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# Wheels and cycles: (sub)optimality and volatility of corrupted economies\*

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## Abstract

We consider a simple economy where production depends on labor supply and social capital. Networking increases the social capital ("greases the wheel") but also the corruption level ("sands the wheel"). Corruption is a negative productive externality. We compare the market economy, where the negative externality is not taken in account by individuals, with a centralized economy, where the planner internalizes the negative effect. We highlight the possible existence of cycles in the market economy and optimal cycles in the planned one. We compare the centralized and the decentralized solutions in the short and in the long run.

## 1 Introduction

Corruption is bad but social networking is good as long as it reduces bureaucratic frictions and socioeconomic transaction costs. If corruption is a by-product of social networking, the question of the price to pay in terms of corruption for a society in order to develop socioeconomic networks can be legitimately raised. In other terms, we can address the question of the "optimal" level of corruption to tolerate in order to reduce these transaction costs. Of course, the definition of an optimal level of corruption rests on the assumption of a kind of complementarity between networking and corruption. Corruption in a market economy is even worse than in a system managed by a benevolent planner who internalizes the negative externality of corruption.

Many authors have considered the impact of corruption on growth. Even if most of the papers, namely empirical, show a negative impact on growth and

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welfare, some paper shows a positive or an ambiguous effect on these variables depending on cultural area of the stage of the growth process.

The idea that corruption can have positive economic effects dates back to Leff (1964), Leys (1965) and Huntington (1968). They introduce the so-called "grease the wheels" hypothesis (see Méon and Weill (2010) for a survey). According to these authors, corruption can compensate an inefficient bureaucracy and, then, help to circumvent administrative obstacles. Conversely, corruption can be viewed as a market imperfection and, therefore, a source of inefficiency. In this case, according to Méon and Weill (2010), corruption "sands the wheels". Hereinafter, let us denote by **GWE** and **SWE** the "**Grease the Wheel**" and the "**Sand the Wheel**" effect.

Empirically, many papers have pointed out the negative effects of corruption on economic growth (Mauro, 1995; Olson et al., 2000; Méon and Sekkat, 2005; Swaleheen, 2011) supporting the SWE hypothesis. However, as stressed empirically by Méon and Weil (2010), the corruption effect on economic growth could differ for a country compared to one another depending upon the quality of their respective institutional framework. In a country with ineffective institutions (i.e. sluggish bureaucracy), corruption can speed up decisions (GWE). Conversely, in a country with an efficient institutional framework, corruption appears to be a market imperfection and slow down economic growth (SWE). In this respect, they find some evidence for the GWE hypothesis. Therefore, corruption seems to have two opposite effects on economic growth: the positive GWE and the negative SWE. Depending upon institutions quality, the GWE can overcompensate the SWE. The present paper aims to develop a simple Ramsey economy taking into account both of these effects.

Often, the interplay between two opposite effects can generate temporary or persistent fluctuations (cycles). In our context, from a dynamic perspective, the GWE enhances the economic growth and, then, works as a repulsive force. Conversely, the SWE hinders the growth and, thus, works as an attractive force. What we expect is that this interaction between opposite forces generates endogenous cycles in the Ramsey model.

Moreover, cycles in corruption are well-documented in the political literature. As pointed out by Bicchieri and Duffy (1997), in many countries, a lasting corrupted era is followed by a clean-up period followed in turn by a new corrupted age and so on.<sup>1</sup> In order to explain those corruption cycles, Bicchieri and Duffy (1997) have developed a theoretical model based on a repeated game. In their framework, corruption cycles arise and are simply explained. Indeed, in order to be reelected, corrupted politicians have to compensate voters with material incentives. To do so, corrupted politicians need resources amassed during a period of honesty. That is, a political turnover arises when the resources are depleted and the emergence of an "honest" period follows. A political cycle takes place: a corruption period is followed by an honest period and viceversa. Our paper aims to propose a new theoretical argument for corruption cycles

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<sup>1</sup>The reader interested in the evidence about the corruption cycles is referred to Gillespie and Okruhlik (1991) for the Middle-East countries and to Sidorkin and Vorobyev (2018) for Russia.

based on the interplay between GWE and SWE within a standard economic growth model.

One of the reasons of a potential positive correlation between corruption and growth is that social networking and coalition rent seeking entails corruption as a collateral effect. Networking is the natural way to reduce the transaction costs (GWE) in a market economy but it promotes also the emergence of coalitions and mafias that extract a rent at the expenses of the rest of society (SWE). There are plenty of examples: religious communities, Freemasonry, lobbies, mafias *stricto sensu*. In some countries where social networking and corruption are complements, a positive correlation between corruption and growth is observed. Of course, the causality is that networking possibly implies corruption and not the converse. What some papers stresses, is just a positive correlation. It should be interesting to disentangle countries where growth mainly rests on natural resources and a resource curse take place (African countries), from those where growth comes from human capital accumulation and industrial production (Japan).

In our paper we address two questions. (1) What is the additional impact of corruption on a market economy with respect to the social optimum? (2) Do corruption cycles arise in a market economy and in a planned economy? The existence of optimal cycles in a planned economy is interesting from methodological point of view. Indeed, few models exhibit such a phenomenon.<sup>2</sup>

We consider a simple economy where production depends on labor supply and social capital. Networking increases the social capital but also the corruption level. Corruption is a negative productive externality. We compare the market economy, where the negative externality is not taken in account by individuals, with a centralized economy, where the planner internalizes the negative effect.

More precisely, we consider a model combining a building block à la Lucas (1988), where the human capital is reconsidered as a social capital à la Bourdieu (1980), with a block à la Romer (1986), where capital is a private input (GWE) and a social externality (SWE), now negative. However, because of the lack of increasing returns, growth remains exogenous in our economy. In this simple framework, we find that the interplay between the GWE and SWE is able to generate the emergence of periodic cycles around the steady state through a flip bifurcation. In this respect, our paper contributes to the literature on corruption cycles by adding a new explanation based on two empirically well documented effects associated with corruption: GWE and SWE. These cycles are robust since they appear both in the market economy and in the planner's solution. The paper is organized as follows. Sections 2 and 3 present respectively the general equilibrium and the planner's solution, while a comparison between these regime is introduced in section 4. Section 5 concludes the paper. All the proofs are gathered in the Appendix.

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<sup>2</sup>The reader interested in models with optimal endogenous cycles is referred to Benhabib and Nishimura (1979), Medio (1987), Dockner and Feichtinger (1991) or Wirl (2004) among others.

## 2 Market economy

Time is discrete. At period  $t$ ,  $n_t$  is the time devoted to networking while  $1 - n_t$  is the time devoted to work. The individual stock of social capital  $s_t$  follows a simple accumulation law:

$$s_{t+1} - (1 - \delta) s_t \leq B n_t \quad (1)$$

where  $\delta$  is the social capital depreciation rate, while  $B$  represents the networking efficiency.

Social capital improves the individual productivity (more personal projects are developed and the GWE takes place) but promotes corruption as well at the social level. Corruption is a negative externality on factor productivity through the so-called SWE:  $A(\bar{s}_t)$  with  $A'(\bar{s}_t) < 0$ , where  $\bar{s}_t$  is the average social capital  $\bar{s}_t \equiv S_t/N_t$  and  $N_t$  is the size of population. For instance, the corruption level reduces the productivity of public services or the quality of public infrastructures, viewed as a public good à la Barro (1990).

The corruption elasticity of production is given by

$$-\alpha(s_t) \equiv \frac{s_t A'(s_t)}{A(s_t)} < 0$$

$\alpha$  captures the magnitude of the SWE.

Labor services  $l_t$  depend on time devoted to work  $1 - n_t$  and social capital  $s_t$  in a simple way:

$$l_t \equiv s_t (1 - n_t)$$

The positive effect of social capital on labor supply captures instead the GWE associated to corruption. In other words, our framework considers the two effects associated with corruption pointed out in the literature: a positive GWE and a negative SWE. Our contribution not only aims to capture their long run impacts on economic variables (steady state), but also their short run effects (fluctuations).

The firm  $i$  employs an amount  $L_{it}$  of labor services and pays  $w_t$  the unit of labor. The production function is linear:  $F(L_{it}) \equiv A(\bar{s}_t) L_{it}$ . Profit maximization leads to a zero profit condition:  $w_t = A(\bar{s}_t)$  (the real wage is a decreasing function of corruption).

The representative household maximizes the life-span utility under a sequence of budget constraints:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t \leq w_t l_t$$

with  $t = 0, \dots, \infty$ .

**Assumption 1** *The utility function  $u$  is given by:*

$$u(c_t) \equiv \frac{c_t^{1-\sigma}}{1-\sigma}$$

where  $1/\sigma$  is the elasticity of intertemporal substitution (with respect to consumption).

**Assumption 2**  $\sigma > 1/2$ .

Notice that Assumption 2 is equivalent to a weak elasticity of intertemporal substitution in consumption:  $1/\sigma < 2$ , including the logarithmic case ( $\sigma = 1$ ). Assumption 2 ensures that the Arrow-Mangasarian conditions are satisfied.

**Proposition 1 (equilibrium)** *Under Assumptions 1 and 2, the utility maximization is well-defined and the market clearing (social capital, labor services and consumption good) entails the following general equilibrium system:*

$$\frac{u'(c_t)}{u'(c_{t+1})} A(s_t) s_t = \beta(1-\delta) A(s_{t+1}) s_{t+1} + \beta B \frac{c_{t+1}}{s_{t+1}} \quad (2)$$

$$s_{t+1} - (1-\delta) s_t = B - \frac{Bc_t}{A(s_t) s_t} \quad (3)$$

jointly with the initial condition  $s_0$  and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t u'(c_t) A(s_t) s_t s_{t+1} = 0$$

where (2) represents the Euler equation, while (3), the social capital accumulation, now binding.

We observe that system (2)-(3) is two-dimensional in the vector  $(s_t, c_t)$  with one predetermined variable  $s_t$ .

## 2.1 Steady state

The steady state is defined by  $(s_t, c_t) = (s_{t+1}, c_{t+1})$ . We obtain

$$s = \frac{\beta B}{1 - \beta + 2\beta\delta} \quad (4)$$

$$c = \frac{1 - \beta + \beta\delta}{1 - \beta + 2\beta\delta} A(s) s \quad (5)$$

and

$$n = \frac{\beta\delta}{1 - \beta + 2\beta\delta} \in (0, 1) \quad (6)$$

We compute the impact of the fundamental parameters on the steady state (comparative statics) in terms of elasticities. The networking effectiveness  $B$

has a positive impact on social capital accumulation as well as the household's patience  $\beta$ :

$$\begin{aligned}\frac{B}{s} \frac{\partial s}{\partial B} &= 1 > 0 \\ \frac{\beta}{s} \frac{\partial s}{\partial \beta} &= \frac{1}{1 - \beta + 2\beta\delta} > 0\end{aligned}$$

The interpretation is straightforward: according to the accumulation law (1), a higher  $B$  means a greater networking efficiency promoting in turn a higher social capital in the long run. Moreover, the higher  $\beta$ , the more patient the agent. To increase her future consumption, this agent spends more time in networking and accumulates more social capital in the end.

The impacts on consumption of  $B$  and  $\beta$  depends on the corruption elasticity of production:

$$\begin{aligned}\frac{B}{c} \frac{\partial c}{\partial B} &= 1 - \alpha > 0 \text{ iff } \alpha < 1 \\ \frac{\beta}{c} \frac{\partial c}{\partial \beta} &= \frac{1 - \beta - \alpha(1 - \beta + \beta\delta)}{(1 - \beta + \beta\delta)(1 - \beta + 2\beta\delta)} > 0 \text{ iff } \alpha < \frac{1 - \beta}{1 - \beta + \beta\delta} (< 1)\end{aligned}$$

As seen above, the social capital increases in  $B$  and  $\beta$ . However, a higher social capital leads to two opposite effects in our economy: (1) it raises the individual productivity (GWE) and, thus, the labour income and (2) it promotes corruption at the social level, that is a negative externality, lowering factor productivity and labor income in turn (SWE). Under a low corruption impact on factor productivity (a sufficiently low value for  $\alpha$ ), the GWE dominates the SWE leading to a higher consumption level.

Consider the explicit form:  $A(s_t) \equiv As_t^{-\alpha}$ . Interestingly, the corruption elasticity of production  $-\alpha$  has no impact on social capital, but the following impact on consumption:

$$\frac{\alpha}{c} \frac{\partial c}{\partial \alpha} = \alpha \ln \frac{1}{s} > 0 \text{ iff } s < 1$$

Let us observe that  $c_t = s_t A(s_t)(1 - n_t)$ . Thus, a higher value of  $\alpha$  exacerbates the negative externality generated by the social capital, entailing two opposite effects on consumption: (1) a decrease in the factor productivity  $A$  (SWE) which reduces the consumption level, and (2) an incentive to reduce the networking  $n$  which increases the consumption. If the level of social capital is sufficiently low ( $s < 1$ ), the positive effect (2) dominates the negative effect (1): in this case, a higher  $\alpha$  raises the consumption level in the long run.

## 2.2 Corruption cycles

In the previous section, we have considered the corruption effects on the steady state (long run). Let us now focus on the corruption effects on local on local dynamics (short run).

**Lemma 2 (local dynamics)** *Dynamics are approximated around the steady state by the following linear system:*

$$\begin{bmatrix} \frac{ds_{t+1}}{s} \\ \frac{dc_{t+1}}{c} \end{bmatrix} = J \begin{bmatrix} \frac{ds_t}{s} \\ \frac{dc_t}{c} \end{bmatrix} \quad (7)$$

where  $J$  is the Jacobian matrix of system (2)-(3) linearized around the steady state  $(s, c)$ , with the following trace ( $T$ ) and determinant ( $D$ ):

$$T = -\frac{\alpha}{\beta} \left[ 1 + \sigma \frac{\beta(1-\delta)}{1-\beta(1-\delta)-\sigma} \right] - \frac{\sigma}{\beta} \frac{1-\beta(1-\delta)+\beta\delta}{1-\beta(1-\delta)-\sigma} + 2(1-\delta) \quad (8)$$

$$D = -\frac{\alpha}{\beta} \left[ 1 + \sigma \frac{\beta(1-\delta)}{1-\beta(1-\delta)-\sigma} \right] + \frac{1}{\beta} \quad (9)$$

where  $\alpha = \alpha(s)$ .

As we will see below, cycles can occur in our economy because of the interplay between the SWE and the GWE.

Let us introduce the following critical values:

$$\alpha_F \equiv \frac{1}{1-\sigma} \left[ \beta(1-\delta) + \frac{1+\beta}{2} - \sigma \frac{1+\beta}{1-\beta(1-\delta)} \right] \quad (10)$$

$$\sigma^* \equiv [1-\beta(1-\delta)] \left[ \frac{1}{2} + \frac{\beta(1-\delta)}{1+\beta} \right] < 1 \quad (11)$$

**Proposition 3 (two-period cycles)** *If  $\sigma < \sigma^*$  or  $\sigma > 1$ , in a neighborhood of  $\alpha = \alpha_F > 0$ , persistent corruption cycles generically arise through a flip bifurcation.*

Because of the Arrow-Mangasarian restriction (Assumption 2),  $\sigma < \sigma^*$  becomes  $1/2 < \sigma < \sigma^*$ . These inequalities make sense only if  $\sigma^* > 1/2$ , which is equivalent to

$$\beta < \frac{1}{1+2(1-\delta)} \quad (12)$$

Inequality (12) holds, for instance, in the case of full social capital depreciation ( $\delta = 1$ ).

Therefore, a flip bifurcation value exists either under dominant intertemporal substitution effects ( $1/2 < \sigma < \sigma^*$ ) or dominant income effects ( $\sigma > 1$ ). It is known that the occurrence of a flip bifurcation entails the existence of period-doubling bifurcations and, eventually, chaos. Thus, dynamics are even richer.

Cycles can occur when the elasticity of intertemporal substitution in consumption is not excessively high ( $(1/\sigma) < 2$ ). Let us explain why. Consider an exogenous rise in the social capital  $s$  at time  $t$ . Since  $c_t = w_t l_t$ , we can disentangle its negative and positive effects on consumption: (1) consumption lowers because corruption sands the wheels and reduces the wage rate according to the zero profit condition  $w_t = A(s_t)$ ; (2) consumption grows because the social

capital greases the wheels and increases the labor supply according to condition  $l_t = s_t(1 - n_t)$ . Under a strong SWE ( $\alpha = \alpha_F$ ), the drop in the wage rate (1) dominates the GWE (2). To maintain the consumption level ( $(1/\sigma) < 2$ ), the household raises her labor supply by reducing the time spent on networking ( $n_t$ ). According to the social capital accumulation law (1), if the capital depreciation rate is sufficiently low, a drop in the next period social capital follows. Thus, a rise in  $s_t$  entails a drop in  $s_{t+1}$  in the end. In other words, the interplay between GWE and SWE generates corruption cycles.

Corruption cycles are documented on the empirical ground but very few papers have examined these cycles from a theoretical perspective.<sup>3</sup> To the best of our knowledge, Bicchieri and Duffy (1997) is the only paper addressing theoretically the emergence of corruption cycles. The authors show the existence of cycles in a repeated game if politicians can compensate voters with material incentives by using resources gained during an "honest" period. We tell an alternative story from a growth-theoretical instead of a game-theoretical perspective. Our approach provides a different rationale to explain the corruption cycles based on the interplay between two empirically established effects: GWE and SWE.

The following proposition excludes the possibility of limit cycles through a Hopf bifurcation.

**Proposition 4 (impossibility of limit cycles)** *Under Assumption 2, Hopf bifurcations are ruled out.*

### 3 Central planner

In the previous section, we have seen that the economy can experience periodic cycles because of the interplay between GWE and SWE. In the current section, we reconsider this issue when the central planner internalizes the two effects. By definition, the central planner solves the following program:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

under the resource constraints

$$N_t c_t \leq \sum_i F(L_{it}) = A(s_t) \sum_i L_{it} \quad (13)$$

$$\sum_i L_{it} = N_t l_t \equiv s_t N_t (1 - n_t) \quad (14)$$

and the (individual) social capital accumulation law.

$$s_{t+1} - (1 - \delta) s_t \leq B n_t \quad (15)$$

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<sup>3</sup>See Gillespie and Okruhlik (1991) or Sidorkin and Vorobyev (2018) among others.

Notice that all agents are identical (representative agent's hypothesis) and, hence, to maximize the social welfare or the utility function is the same. We observe also that the external effect  $A(s_t)$  is now internalized, that is, in the program,  $\bar{s}_t = s_t$  becomes a choice variable, no longer taken as given. Finally, the social capital accumulation (15) remains an individual trajectory but, now, decided by the social planner.

Constraints (13) and (14) yield:

$$c_t \leq s_t A(s_t) (1 - n_t)$$

As we will see below, the existence of a steady state requires the following restriction.

**Assumption 3**  $\alpha < 1$ .

Assumption 3 means that, in order to preserve the convexity of the planner's program, the external effects of corruption (SWE) should not be excessive.

Finally, in order to solve the central planner's program and satisfy the Arrow-Mangasarian conditions, we introduce an additional assumption.

**Assumption 4** (*Arrow-Mangasarian*) *The function*

$$v(s_t, n_t) \equiv u(s_t A(s_t) (1 - n_t))$$

*is strictly concave.*

**Lemma 5** *Under Assumption 3, in the isoelastic case:*

$$A(s_t) \equiv A s_t^{-\alpha} \text{ and } u(c_t) \equiv \frac{c_t^{1-\sigma}}{1-\sigma}$$

*Assumption 4 is equivalent to:*

$$(\sigma < 1 \text{ and } \frac{1-2\sigma}{1-\sigma} < \alpha < 1) \text{ or } (\sigma > 1 \text{ and } \alpha < 1) \quad (16)$$

Unsurprisingly, in the social planner's case, the Arrow-Mangasarian conditions are more restrictive. The central planner internalizes the SWE and  $\alpha$  can not exceed a critical value to preserve the program convexity, while, in the competitive case, one needs no restriction on  $\alpha$  since the SWE is an external effect out of individuals' control. As we will see, the emergence of cycles requires a larger negative externality in the centralized economy than in the decentralized one, but the more restrictive Arrow-Mangasarian conditions on  $\alpha$  does not rule out the possibility of cycles in the planner's case.

**Proposition 6 (social optimum)** *Under Assumption 4, the planner's solution is given by the following two-dimensional dynamic system:*

$$\frac{u'(c_t)}{u'(c_{t+1})} A(s_t) s_t = \beta(1-\delta) A(s_{t+1}) s_{t+1} + \beta B \frac{c_{t+1}}{s_{t+1}} [1 - \alpha(s_{t+1})] \quad (17)$$

$$s_{t+1} - (1-\delta) s_t = B - \frac{B c_t}{s_t A(s_t)} \quad (18)$$

jointly with the initial condition  $s_0$  and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t u'(c_t) A(s_t) s_t s_{t+1} = 0$$

where (17) represents the Euler equation, while (18), the social capital accumulation, now binding.

### 3.1 Steady state

As above, the steady state is defined by  $(s_t, c_t) = (s_{t+1}, c_{t+1})$ . We obtain

$$\begin{aligned} s^* &= \frac{\beta - \alpha\beta}{1 - \beta + 2\beta\delta - \alpha\beta\delta} B \\ c^* &= \frac{1 - \beta + \beta\delta}{1 - \beta + 2\beta\delta - \alpha\beta\delta} A(s^*) s^* \end{aligned}$$

and

$$n^* = \frac{(1 - \alpha)\beta\delta}{1 - \beta + 2\beta\delta - \alpha\beta\delta}$$

where  $\alpha = \alpha(s^*)$  and  $*$  denotes the planner's solution. The steady state is positive ( $s^*, c^* > 0$ ) if and only if  $\alpha < 1$ .

Interestingly, the social capital is positive at the steady state. It would be not the case if corruption has only a negative impact on the TFP through the SWE. But, in our model, corruption is a by-product of networking, which has a positive effect on labor productivity through the GWE, and the central planner has to control the social capital accumulation to balance these opposite effects. The social capital is nonzero precisely because of the GWE. We will compare the steady states of the market economy and the planner's solution in the last section of the paper before concluding.

Consider now the explicit form:  $A(s_t) \equiv A s_t^{-\alpha}$ . Since  $\alpha < 1$ , the networking effectiveness  $B$  has a positive impact on social capital accumulation as well as the household's patience  $\beta$ , while the absolute value of the corruption elasticity of production  $\alpha$  has a negative effect:

$$\begin{aligned} \frac{B}{s^*} \frac{\partial s^*}{\partial B} &= 1 \\ \frac{\beta}{s^*} \frac{\partial s^*}{\partial \beta} &= \frac{1}{1 - \beta + (2 - \alpha)\beta\delta} > 0 \\ \frac{\alpha}{s^*} \frac{\partial s^*}{\partial \alpha} &= -\frac{\alpha(1 - \beta + \beta\delta)}{(1 - \alpha)(1 - \beta + 2\beta\delta - \alpha\beta\delta)} < 0 \end{aligned}$$

Moreover,

$$\frac{\alpha}{n^*} \frac{\partial n^*}{\partial \alpha} = -\frac{\alpha}{1 - \alpha} \frac{1 - \beta(1 - \delta)}{1 - \beta[1 - \delta(2 - \alpha)]} < 0$$

Under Assumption 3, the interpretations about the effects of  $B$  and  $\beta$  on  $s^*$  provided in the case of a market economy, remain the same. However, now, the

central planner internalizes the negative externality (SWE) induced by the social capital. A higher  $\alpha$  reinforces the negative effect of the social capital on factor productivity. To mitigate this negative effect, the central planner dampens the accumulation of social capital by reducing the time spent in networking. Therefore, a rise of  $\alpha$  entails a drop both in  $s^*$  and in  $n^*$  in the long run.

The impacts on consumption of  $B$  and  $\beta$  depends on the corruption elasticity of production:

$$\begin{aligned}\frac{B}{c^*} \frac{\partial c^*}{\partial B} &= 1 - \alpha \\ \frac{\beta}{c^*} \frac{\partial c^*}{\partial \beta} &= \frac{(1 - \alpha)(1 - \beta)}{(1 - \beta + \beta\delta)[1 - \beta + \beta\delta(2 - \alpha)]} > 0 \\ \frac{\alpha}{c^*} \frac{\partial c^*}{\partial \alpha} &= \alpha \ln \frac{1}{s^*} + \frac{\alpha\beta\delta}{1 - \beta + \beta\delta(2 - \alpha)}\end{aligned}$$

Under Assumption 3, the effects of  $B$ ,  $\beta$  and  $\alpha$  on consumption are analogous to those we have observed in the market economy, and the interpretations remain the same. However,  $\alpha$  has a larger positive effect on consumption in a centralized economy because of the positive additional term on the right. Unsurprisingly, the central planner improves the welfare through an increase in the consumption level, by internalizing the negative externality.

### 3.2 Optimal cycles

As above, we study the occurrence of cycles, now optimal by definition of central planner. We consider the same isoelastic fundamentals of the market economy.

$$A(s_t) \equiv As_t^{-\alpha} \text{ and } u(c_t) \equiv \frac{c_t^{1-\sigma}}{1-\sigma}$$

with  $\alpha < 1$  to ensure the steady state existence.

**Lemma 7 (local dynamics)** *Dynamics are approximated around the steady state by the following linear system:*

$$\begin{bmatrix} \frac{ds_{t+1}}{s} \\ \frac{dc_{t+1}}{c} \end{bmatrix} = J \begin{bmatrix} \frac{ds_t}{s} \\ \frac{dc_t}{c} \end{bmatrix}$$

where  $J$  is the Jacobian matrix of system (17)-(18) linearized around the steady state  $(s, c)$ , with the following trace and determinant:

$$T = \frac{1}{\beta} + \frac{1}{\sigma - 1 + \beta(1 - \delta)} \left[ \sigma + \frac{[1 - \beta(1 - \delta)][1 - \beta(1 - \delta)(2 - \alpha)]}{\beta(1 - \alpha)} \right] \quad (19)$$

$$D = \frac{1}{\beta} \quad (20)$$

Let us introduce the following values:

$$\begin{aligned}
\sigma_1 &\equiv \frac{1 - \beta(1 - \delta)}{1 + \beta} \frac{1 + \beta(3 - 2\delta)}{2} \\
\sigma_2 &\equiv \frac{1 - \beta(1 - \delta)}{1 + \beta} \frac{2 + \beta(2 - \delta)}{2} > \sigma_1 \\
\sigma_{\pm} &\equiv \frac{2\sigma_2 - \sigma_1 \pm \sqrt{(2\sigma_2 - \sigma_1)^2 - 4(\sigma_2 - \sigma_1)}}{2}
\end{aligned} \tag{21}$$

**Proposition 8** *When  $\alpha$  crosses the critical value*

$$\alpha^* \equiv \frac{\sigma - \sigma_1}{\sigma - \sigma_2}$$

*the system generically undergoes a flip bifurcation and a two-period cycle arises. There exists a parameter configuration such that  $\alpha^*$  satisfies Assumptions 3 and 4. More precisely, there exists  $\beta^* > 0$  such that, for any  $\beta \in (0, \beta^*)$ , the interval  $I \equiv (\sigma_-, \min\{1, \sigma_2, \sigma_+\})$  is nonempty and  $\sigma \in I$  is equivalent to Assumptions 3 and 4.*

Interestingly, we recover the possible occurrence of periodic cycles as in the market economy. In other words, periodic cycles generated by the interplay between GWE and SWE seem to be a robust feature.

**Proposition 9** *There is no room for Hopf bifurcations.*

As was the case in the market economy (Proposition 1), even in the case of the central planner, limit cycles are ruled out.

## 4 Market solution vs social optimum

Comparing the dynamic systems (2)-(3) and (17)-(18), we see that the social capital accumulation law remains the same, while the Euler equations differ:

$$\begin{aligned}
\frac{u'(c_t)}{u'(c_{t+1})} A(s_t) s_t &= \beta(1 - \delta) A(s_{t+1}) s_{t+1} + \beta B \frac{c_{t+1}}{s_{t+1}} \\
\frac{u'(c_t)}{u'(c_{t+1})} A(s_t) s_t &= \beta(1 - \delta) A(s_{t+1}) s_{t+1} + \beta B \frac{c_{t+1}}{s_{t+1}} [1 - \alpha(s_{t+1})]
\end{aligned}$$

The planner now internalizes the external effect of corruption by introducing the corrective factor  $1 - \alpha(s_{t+1})$ .

Focus now on the steady state. Let  $A(s_t) \equiv A s_t^{-\alpha}$ , with  $\alpha < 1$  to ensure the steady state existence.

Comparing the market solution  $(s, c, n)^*$  with the planner's one  $(s, c, n)$  (see equations (4), (5) and (6)), we observe that

$$\begin{aligned}
s^* &= \frac{\beta - \alpha\beta}{1 - \beta + 2\beta\delta - \alpha\beta\delta} B < \frac{\beta}{1 - \beta + 2\beta\delta} B = s \\
n^* &= \frac{(1 - \alpha)\beta\delta}{1 - \beta + 2\beta\delta - \alpha\beta\delta} < \frac{\beta\delta}{1 - \beta + 2\beta\delta} = n
\end{aligned}$$

Moreover,

$$\begin{aligned}
c^* &= \frac{1 - \beta + \beta\delta}{1 - \beta + 2\beta\delta - \alpha\beta\delta} A(s^*) s^* < \frac{1 - \beta + \beta\delta}{1 - \beta + 2\beta\delta} A(s) s = c \\
\Leftrightarrow \beta &< \frac{1}{1 + \delta \left[ \frac{\alpha}{1 - (1 - \alpha)^{\frac{1 - \alpha}{2 - \alpha}}} - 2 \right]} \equiv \beta^* \in (0, 1)
\end{aligned}$$

Notice that  $\alpha \in (0, 1)$  implies

$$\frac{\alpha}{1 - (1 - \alpha)^{\frac{1 - \alpha}{2 - \alpha}}} - 2 > 0$$

Clearly, it is meaningless to compare the welfare functions at the steady state  $\sum_{t=0}^{\infty} \beta^t u(c) = u(c) / (1 - \beta)$ , because the initial conditions are not the same:  $c^* \neq c$ . However we can see that, if agents are sufficiently impatient ( $\beta < \beta^*$ ), asymptotically, the consumption in the market economy exceeds the consumption in the centralized economy. This implies that at some initial date, the consumption in the centralized economy exceeds the one in the market economy. Indeed, by definition of social optimum,  $\sum_{t=0}^{\infty} \beta^t u(c^*) \geq \sum_{t=0}^{\infty} \beta^t u(c)$ .

Agents accumulate more social capital in the market economy at the steady state ( $s > s^*$ ), because they do not take in account the negative impact of corruption. This leads to a lower consumption at the steady state when they are patient ( $\beta > \beta^*$ ). However, if they become impatient ( $\beta < \beta^*$ ), the planner increases more the initial consumption at the beginning by internalizing the negative impact of corruption through an additional reduction of social capital. This leads to a lower consumption level in the long run with respect to the market economy.

Finally, we can compare the flip bifurcation values in the cases of market economy and central planner. Interestingly, we find

$$\alpha^* - \alpha_F = \alpha^* \frac{\beta(2 - \delta)[2\sigma - 1 + \beta(1 - \delta)]}{2(1 - \sigma)(1 - \beta + \beta\delta)}$$

If  $\sigma > 1/2$  (implying Assumption 4 under Assumption 3 according to (16)), then  $\alpha^* > \alpha_F$ . The emergence of cycles requires a larger negative externality in the centralized economy than in the decentralized.

The interpretation is straightforward. As seen above,  $s^* < s$ . Therefore, the sensitivity of  $A$  with respect to  $s$  (captured by  $\alpha$ ) has to be higher in the centralized economy than in the market one to ensure a sufficiently large negative externality (SWE) to exceed the positive effect (GWE) and generate cycles.

## 5 Conclusion

We have mixed three theoretical blocks (Ramsey, 1928; Romer, 1986; and Lucas, 1988) in a model where a representative household decides how much time to

spend in networking, entailing corruption as an unpleasant collateral effect. Networking has a positive effect at the individual level, greasing the wheels, facilitating projects and enhancing the labor productivity in the end. However, corruption is a by-product of networking with a negative social effect: impairing the quality of public services, it lowers the total factor productivity and sands the wheels. Both these effects are supported by a solid empirical evidence.

We have provided conditions for the existence of a unique steady state either in the market or in the planner's solution. In the short run, the interplay between the "sand the wheels" and the "grease the wheels" effects promotes the occurrence of persistent two-period cycles. More precisely, not only a flip bifurcation but also period-doubling bifurcations and chaos are possible.

Comparing the market economy and the planner's solution, we observe that cycles are more likely to occur in a market economy because the central planner internalizes the negative externality of corruption, the "sand the wheels" effect.

## 6 Appendix

### Proof of Proposition 1

Since the utility function is strictly increasing (Assumption 1), the budget constraint holds with equality  $c_t = w_t s_t (1 - n_t)$ . In this case, the maximization of the Lagrangian function for the market economy:

$$L_M \equiv \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \mu_t [Bn_t - s_{t+1} + (1 - \delta) s_t]$$

leads to the following first-order conditions.

$$\frac{\partial L_M}{\partial s_t} = \beta^t u'(c_t) w_t (1 - n_t) + (1 - \delta) \mu_t - \mu_{t-1} = 0 \quad (22)$$

$$\frac{\partial L_M}{\partial n_t} = -\beta^t u'(c_t) w_t s_t + \mu_t B = 0 \quad (23)$$

$$\frac{\partial L_M}{\partial \mu_t} = Bn_t - s_{t+1} + (1 - \delta) s_t = 0 \quad (24)$$

jointly with the transversality condition  $\lim_{t \rightarrow \infty} \mu_t s_{t+1} = 0$ .

We need to introduce second-order restrictions to ensure that the maximization is well-defined.

Assumption 1 allows us to redefine the utility function as follows  $w_t^{1-\sigma} v(n_t, s_t) = u(w_t s_t (1 - n_t))$  where

$$v(n_t, s_t) \equiv \frac{[s_t (1 - n_t)]^{1-\sigma}}{1 - \sigma}$$

The Hessian matrix of  $v$  is given by

$$H(n_t, s_t) \equiv \begin{bmatrix} \frac{\partial^2 v}{\partial n_t^2} & \frac{\partial^2 v}{\partial s_t \partial n_t} \\ \frac{\partial^2 v}{\partial n_t \partial s_t} & \frac{\partial^2 v}{\partial s_t^2} \end{bmatrix}$$

$$= \begin{bmatrix} -\sigma [s_t (1 - n_t)]^{-\sigma} \frac{s_t}{1 - n_t} & (\sigma - 1) [s_t (1 - n_t)]^{-\sigma} \\ (\sigma - 1) [s_t (1 - n_t)]^{-\sigma} & -\sigma [s_t (1 - n_t)]^{-\sigma} \frac{1 - n_t}{s_t} \end{bmatrix}$$

$v$  is strictly concave if and only if

$$\frac{\partial^2 v}{\partial n_t^2} = -\sigma [s_t (1 - n_t)]^{-\sigma} \frac{s_t}{1 - n_t} < 0 \text{ and } \det H = (2\sigma - 1) [s_t (1 - n_t)]^{-2\sigma} > 0$$

that is, if and only if Assumption 2 holds.

These conditions are necessary and sufficient for maximization under Assumption 2 (Arrow-Mangasarian).

Replacing  $\mu_t = \beta^t u'(c_t) w_t s_t / B$  in (22), we get the system

$$\frac{u'(c_t)}{u'(c_{t+1})} \frac{w_t}{w_{t+1}} s_t = \beta B (1 - n_{t+1}) + \beta (1 - \delta) s_{t+1} \quad (25)$$

$$s_{t+1} - (1 - \delta) s_t = B n_t \quad (26)$$

jointly with the transversality condition  $\lim_{t \rightarrow \infty} \beta^t u'(c_t) A(s_t) s_t s_{t+1} = 0$ .

In the labor market, the aggregate demand is equal to the aggregate supply:  $\sum_i L_{it} = N_t l_t$ .

The equilibrium in the social capital market entails:  $\bar{s}_t = s_t$ .

The equilibrium in the goods market implies

$$n_t = 1 - \frac{c_t}{A(s_t) s_t}$$

Replacing these equations in (25) and (26), we obtain the dynamical system (2)-(3). ■

### Proof of Lemma 2

We linearize the dynamical system (2)-(3) around the steady state  $(s, c)$ :

$$[\beta(1 - \delta)(2 - \alpha) - 1] \frac{ds_{t+1}}{s} + [1 - \beta(1 - \delta) - \sigma] \frac{dc_{t+1}}{c} = (1 - \alpha) \frac{ds_t}{s} - \sigma \frac{dc_t}{c}$$

$$\beta \frac{ds_{t+1}}{s} = [1 - \alpha + \alpha\beta(1 - \delta)] \frac{ds_t}{s} + [\beta(1 - \delta) - 1] \frac{dc_t}{c}$$

In matrix terms, we get (7), where

$$J = \begin{bmatrix} \beta(1 - \delta)(2 - \alpha) - 1 & 1 - \beta(1 - \delta) - \sigma \\ \beta & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 - \alpha & -\sigma \\ 1 - \alpha + \alpha\beta(1 - \delta) & \beta(1 - \delta) - 1 \end{bmatrix}$$

denotes the Jacobian matrix. ■

### Proof of Proposition 3

A flip bifurcation generically arises iff  $D = -T - 1$ , where  $T$  and  $D$  are given by (8) and (9). Replacing  $T$  and  $D$  in this equation, we get the flip bifurcation

value (10). In a neighborhood of  $\alpha = \alpha_F$ , persistent corruption cycles arise. Clearly,  $\alpha_F > 0$  if and only if  $\sigma < \sigma^*$  or  $\sigma > 1$ , where  $\sigma^*$  is given by (11). ■

**Proof of Proposition 4**

This bifurcation generically arises iff  $D = 1$  and  $-2 < T < 2$ . We observe that

$$T = D - \frac{1}{\beta} - \frac{\sigma}{\beta} \frac{1 - \beta(1 - \delta) + \beta\delta}{1 - \beta(1 - \delta) - \sigma} + 2(1 - \delta)$$

$D = 1$  is equivalent to

$$\alpha_H \equiv \frac{1 - \beta}{1 - \sigma} \left[ 1 - \frac{\sigma}{1 - \beta(1 - \delta)} \right]$$

Moreover,  $D = 1$  implies

$$T = 1 - \frac{1}{\beta} + 2(1 - \delta) - \frac{\sigma}{\beta} \frac{1 - \beta(1 - \delta) + \beta\delta}{1 - \beta(1 - \delta) - \sigma}$$

Then,  $T < 2 \Leftrightarrow \sigma < 1 - \beta(1 - \delta)$ .

If  $\sigma < 1 - \beta(1 - \delta)$ , we have  $-2 < T \Leftrightarrow \sigma < \sigma^{**}$ , where

$$\sigma^{**} \equiv [1 - \beta(1 - \delta)] \left[ 1 - \frac{1 - \beta(1 - 2\delta)}{4\beta} \right] < 1 - \beta(1 - \delta)$$

Therefore,  $-2 < T < 2 \Leftrightarrow \sigma < \sigma^{**}$ . It is possible to show that  $\sigma^{**} < 1/2$  against Assumption 2.

Therefore, under Assumption 2,  $\sigma > 1/2 > \sigma^{**}$  and, thus, if  $D = 1$ ,  $T < -2$ : there is no room for Hopf bifurcations. ■

**Proof of Lemma 5**

We observe that

$$v(s_t, n_t) = \frac{[s_t A(s_t)(1 - n_t)]^{1 - \sigma}}{1 - \sigma}$$

with

$$\frac{s_t A'(s_t)}{A(s_t)} = -\alpha \text{ and } \frac{s_t A''(s_t)}{A'(s_t)} = -1 - \alpha$$

We can compute the Hessian matrix:

$$H(s_t, n_t) \equiv \begin{bmatrix} \frac{\partial^2 v}{\partial n_t^2} & \frac{\partial^2 v}{\partial s_t \partial n_t} \\ \frac{\partial^2 v}{\partial n_t \partial s_t} & \frac{\partial^2 v}{\partial s_t^2} \end{bmatrix}$$

with

$$\begin{aligned} \frac{\partial^2 v}{\partial n_t^2} &= -\sigma [s_t A(s_t)(1 - n_t)]^{-\sigma} \frac{s_t A(s_t)}{1 - n_t} \\ \frac{\partial^2 v}{\partial s_t^2} &= -(1 - \alpha) [\alpha + (1 - \alpha)\sigma] A(s_t)^2 [s_t A(s_t)(1 - n_t)]^{-\sigma} \frac{1 - n_t}{s_t A(s_t)} \\ \frac{\partial^2 v}{\partial n_t \partial s_t} &= (1 - \alpha)(\sigma - 1) A(s_t) [s_t A(s_t)(1 - n_t)]^{-\sigma} \end{aligned}$$

This Hessian matrix is negative definite iff

$$\frac{\partial^2 v}{\partial n_t^2} = -\sigma [s_t A(s_t) (1 - n_t)]^{-\sigma} \frac{s_t A(s_t)}{1 - n_t} < 0$$

and

$$\det H = (1 - \alpha) [\alpha (1 - \sigma) + 2\sigma - 1] A(s_t)^2 [s_t A(s_t) (1 - n_t)]^{-2\sigma} > 0$$

Therefore, the Hessian matrix is negative definite iff

$$(1 - \alpha) [\alpha (1 - \sigma) + 2\sigma - 1] > 0$$

or, more explicitly, iff

$$\begin{aligned} \sigma &< 1 \text{ and } \frac{1 - 2\sigma}{1 - \sigma} < \alpha < 1 \\ \text{or} \\ \sigma &> 1 \text{ and } (\alpha < 1 \text{ or } \alpha > \frac{2\sigma - 1}{\sigma - 1}) \end{aligned}$$

or, equivalently, under Assumption 3, (16) holds. ■

#### **Proof of Proposition 6**

The central planner's Lagrangian function becomes

$$L_P \equiv \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \lambda_t [Bn_t - s_{t+1} + (1 - \delta) s_t]$$

(with, under Assumption 1,  $c_t = s_t A(s_t) (1 - n_t)$ ) and maximization leads to the following first-order conditions:

$$\frac{\partial L_P}{\partial s_t} = \beta^t u'(c_t) [A(s_t) + s_t A'(s_t)] (1 - n_t) + (1 - \delta) \lambda_t - \lambda_{t-1} = 0 \quad (27)$$

$$\frac{\partial L_P}{\partial n_t} = -\beta^t u'(c_t) s_t A(s_t) + \lambda_t B = 0 \quad (28)$$

$$\frac{\partial L_P}{\partial \lambda_t} = Bn_t - s_{t+1} + (1 - \delta) s_t = 0 \quad (29)$$

jointly with the transversality condition

$$\lim_{t \rightarrow \infty} \lambda_t s_{t+1} = \lim_{t \rightarrow \infty} \beta^t u'(c_t) A(s_t) s_t s_{t+1} / B = 0$$

Under Assumption 3, these first-order conditions are not only necessary but also sufficient for maximizing the social welfare.

Notice that, the main difference with respect to the first-order conditions (22)-(24) is the additional term  $s_t A'(s_t)$  internalizing the negative externality of corruption.

Eliminating the multipliers and replacing in system (27)-(29)

$$n_t = 1 - \frac{c_t}{s_t A(s_t)}$$

we obtain system (17)-(18). ■

**Proof of Proposition 7**

We linearize the system (17)-(18) around the steady state:

$$\begin{aligned} [\beta(1-\delta)(2-\alpha)-1] \frac{ds_{t+1}}{s} + [1-\beta(1-\delta)-\sigma] \frac{dc_{t+1}}{c} &= (1-\alpha) \frac{ds_t}{s} - \sigma \frac{dc_t}{c} \\ \beta(1-\alpha) \frac{ds_{t+1}}{s} &= (1-\alpha) \frac{ds_t}{s} - [1-\beta(1-\delta)] \frac{dc_t}{c} \end{aligned}$$

where  $\sigma = \sigma(c)$ .

The Jacobian matrix  $J$  is given by:

$$J = \begin{bmatrix} \beta(1-\delta)(2-\alpha)-1 & 1-\beta(1-\delta)-\sigma \\ \beta(1-\alpha) & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1-\alpha & -\sigma \\ 1-\alpha & \beta(1-\delta)-1 \end{bmatrix}$$

We obtain the trace and the determinant (19)-(20). ■

**Proof of Proposition 8**

The locus of flip bifurcations is given by  $D + T + 1 = 0$ . Solving for  $\alpha$  we get the flip bifurcation value  $\alpha^*$ .

(1)  $\sigma < 1$ . According to Lemma 5, we require

$$\frac{1-2\sigma}{1-\sigma} < \alpha^* < 1$$

that is

$$\frac{1-2\sigma}{1-\sigma} < \frac{\sigma-\sigma_1}{\sigma-\sigma_2} < 1 \quad (30)$$

(1.1) If  $\sigma < \sigma_2$ , (30) is equivalent to

$$(1-2\sigma)(\sigma_2-\sigma) < (1-\sigma)(\sigma_1-\sigma) < (1-\sigma)(\sigma_2-\sigma)$$

that is to

$$(1-2\sigma)(\sigma_2-\sigma) < (1-\sigma)(\sigma_1-\sigma)$$

that is to  $\sigma_- < \sigma < \sigma_+$  where  $\sigma_-$  and  $\sigma_+$  are given by (21).

According to (21), if  $(2\sigma_2 - \sigma_1)^2 - 4(\sigma_2 - \sigma_1) > 0$ , then

$$0 < \sigma_- < \frac{2\sigma_2 - \sigma_1}{2} < \sigma_+ < 2\sigma_2 - \sigma_1$$

Therefore,

$$\alpha^* \in \left( \frac{1-2\sigma}{1-\sigma}, 1 \right) \quad (31)$$

iff  $\sigma < 1$ ,  $\sigma < \sigma_2$  and  $\sigma_- < \sigma < \sigma_+$ .

Consider  $\beta$  close to zero:  $\lim_{\beta \rightarrow 0} \sigma_1 = 1/2$ ,  $\lim_{\beta \rightarrow 0} \sigma_2 = 1$ ,  $\lim_{\beta \rightarrow 0} \sigma_- = 1/2$ ,  $\lim_{\beta \rightarrow 0} \sigma_+ = 1$ . Thus, if  $\beta$  is close to zero, we get that (31) is satisfied (and, according to Lemma 5, Assumptions 3 and 4) iff  $1/2 < \sigma < 1$ . By continuity, we get the conclusion of Proposition 8.

(1.2) If  $\sigma > \sigma_2$ , (30) is equivalent to

$$(1 - 2\sigma)(\sigma - \sigma_2) < (1 - \sigma)(\sigma - \sigma_1) < (1 - \sigma)(\sigma - \sigma_2)$$

a contradiction because  $\sigma_1 < \sigma_2$ .

(2)  $\sigma > 1$ . According to Lemma 5, we require  $0 < \alpha^* < 1$ , that is

$$0 < \frac{\sigma - \sigma_1}{\sigma - \sigma_2} < 1$$

(2.1) If  $\sigma < \sigma_2$ , then  $0 < \alpha^* < 1$  is equivalent to

$$0 < \sigma_1 - \sigma < \sigma_2 - \sigma$$

that is to  $\sigma < \sigma_1$ . But  $\sigma_1 < 1$ , a contradiction.

(2.2) If  $\sigma > \sigma_2$ , then  $0 < \alpha^* < 1$  is equivalent to  $0 < \sigma - \sigma_1 < \sigma - \sigma_2$ , a contradiction because  $\sigma_1 < \sigma_2$ .

Thus,  $\sigma > 1$  rules out the existence of  $\alpha^* \in (0, 1)$  and there is no room for a flip bifurcation.

Summing up, putting together points (2) and (3), we find that there exists  $\beta^*$  such that  $\beta \in (0, \beta^*)$  implies  $\sigma_- < \min\{1, \sigma_2, \sigma_+\}$ , that is, there exists a nonempty interval  $I$  for  $\sigma$  such that for any value inside a critical flip bifurcation value  $\alpha^*$  satisfying Assumptions 3 and 4 exists. ■

#### **Proof of Proposition 9**

We observe that  $D = 1/\beta > 1$ . ■

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