription). Activation polarization can
 6 be calculated as [43]:

$$\begin{aligned} v_{Act} &= v_{Act,An} + v_{Act,Ca} \\ &= 2RT/(n_e Fr) \sinh^{-1}(i/(2i_{0,An})) \\ &\quad + 2RT/(n_e Fr) \sinh^{-1}(i/(2i_{0,Ca})) \end{aligned} \quad (20)$$

$$i_{0,An} = Y_{An}(P^{H_2}/P_0)(P^{H_2O}/P_0) \exp(-E_{Act,An}/(RT)) \quad (21)$$

$$i_{0,Ca} = Y_{Ca}(P^{O_2}/P_0)^{0.25} \exp(-E_{Act,Ca}/(RT)) \quad (22)$$

7 Y_i are exchange current density pre-exponential factor and $E_{Act,i}$ are activation energies [46].
 8 Partial pressures of gasses are related to their molar concentrations in *An4* that were already
 9 calculated in section 3.1. Then for ohmic and concentration polarization following equations
 10 can be written [47,48].

$$v_{Ohm} = \sum ir = \sum i \delta \underline{A} \times \exp(\underline{B}/T) \quad (23)$$

$$v_{Conc} = (RT/(n_e Fr))(1 - i/i_l) \quad (24)$$

11 Constants to calculate ohmic polarization are presented in Table 3.

12 *Table 3: Constants to calculate ohmic polarization [47,48]*

	$\underline{A}(\Omega. cm)$	$\underline{B}(K)$	$\delta(cm)$
Anode	0.00298	1392	0.01
Cathode	0.00814	-600	0.19
Electrolyte	0.00294	-10350	0.004
Internal connections	0.1256	-4690	0.0085

13 Now to calculate cell voltage, at first based on equations (20) - (22) activation polarization
 14 (v_{Act}), based on equation (23), ohmic polarization (v_{Ohm}), and based on equation (24),
 15 concentration polarization (v_{Conc}) should be calculated. Then these values based on equation
 16 (19) are extracted from the reversible voltage (v_{rev}) that can be calculated based on equation
 17 (18) to calculate cell voltage (v).

18 3.3. IRSOFC Temperature

19 IRSOFC temperature can be calculated based on a trial and error method. The energy balance
 20 is based on the idea that the heat generation inside IRSOFC should be equal to the energy
 21 increment of gasses pass through it. Thus, thermal energy generation/consumption due to three
 22 main equations inside IRSOFC (Eqs. (1)-(3)) and due to irreversibilities of polarizations should
 23 be calculated and compared to the required energy to increase the temperature of reactants and
 24 products to the cell temperature. Based on this idea, the temperature of the IRSOFC is
 25 calculated [49]. The heat production due to irreversibilities and also electrochemical reaction
 26 inside IRSOFC are as follows:

$$Q_{rxn} = i_{cell} A_{cell} (v_{Act} + v_{Ohm} + v_{Conc}) + T_{cell} \Delta S \quad (25)$$

$$\Delta S = (S_0^{H_2O} - S_0^{H_2} - (1/2) S_0^{O_2}) - R \ln(P^{H_2} (P^{O_2})^{0.5} / P^{H_2O}) \quad (26)$$

1 The required heat for reforming and shifting processes can also be calculated as:

$$Q_r = a(H^{CO} + 3H^{H_2} - H^{H_2O} - H^{CH_4}) \quad (27)$$

$$Q_s = b(H^{CO_2} + H^{H_2} - H^{H_2O} - H^{CO}) \quad (28)$$

2 The produced heat (Eq. (25)) is used to provide required heat for the reforming and shifting
3 reactions (Eqs. (27) and (28)), and also, to heat the gas streams, that can be calculated as:

$$Q_{heat\ up} = \left(\dot{n}_{An, re} \int_{T_{An}}^{T_{cell}} Cp\ dT \right)_{re} + \left(\dot{n}_{An, pr} \int_{T_{An}}^{T_{cell}} Cp\ dT \right)_{pr} \quad (29)$$

$$+ \left(\dot{n}_{Ca, re} \int_{T_{Ca}}^{T_{cell}} Cp\ dT \right)_{re} + \left(\dot{n}_{Ca, pr} \int_{T_{Ca}}^{T_{cell}} Cp\ dT \right)_{pr}$$

4 Now, the remaining thermal energy of the reactions are compared to the required energy for
5 the heat up to calculate the cell temperature through a trial and error procedure. The heat transfer
6 with the surrounding is also ignored.

$$Error = (Q_{heat\ up} - (Q_{rxn} - Q_r - Q_s)) / (Q_{rxn} - Q_r - Q_s) < 1\% \quad (30)$$

7 The procedure of the temperature calculation is shown in Figure 4.

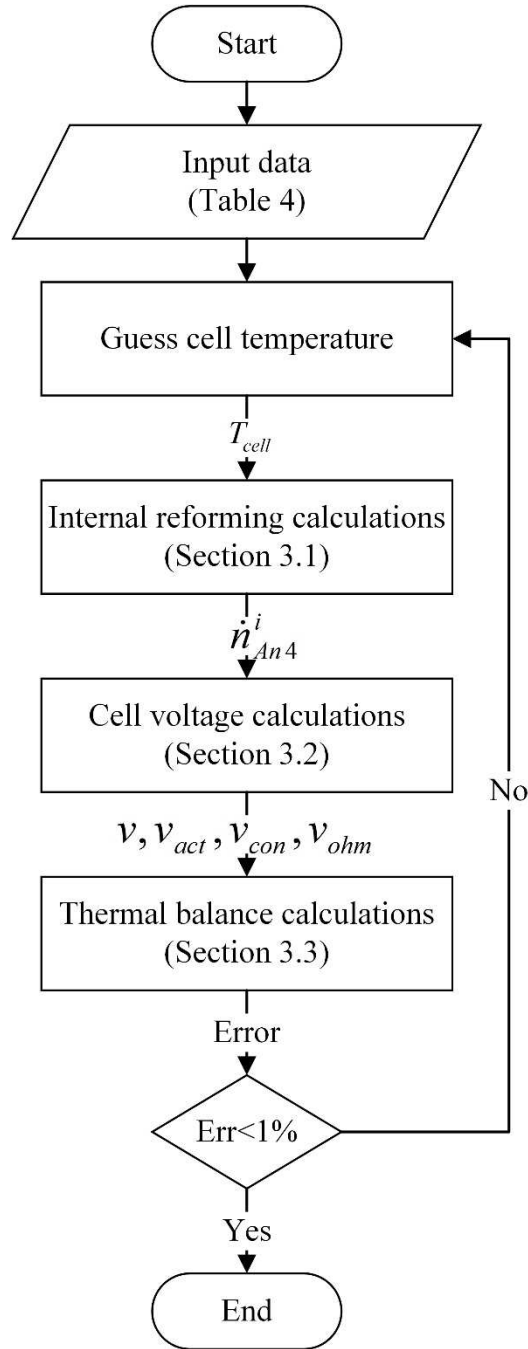


Figure 4: Flowchart of trial and error procedure for IRSOFC temperature calculation

3.4. IRSOFC Results and Validation

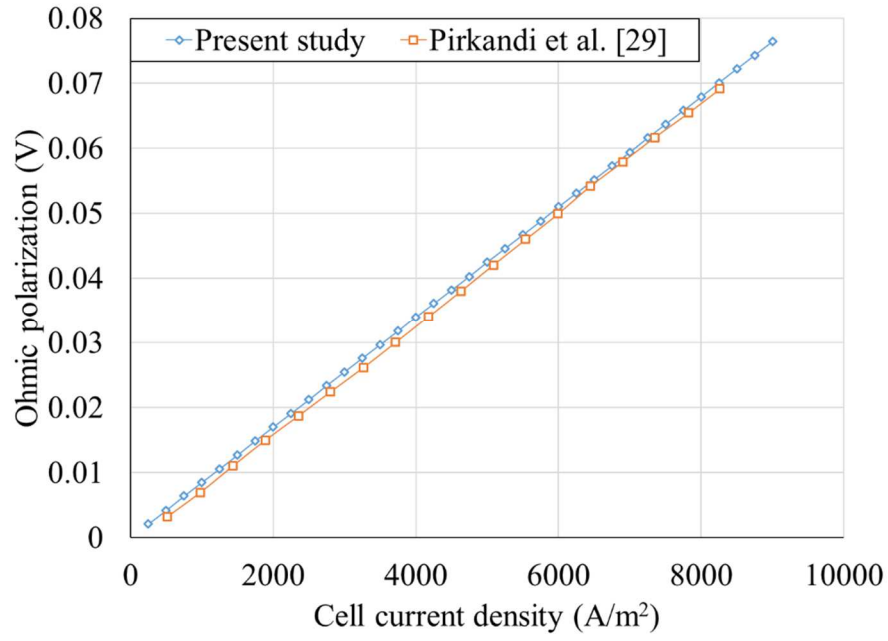
The IRSOFC is modeled in MATLAB® with the equations presented in the previous section. In order to validate it, the results of the modeling are compared with Pirkandi et al. [49] ones (Figure 5 - Figure 9), considering the same input data, which is shown in Table 4.

Table 4: IRSOFC input data for validation

U_f	0.85
U_{ox}	0.25
Re	60 %

P	1 bar
A_{cell}	0.10362 m ²
i_l	9000 A/m ²
Number of cells	8792

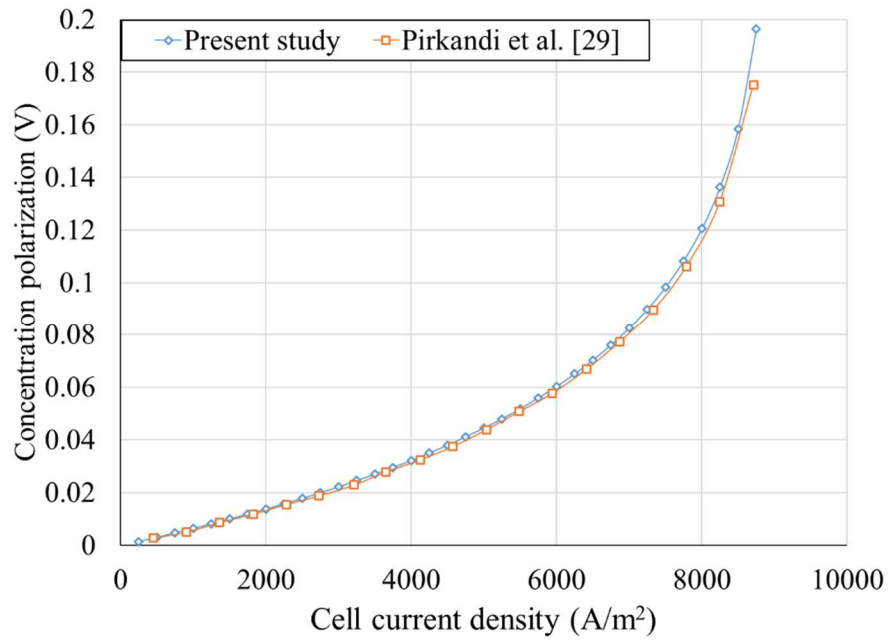
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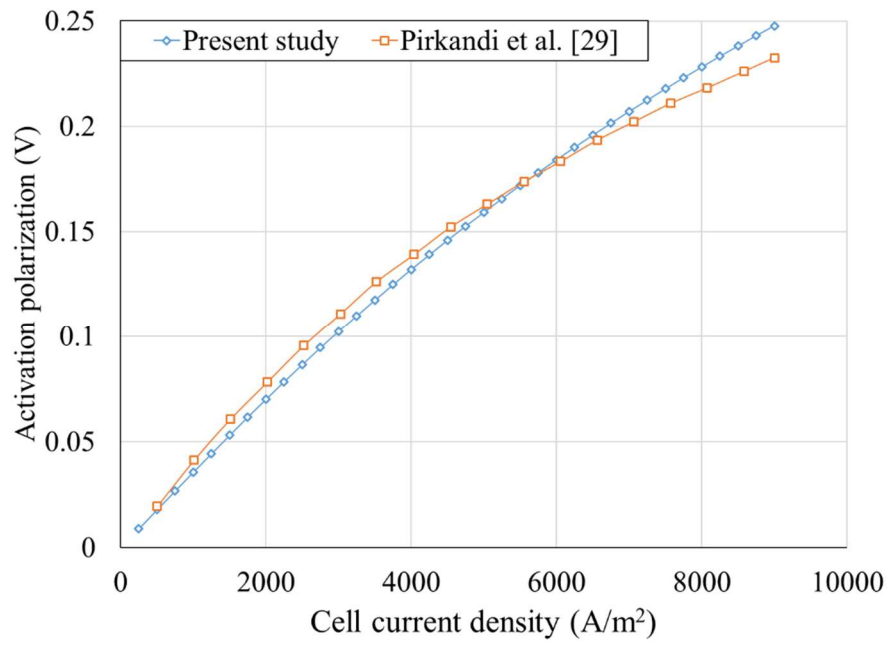
Figure 5: Validation of ohmic polarization



4

5

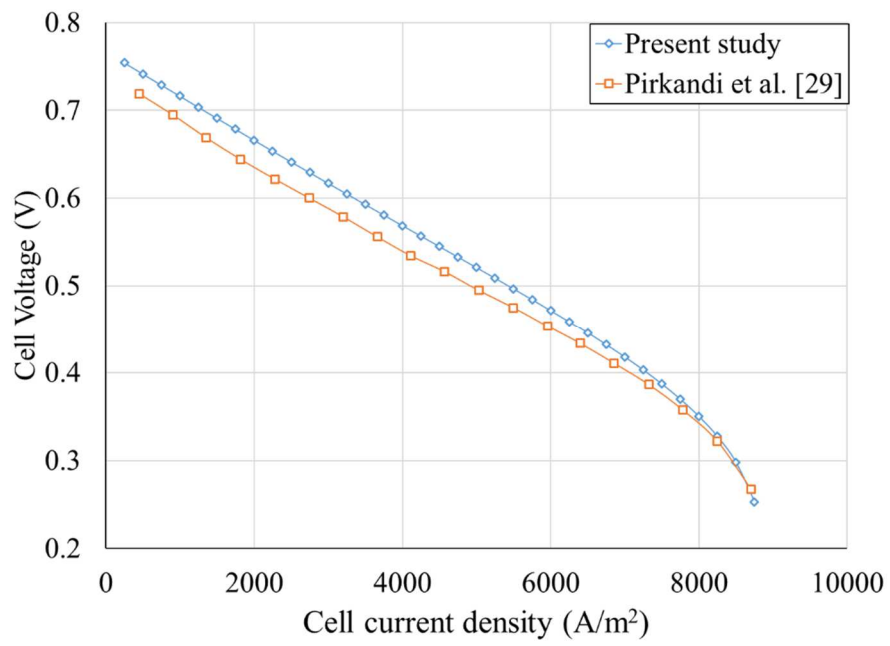
Figure 6: Validation of concentration polarization



1

2

Figure 7: Validation of activation polarization



3

4

Figure 8: Validation of cell voltage

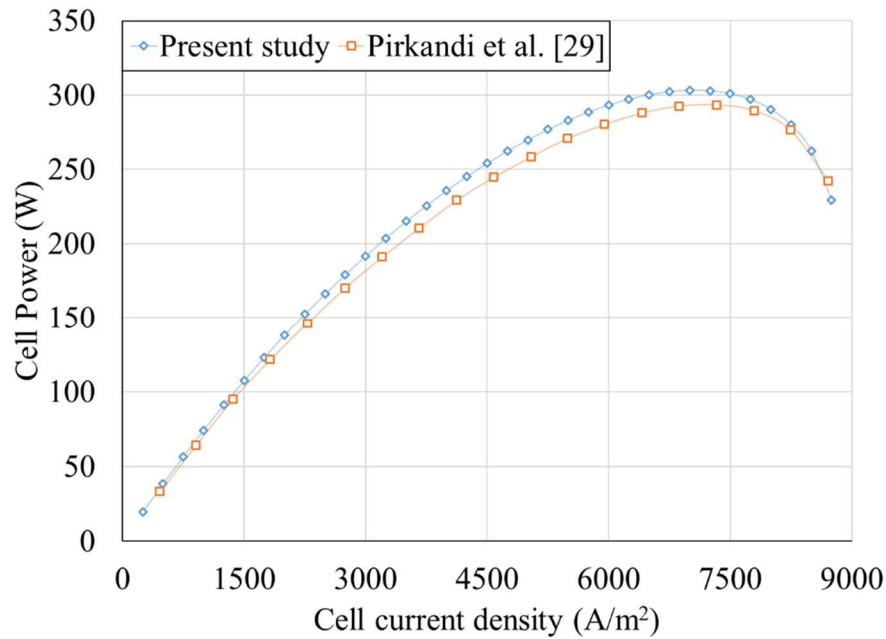


Figure 9: Validation of cell power

As can be seen in Figure 5 - Figure 9, there is a good agreement between the present study and Pirkandi et al. [49] results that shows the IRSOFC model is valid. The effect of the current density on the power and voltage is also shown. By increasing the current density, the voltage of the cell is continuously decreasing. Also, this increment increases the power by a point, and after that (around 7500 A/m²), the power decreases. It is important to choose an appropriate working current density for the system to satisfy the requirements.

4. FPSE-PMLSM Analysis

There are three types of Stirling engines: alpha, beta, and gamma. For the present study, due to the higher power density of the beta-type compared to alpha and gamma-type Stirling engines, the beta-type Stirling engine is chosen [50]. The schematic of the FPSE combined with the PMLSM is shown in Figure 10.

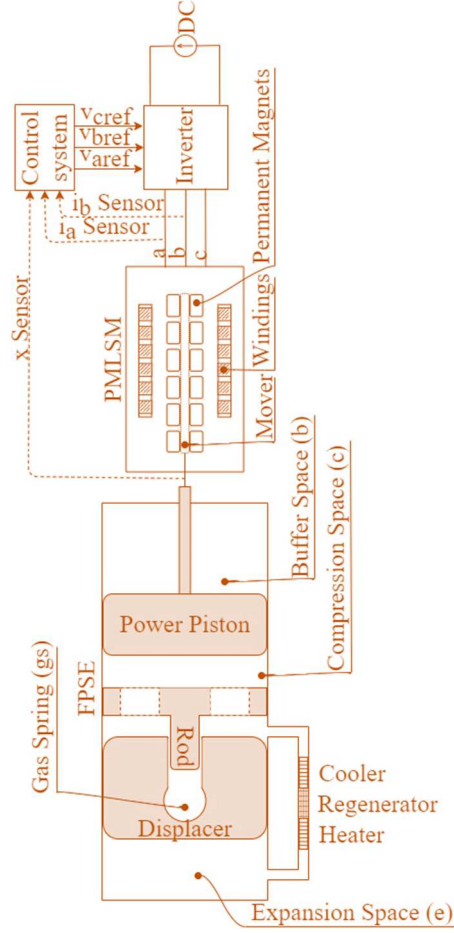


Figure 10: FPSE-PMLSM schematic [40]

The FPSE system is a beta type Sunpower RE-1000 engine, and the PMLSM is a three-phase one. The FPSE converts thermal energy into linear movement. This produced linear movement in the power piston is transferring to the PMLSM mover through a rod. In PMLSM, the linear movement of the rod is converting to electrical power. A control system is also used beside an inverter to control the combined system.

Based on the main spaces of the FPSE dynamic equations for the combined system around power piston (p subscript) and displacer piston (D subscript) can be written as [40]:

$$(m_p + m_m)\ddot{x}_p = A_p(P_c - P_b) + F_{em} - B_v\dot{x}_p \quad (31)$$

$$m_D\ddot{x}_D = A_D P_e - (A_D - A_{rod})P_c - A_{rod}(P_{gs}) = A_D(P_e - P_c) + A_{rod}(P_c - P_{gs}) \quad (32)$$

In these equations, at first, pressures and electromagnetic force should be calculated to be able to calculate \ddot{x}_i , \dot{x}_i , and finally x_i . Pressures of buffer space (P_b) and displacer gas spring space (P_{gs}) can be calculated based on the adiabatic assumption [40]:

$$P_i = P^{mean} (V_i^{mean} / V_i^{instantaneous})^\lambda \quad i: b, gs \quad (33)$$

Compression pressure at each moment can be calculated as [40]:

$$P_c = MR \left(\frac{V_c}{T_c} + \frac{V_e}{T_e} + \frac{V_h}{T_h} + \frac{V_k}{T_k} + \frac{V_{R1}}{T_{R1}} + \frac{V_{R2}}{T_{R2}} \right)^{-1} \quad (34)$$

1 In equation (34) V_i and T_i are instantaneous values. M is total mass, which is constant. As
 2 can be seen, to have a more precise model of the regenerator, it is divided into two parts
 3 ($R1, R2$). Other pressures can be calculated based on P_c and pressure drops in the heater, cooler,
 4 and regenerator. Details of the pressure drops calculations have been already discussed by
 5 Majidniya et al. [40], which are based on the following nonlinear equation:

$$\Delta P = \frac{1}{2} \rho \left(\frac{C_f l}{d_{hydraulic}} \right) u |u| \quad (35)$$

6 Also, T_i of equation (34) can be calculated based on the energy equation in each space:

$$\dot{Q}_{in} + (\dot{m}CpT)_{in} - (\dot{m}CpT)_{out} - \dot{W}_{out} = Cv \frac{d}{dt} (mT) \quad (36)$$

7 For spaces with constant volumes (heater, cooler, and regenerator), there is no work ($\dot{W}_{out} =$
 8 0) and for expansion and compression spaces, it is assumed that there is no heat transfer with
 9 the environment ($\dot{Q}_{in} = 0$). Here two key parameters are $\dot{m}_{in \& out}$ and $T_{in \& out}$. For
 10 temperatures, it is assumed that when the mass flows from space i to space j , the temperature
 11 in the boundary is equal to T_i and vice versa. To calculate the mass flow rates in boundaries, at
 12 first, the mass variation at each space is calculated. Then, based on these mass variations, the
 13 mass flow rate at each boundary can be calculated. For example, the mass flow rate from cooler
 14 to compression space is equal to mass variation in compression space. It means that the
 15 summation of mass variations before/after each boundary is equal to the mass flow rate through
 16 that boundary. For example, the summation of mass variations in cooler and compression space
 17 is the mass flow rate from the first part of the regenerator ($R1$) to cooler. In order to calculate
 18 heat flow in heater, cooler, and regenerator, the following equation can be used:

$$\dot{Q}_i = h_i A_i^{wetted} (T_i^{wall} - T_i) \quad i: h, k, R1, R2 \quad (37)$$

19 After solving equation (36), instantaneous gas temperatures in each space can be calculated.
 20 A more detailed thermodynamic model of the FPSE was presented in [41]. Furthermore,
 21 PMLSM equations in $d - q$ frame to model the combined system are as [40]:

$$v_d = L_d \frac{di_d}{dt} + r i_d - \frac{\pi}{\tau} \dot{x} L_q i_q \quad (38)$$

$$v_q = L_q \frac{di_q}{dt} + r i_q + \frac{\pi}{\tau} \dot{x} (L_d i_d + \sqrt{3/2} \psi_f) \quad (39)$$

$$F_{em} = \frac{\pi}{\tau} (\sqrt{3/2} \psi_f i_q + (L_d - L_q) i_d i_q) \quad (40)$$

$$Power_{em} = F_{em} \dot{x}_p \quad (41)$$

$$Power_{el} = v_q i_q + v_d i_d \quad (42)$$

22 The input parameters of PMLSM are presented in Table 5.

23

Table 5: Input parameters of PMLSM

r	0.1 (Ω)	L_q	3.01 (mH)
L_d	1.77 (mH)	ψ_f	0.0513 (Wb)

24 To control the combined system, two PR controllers (for i_q and \dot{x}_p) and one PI controller
 25 (for i_d) are used. The model of the FPSE-PMLSM system is developed in MATLAB
 26 Simulink®.

1 **4.1. FPSE-PMLSM Results and Validation**

2 Before coupling FPSE with PMLSM, its equations based on Schreiber [51] input data
 3 (shown in Table 6) are solved, and the results are presented in Table 7. These input data and
 4 experimental results have also been used by the recently published articles [40,41] to validate
 5 the FPSE model.

6

Table 6: FPSE input data for validation

T_h^{wall}	814.3 K
T_k^{wall}	322.8 K
T_{R1}^{wall}	418.4 K
T_{R2}^{wall}	662.7 K
p^{mean}	71 bars
<i>Porosity</i>	75.9 %
m_D	0.426 kg
m_p	6.2 kg
d_p	5.718 cm
d_D	5.67 cm
d_{rod}	1.663 cm
d_h	0.2362 cm
d_w	0.00889 cm
L_k	7.92 cm
L_h	18.34 cm
L_R	6.44 cm
A_h	1.4898 cm ²
A_k	2.6163 cm ²
$A_{R1,R2}$	8.745 cm ²
C_c	1.83 cm
C_e	1.861 cm
V_b^{mean}	2615 cm ³
V_{gs}^{mean}	37.97 cm ³
$V_{R1,R2}$	28.185 cm ³
A_k^{wetted}	115.2 cm ²

7

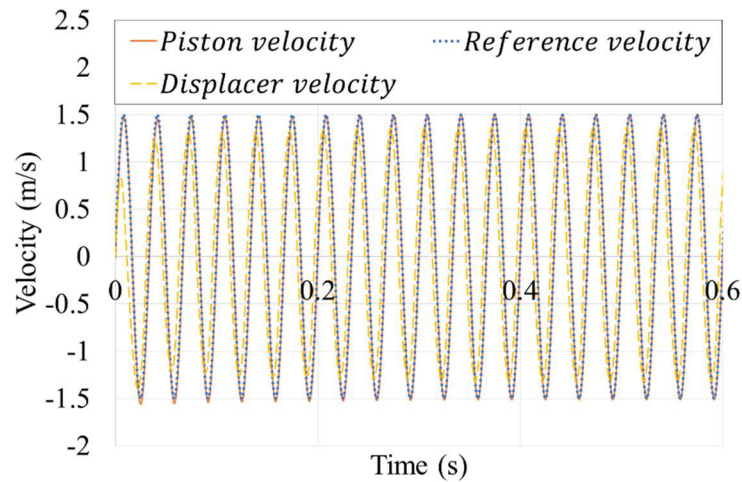
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Table 7: Validation of FPSE thermodynamic model

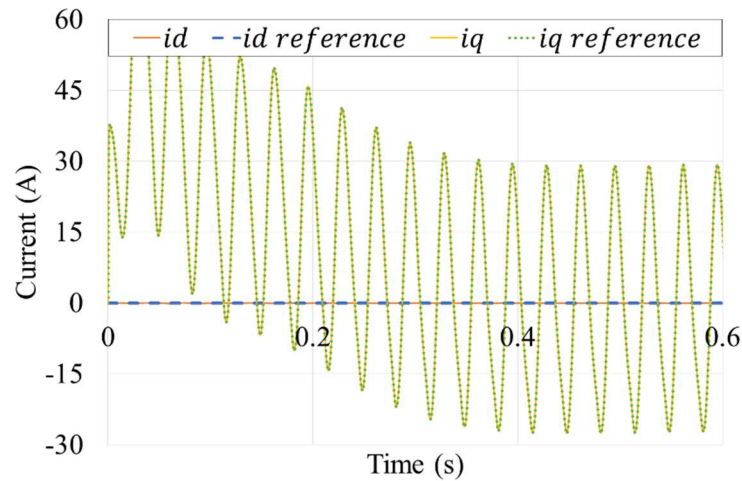
Exp. [51]	Theoretical	Error (%)
-----------	-------------	-----------

Power (W)	1000	1003.9	0.39
Frequency (Hz)	30.2	28.57	5.4
Phase Shift (°)	47.6	32.94	30.8
Stroke _p (cm)	2.32	2.746	18.4
Stroke _d (cm)	2.55	2.379	6.7
Efficiency (%)	27.4	30.02	9.56

1 As can be seen, the FPSE model is valid. After that, the thermodynamic-electric coupled
2 model of FPSE-PMLSM is solved, and the combined system is controlled. The results of the
3 combined model are presented in Figure 11 - Figure 14.



4
5 *Figure 11: Velocity variations*



6
7 *Figure 12: Current variations*

8 As shown in Figure 11 and Figure 12, the system based on one PI and two PR controllers is
9 well controlled. All the parameters are precisely following their reference values.

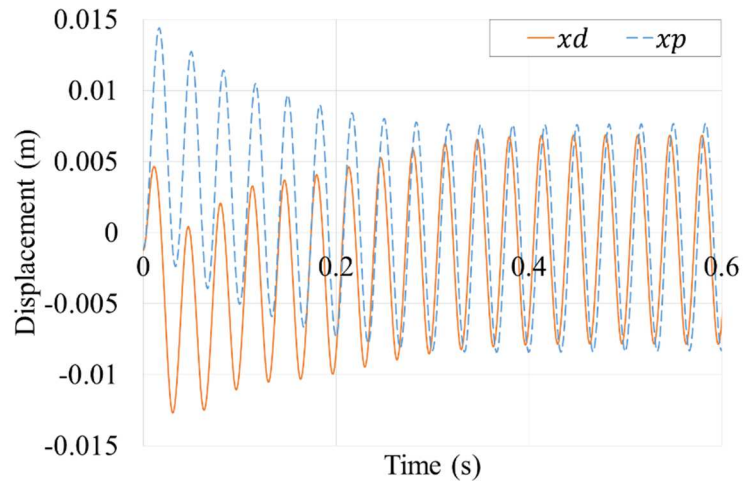


Figure 13: Displacement variations

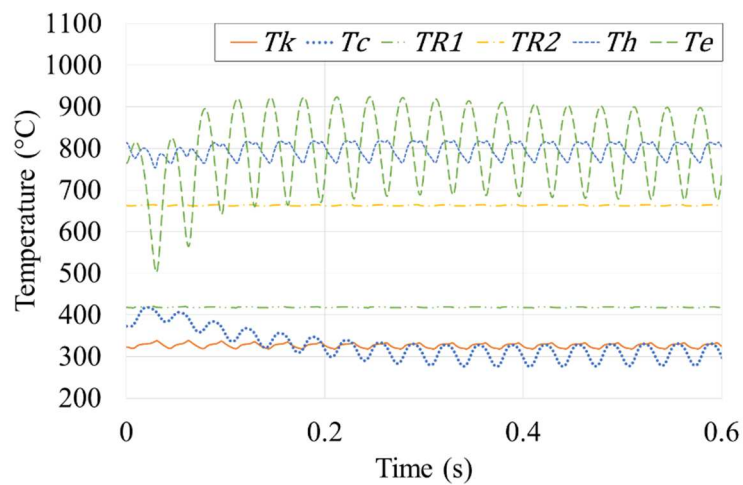


Figure 14: Temperature variations

In Figure 13 and Figure 14, the variations of thermic and dynamic properties of the system are shown. As can be seen, the system is stabilized after around 0.4 (s). The variations of temperatures in Figure 14 in steady-state are always around a specific value. The periodic behavior of the system causes the periodic fluctuations of the temperatures. Regenerators' temperatures are almost constant, and the expansion temperature has the maximum range of variations.

5. DEACH

The Schematic of the DEACH is shown in Figure 15. The refrigerant and the absorbents are water and lithium bromide, respectively. The following assumptions are made to model the DEACH system:

- The system operates at steady-state
- The refrigerant that leaves the condenser is saturated liquid
- The refrigerant that leaves the evaporator is saturated vapor
- The pressure drop inside tubes is neglected
- The pump work is neglected
- The heat transfer with outside happens only in the evaporator, condenser, absorber, and high-pressure generator
- The processes in expansion valves are enthalpy constant

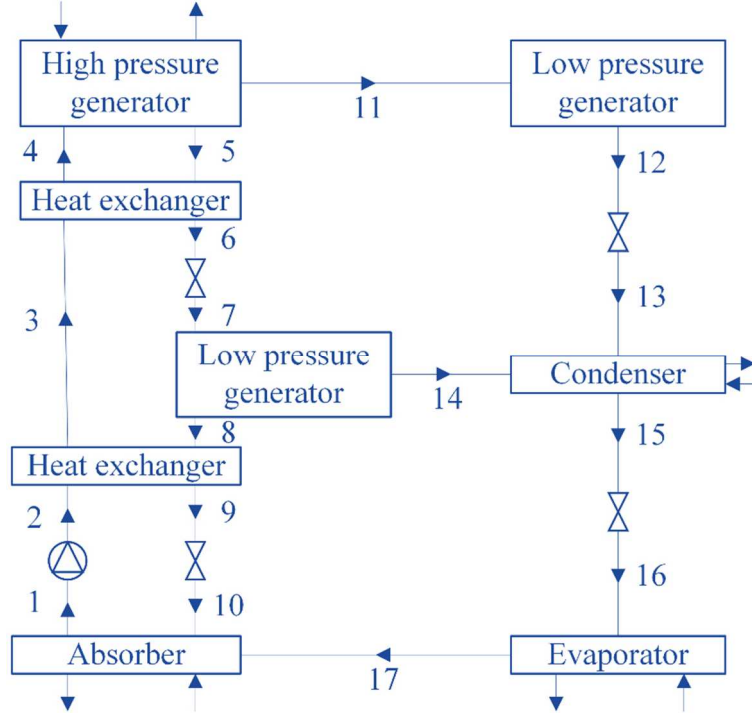


Figure 15: Schematic of DEACH

Components of the DEACH are in three pressure levels and five temperature levels. For each component, the laws of conservation of mass and energy are applied by calculating each flow's properties. For a lithium bromide solution, the enthalpy can be calculated as [52]:

$$H = 2.326\{\underline{A}_1 + \underline{A}_2(1.8T + 32) + \underline{A}_3(1.8T + 32)^2\} \quad (43)$$

A_i can be calculated as [52]:

$$\underline{A}_i = \underline{a}_0 + \underline{a}_1Xm + \underline{a}_2Xm^2 + \underline{a}_3Xm^3 + \underline{a}_4Xm^4 \quad (44)$$

Table 8: Constants to calculate the enthalpy of lithium bromide solution [52]

	\underline{A}_1	\underline{A}_2	\underline{A}_3
\underline{a}_0	-1015.07	4.68108	-4.9107×10^{-3}
\underline{a}_1	79.5387	-3.037766×10^{-1}	3.83184×10^{-4}
\underline{a}_2	-2.358016	8.44845×10^{-3}	-1.078963×10^{-5}
\underline{a}_3	0.03031583	-1.047721×10^{-4}	1.3152×10^{-7}
\underline{a}_4	-1.400261×10^{-4}	4.80097×10^{-7}	-5.897×10^{-4}

Xm is the mass concentration of lithium bromide in the solution. T is the solution temperature, which can be calculated as [52]:

$$T = T_{Dew\ Point} \sum_{n=0}^3 \underline{A}_n Xm^n + \sum_{n=0}^3 \underline{B}_n Xm^n \quad (45)$$

A_n and B_n are constants that are shown in Table 9.

1

Table 9: Constants of solution temperature formula [52]

n	A_n	B_n
0	-2.00755	124.934
1	0.16976	-7.7165
2	-3.13336×10^{-3}	0.152286
3	1.97668×10^{-5}	-7.9509×10^{-4}

2 To validate the DEACH, it is run based on Gebreslassie et al. [53] conditions shown in Table
3 10, and the results are presented in Table 11.

4

Table 10: Five-level temperatures of DEACH

Evaporator	280.15 K
Absorber	305.15 K
Low-pressure generator	340.95 K
High-pressure generator	381.05 K
Condenser	305.35 K

5

6

Table 11: DEACH validation

Flow number	Temperature (K)	
	Present study	Gebreslassie et al. [53]
1	305.15	305.15
2	305.15	305.15
3	337.055	337.95
4	378.39	379.15
5	381.05	382.95
6	337.055	338.15
7	337.055	338.15
8	340.95	343.75
9	305.15	305.35
10	305.15	305.35
11	381.05	381.05
12	343.426	343.95
13	305.35	305.35
14	340.95	340.95
15	305.35	305.35

16	280.15	280.15
17	280.15	280.15
Flow number	Pressure (kPa)	
	Present study	Present study
1	1.0021	1.0021
2	31.575	31.575
3	31.575	31.575
4	31.575	31.575
5	31.575	31.575
6	31.575	31.575
7	4.8133	4.8133
8	4.8133	4.8133
9	4.8133	4.8133
10	1.0021	1.0021
11	31.575	31.575
12	31.575	31.575
13	4.8133	4.8133
14	4.8133	4.8133
15	4.8133	4.8133
16	1.0021	1.0021
17	1.0021	1.0021
Flow number	Concentration (%)	
	Present study	Present study
1	53.95	53.95
2	53.95	53.95
3	53.95	53.95
4	53.95	53.95
5	55.92	55.92
6	55.92	55.92
7	55.92	55.92
8	57.65	57.65
9	57.65	57.65

10	57.65	57.65
11	-	-
12	-	-
13	-	-
14	-	-
15	-	-
16	-	-
17	-	-
Flow number	Enthalpy (kJ/kg)	
	Present study	Gebreslassie et al. [53]
1	73.583	73.1
2	73.583	73.2
3	140.46	142.3
4	227.43	230.9
5	235.96	241.4
6	145.82	148.3
7	145.82	148.3
8	159.61	167
9	88.152	91.1
10	88.152	91.1
11	2700.1	2700
12	294.17	296.4
13	294.17	296.4
14	2627.1	2627.1
15	134.94	134.9
16	134.94	134.9
17	2513.7	2513.7

1 Also, the *COP* of the Gebreslassie et al. [53] model was 1.655, while the present study has
2 a *COP* of 1.6012. As can be seen, the present model of the DEACH is valid and presents almost
3 the same results as Gebreslassie et al. [53] study. The model of the DEACH is developed in
4 MATLAB®.

5 **6. Combined system analysis**

6 After modeling, presenting the results, and validating each system, they are coupled based
7 on Figure 1. At first, the high-quality output heat of the IRSOFC is used for running the FPSE-

1 PMLSM system. Then to increase the temperature and burn the left output gas of the IRSOFC,
 2 it passes through a combustion chamber. After that, the reheated gas is used for preheating the
 3 input air and fuel of the IRSOFC. Then, it provides the required heat of the DEACH. The final
 4 low-quality heat is used to produce the required hot water.

5 The input parameters of the combined system are presented in Table 12. For a better
 6 understanding of FPSE input data, its schematic is shown in Figure 16, that all the spaces and
 7 parameters can be located.

8

Table 12: Combined system input parameters

FPSE input parameters	
T_h^{wall}	883.7 K
T_k^{wall}	322.8 K
T_{R1}^{wall}	429.3 K
T_{R2}^{wall}	707.8 K
p^{mean}	71 bars
Porosity	75.9 %
m_D	0.426 kg
m_p	6.2 kg
d_p	5.718 cm
d_D	5.67 cm
d_{rod}	1.663 cm
d_h	0.2362 cm
d_w	0.00889 cm
L_k	7.92 cm
L_h	18.34 cm
L_R	6.44 cm
A_h	1.4898 cm ²
A_k	2.6163 cm ²
$A_{R1,R2}$	8.745 cm ²
C_c	1.83 cm
C_e	1.861 cm
V_b^{mean}	2615 cm ³
V_{gs}^{mean}	37.97 cm ³
$V_{R1,R2}$	28.185 cm ³
A_k^{wetted}	115.2 cm ²
IRSOFC input parameters	

U_f	0.85
U_{ox}	0.25
Re	60 %
P	1 bar
A_{cell}	0.0834 m ²
i_{cell}	2000 A/m ²
i_l	9000 A/m ²
Number of cells	26
PMLSM input parameters	
ψ_f	0.0513 Wb
B_v	10
m_m	0.824 kg
r	0.1 Ω
L_d	1.77 mH
L_q	3.01 mH

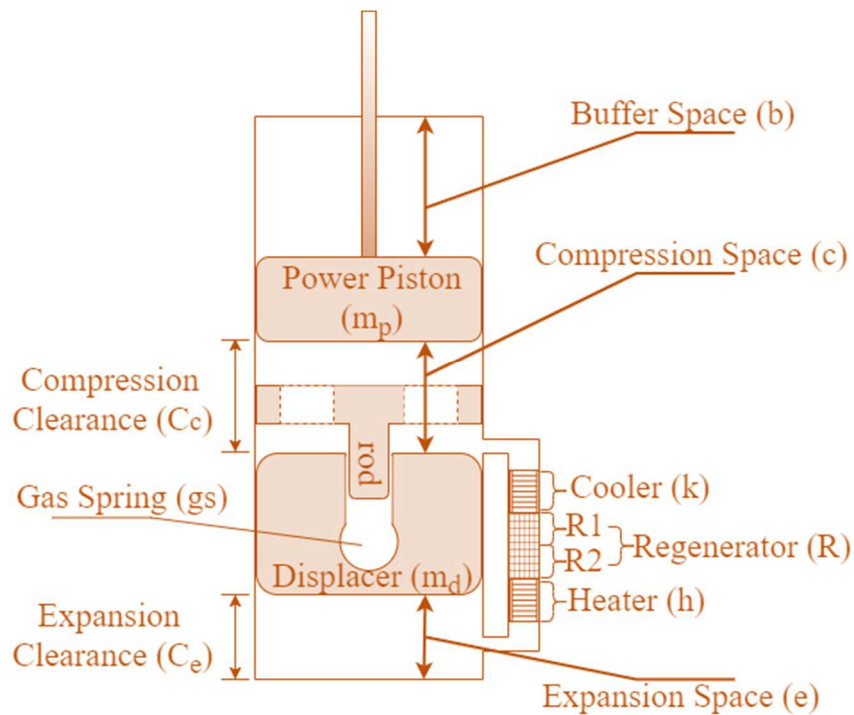


Figure 16: FPSE schematic

Based on these input data and DEACH temperature levels of Table 10, the systems are combined. For IRSOFC, based on input data of Table 12, the calculated operating temperature is 1000 K. The output of each system in the combined mode is shown in Table 13.

1

IRSOFC	1981 W
FPSE-PMLSM	279.4 W
DEACH	1366.6 W
Hot water	0.27 kg/s
Input heat	6165.4 W

2 As can be seen, IRSOFC and FPSE-PMLSM electricity production, DEACH cooling
3 production, and produced hot water flow rate are shown in Table 13. The equation which is
4 used to calculate IRSOFC power production is as follows:

$$Power_{IRSOFC} = (\text{Number of Cells}) \times i_{cell} A_{cell} v \quad (46)$$

5 FPSE-PMLSM power production is also calculated based on equation (42). The hot water
6 thermal flowrate also can be calculated as:

$$\dot{Q}_{Hotwater} = \dot{m}_{water} C_{p_{water}} \times (T_{water\ hot} - T_{water\ cold}) \quad (47)$$

7 It is assumed that the hot water temperature ($T_{water\ hot}$) is 333.15 K that should be achieved
8 from the cold water ($T_{water\ cold}$) at 298.15 K. Also, the input thermal energy is calculate based
9 on the input fuel (CH_4) flow rate to IRSOFC and its LHV (Lower Heating Value).

$$\dot{Q}_{Input} = \dot{m}_{CH_4} LHV \quad (48)$$

10 Now, the efficiency of this combined system can be calculated. To calculate efficiency, two
11 methods can be defined based on the following equation:

$$\eta = \frac{Power_{IRSOFC} + Power_{FPSE-PMLSM} + \dot{Q}_{DEACH} + \dot{Q}_{Hotwater}}{\dot{Q}_{Input}} \quad (49)$$

12 In this equation, powers are output electrical powers, and $\dot{Q}_{Hotwater}$ is the amount of heat
13 that was recovered by heating the water. \dot{Q}_{DEACH} can be defined as the heat that is used in
14 DEACH or the cooling that is produced by it. If it is assumed that \dot{Q}_{DEACH} is the consumed heat
15 in DEACH, the whole combined system efficiency is equal to 63.82 %, but if \dot{Q}_{DEACH} is the
16 cooling production, the efficiency of the combined system is equal to 72.14%. Comparing this
17 system with a standalone IRSOFC, which has an efficiency of 32.13%, shows that the total
18 efficiency of the system based on the first and the second method of efficiency calculation is
19 increased by 31.7% and 40%, respectively. Also, based on the electrical efficiency of the
20 system, according to Table 13, the efficiency of the combined system is 36.66%, which shows
21 around 14.1% increment compared to the standalone IRSOFC. All these results show a
22 significant improvement of the IRSOFC performance when it works in a combined system with
23 an FPSE-PMLSM, a DEACH, and a heat exchanger to produce hot water.

24 7. Conclusion

25 In the present study, a new power production system, which was an FPSE-PMLSM one, was
26 proposed as a heat recovery option for an IRSOFC. To recover the final residual heat of the
27 IRSOFC, a DEACH, and a heat exchanger to produce cooling and hot water, respectively, were
28 also added to the system. At first, the IRSOFC was modeled and validated using polarization
29 and power curves at different current densities. Then, a nonlinear thermodynamic model of the
30 FPSE was presented and validated at 1 kW of power production. After that, the PMLSM was
31 modeled and coupled with the FPSE, and the combined system was controlled using two PR

1 and one PI controller. After presenting each system's results in different sections, they were
2 combined, and the results of the combined system were presented. The goal of this combined
3 system was to produce electrical power (by IRSOFC and FPSE-PMLSM), cooling (by
4 DEACH), and hot water (by heat exchanger). Two methods were applied to calculate the system
5 efficiency. In the first method, the consumed heat of DEACH was assumed as the useful energy,
6 and in the second method, the cooling production of the DEACH was used to calculate the
7 efficiency.

8 The results showed that the combined system produced 1981 W power by IRSOFC and
9 279.4 W by FPSE-PMLSM system. Thus, the total produced electrical power was 2260.4 W,
10 which was around 14% more than the standalone IRSOFC system. This meant around a 4.5%
11 increment in electrical efficiency. The combined system also produced around 1367 W of
12 cooling by the DEACH, and 0.27 kg/s of hot water at 333.15 K by the heat exchanger. Based
13 on these data and two different methods, the efficiency of the system was calculated. At first, it
14 was assumed that all the productions (electrical power, cooling, required heat for hot water
15 production) were the system's output that was used for efficiency calculation. In this case, the
16 system efficiency was around 72%, which was 40% higher than the standalone IRSOFC with
17 around 32% of efficiency. Then, only the recovered heat (by the heat exchanger and DEACH)
18 was used besides the produced electrical power (by IRSOFC and FPSE-PMLSM) for efficiency
19 calculation. In this case, the total efficiency of the system was around 63%, which was around
20 32% higher than the standalone IRSOFC. In any case, these results showed the significant effect
21 of adding the FPSE and other auxiliary heat recovery systems to the IRSOFC system.

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