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Thomas Huchard, Guillaume Robin, Marc Ponçot, Sandrine Hoppe, El Mostafa Daya. Elaboration, Characterization and Modelling of Periodic Viscoelastic Sandwich Beams for Lightning and Vibration Damping. *Mechanics Research Communications*, 2022, 121, pp.103863. <10.1016/j.mechrescom.2022.103863>. <hal-04142889>

HAL Id: hal-04142889

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Submitted on 27 Jun 2023

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Elaboration, Characterization and Modelling of Periodic Viscoelastic Sandwich Beams for Lightning and Vibration Damping

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Abstract

In order to maximize the properties of viscoelastic materials, they are often used as an interlayer between two elastic layers inducing shearing in the core layer and thus passive damping. This paper focuses on the design, characterization and modelling of sandwich structures with periodic and viscoelastic core. The viscoelastic polymers used are blends of isotactic polypropylene grafted maleic anhydride (iPP-g-MAH), selected for its adhesion properties to metal surfaces, with other polymers in order to adapt the mechanical properties. The periodicity of the sandwich structure, is obtained by alternating two materials with specific viscoelastic properties into the core, this allows the resonance frequencies to be located in bundles creating frequency ranges without any resonance peaks. The latter, called "Band Gaps", are simulated using finite element modelling. Parametric studies are then conducted to study the impact of variations in geometric and mechanical properties on the width and position of these Band Gaps. Samples of periodic sandwich composites developed in the laboratory are tested on a vibrating pot in order to experimentally determine the damping properties of the sandwich. The results obtained are then compared with those obtained numerically in order to validate the parameters used in the modelling.

Keywords: Sandwich, Composites, Viscoelastic, Periodic, Damping

1. Introduction

Noise reduction and vibration control are objectives of many industrial companies in various fields ranging from aerospace, aeronautics to automotive and even civil engineering. Coupled with a view to reducing the mass of structures, these objectives have led to the development of viscoelastic damped structures [1–3]. The most classical viscoelastic damped structures are sandwich composites in which a flexible viscoelastic layer is sandwiched between two rigid elastic faces. In addition to the lightning and the flexibility, this structure allows an effective passive damping by the difference in displacement of the two elastic faces inducing a strong shear in the central layer. [4,5]

The vibration modelling of those sandwich composites has been widely studied for the past decades. In his work, Rao [6] gave an exact analytical formula to calculate the eigenfrequencies and loss factors for a complex linear model for various boundary conditions. Then, many studies were carried out to develop numerical models for this type of problem [7–9]. A review of various theories for sandwich structures modelling is provided by Hu *et al.* [10] As for the non-linear domain, different methods were developed, such as an asymptotical numerical method [11], iterative algorithms [12], resolution of complex non-linear equations [13,14].

Furthermore, the growing number of studies [15–19] on phonic crystals have led to the appearance of periodic

structures for vibration control. The repetition of periodic patterns enables the appearance of a filtering effect. The resonant frequencies regroup into pass bands where the acoustic waves propagate freely leaving frequency ranges without any resonance called Band Gaps (BG).

To couple this band gap phenomenon with the passive damping of a viscoelastic sandwich composite, structures presenting the repetition of a periodic pattern have appeared. The different periodic structures are numerous. Periodicity in the boundary conditions have been studied by Daya *et al.* [20–22] and Lougou *et al.* [23]. This type of periodicity can present two kinds of vibration modes: localized modes and modulated ones. Thus, two scale methods can be used for this problem. Ruzzene *et al.* [24] studied periodic sandwich plates exhibiting an auxetic structure which behave as directional mechanical filters. Sandwich structures with a honeycomb core were widely studied. Ruzzene [25] developed a numerical model where Boudjemai *et al.* [26] studied this type of structure experimentally. In a more recent work, Zhou *et al.* [27] studied a honeycomb structure filled with a viscoelastic damping material. A corrugation in the core layer was investigated by Buannic *et al.* [28]. Additionally, many works combining different kind of the periodic structures presented before were carried out [24,29,30]. Finally, another type of periodic

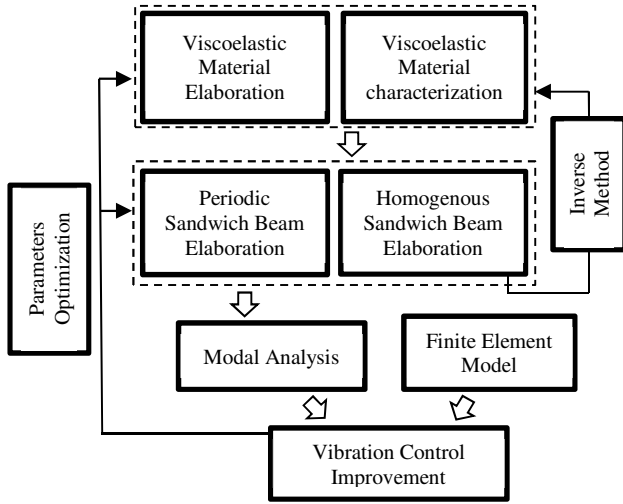


Fig. 1. Outline of the optimizing process of viscoelastic sandwich composite with a periodic structure

structure is an alternation of two different materials inside the central layer. This type of structure, which will be studied in this paper, was previously studied by different authors. In particular, Guo *et al.* [31] developed a numerical model to calculate the flexural waves in a periodic sandwich beam. Sheng *et al.* [32] and Jiang *et al.* [33] studied numerically and experimentally periodic sandwich plates composed of foam / PMMA and silicone rubber / epoxy, respectively. These works permitted to highlight both the damping and filtering properties of this type of periodicity.

Other studies investigate the impact of a functional gradient and geometric heterogeneities on the mechanical and vibrational properties of sandwich structures [34,35]. It is shown that these inhomogeneities play an important role in the determination of resonance frequencies.

To go further in the understanding of the role of the viscoelastic cores inside a periodic sandwich beam, chemically modified thermoplastic polymers can be used as core materials. Isotactic polypropylene can be grafted with maleic anhydride (iPP-g-MAH) to enhance the adhesive properties. This grafting was studied by Minoura *et al.* [36], but it was found that grafting maleic anhydride deteriorates the mechanical properties of the polymer [37]. Therefore, L  toff   *et al.* [38–40] studied the crosslinking of iPP-g-MAH with a polyether diamine and a polyether triamine.

The objective of this paper is to propose a new approach focusing on the global study of periodic viscoelastic sandwich beams. It goes from the elaboration of the sandwich composites, to the experimental vibrational characterization through numerical modelling (Fig. 1), in order to optimize this type

of structure. A finite element method is adapted from a homogenous to a periodic structure. Chemically modified viscoelastic polymers are elaborated and characterized to then elaborate some periodic sandwich beam samples exhibiting vibration damping and filtering, mechanical strength and strong bonding. The vibrational properties of these samples are determined experimentally and compared to the numerical model. Finally, in order to study and optimize the placement and width of Band Gaps, parametric studies were conducted on the numerical model. This helps to better understand and improve the filter properties of sandwich composites.

2. Numerical modelling

2.1. Modelling of viscoelastic sandwich beams

A viscoelastic sandwich beam is composed of a viscoelastic core in between two elastic faces. In this paper, the viscoelastic core is composed of thermoplastic polymers and the elastic faces are aluminium sheets. The kinematic model used is Rao’s zig-zag model [6] schematized on Fig. 2. Diverse assumptions used by several authors are made, such as:

- Plane transverse sections to the middle plane remain plane after bending
- Shearing is neglected in the elastic faces meaning that only the shearing of the core is considered
- The three layers undergo the same transverse deflection (w)
- The interfaces are considered as perfect meaning that no slipping occurs
- Both the materials of the core and the faces are considered linear homogeneous and isotropic

Bilasse *et al.* [14] showed that the linear vibration analysis with the finite element method is formulated by the following elementary eigenvalue problem:

$$([K^e(\omega)] - \omega^2[M^e])\{U^e\} = \{F\} \quad (1)$$

where $[M^e]$ and $[K^e]$ are element mass and stiffness matrices, respectively. When assembled, these matrices give the following complex nonlinear eigenvalue problem:

$$([K(\omega)] - \omega^2[M])\{U\} = \{F\} \quad (2)$$

where $[M]$ and $[K]$ are the assembled mass and stiffness matrices, respectively, $\{U\}$ the total displacement vector and $\{F\}$ the nodal force vector.

The frequency dependence of the stiffness matrix makes the problem nonlinear. This nonlinearity implies that it cannot be resolved by classical eigenvalue procedures. Many methods have been developed in the literature to remedy this problem in free vibrations ($F = 0$). [7,8,12,41–43]

In order to have a reduced computational cost, the asymptotic numerical method (ANM) developed by Abdoun *et al.* [9], which is adapted from the model of Azrar *et al.* [11,44], is used in this study. This method allows direct calculation of the vibrational response by splitting $[K(\omega)]$:

$$\begin{cases} [K(\omega)] = [K(0)] + E(\omega)[K_v] \\ [K(0)] = E^*(0)[K_v] \\ E(\omega) = E^*(\omega) - E^*(0) \end{cases} \quad (3)$$

where $E^*(\omega)$ is the complex Young modulus and $[K_v]$ does not

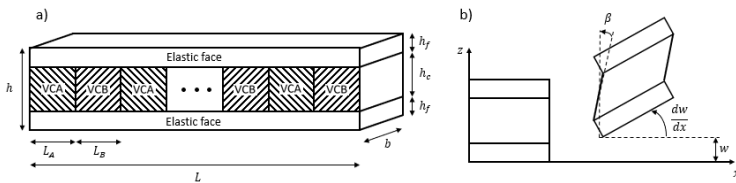


Fig. 2. Periodic sandwich beam (a) and the kinematic zig-zag model (b) (VCA: viscoelastic core A; VCB: viscoelastic core B)

depend on ω . Eqn.2 thus becomes:

$$([K(0)] + E(\omega)[K_v] - \omega^2[M])\{U\} = \{F\} \quad (4)$$

Then, to calculate the response curve, harmonic excitations $\{F\}$ are applied in such a way that all modes of interest are solicited. The displacement response curves $\{U\}$ are obtained by assembling and inverting the complex dynamic matrix $([K(0)] + E(\omega)[K_v] - \omega^2[M])$ for each pulse ω . For each mode, the resonant frequency Ω and the loss factor η can thus be determined, at the resonance peak and by the 3dB method [7], respectively.

2.2. Modelling of periodic viscoelastic sandwich beams

In order to analyse the damping of periodic viscoelastic sandwich beams, the previously presented model must be adapted to a periodic structure. The periodic structure studied in this paper, schematized in Fig.2, is composed of a unit cell containing symmetrically two different viscoelastic materials.

The transition from the simple core sandwich model to a periodic one relies on the assembly of different element matrices: $[M^e]$, $[K^e(0)]$ and $[K_v^e]$. Two different core materials imply two different matrices for each element matrices: $[M^{eA}]$, $[M^{eB}]$, $[K^{eA}(0)]$, $[K^{eB}(0)]$, $[K_v^{eA}]$ and $[K_v^{eB}]$. In this model, each element contains two nodes and each node has three degrees of freedom which are the transversal displacement w , the slope w' and the rotation β . Thus, the elementary matrices are of dimension 6 by 6 and are detailed after:

$$[M^{eA}] = (2\rho_f S_f + \rho_{cA} S_c)[m^e] \quad (5)$$

$$[M^{eB}] = (2\rho_f S_f + \rho_{cB} S_c)[m^e] \quad (6)$$

$$[K_v^{eA}] = 2(1 + \nu_{cA})I_c[k_1^e] + S_c[k_2^e] \quad (7)$$

$$[K_v^{eB}] = 2(1 + \nu_{cB})I_c[k_1^e] + S_c[k_2^e] \quad (8)$$

$$[K^{eA}(0)] = \left(\frac{1}{2}E_f S_f h_c^2 + I_c E_{cA}^0\right)[k_1^e] - \frac{1}{2}E_f S_f h_f h_c[k_2^e] + \frac{1}{2}E_f(4I_f + S_f h_f^2)[k_3^e] + \frac{E_{cA}^0 S_c}{2(1 + \nu_{cA})}[k_4^e] \quad (9)$$

$$[K^{eB}(0)] = \left(\frac{1}{2}E_f S_f h_c^2 + I_c E_{cB}^0\right)[k_1^e] - \frac{1}{2}E_f S_f h_f h_c[k_2^e] + \frac{1}{2}E_f(4I_f + S_f h_f^2)[k_3^e] + \frac{E_{cB}^0 S_c}{2(1 + \nu_{cB})}[k_4^e] \quad (10)$$

The matrices $[m^e]$, $[k_1^e]$, $[k_2^e]$, $[k_3^e]$ and $[k_4^e]$ are integration matrices of the shape functions and are given in appendix.

Then, the global matrices $[M]$, $[K(0)]$, and $[K_v]$ are obtained by a dichotomous assembly. This is a classical assembly when the finite elements are composed of the same material. But at the interface between two finite elements of different materials, a superposition of the element matrices must be made due to the common node for both elements.

To validate this periodic sandwich beam model, a comparison has been made with the results obtained by Guo *et al.* [31] and the calculation made by the present model with the same parameters. The results are similar, thus validating the model presented in this study.

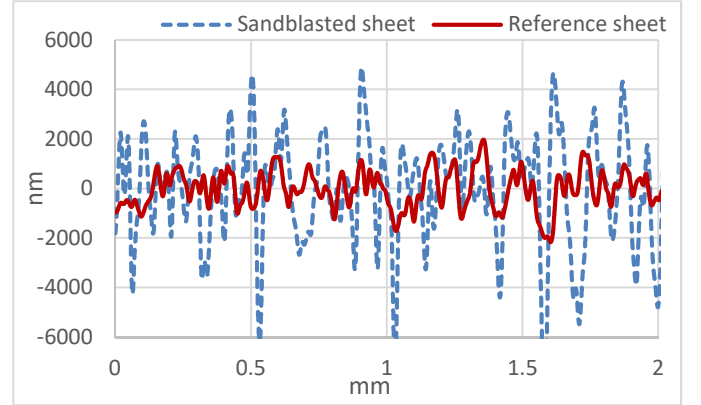


Fig. 3. Surface profile of a sandblasted aluminium sheet and a reference aluminium sheet

3. Sandwich composite structure

3.1. Core materials

The viscoelastic materials used are divided into two categories for this paper: stiff and soft cores differentiated by their shear modulus values. In order to avoid the use of glue or adhesive, the core materials are chosen for their metal bonding properties in addition to their viscoelasticity. This is why the viscoelastic materials used in this study are polymers grafted with maleic anhydride which allows a good adhesion to metals and especially to aluminium [38,39,45].

To have the distinction between the stiff and soft cores, semi crystalline polymers and elastomers are designated. The semi crystalline material is a blend of isotactic polypropylene (iPP) with isotactic polypropylene grafted with 1 wt% of maleic anhydride (iPP-g-MAH). The iPP-g-MAH is blended into a virgin iPP in order to compensate the deterioration of its mechanical properties due to maleic anhydride grafting. Whereas for the softer materials, two polymers are selected. The first is a thermoplastic elastomer maleated styrene-ethylene-butylene-styrene (SEBS-g-MAH) and the second is a blend of an elastomer: the ethylene propylene diene monomer rubber (EPDM) with iPP-g-MAH.

The virgin iPP used is the PP 3050 MN1 provided by Total Petrochemical. This polymer has been well studied in the literature [46]. Its average molecular weight is estimated at $\overline{M}_n = 76\,000 \text{ g.mol}^{-1}$. The iPP-g-MAH was supplied by Arkema under the commercial name Orevac® CA100. Its yield strength is 22 MPa, and the engineering elongation at break is 12%. Its average molecular weight is estimated at $\overline{M}_n = 25\,000 \text{ g.mol}^{-1}$ [47,48]. The EPDM used is composed of 23.7% polyethylene, 71.8% polypropylene and 4.5% ethylidene norbornene, in weight percentage. It was supplied by Kraiburg under the reference HTC8364/43. Finally, the SEBS-g-MAH as provided by Sigma Aldrich. According to the information given by Sigma Aldrich, it contains around 2 wt% of maleic anhydride.

The different blends were made in the LRGP laboratory (CNRS - University of Lorraine, Nancy, France) using a twin-screw extruder. The proportion of the blends are: 80 wt% /



Fig. 4. a) Top view and b) side view of a sandwich composite with a 4 cells periodic structure

20 wt% for the iPP / iPP-g-MAH and 95 wt% / 5 wt% for the EPDM / iPP-g-MAH. The weight percentage of iPP-g-MAH is kept under 20 wt% because a higher proportion would decrease the adhesion strength [49].

3.2. Sandwich composites elaboration

The elastic faces of the sandwich composites are made out of 1050A aluminium sheets. These sheets are sandblasted in order to increase rugosity which allows a better adhesion with the core. The rugosity coefficient Ra of the aluminum sheets, measured with a profilometer (Fig.3), went from 270 nm for the reference sheet to 3250 nm for the sandblasted one.

The sandwich composites were made by using a stainless-steel mould, where the polymer cores were placed in the form of plates between the two aluminium sheets. The assembly was then hot pressed at 190 °C for 10 minutes to melt entirely the two polymers so that they adhere well to the aluminium sheets and to each other. The mould is then cooled to room temperature in the open air. An example of a periodic sandwich composite is shown on Fig. 4, where the periodicity in the polymer core is visible.

3.3. Identification of composite properties

The elastic faces properties are standard aluminium sheets, so their properties are well known. The core thickness may varies depending on the samples because of the elaboration process. For each sandwich composite beam, the total thickness is measured and the corresponding numerical simulation takes the measured thickness as input. The core thickness may varies depending on the samples because of the elaboration process. For each sandwich composite beam, the total thickness is measured and the corresponding numerical simulation takes the measured thickness as input. All the properties needed as input parameters for the finite element calculation are listed in Table 1.

The mechanical properties (G and η) of polymer cores can differ depending on the solicitation and forming conditions, in particular for elastomers where this variation represents a higher percentage because of the low values of G . For this reason, instead of using the DMA results as input parameters in the numerical calculations, an inverse method is used to identify the mechanical properties of the viscoelastic cores B and B bis of simple core sandwich composite.

The identification method used was developed by Ledi *et al.* [50]. This method consists in extrapolating the frequency dependence of the shear modulus and the loss factor from experimental results on simple core sandwich composites. These experimental results are obtained during vibration tests by determining resonant frequencies and loss factors for different bending modes. Then, the algorithm calculates the mode shapes and computes the viscoelastic properties from the modes using a Rayleigh quotient until convergence on the material properties is met. Thus, a point cloud of $G(\omega)$ and $\eta(\omega)$ is obtained by this method. The frequency dependent laws of $G(\omega)$ and $\eta(\omega)$ can then be deduced by an interpolation depending on the chosen rheological model.

Differential mechanical analysis (DMA) results are chosen as starting values and experimental results of simple core sandwich composites are used in the algorithm for viscoelastic core B and B bis. A polynomial distribution of order 2 is then extrapolated allowing the frequency dependence of G and η in the form of:

$$G(\omega) = a_1\omega^2 + b_1\omega + c_1 \quad (11)$$

$$\eta(\omega) = a_2\omega^2 + b_2\omega + c_2 \quad (12)$$

These identified dependencies (Table 2) are then considered as input parameters for the numerical calculation of the vibrational spectra of sandwich composites.

Table 1. List of the parameters of the sandwich beams

Elastic faces: Aluminium	Young Modulus	$E_f = 6.9 * 10^{10} Pa$
	Poisson Ratio	$\nu_f = 0.3$
	Mass density	$\rho_f = 2766 kg.m^{-3}$
Viscoelastic core A: iPP / iPP-g-MAH 80 % / 20 %	Shear Modulus	$G_0 = 6.52 * 10^8 Pa$
	Loss factor	$\eta_c = 0.076$
	Poisson Ratio	$\nu_c = 0.42$
	Mass density	$\rho_c = 938 kg.m^{-3}$
Viscoelastic core B: SEBS-g-MAH	Shear Modulus	Table 2
	Loss factor	Table 2
	Poisson Ratio	$\nu_c = 0.45$
	Mass density	$\rho_c = 910 kg.m^{-3}$
Viscoelastic core B bis: EPDM / iPP-g-MAH 5 % / 95 %	Shear Modulus	Table 2
	Loss factor	Table 2
	Poisson Ratio	$\nu_c = 0.33$
	Mass density	$\rho_c = 900 kg.m^{-3}$
Beam geometry	Core Thickness	$h_c = 2.63 mm$
	Faces Thickness	$h_f = 1.2 mm$
	Length	$L = 263 mm$
	Width	$b = 22 mm$

Table 2. Identification of the frequency dependent viscoelastic laws

Viscoelastic core B: SEBS-g-MAH		
$a_1 (Pa s^{-2})$	$b_1 (Pa s^{-1})$	$c_1 (Pa)$
-1.20E-02	1.67E+02	4.65E+06
$a_2 (s^{-2})$	$b_2 (s^{-1})$	c_2
-1.27E-10	7.76E-06	7.83E-02
Viscoelastic core B bis: EPDM/iPP-g-MAH 5 % / 95 %		
$a_1 (Pa s^{-2})$	$b_1 (Pa s^{-1})$	$c_1 (Pa)$
-7.95E-03	3.12E+02	1.53E+07
$a_2 (s^{-2})$	$b_2 (s^{-1})$	c_2
-4.06E-10	1.05E-05	6.00E-02

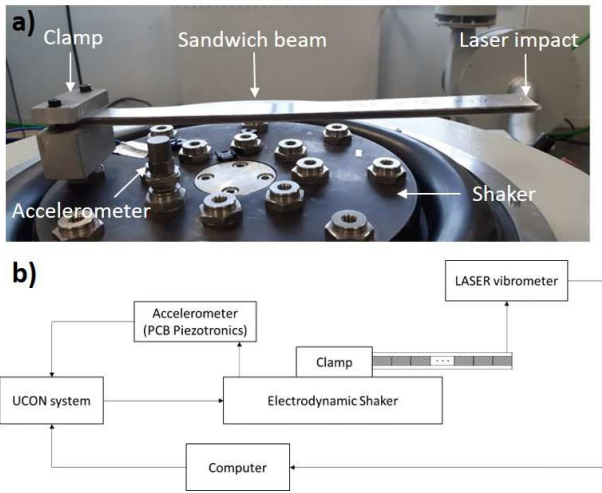


Fig. 5. Experimental setup picture (a) and diagram (b) of the vibration test

4. Experimental measurement of the vibrational properties

4.1. Experimental set up

Fig. 5 shows the experimental setup. The setup is composed by a shaker, a laser vibrometer and a boundary structure to attach the study beam on the shaker. The generation of the vibrations is done with the shaker which are controlled and measured by a controller device (UCON system). The vibration amplitude of the shaker is controlled using an accelerometer (PCB Piezotronics). The range of frequency excitation is between 10 and 3000 Hz. The amplitude of the beam's vibration is measured by a laser vibrometer at the free extremity and **in the middle of the width in order to avoid torsion modes**.

The amplitude of the shaker is chosen to minimize nonlinear vibrations typically 0.002 mm displacement between 10 and 357 Hz and a 1g acceleration between 357 and 3000 Hz. The response spectrum using FRF (Frequency Response Function) data allows us to determine the resonant frequencies and the loss factors.

4.2. Vibrational testing of sandwich composites with different core materials

The results obtained for different periodic core sandwich beams with a 3-cells structure are shown on Fig. 6. For each beam, the experimental results are compared with the results obtained numerically using the finite element model presented previously. The objective of this paper being the study of the damping of periodic core sandwich beams, the amplitudes were normalized logarithmically to facilitate the comparison between the experiment and the simulation.

Fig. 6 a) and b) are results obtained with periodic core sandwich beams having the same structure and materials in order to have slight repeatability. A good correlation between the experiment and the simulation can be observed for all the resonant frequencies except the fourth one which may be due to an artefact caused by the experimental set up. **Indeed, a relative error obtained is mostly less than 12%. It could be caused by some experimental difficulties such as geometrical inhomogeneity in the sample created during the elaboration process; imprecision of the identification of the rheological models.** Also, two different band gaps can be identified: the first one is located between the third and the fourth resonant frequencies (620 Hz and 1130 Hz) and the second one between the sixth and seventh resonant frequencies (1680 Hz and 2720 Hz).

The changes due to a different material can be observed on Fig. 6 c), where the viscoelastic core B is replaced from SEBS-g-MAH to EPDM / iPP-g-MAH 5 % / 95 %. The shear modulus is thus slightly increased which results in an increase of the resonant frequencies **as expected**. The band gaps are then located between 740 Hz and 1250 Hz for the first one and between 1825 Hz and more than 3000 Hz for the second one. It can thus be observed that the band gaps move towards the higher frequencies when the modulus of the elastomeric core increase.

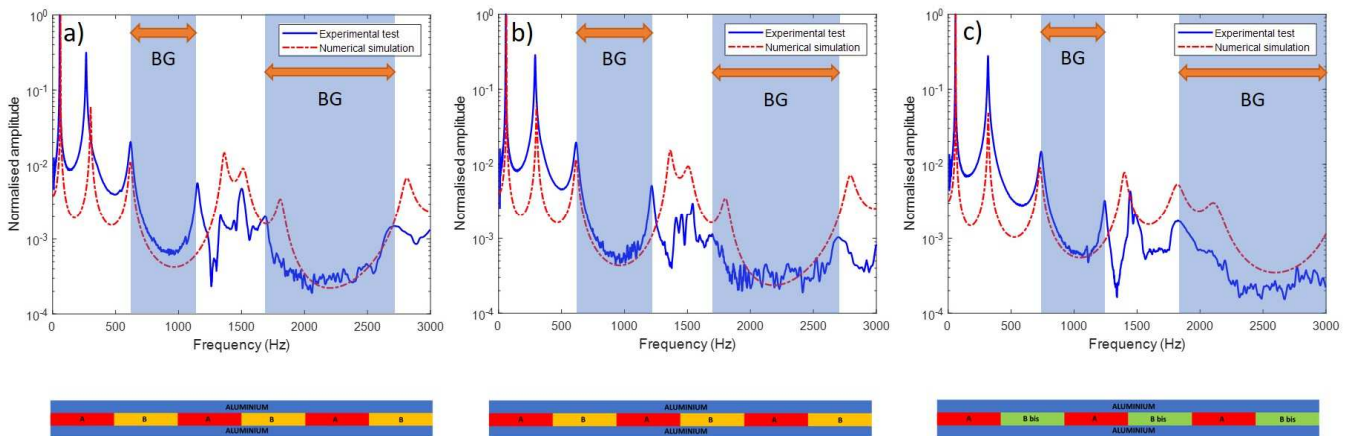


Fig. 6. Vibrational spectra of computed and experimental behaviour of 3-cell sandwich beams with viscoelastic core A & B (a & b), viscoelastic core A & B bis (c) and their respective schematic

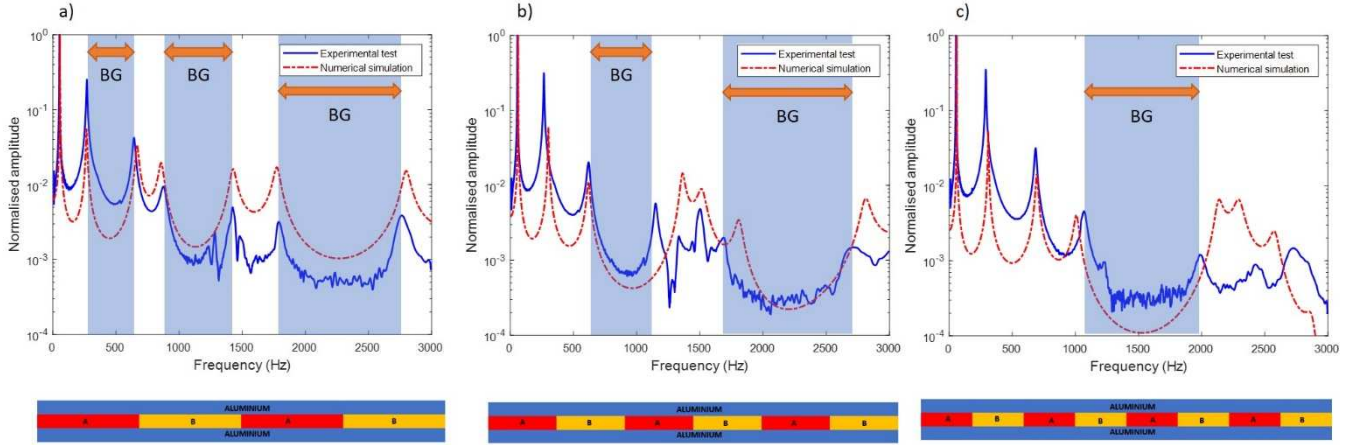


Fig. 7. Vibrational spectra of computed and experimental behaviour of a 2-cell (a), a 3-cell (b) and a 4-cell (c) sandwich beam and their respective schematic

4.3. Vibrational testing of sandwich composites with different periodicity

Two ways of changing the number of unit cells composing the periodic core sandwich beams. The way studied here is to keep the same total length for the different beams, meaning that when the number of unit cells increases, the length of a unit cell decreases and vice versa. No matter the number of cells, the total mass and length of the periodic core sandwich beams are equal.

Three different configurations are studied here and correspond to a 2-, 3- and 4-unit cells repetition. The results for these different configurations are shown on Fig. 7 a), b) and c) respectively where the numerical simulations correlate well with the experimental results. It can be observed that the first band gap moves from 270 – 640 Hz for a 2-cells structure, to 620 – 1130 Hz for a 3-cells structure and to 1070 – 2000 Hz for a 4-cells structure. Adding the fact that the number of band gaps observed below 3000 Hz changes from 3 to 1, it can be extrapolated that band gaps move towards the higher frequencies when the number of unit cells increases and vice versa.

As for the width of the first band gap, it goes from 370 Hz for a 2-cells structure, to 510 Hz for a 3-cells structure and finally to 930 Hz for a 4-cells structure. Thus, the band gap width also increases when the number of unit cells increases and vice versa.

5. Parametric Studies

In order to study the impact of the geometrical parameters and the mechanical parameters of the viscoelastic materials on the damping properties and thus the band gaps, parametric studies were carried out by varying different variables in comparison to the reference model. Two results are presented in this paper. To measure a relative band gap width independently of the spectral position, a band gap ratio R_{BG} is defined as:

$$R_{BG} = \frac{f_{k+1} - f_k}{f_k - f_{k-(N-1)}} \quad (13)$$

where f_k is the k-th resonance where the Band Gap begins and N is the number of periodic patterns.

The impact of variations in the stiff material modulus is shown on Fig. 8. (a) where the shear modulus of the stiff core G_{c1} varies from 10^6 to 10^{10} Pa and the soft core G_{c2} keeps its reference value of 4.65×10^6 Pa. The resonant frequencies

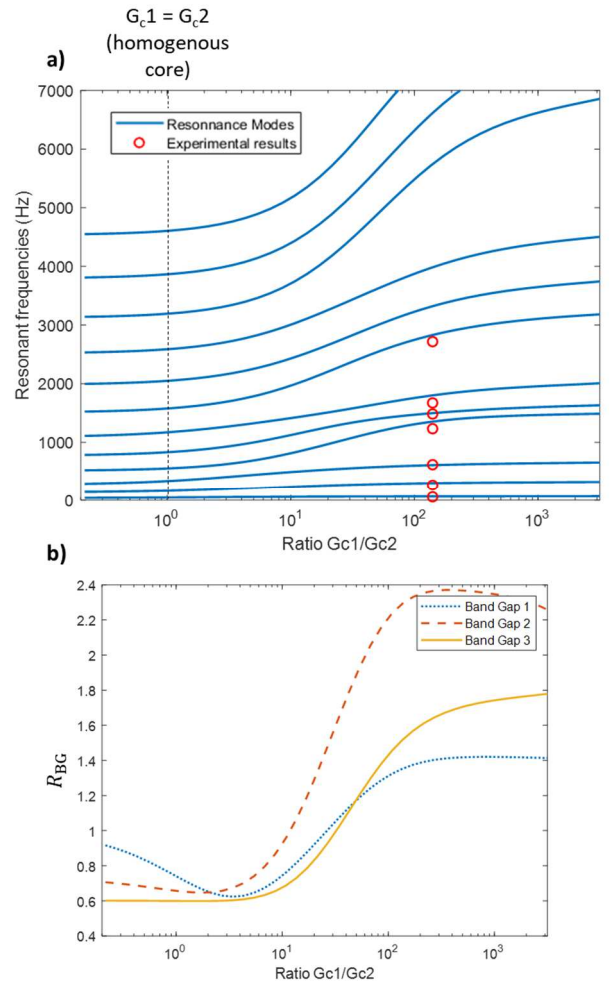


Fig. 8. Effect of the variation of the ratio between the stiff and soft shear moduli on the resonant frequencies (a) and on the band gap ratio (b). The experimental results are highlighted in red circles.

and the band gap ratios calculated are then plotted as a function of the ratio G_{c1}/G_{c2} . These variations show that the band gap width increases between $G_{c1} = G_{c2}$ and $G_{c1} = 100 \times G_{c2}$ and then stabilizes. It can be deduced that 2 orders of magnitude are needed between the two moduli in order to have the optimal band gaps.

The evolution of resonant frequencies and relative band gap width as a function of the thickness ratio h_c/h_f is shown in Fig.9 (a) and (b) respectively. The total thickness of the composite is constant at a value of 5.4 mm. A maximum band gap width can be observed between thickness ratios of 2 and 4 meaning that, in order to have wide band gaps, the optimal structure must have a core between 2 and 3 times thicker than the elastic faces.

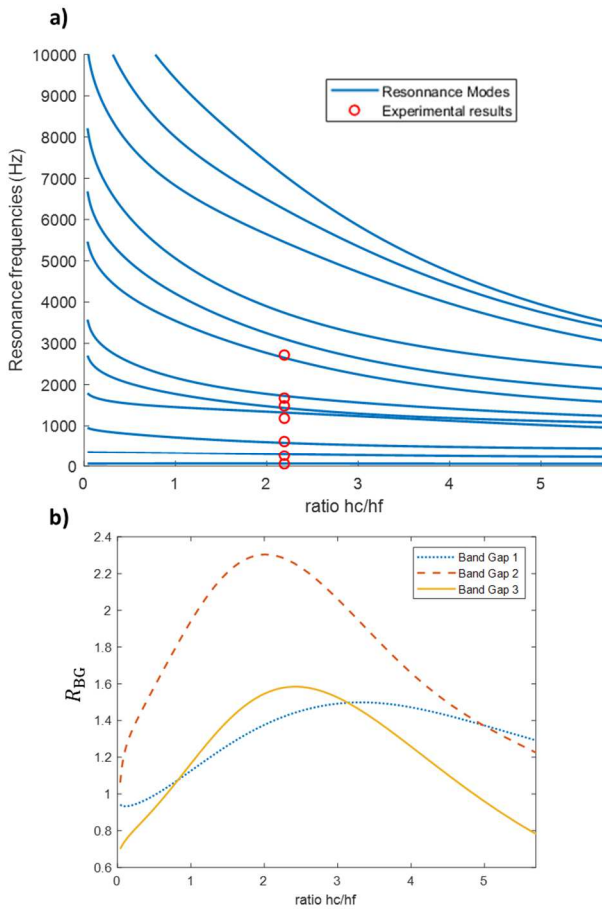


Fig. 9. Effect of the variation of thickness ratio on the resonant frequencies (a) and on the band gap ratio (b). The experimental results are highlighted in red circles.

6. Conclusion

The optimization of viscoelastic sandwich beams with a periodic structure for noise reduction and weight reduction was examined. Samples were elaborated with both semi crystalline polymers and elastomers to have periodic and viscoelastic cores. Those polymers were grafted with maleic anhydride to enhance the bonding with the aluminium faces without any use of adhesive. The damping and band gap properties were

determined experimentally and compared to numerical results based on a finite element model developed from a simple core viscoelastic sandwich model.

The experimental results showed good correspondence with the numerical simulations. The sandwich beams exhibited a filtering behaviour with the presence of band gap and also vibration absorption within the pass bands. The changes in position and width of the band gaps were studied experimentally with different periodic structure and different material, but also numerically where optimums in geometrical and mechanical parameters were exhibited.

At this point, only flexural vibrations were studied, leaving torsion modes to future work on viscoelastic sandwich plates with a periodic structure. A change in the periodic structure would be an interesting perspective as well. A more complex structure produced by additive manufacturing could increase damping while keeping a band gap structure. The adhesion between aluminium and the different viscoelastic polymers and its influence on the damping properties would also justify another study.

Acknowledgements

The French Ministry of Research (DRRT), the regional Council “Région Lorraine”, the European Regional Development Fund (FEDER) has contributed to the funding of the *vibration and wave propagation platform*.

Appendix

Denoting by L^e the element length and setting the variable $\xi = 2x/L^e - 1$, for $x \in [0 L^e]$ so that $\xi \in [-1 1]$. The classical polynomial shape functions used in the finite element model are defined as:

$$n_1(\xi) = \frac{(1 - \xi)^2(2 + \xi)}{4} \quad (14)$$

$$n_2(\xi) = \frac{L^e(1 - \xi)^2(1 + \xi)}{8} \quad (15)$$

$$n_3(\xi) = \frac{(1 + \xi)^2(2 - \xi)}{4} \quad (16)$$

$$n_4(\xi) = \frac{L^e(1 + \xi)^2(1 - \xi)}{8} \quad (17)$$

$$n_5(\xi) = \frac{1 - \xi}{2} \quad (18)$$

$$n_6(\xi) = \frac{1 + \xi}{2} \quad (19)$$

These shape functions are then integrated:

$$[m^e] = \int_{-1}^1 J^T [N_w] [N_w] d\xi \quad (20)$$

$$[k_1^e] = \int_{-1}^1 J^{-1 T} [N'_\beta] [N'_\beta] d\xi \quad (21)$$

$$[k_2^e] = \int_{-1}^1 J^{-2} ({}^T [N'_\beta] [N''_w] + {}^T [N''_w] [N'_\beta]) d\xi \quad (22)$$

$$[k_3^e] = \int_{-1}^1 J^{-3 T} [N''_w] [N''_w] d\xi \quad (23)$$

$$[k_4^e] = \int_{-1}^1 (J^{-1 T} [N'_w] [N'_w] + {}^T [N_\beta] [N'_w]) d\xi + \int_{-1}^1 (J^{-1 T} [N'_w] [N_\beta] + J^T [N_\beta] [N'_w]) d\xi \quad (24)$$

where $J = L^e/2$, $[N_w] = [n_1(\xi) n_2(\xi) 0 n_3(\xi) n_4(\xi) 0]$ and $[N_\beta] = [0 0 n_5(\xi) 0 0 n_6(\xi)]$.

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