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# Instrumentation study for aspersions cooling experiments aiming optimal estimation of the heat flux dissipation by 2D inverse heat conduction problems

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## Abstract

Cooling experiments usually require using many thermocouples to better characterize the heat transfer processes, but there is usually a limitation on the maximum number of thermocouples to be used, because of either limited acquisition channels or limited budget. This paper presents an experiment-design process to guide choosing the number of thermocouples to ensure a high-quality inversion and accurate heat flux estimations. Two main criteria define the quality of the instrumentation: 1) the agreement between the simulated heat flux (imposed in the numerical simulations) and that obtained by an inverse method, and 2) the agreement between the calculated temperature profiles along the sample thickness using the simulated and estimated heat fluxes. We found that large errors in the estimated heat flux can be captured by estimating the central thermocouples measurements, but not if the difference between estimated and simulated boundary conditions is small.

## Keywords

Quenching by water jets, inverse method, instrumentation, heat conduction, validation, optimization

## 1 Introduction

In any quenching or metal cooling experiment, it is necessary to instrument the test sample to characterize the temperature evolution throughout the test. We focus here on quenching by water jets and sprays. Although back-surface infrared thermography is an option in cases using thinner samples [Labergue et al., 2017; Gradeck et al., 2012], most of the experimental campaigns use thermocouples inserted in one of several locations in the part [Karwa, and Stephan, 2013; Tsukamoto et al., 2020]. Besides the temperature evolution, these experiments usually aim to find the boundary condition, i.e. the heat flux or the heat transfer coefficient on the surface, which are obtained by inverse methods. In this case, the instrumentation becomes more important, because the appropriate location and number of thermocouples may optimize the quality of the inversion.

The instrumentation planning must be aligned with the inverse method to be used. On the one hand, numerical inverse methods require few thermocouples [Leocadio et al., 2018; Gomez et al., 2021], but are time-consuming. On the other hand, analytical and pseudo-analytical methods need more

thermocouples [Gradeck et al., 2009] because they usually make use of the Fourier transform, thus using few thermocouples may limit the number of harmonics that can be used. Consequently, the model may reproduce poorly the boundary condition profile. The aforementioned infrared thermography does not have this issue of local measurements because it obtains a temperature field, which allows using integral transforms very accurately [Naveira-Cotta et al., 2011].

While using more thermocouples would improve the quality of the inversion, the instrumentation becomes as well more intrusive in the test sample, which may disturb the system [Moffat, 1988], and the experimental campaign comes to be more expensive. Also, there is usually a limit on the maximum number of thermocouples recorded by the data acquisition system because of a limited number of channels. Moreover, instrumenting large samples results in high machining costs during drilling and welding. Hence, for both technical and economical reasons, one may be motivated to use the least thermocouples possible that still provides accurate estimates of the boundary condition using an inverse method.

In this paper, we present a preliminary study of the two-dimensional inversion quality according to the sample instrumentation with thermocouples. The heat equation is solved for a planar wall and temperature evolutions at several locations are extracted to simulate the inserted thermocouples, this step being named direct problem. Then, these data were used in an inverse method to estimate the input heat flux and the result was compared with the heat flux imposed in the direct problem. We evaluated herein the effect of the number of thermocouples for three different heat flux profiles applied on the surface. We also propose the use of a thermocouple in a remote position of those used in the inverse problem to serve as a validation of the estimated heat flux in experimental campaigns.

## 2 Method presentation

Figure 1 presents our test case, which is a two-dimensional plate 20-mm thick and 800-mm long. At the upper surface, there is a Gaussian-shaped heat flux  $\varphi(x, t)$  centred at the middle of the wall and with a profile described by the following expression:

$$\varphi(x, t) = \varphi_M \exp \left\{ -0.5 \left[ \frac{\left(x - \frac{L}{2}\right)}{\sigma} \right]^2 \right\} \exp \left( -\frac{t}{t_c} \right) \quad (1)$$

where  $\varphi_M$  is the maximum heat flux (at  $t = 0$ ),  $L$  if the plate length,  $\sigma$  is the standard deviation of the Gaussian distribution and  $t_c$  is the characteristic time of the peak heat flux exponential decay. In the present study, we kept  $\varphi_M = 5 \text{ MW/m}^2$  and  $t_c = 30 \text{ s}$ , and we tested three different standard deviations: 20, 100 and 500 mm. The other surface is considered thermally insulated, as well as both boundaries in the x-direction.

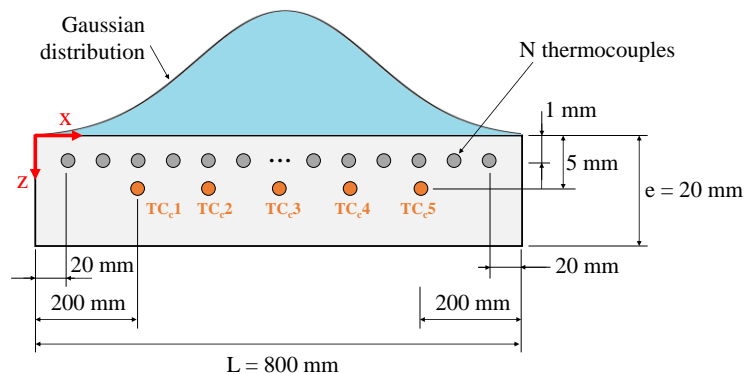


Figure 1: Test case – two-dimensional planar wall with Gaussian-shaped heat flux

There are  $N$  simulated thermocouples at 1 mm from the heat-exchange surface (from now on named “surface thermocouples”) that were used in the inverse method to estimate the heat flux at  $z = 0$ . In this study, we tested cases with 5, 9 and 15 surface thermocouples. Moreover, five other thermocouples were simulated at  $z = 5 \text{ mm}$  (named “central thermocouples” or TC<sub>c</sub>) to aid the inverse method validation by comparing the temperature evolution in the direct problem and the one simulated using the heat flux found with the inverse method. Because this is a simulation study, we can compare directly the inverse heat flux with the true imposed one; however, this is not possible in a true cooling experiment.

Figure 2 presents the calculations and validation steps in more detail. First, we solve the direct heat conduction problem with the known imposed heat flux, obtaining the simulation of the surface thermocouples’ measurements. Then, we solve the inverse problem using these simulated thermocouple measurements and estimate the boundary heat flux. At this point, we can compare the previously imposed and the estimated heat fluxes to evaluate the inversion quality. However, as mentioned, a second validation process is performed. In the first direct problem solution, we also simulate the temperature response of the central thermocouples, which represents the temperature measurements in a true cooling experiment. The estimated heat flux (using the surface thermocouples) is then used as a boundary condition in a direct heat conduction problem to estimate the response of the central thermocouples. Therefore, we can also compare their simulated and estimated responses to evaluate the inversion quality in a procedure that is actually feasible in cooling experiments.

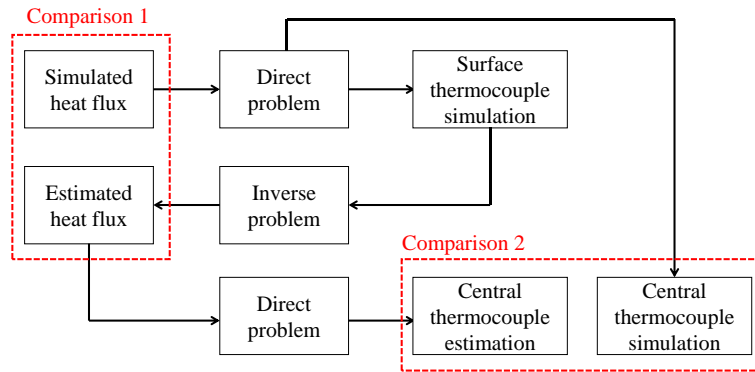


Figure 2: Flowchart showing the direct problem, inverse problem and validations

Both the direct and inverse problems are solved using an analytical solution of the transient two-dimensional heat equation and using the Laplace and Fourier cosine transforms [Hahn, and Ozisik, 2012]. In summary, the original 2D Cartesian heat equation of this problem, considering constant thermophysical properties and no heat source, is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a} \frac{\partial T}{\partial t} \quad (2)$$

where  $T$  is the temperature and  $a$  the thermal diffusivity. After applying the Laplace transform and the Fourier cosine transform on the  $x$ -direction, we have the following expression:

$$\frac{\partial^2 \bar{\theta}}{\partial z^2} - \left( \frac{p}{a} + \omega^2 \right) \bar{\theta} = 0 \quad (3)$$

where  $\theta = T - T_0$  with  $T_0$  being the initial uniform temperature of the body (which is arbitrary in this study as the thermophysical properties are constant),  $p$  is the Laplace variable, and  $\omega = n\pi/L$  with  $n = 0, 1, 2, \dots$ . The tilde and the line over the variable (in this equation, over  $\theta$ ) indicates that the variable is in the Fourier and Laplace domains, respectively. Considering the problem’s boundary conditions, the solution of this equation, i.e. of the direct problem, in the Fourier domain is:

$$\tilde{\theta}_n(z, t) = -\frac{1}{\lambda} \int_0^t e^{-a\omega_n^2(t-\tau)} Z_n(z, t - \tau) \tilde{\varphi}_n(\tau) d\tau \quad (4)$$

with:

$$Z_n(z, t) = \mathcal{L}^{-1}[\bar{Z}_n(z, p)] = \mathcal{L}^{-1} \left\{ \frac{1}{\sqrt{\frac{p}{a}}} \left[ \frac{\cosh\left(\sqrt{\frac{p}{a}} \cdot z\right)}{\tanh\left(\sqrt{\frac{p}{a}} \cdot e\right)} - \sinh\left(\sqrt{\frac{p}{a}} \cdot z\right) \right] \right\} \quad (5)$$

where  $e$  is the plate thickness,  $\lambda$  is the material thermal conductivity and  $\tilde{\varphi}_n$  is the  $n$ -th harmonic of the heat flux in the Fourier domain. Equation 5 cannot be calculated analytically, so we used the Gaver-Stehfest algorithm [Stehfest, 1970] to perform the Laplace transform inversion. This convolution product is more easily calculated if we consider the heat flux is constant by parts (hence, we are discretizing the equation), so:

$$\tilde{\theta}_n(x, t_i) = \sum_{k=0}^{i-1} X_{n,i,k+1} \tilde{\varphi}_{n,k} \quad (6)$$

with:

$$X_{n,i,k+1} = -\frac{1}{\lambda} \int_{t_k}^{t_{k+1}} e^{-a\omega_n^2(t-\tau)} Z_n(z, t - \tau) d\tau \quad (7)$$

Computing both the temperature and the heat flux in the real domain requires only calculating the Fourier series using their corresponding harmonics. Using Eq. 6 and knowing the simulated heat flux, we can solve the direct problem and calculate the body temperature at each position  $(x, z)$  and for each instant  $t$ . The inverse problem is solved using the function specification method proposed by Beck (*apud* [Beck et al., 1996]), where  $n_{fts}$  future time steps are used as a regularization tool to overcome the ill-posed nature of the sensitivity matrix built with  $X_{n,i,k+1}$ . In the present study, we used  $n_{fts} = 5$  for all the test cases.

### 3 Results and discussion

Before comparing results with different numbers of thermocouples and different heat flux profiles (i.e. standard deviations), let us first introduce results for  $\sigma = 20 \text{ mm}$  and  $N = 15$  thermocouples to explain the results analysis and some data reduction parameters to later compare different conditions. Figure 3 presents direct solution results, in (a) the temperature evolution of the 15 thermocouples after imposing the Gaussian heat flux profile in (b). The temperature results demonstrate that this very concentrated heat flux profile decreases the plate temperature mainly at the centre ( $x = 400 \text{ mm}$ ), while most of the thermocouples are practically insensitive to the imposed boundary condition. The inverse solution (dashed lines in Figure 3b) represents very well the simulated heat flux (solid lines in Figure 3b), although we find a slight difference beside the Gaussian distribution. Analysing each inverse heat flux profile for all the test cases would be difficult to compare the results, so we reduced the data to a mean heat flux error  $|\overline{\Delta\varphi}|$  defined by:

$$|\overline{\Delta\varphi}|(t) = \frac{1}{L} \int_0^L |\varphi_{est} - \varphi_{sim}| dx \quad (7)$$

$\varphi_{est}$  and  $\varphi_{sim}$  being, respectively, the estimated and the simulated heat flux. The lower  $|\overline{\Delta\varphi}|$ , the better the estimated heat flux by the inverse method. Figure 4 presents the result of this parameter for the case shown in Figure 3b, where we see a larger deviation at the beginning of the simulation caused exactly by the less-accurate estimates beside the Gaussian shape in the first seconds. By the end of the simulation, the mean error becomes negligible.

However, as mentioned before, we do not know the imposed heat flux during cooling experiments, so we performed a second validation comparing the simulated and the estimated central thermocouple measurements, as described in Figure 2. Thus, Figure 5a presents the temperature evolutions for each central thermocouple, both simulated (solid lines) and estimated (dashed lines), where we see that only the central thermocouple temperature is well estimated, the others being affected by the heat flux errors in the first seconds. To better visualise this deviation, Figure 5b presents the difference between the estimated and simulated temperatures for each central thermocouple. For better comparison with other conditions, we also present in this figure the absolute mean temperature difference  $|\overline{\Delta T_c}|$  considering all the central thermocouples. The largest temperature difference is also observed a little after the first seconds where the estimated heat flux was less accurate. Hence, using these central thermocouples may be very helpful in cooling experiments to validate the estimated heat flux, although, unfortunately, it means increasing the sample instrumentation – the contrary of what we desire because of the reasons cited in the introduction.

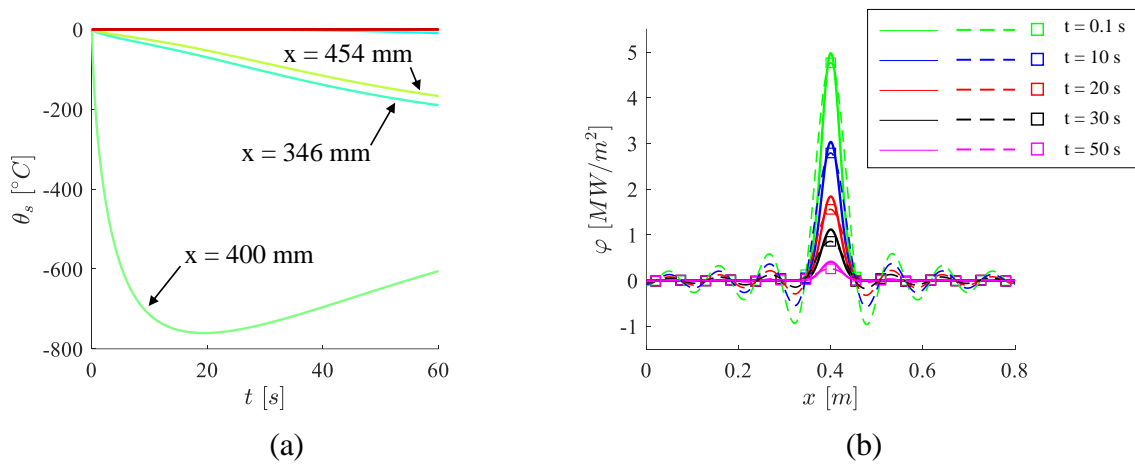


Figure 3: Direct and inverse solutions for  $\sigma = 20 \text{ mm}$  and  $N = 15$  thermocouples: (a) Temperature evolution and (b) heat flux profiles at different times. In (b), solid lines (—) correspond to the simulated heat flux, dashed lines (---) to estimated heat flux, and squares ( $\square$ ) to the heat flux on the surface thermocouple position.

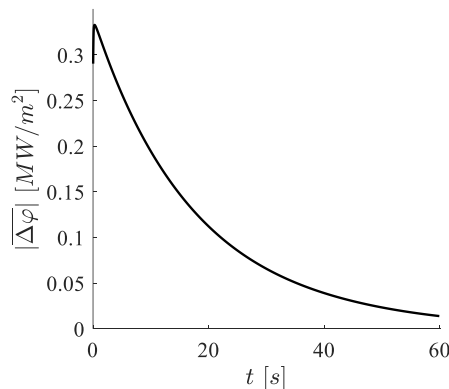


Figure 4: Mean heat flux error for  $\sigma = 20 \text{ mm}$  and  $N = 15$  thermocouples.

Figure 6 presents the simulated and estimated heat flux profiles for different numbers of thermocouples and two different standard deviations for the Gaussian heat flux profile. When the heat flux profile is more concentrated, it requires more harmonics to describe it in the Fourier series because fast transitions are represented by higher frequencies (i.e. higher harmonics). In fact, the number of thermocouples limits the number of harmonics that can be used in the Fourier system. If we had more harmonics than thermocouples, we have an unconstrained system of equations so the heat flux harmonics cannot be

found. Consequently, we cannot reproduce the heat flux profile using only 5 thermocouples if its standard deviation is 20 mm, as shown in Figure 6. On the other hand, if the heat flux is more uniformly distributed, there are no steep transitions in the heat flux profiles, so using very few heat flux harmonics is already capable of reproducing its profile. Therefore, we do not see substantial differences between using 5 or 15 thermocouples in this case. Thus, we can conclude that the steeper the heat flux profile transitions are, the more thermocouples are necessary to better estimate the boundary condition.

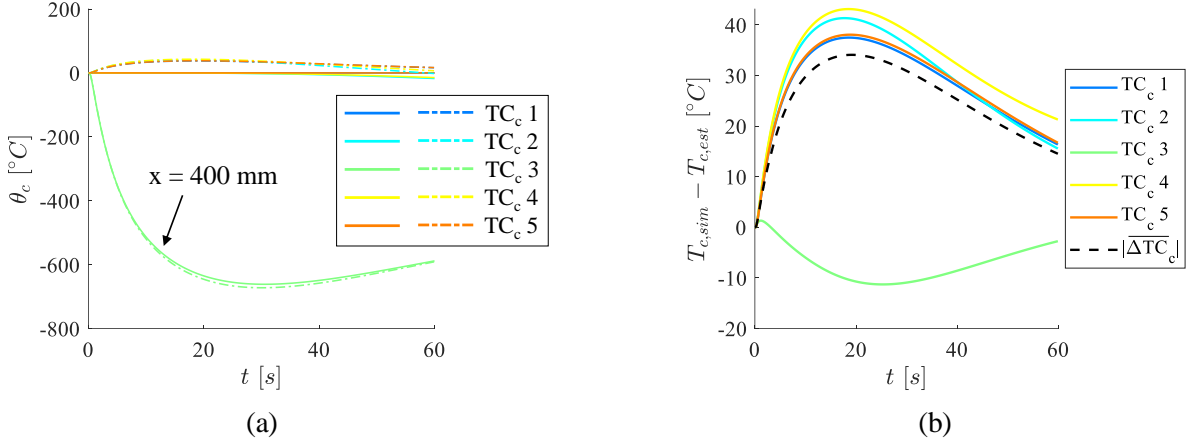


Figure 5: Temperature at the central thermocouple positions: (a) simulated and estimated temperature evolutions and (b) evolution in the difference between simulated and estimated temperatures for each thermocouple and the mean value. In (a), In (b), solid lines (—) correspond to the simulated temperatures and dashed lines (---) to estimated temperatures.

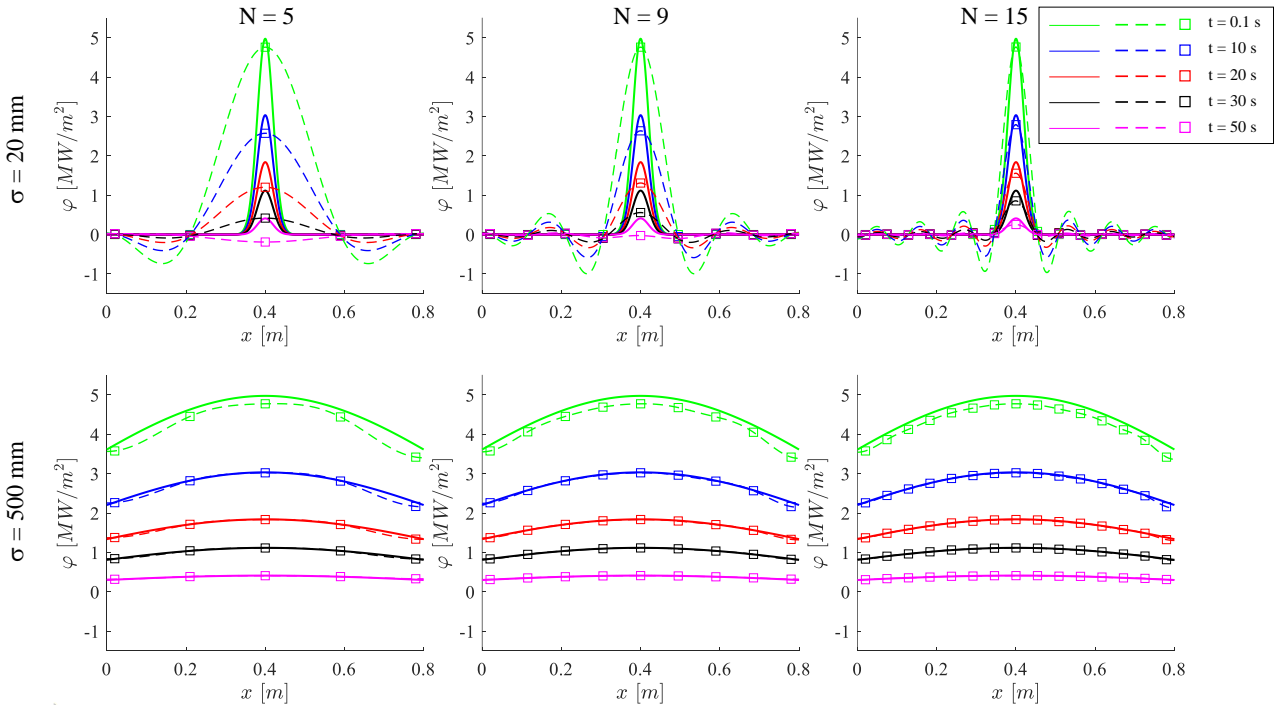


Figure 6: Simulated and estimated heat flux for different numbers of thermocouples and different standard deviations for the Gaussian heat flux. Solid lines (—) correspond to the simulated heat flux, dashed lines (---) to estimated heat flux, and squares ( $\square$ ) to the heat flux on the surface thermocouple position.

Finally, we compare the mean heat flux error  $|\overline{\Delta\varphi}|$  and the absolute mean temperature difference  $|\overline{\Delta T_c}|$  for all the nine tested cases (recalling:  $\sigma = 20, 100$  and  $500$  mm and  $N = 5, 9$  and  $15$  thermocouples)

in Figure 7. As already observed in Figure 6, the more concentrated the heat flux profile, the higher the estimated heat flux error. This is verified by looking at both  $|\overline{\Delta\varphi}|$  and  $|\overline{\Delta T_c}|$  evolutions. Moreover, we observe that the heat flux estimation is less accurate at the beginning of the cooling. In the course of the simulation, the heat flux peak decreases, so the transitions in the profile are less and less steep and the inversion quality is improved. Finally, the central thermocouples can only verify well the quality of the estimated heat flux if the estimated errors are large. For example, with  $\sigma = 20$  mm, we saw in Figure 6 that the estimated heat flux is not very accurate with 9 thermocouples. However, the absolute mean temperature differences with 9 or 15 thermocouples are very similar (Figure 7), which could induce to believe that the estimates with 9 or 15 thermocouples are equally accurate in a cooling experiment. Hence, more studies are necessary to understand how to place the central thermocouples and find a better criterion than the proposed  $|\overline{\Delta T_c}|$  to validate the estimated heat flux in cooling experiments.

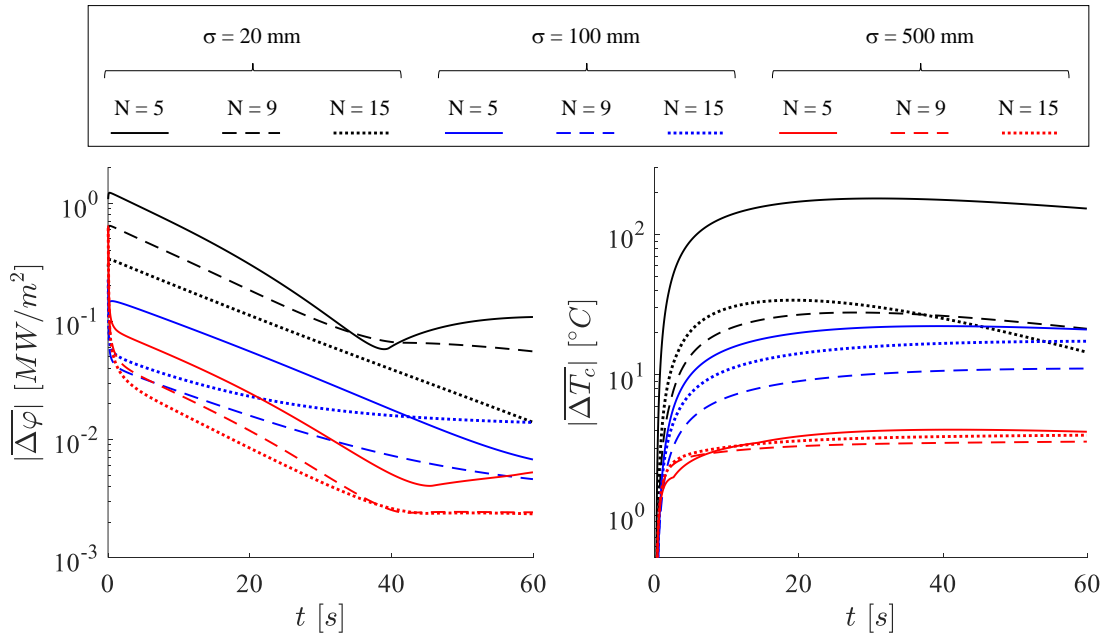


Figure 7: Comparison of the mean heat flux error and the absolute mean temperature difference for standard deviations of different heat flux profiles and different numbers of thermocouples used in the inverse solution.

## 4 Conclusions

In the present study, the quality of the estimated boundary condition using an inverse method was discussed using an example of a plate cooled by a Gaussian-shaped heat flux at one boundary. Different standard deviations for the Gaussian distribution and different numbers of thermocouples for the inverse solution were tested. The quality of the estimated heat flux was evaluated by comparing the estimated and simulated heat fluxes, as well as comparing the simulated and estimated temperature measurement by “central” thermocouples. This analysis showed the importance of using more thermocouples when there are steep transitions in the heat flux shape. Finally, a parameter called absolute mean temperature difference was proposed to validate the estimated heat flux using central thermocouples, which proved to be useful to indicate if the estimated boundary condition is very badly estimated, as the tested case with  $\sigma = 20$  mm and 5 thermocouples. However, it was not sensitive enough for a case where the estimated heat flux was not accurate enough ( $\sigma = 20$  mm and 9 thermocouples), which could induce misleading conclusions in cooling experiments. More investigation is necessary to understand the best approach to validate experimentally the estimated heat flux using central thermocouples.

Although the present analysis considers constant thermophysical properties and no heat sources due to solid-phase transformation, both phenomena will be included in future analyses.



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