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Numerical simulation and modelling of the hydrodynamic forces and torque acting on individual oblate spheroids

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Computation of a three dimensional uniform, steady Newtonian flow past oblate spheroidal particles is undertaken. The main objective of the present study is to compute the hydrodynamic forces on oblate spheroidal particles as a function of the particle orientation, for different particle aspect ratio and a large range of particle Reynolds number. Results of simulations are used to provide a new complete set of correlations for drag, lift and torque coefficients. These correlations are derived for an aspect ratio ranging from 0.2 to 1, for particle Reynolds number up to 100 and for all orientation. In addition, it is found that the Stokesian evolution of the drag and lift coefficient as a function of the incidence remains still valid at moderate particle Reynolds number, i.e., drag coefficient evolves as a sine squared and lift coefficient evolves as $(\sin \phi \cos \phi)$.

Keywords: Drag, Lift, Pitching torque, Body fitted.

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I. INTRODUCTION

Fluid-solid flows carrying non-spherical particles are widely found in many environmental and industrial applications such as pollen transport¹ micro-organisms², pulp and paper³ or pulverized solid fuels⁴. Most methods for modeling a gas-solid flows require information on the behavior of an individual solid particle in a gaseous flow. These particles can have an indefinite possibility of shapes and sizes such as ellipsoids, cylinder, discs or more often irregular in shape. Owing to the importance of the particle shape on the particle motion, the non-sphericity is more and more considered in the euler-lagrange modeling and simulations of particles transport in fluid flows^{5–15}. However, in the listed works, the hydrodynamic forces are estimated under the Stoksonian assumption using the Brenner's models¹⁶ to determine the hydrodynamic forces which are in principle strictly valid in the absence of slip between the particle and the fluid flow. Indeed, while the Brenner's formulas are accurate in Stokes flow, significant deviations can be observed when the particle Reynolds numbers is greater than 1^{17–20}.

Until now, investigations of the hydrodynamic forces acting on non-spherical particles were mainly devoted to the drag force^{18,19,21–26}. Only few works have been focused on the lift force as well as the torques^{27–35}. Furthermore, the developed expressions, characterizing one of the mentioned hydrodynamic coefficients (C_D , C_L and C_T) remain specific to a particular aspect ratio and to a definite interval of Reynolds numbers.

Restricting the discussion to the oblate spheroids, there are few works devoted to the hydrodynamic forces acting on such particles. Using a finite difference technique, Rimon and Lugt³⁶ obtained time-dependent viscous flow past an oblate spheroid of aspect ratio 0.05 and 0.2 at Reynolds numbers of 10 and 100. Masliyah and Epstein³⁷ and Comer and Kleinstreuer³⁸ have numerically solved the steady ow past an oblate spheroid of aspect ratio $\lambda \in [0.2-1]$ up to Reynolds numbers of 100. These studies examined the effects of the aspect ratio and the Reynolds number on streamlines and vorticity contours, surface pressure and drag coefficients. Pitter et al.³⁹ performed numerical simulation of a steady ow around oblate spheroids of aspect $\lambda \in [0.05-0.2]$. In addition, the authors performed some experiment as a validation cases for two aspect ratios 0.3 and 0.5. However, both works were focused on

a specific incidence angle ($\phi = 0^\circ$), i.e., the revolution axis is parallel to the flow. Unfortunately, no formula were proposed in these studies^{37,39}. O'Donnell & Helenbrook¹⁷ extend these earlier reported works to develop a quadratic correlation for the drag correlation for a particle Reynolds number up to 200 and aspect ratio $\lambda \in [0.2-1]$. Li et al.⁴⁰ investigate numerically the interfacial transport characteristics of spheroid bubbles and solid particles for the range of Reynolds numbers up to 100. The authors focused the analysis on comparisons of the drag, wake structures, and mass transfer rates between spheroid solid particles and spheroid inviscid bubbles. Only few informations were provided on individual and total drag coefficients of spheroid particles. Compiling the experimental and numerical drag values provided in the literature, Militzer et al.²¹ proposed a new correlation for the drag coefficient of ellipsoidal particles in the range of aspect $\lambda \in [0.05-5]$ and Reynolds number up to 200. However, this correlation does not take particle orientation into account. In addition, their results overestimate the drag values both at low and high Reynolds numbers in the case of a sphere $\lambda = 1$. Kishore & Gu⁴¹ used data issued from 2D simulation to propose a drag model for an aspect ratio $0.25 \leq \lambda \leq 2.5$ and moderate range of particle Reynolds numbers up to 200. Recently, Ke et al.⁴² used the IB-LBM simulation on a classic problem where a hot stationary ellipsoidal particle is passed by continuous cold fluid flows. Despite the wide range of working conditions ($0.25 \leq \lambda \leq 2.5, 0^\circ \leq \phi \leq 90^\circ, 10 \leq Re_p \leq 200$ and $Pr = 0.744$), the authors focused only on providing correlations for drag coefficient and average Nusselt number.

To sum up, based on the aforementioned discussion, it is clearly noticeable that all these works were focused on the drag coefficient and no complete set of correlations was proposed to quantify the drag lift and torque coefficient for different incidence angles.

Up to now, only investigations performed by Zastawny et al.³⁰ using an immersed boundary method and Sanjeevi et al.³² using Lattice Boltzmann Method provided a complete set of correlation for oblate spheroidal particles. However, both studies focused on specific aspect ratios, $\lambda = 0.2$ and $\lambda = 0.4$, respectively.

According to this literature review, it seems undeniable that further efforts are necessary to develop a complete set of correlations for the hydrodynamic coefficients (C_D , C_L and C_T). Therefore, the main focus of this work is to use the results of numerical simulations to develop new correlations for drag, lift and pitching torque coefficients for oblate spheroids

of an aspect ratio ranging from 0.2 to 1 and a particle Reynolds number up to 100 and for all orientations.

The paper is organized as follows. The motion of non-spherical particles are reported in the second section. The third section is devoted to the numerical simulation technique and the validation cases. The results obtained for the hydrodynamic coefficients and the proposed correlations are given in the fourth section before the conclusion.

II. MOTION OF NON-SPHERICAL PARTICLE

To describe the motion of rigid non-spherical particles (see fig.1), the translational and rotational dynamic have to be solved. The translational dynamic of a heavy non-spherical particle is given in the inertial frame by the following set of differential equations^{43,44}:

$$\rho_p V_p \frac{d\mathbf{v}_p}{dt} = \mathbf{F}_D(Re_p, \lambda, \phi) + \mathbf{F}_L(Re_p, \lambda, \phi) + (\rho_p - \rho_f) V_p \mathbf{g} \quad (1)$$

Here, ρ_f , ρ_p , and V_p represent respectively the fluid density, particle density and the volume of the particle; \mathbf{F}_D and \mathbf{F}_L are respectively the drag and lift forces.

The rotational dynamic of an oblate spheroid is given by Euler's equation²⁹ and reads in the particle frame as:

$$\mathbf{I} \frac{d\boldsymbol{\omega}_p}{dt} + \boldsymbol{\omega}_p \mathbf{I} \times \boldsymbol{\omega}_p = \mathbf{T}_R + \mathbf{T}_P, \quad (2)$$

where, \mathbf{I} is the inertia matrix which is diagonal in the particle frame, $\boldsymbol{\omega}_p = (\omega_{p,x''}, \omega_{p,y''}, \omega_{p,z''})$ is the particle angular velocity vector, while $\mathbf{T}_R = (T_{R,x''}, T_{R,y''}, T_{R,z''})$ and $\mathbf{T}_P = (T_{P,x''}, T_{P,y''}, T_{P,z''})$ denote respectively the rotational and pitching torques with respect to particle axes.

The hydrodynamic forces and torques acting on a non-spherical particle are characterized by the dimensionless coefficients drag C_D , lift C_L and torque C_T coefficients that can be obtained using the following expressions.

$$C_D = \frac{\|\mathbf{F}_D\|}{0.5\rho_f \|u_R\|^2 \frac{\pi d_p^2}{4}}, \quad (3)$$

$$C_L = \frac{\|\mathbf{F}_L\|}{0.5\rho_f \|u_R\|^2 \frac{\pi d_p^2}{4}}, \quad (4)$$

Table I. Parameters of the simulations

Re_p	$\lambda = \frac{c}{a}$	ϕ
0.1 to 100	0.2, 0.5, 0.8 and 1	0° to 90°

$$C_T = \frac{\|\mathbf{T}_P\|}{\rho_f \|u_R\|^2 \frac{\pi d_p^3}{8}}, \quad (5)$$

where d_p is the diameter of the volume equivalent sphere and $\mathbf{u}_R = \mathbf{u}_f - \mathbf{u}_p$ is the relative velocity between the particle \mathbf{u}_p and the fluid \mathbf{u}_f .

In the present work, all simulations were conducted under the assumption of a fixed particle. Thus, the rotational torque is not considered.

III. NUMERICAL SOLUTION METHODOLOGY

A. Parameters of the simulation

The present work is undertaken to fill the gap in the literature. Thus, quantifying the influence of shape and orientation of spheroids on the hydrodynamic coefficients (C_D , C_L and C_T) and providing a complete set of correlations for these coefficients are the main objectives of this study. In Table I, the aspect ratios λ and particle Reynolds numbers Re_p are summarized. Here, the aspect ratio is defined as the ratio between the revolution axis and the major axis $\lambda = \frac{c}{a}$, the incidence angle ϕ is the angle between the revolution axis and the relative velocity, whereas the particle Reynolds number is $Re_p = \frac{\|\mathbf{u}_R\| d_p}{\nu}$, where $\mathbf{u}_R = \mathbf{u}_f - \mathbf{u}_p$ is the relative velocity between the particle \mathbf{u}_p and the fluid \mathbf{u}_f . d_p is the diameter of the volume equivalent sphere, and ν is the kinematic viscosity.

Owing to the asymmetry of wake structures formed behind the inclined spheroids, a complete 3D configuration is considered in all simulations. Thus, the particle is supposed to be fixed and immersed in a tridimensional uniform flow.

B. Governing equation and numerical details

In the present study, it is assumed that the flow is isothermal, incompressible and the fluid is considered as Newtonian. In addition, it is worthwhile mentioning that the flow

fields behind the oblate spheroids when $0.2 \leq \lambda \leq 1$ and $Re_p \leq 100$ show steady regimes. Indeed, Zastawny et al.³⁰ showed that the flow remains steady event for wider particle Reynolds number 300, and inclined spheroid $\phi = 10^\circ$ and $\phi = 40^\circ$. Moreover, Sanjeevi et al.³², noticed a steady flow behavior behind the oblate spheroids with aspect ratio $\lambda = 0.4$ and $Re \leq 100$, whatever the incidence angle. Recently, Ke et al.⁴² noticed that the flow fields behind the particles show unsteady regimes when ($Re_p = 200, \lambda = 0.25, \phi = 0^\circ$) and ($Re_p = 200, \lambda = 0.25, \phi = 30^\circ$). Thus, the unsteady term in the Navier–Stokes equations has been neglected and the final form of governing equations reads as:

$$\nabla \cdot \mathbf{u} = 0, \quad (6)$$

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho_f}\nabla p + \nu\Delta\mathbf{u}, \quad (7)$$

where p is the pressure.

The drag and lift are the components of the fluid forces acting on the considered particle in the three principal directions. These forces are obtained by integrating the total stress acting on the particle surface area:

$$\mathbf{F} = \oint_S [-p\mathbf{n} + \mu(\nabla\mathbf{u} + \nabla\mathbf{u}^T) \cdot \mathbf{n}] dS. \quad (8)$$

Here, the drag force $F_D = \mathbf{n}_x \cdot \mathbf{F}$ is the force acting in the x direction and $F_L = \mathbf{n}_y \cdot \mathbf{F}$ is the force acting in the y direction.

Since the pitching torque is resulted from the resistance force acting the pressure center, the general formula reads as:

$$\mathbf{T} = \oint_S [\mathbf{r} \times (-p\mathbf{n} + \mu(\nabla\mathbf{u} + \nabla\mathbf{u}^T) \cdot \mathbf{n})] dS, \quad (9)$$

where, the vector \mathbf{r} is defined as the vector between the specific wall cell and the center point and the pitching torque $T_p = \mathbf{n}_z \cdot \mathbf{T}$ acts in the z direction.

Simulations are performed using the CFD code Ansys Fluent© 14.5. Centered scheme is used for discretizing all the numerical fluxes in the Navier–Stokes equations. The problem of the coupling between velocity and pressure is tackled with a Semi-Implicit Method for Pressure-Linked Equations (SIMPLE)⁴⁵. The geometrical configuration (figure 2) is a parallelepiped fluid box surrounding the particle. A constant velocity flat profile is imposed in

Table II. Influence of the domain size on the drag coefficient for the sphere and for $Re_p = 0.1$

Mesh	$\frac{h}{d_p}$	C_D	Deviation from the analytical solution (%)
1	20	247.67	3.20
2	40	245.43	2.26
3	60	245.09	2.12

the inlet boundary, Homogeneous Neumann conditions are chosen at the outlet and a no-slip condition is specified on the lateral boundaries. The dimensions of the box (figure 2) were determined to prevent any disturbance of the flow due to boundary conditions, particularly in the boundary layer. From a practical point of view, these dimensions were imposed by the case $Re_p = 0.1$ resulting in a greater boundary layer where the revolution axis is parallel to the flow ($\phi = 0^\circ$). As a test of the influence of the boundary conditions, three dimensions were tested and the results of the drag coefficient are reported in table II. It can be seen that the geometrical configuration (domain 2) is accurate to perform the simulations since the deviation of the analytical solution of the drag coefficient is not significant compared to that obtained for the domain 3. In addition, the length of the computational box was set to have an adequate distance for the development of the wake. Thus, dimension of the numerical domain are of $120d_p * 80d_p * 80d_p$. Note that, in order to maintain the size of the box accurate for the simulations, the major axis are fixed to be equal to the sphere diameter used to fix the box size, whatever the aspect ratio.

C. Mesh dependence

The mesh of the fluid domain consists of tetrahedral elements (fig.3). The influence of the number of mesh cells on the drag coefficient for an oblate spheroid ($\lambda = 0.2$) and an incidence angle equals to $\phi = 0^\circ$ at $Re_p = 100$ is shown in TabIII. As it can be seen, the drag coefficient can be considered mesh independent with a number of cells greater than 900000. The difference with the finest grid result is less than 0.27 %. For this aspect ratio and incidence angle, the computed values of the lift and torque coefficients are close to zero.

It has to be noted, that the number of cells chosen for the entire simulations is always greater than 900000 (this number varies with λ and ϕ). In addition, for the particle Reynolds

Table III. Influence of the mesh resolution on drag coefficient, for $\lambda = 0.2$, $\phi = 0^\circ$ and $Re_p = 100$

Mesh	Cells number	C_D	Deviation from finest grid (%)
1	739749	1.126	0.626
2	993488	1.122	0.268
3	3039817	1.119	-

number greater than unity, the boundary layer is always represented by at least 20 nodes³¹. Because of the high number of nodes, the calculations are performed by MPI parallel programming (8 processors). The CPU time ranges between 5 and 7h.

D. Validation cases

To improve the numerical technique, different validation tests were conducted for both small and a high particle Reynolds number.

The first test case concerns the ability of the numerical technique to compute the drag coefficient for a spherical particle at higher particle Reynolds number. This can be done by comparing the numerical results with the well known correlation given by Schiller & Naumann⁴⁶:

$$C_D = \frac{24}{Re_p} [1 + 0.15Re_p^{0.687}]. \quad (10)$$

This comparison is reported in a previous work²⁰ for a large range of particle Reynolds number than of that concerned in the present study (Re_p up to 240). The mean relative deviation from Schiller & Naumann's correlation (Eq.10) was found less than 2 %.

The second test case concerns the ability of the numerical technique to compute the drag coefficient for the oblate spheroids at small particle Reynolds number. Results of simulations at $Re_p = 0.1$ are reported in table IV for the drag of both extremums $\phi = 0^\circ$ and $\phi = 90^\circ$. These results are compared to the analytical solution given by Happel & Brenner¹⁶:

$$C_{D,\phi=0^\circ} = K_{D,\phi=0^\circ} C_{D,sphere}, \quad (11)$$

$$C_{D,\phi=90^\circ} = K_{D,\phi=90^\circ} C_{D,sphere}, \quad (12)$$

Table IV. Drag coefficient at $Re_p = 0.1$. Comparison between the results of simulation and the analytical solution (Eqs. 11 and 12)

λ	$C_{D,\phi=0^\circ}$			$C_{D,\phi=90^\circ}$		
	Eq:11	Simulation	Deviation	Eq:12	Simulation	Deviation
-						
0.2	353.54	372.15	5.27	270.69	281.75	4.09
0.5	273.75	287.76	5.20	239.70	245.56	2.44
0.8	248.34	249.92	0.64	237.57	237.17	0.17

where,

$$K_{\phi=0^\circ} = \frac{8}{3} \lambda^{-\frac{1}{3}} \left[\frac{2\lambda}{1-\lambda^2} + \frac{2(1-2\lambda^2)}{(1-\lambda^2)^{\frac{3}{2}}} \tan^{-1}\left(\frac{\sqrt{1-\lambda^2}}{\lambda}\right) \right]^{-1}, \quad (13)$$

$$K_{\phi=90^\circ} = \frac{8}{3} \lambda^{-\frac{1}{3}} \left[\frac{-\lambda}{1-\lambda^2} - \frac{2\lambda^2-3}{(1-\lambda^2)^{\frac{3}{2}}} \sin^{-1}(\sqrt{1-\lambda^2}) \right]^{-1}, \quad (14)$$

and,

$$C_D = C_{D,\phi=0^\circ} + (C_{D,\phi=90^\circ} - C_{D,\phi=0^\circ}) \sin^2 \phi, \quad (15)$$

$$C_L = (C_{D,\phi=0^\circ} - C_{D,\phi=90^\circ}) \sin \phi \cos \phi. \quad (16)$$

It can be seen that the results of simulations are in good accordance with the theoretical results since the mean relative deviation never exceeded 5%. A possible reason for such deviations is that the particle Reynolds number $Re_p = 0.1$ is not small enough. Compared to the earlier investigations, Zastawny et al.³⁰ and Sanjeevi et al.³² found a deviation of about 17% and 13% for $\lambda = 0.2$ and $\lambda = 0.4$ respectively. Therefore, it is fair to conclude that these deviations can be considered as acceptable.

The third test case concerns the accuracy of the numerical technique to compute the drag coefficient for the oblate spheroids at higher particle Reynolds number. Due to the scarcity of experimental results, the validation case concerns only the spheroid with an aspect ratio of $\lambda = 0.5$ and incidence angle of $\phi = 0^\circ$ for $Re_p = 100$. The computed value is $C_{D,simulation} = 1.634$ agree well with the experimental value given by Stringham et al.⁴⁷, $C_{D,experiment} = 1.592$. The relative deviation is 2.652%.

According to the different validation tests, i.e., domain size dependence, mesh influence and the comparison with the theoretical; empirical and experimental results, the numerical technique can be used to perform simulations of the flow around the oblate spheroidal particle at higher range of the particle Reynolds number.

IV. RESULTS AND DISCUSSION

The main focus of this study is to develop models for the hydrodynamic coefficients (C_D , C_L and C_T). Because of the number of parameters (Re_p , λ and ϕ) and in order to provide a wide database for the best fitting, 126 cases are investigated in this work.

Before showing the different results, it is interesting to illustrate the flow past the spheroids at $\phi = 0^\circ$ and $\phi = 90^\circ$. It is a well-known for the sphere that, at $Re_p = 100$, the recirculation domain exists in the wake of the sphere and remain symmetric⁴⁸ even for $Re_p = 200$. It can be expected that similar wake structures can be observed for the spheroids. Figure 4 illustrate the flow around spheroids when the revolution axis is parallel to the flow ($\phi = 0^\circ$). It can be seen in figure 4 that the recirculation domain increases with decreasing aspect ratio. In addition the flow remain symmetric for the spheroids with $\lambda = 0.5$ and $\lambda = 0.8$ while the symmetry in the wake is broken for the spheroid with an aspect ratio of $\lambda = 0.2$. This is consistent with the earlier finding by Militzer et al.²¹ who observe a substantial effect of the aspect ratio on the separation start and the size of recirculation wake. Since the frontal area is the same for all spheroids, only the shape (curvature) can be expected behind this behavior. In contrast to $\phi = 0^\circ$, when the revolution axis is perpendicular to the flow ($\phi = 90^\circ$), the recirculation domain increases with increasing aspect ratio. In addition, the recirculation wake remains symmetric whatever the aspect ratio, which is in accordance with the observation of Kishore and Gu⁴¹. One can explain such a behavior by the size of the frontal area which acts as an obstacle in the uniform flow. In summary, when the revolution axis is aligned with the flow, the recirculation wake size decreases with increasing aspect ratio and increases with increasing particle Reynolds number. When the revolution axis is perpendicular to the flow, size of recirculation wake increases as the values of the aspect ratio and the particle Reynolds number increase.

In this section, the methodology used to develop the new correlations is presented. Thus, a comparison between the results obtained by numerical simulations and the proposed correlations are depicted in different graphs.

A. Drag coefficient

Drag coefficient as a function of the particles Reynolds number is reported in figure 6 for the three aspect ratios and at both extremums incidence angles ($\phi = 0^\circ$ and $\phi = 90^\circ$). It is seen that drag coefficient decreases with increasing particle Reynolds number whatever the aspect ratio and for both incidence angles. In contrast to the prolate ellipsoids^{30, 31}, drag coefficient is larger when the revolution axis is aligned with the flow ($\phi = 0^\circ$) compared to the case of a revolution axis perpendicular to the flow ($\phi = 90^\circ$). Regarding the evolution of the drag coefficient as a function of aspect ratio, it is observed that the drag coefficient decreases with increasing the aspect ratio when the revolution axis is aligned with the flow. Unlike at $\phi = 90^\circ$, i.e., when the revolution axis is perpendicular to the flow, a non-monotonically evolution of the drag coefficient as a function of the aspect ratio is observed. This observation is qualitatively consistent with the results of an earlier study by O'Donnell & Helenbrook¹⁷. According, to these evolutions of the drag coefficient as a function of the particle Reynolds number, aspect ratio and incidence angle, the general form of the correlation can be inspired from the earlier study³¹:

1. The drag coefficients $\phi = 0^\circ$ and $\phi = 90^\circ$ evolve as a function of the particle Reynolds number.
2. The aspect ratio λ is taken to be the main shape parameter of the spheroids.
3. The correlation has to degenerate to Schiller & Naumann correlation⁴⁶ for $\lambda = 1$ (sphere Eq. 10).
4. For small particle Reynolds numbers, the correlation tends towards the Brenner's¹⁶ analytical solution (Eqs.11 and 12).

Using those criteria, results were fitted using the robust regression residuals proposed in Matlab. The algorithm using the bisquare weight method⁴⁹ which provides an effective alternative to deleting specific points was chosen. To analyze how robust is the fitted parameter

values, a statistical measure named R-squared (R^2) is computed:

$$R^2 = 1 - \frac{\text{Unexplained Variation}}{\text{Total Variation}} \quad (17)$$

In the best case, the modeled values exactly match the simulation values, which results in $R^2 = 1$.

The best fitting of the results with $R^2 = 0.9996$ and $R^2 = 0.9995$, respectively for $\phi = 0^\circ$ and $\phi = 90^\circ$, gives the new following formula:

$$C_{D,\phi=0^\circ} = \frac{24}{Re_p} (K_{\phi=0^\circ} + 0.15\lambda^{95.91} Re_p^{0.687} + 0.2927(1 - \lambda)^{0.4374} Re_p^{0.7512}), \quad (18)$$

$$C_{D,\phi=90^\circ} = \frac{24}{Re_p} (K_{\phi=90^\circ} + 0.15\lambda^{100.7} Re_p^{0.687} + 0.1411(1 - \lambda^{24.75}) Re_p^{0.7143}). \quad (19)$$

It is noteworthy that the functional form of the present correlation is not trivial. Indeed, the present correlation tends to the theoretical results given by Happel & Brenner (Eqs. 14 and 13). In addition, for an aspect ratio $\lambda = 1$, the correlation degenerates to Schiller and Naumann correlation (Eq.10). Thus, this correlation can be considered as an extension of Schiller & Naumann correlation to the oblate spheroidal particles.

Figure 6 shows a qualitative comparison between the results of simulation and the proposed formulas (Eqs. 18 and 19). For comparison, correlation provided by Zastawny et al.³⁰ for $\lambda = 0.2$ is also plotted in figure 6. Qualitatively, it can be seen that the drag coefficient is well estimated by the proposed formula, whatever the aspect ratio. This is confirmed by computing the mean relative deviation $\frac{|C_{D,Correlation} - C_{D,simulation}| * 100}{C_{D,simulation}}$ which varies between 4% and 10.87%. In addition, the present correlation gives a better estimation of $C_{D,\phi=0^\circ}$ and $C_{D,\phi=90^\circ}$ compared to Zastawny et al.³⁰. This observation is confirmed by computing the mean relative deviation $\frac{|C_{D,Zastawny} - C_{D,simulation}| * 100}{C_{D,simulation}}$, which was found to be about 12.5% and 26.5% respectively for $C_{D,\phi=0^\circ}$ and $C_{D,\phi=90^\circ}$.

To confirm the accuracy of the present correlation, an independent test case is performed (see fig.7). The results obtained using the equation Eq: 18 are compared with the experimental results given by Pitter et al.³⁹ for the spheroid with an aspect ratio $\lambda = 0.3$. It is seen from figure 7 that Eq. 18 agree well with the experimental results. the mean relative deviation confirms this qualitative observation since it is found to be approximately 2%.

To show the effect of orientation, the drag coefficient normalized by $C_D = 0^\circ$ is depicted in figure 8. It can be seen that the drag coefficient of the oblate spheroids decreases with

increasing incidence angle whatever the particle Reynolds number and aspect ratio. However, the influence of incidence angle is seen to be less important for the higher aspect ratio $\lambda = 0.8$. This is consistent with the earlier finding for the prolate ellipsoids for which the incidence angle has less influence when the aspect ratio is close to $\lambda = 1$ (sphere case). In addition, we observe that the influence of the incidence angle increases with an increase of the particle Reynolds number. In addition, as in the creeping flow, drag coefficient evolves as a sinus function with respect to the incidence angle. According to these observations, an extension of Brenner's formula to a higher particle Reynolds number is tested. Figure 8 shows a comparison of the normalized drag coefficient with the analytical solution (Eq.15). It can be seen that the results of simulation are in a good accordance with the analytical formula (15) (the mean relative deviation is less than 2 %). This means that, the evolution of the drag coefficient as sine squared with respect to the incidence angle, exhibited in the creeping flow, remains valid for higher particle Reynolds numbers up to 100. Thereby, the final form of the new correlation for the drag coefficient is:

$$\left\{ \begin{array}{l} C_{D,\phi=0^\circ} = \frac{24}{Re_p} (K_{\phi=0^\circ} + 0.15\lambda^{95.91} Re_p^{0.687} + 0.2927(1-\lambda)^{0.4374} Re_p^{0.7512}), \\ C_{D,\phi=90^\circ} = \frac{24}{Re_p} (K_{\phi=90^\circ} + 0.15\lambda^{100.7} Re_p^{0.687} + 0.1411(1-\lambda^{24.75}) Re_p^{0.7143}), \end{array} \right. \quad (20)$$

$$C_D = C_{D,\phi=0^\circ} + (C_{D,\phi=90^\circ} - C_{D,\phi=0^\circ}) \sin^2 \phi.$$

B. Lift coefficient

In the present work, the particle is supposed to be fixed and immersed in a uniform flow. Thus, the only lift force calculated is the hydrodynamic lift, i.e., the lift force arising from the orientability of the particle. This lift appears when the revolution axis is not parallel or perpendicular to the flow. There are few works dedicated to the hydrodynamic lift in the literature. Recently, some efforts have been made to propose new models for the hydrodynamic forces^{29, 30, 50, 31}. Unfortunately, most of these works are focused on the elongated particle. The only works which proposes a model of the hydrodynamic lift for an oblate spheroid is performed by Zastawny et al.³⁰ and Sanjeevi et al.³². However, the proposed models concerns a single aspect ratio, namely $\lambda = 0.2$ and $\lambda = 0.4$, respectively. From this point of view, the second aim of this work is to develop a new correlation for the hy-

hydrodynamic lift for the spheroid concerned in the present study. Figure 9 shows the profile of the lift coefficient as a function of incidence angle. For the sake of brevity and clarity, only results of the extremum particle Reynolds number concerned in this work ($Re_p = 1$ and $Re_p = 100$) are reported. It is seen that the lift coefficient is zero when the revolution axis is parallel or perpendicular to the flow while it is non-zero elsewhere. In addition, as in the creeping flow, the profile of the lift coefficient exhibit a maximum lift at $\phi = 45^\circ$. Since the maximum C_L generated at a given Re_p is a function of the particle aspect ratio and also of the projected area of the particle at $\phi = 45^\circ$, evolution of the maximum C_L as a function of Re_p is reported in figure 10. It can be seen that, as for the drag coefficient, the lift force decreases with increasing of the particle Reynolds number and aspect ratio. However, in contrast to the creeping flow, the maximum lift coefficient do not coincide with $\frac{C_{D,\phi=0^\circ} - C_{D,\phi=90^\circ}}{2}$. This results is in accordance with the observation of Sanjeevi et al.⁵¹ and can be explained by non-linear behavior of the lift force for the higher particle Reynolds number.

According those observations, the approach used to determine the new formula for the lift coefficient is based on the four following points:

1. The lift coefficient has a similar profile to that of the creeping flow ($\sin \phi \cos \phi$).
2. The aspect ratio λ is taken to be the main shape parameter of the spheroids.
3. The lift coefficient exhibits a maximum at $\phi = 45^\circ$.
4. For small particle Reynolds numbers, the correlation tends towards the analytical solution proposed by Happel & Brenner¹⁶.

Based on the listed criteria, the proposed general form of the correlation for the lift coefficient reads as :

$$C_L = [(C_{D,\phi=0^\circ} - C_{D,\phi=90^\circ}) + F(\lambda, Re_p)] \sin \phi \cos \phi, \quad (21)$$

where $F(\lambda, Re_p)$ is the correction function of the lift coefficient over the creeping flow. This function is obtained by fitting the entire results using the same methodology as for the drag coefficient with $R^2 = 0.9974$:

$$F(\lambda, Re_p) = \frac{2\left(\frac{1-\lambda}{\lambda}\right)^{0.542}}{(1+\lambda)^{7.85}} Re_p^{0.1516}. \quad (22)$$

Thus, the final form of the lift coefficient as a function of the aspect ratio, incidence angle

and particle Reynolds number reads as:

$$C_L = \left[(C_{D,\phi=0^\circ} - C_{D,\phi=90^\circ}) + \frac{2\left(\frac{1-\lambda}{\lambda}\right)^{0.542}}{(1+\lambda)^{7.85}} Re_p^{0.1516} \right] \sin \phi \cos \phi. \quad (23)$$

To show the qualitative accuracy of the present correlation Eq. 23, it was plotted with results of simulations in figure 10. Correlation proposed by Zastawny et al.³⁰ is also reported for comparison. It can be seen that the profile of the present correlation (Eq.23) is in a good accordance with the profile of the lift coefficient, whatever the particle Reynolds number and aspect ratio. Compared to correlation proposed by Zastawny et al.³⁰, equation 23 gives a better estimation of the lift coefficient for $\lambda = 0.2$. Quantitatively, the mean relative deviations are 1.67, 5.11 and 10.31 respectively for $\lambda = 0.2, 0.5$ and 0.8 while it is of about 13% for the correlation proposed by Zastawny et al.³⁰. Nonetheless, it is difficult to achieve a better fit due to the fact that C_L is a function of C_D and any small deviation is amplified. Interestingly, the mean relative deviations decreases with increasing aspect ratio and allow us to consider this new correlation Eq.23 as trustworthy.

C. Pitching torque coefficient

The pitching torque is one of the contributions to the rotational motion of the spheroids. It is inherent to the orientability of the particle with respect to the relative motion of the particle. Indeed, when the revolution axis is not parallel or perpendicular to the flow, drag and lift forces appears at the center of pressure rather than the gravity center, thus inducing a hydrodynamic torque which is perpendicular to the plane defined by the flow velocity vector and the revolution axis of the spheroid. Despite a numerous works on the non-spherical particle motion, the pitching torque is less investigated for the oblate spheroids. Indeed, the only correlation of the pitching torque coefficient is that proposed by Zastawny et al.³⁰ which, unfortunately, concerns a single aspect ratio $\lambda = 0.2$. According to this scarcity of data and correlations for the pitching torque, the thirds part of the present work is devoted to propose a new correlation for the pitching torque taking the particle Reynolds number, the aspect ratio and the incidence angle into account. Results of simulations are depicted in figure 11 as the pitching torque coefficient with respect to the incidence angle. For the sake of clarity, only results of the extremums of particle Reynolds number ($Re_p = 1$ and $Re_p = 100$) are shown. It is seen that the pitching torque coefficient has a similar trends as

lift coefficient and exhibit a maximum at $\phi = 45^\circ$. Hence, in order to reveal the effect of the aspect ratio and the particle Reynolds number, the evolution of the maximum torque coefficient is reported in figure 12. It is seen that the pitching torque coefficient decreases with increasing the particle Reynolds number and aspect ratio. In addition, as for the lift coefficient, the profiles of pitching torque exhibit a maximum located at $\phi = 45^\circ$ and seems to evolve as a $(\sin \phi \cos \phi)$ function. Nevertheless, there is no analytical formula for the pitching torque in the creeping flow. Therefore, the new formula is obtained according only to the mentioned observations. Hence, the relevant criteria considered in the fitting for the proposed formula are:

1. The aspect ratio λ is taken to be the main shape parameter of the spheroids.
2. The pitching torque coefficient exhibit a maximum at $\phi = 45^\circ$ and seems to evolves as $(\sin \phi \cos \phi)$.
3. The pitching coefficient decreases with increasing the particle Reynolds number Re_p .

According these criteria and the profile of the pitching torque, the general form of the new formula reads as:

$$C_T = \frac{1.85 \left[\frac{1-\lambda}{\lambda} \right]^{0.832}}{Re_p^{0.146}} \sin \phi \cos \phi. \quad (24)$$

As for the lift coefficient, the accuracy of the new correlation is analyzed both qualitatively and quantitatively. Figure 12 shows that the present correlation is in a good accordance with the results of simulations whatever the particle Reynolds number, aspect ratio and incidence angle. The quantitative accuracy is shown by computing the mean relative deviations for all cases concerned in this study. Those deviations are 7.09%, 4.99% and 5.58% respectively for $\lambda = 0.2, 0.5$ and 0.8 . For a comparison correlation provided by Zastawny et al.³⁰ is plotted for $\lambda = 0.2$ and mean relative deviation of this correlation is found to be 20%. This shows clearly that equation 24 gives a significantly better estimation of the pitching torque coefficient even when it is compared to a correlation provided for a single aspect ratio.

V. CONCLUSION

In the present work, numerical simulations were successfully performed for the uniform flow around oblate spheroids $\lambda \in [0.2-1]$ for $Re_p \leq 100$. Different validation tests have been

conducted to assess the numerical methodology and to ensure the quality of the results. On the basis of extensive results of hydrodynamic coefficients (C_D , C_L and C_T) a new simple, complete set of correlations taking the shape and the orientation into account for a moderate particle Reynolds number $Re_p \leq 100$ is provided. The accuracy of the proposed correlations is discussed through the mean relative deviations and R^2 . It shows clearly that the correlations are trustworthy since the mean relative deviation remains in the order or less than 10% for the drag and lift coefficient, and less than 8% for the pitching torque coefficients. In addition, it is noteworthy that the proposed correlations of the drag and lift coefficients are well recorded to the analytical solution given by Happel & Brenner¹⁶ in the creeping flow. Furthermore, it is noticed that the evolution of the hydrodynamic coefficient (C_D , C_L) as a function of the particle orientation is not significantly altered by the particle Reynolds number. Indeed, it is found that the evolution of the drag coefficient as sine squared and the lift coefficient as $(\sin \phi \cos \phi)$ in the creeping flow remains adequate despite the high particle Reynolds number. Regarding the pitching torque coefficient, there is, unfortunately, not an analytical formula. Thus, the correlation is valid only in the range of $Re_p \in [0.2-1]$.

The correlations provided in this study can be useful in the Eulerian-Lagrangian simulations of the dispersion of spheroids. In the case of prolate ellipsoid, it is shown in^{52,53} that the non-linearity of the hydrodynamic forces and torque has a substantial influence on the collective dynamic of the ellipsoids. Thus, using the present correlations, the effects of non-linear hydrodynamic forces and torques on the oblate spheroids can be included in the numerical simulation. This is the aim of the future investigations.

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VII. DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

- ¹D. Mourelle and A. R. Prieto, “Pollen and spores from surface samples in the campos region of uruguay and their paleoecological implications,” *Acta Botanica Brasilica* **30**, 351–370 (2016).
- ²D. L. Koch and G. Subramanian, “Collective hydrodynamics of swimming microorganisms: living fluids,” *Annual Review of Fluid Mechanics* **43**, 637–659 (2011).
- ³F. Lundell, L. Söderberg, Daniel, and P. H. Alfredsson, “Fluid mechanics of papermaking,” *Annual Review of Fluid Mechanics* **43**, 195–217 (2011).
- ⁴M. Mandø, “Turbulence modulation by non-spherical particles,” (2009).
- ⁵H. Zhang, A. Goodarz, F. Fan, and J. B. McLaughlin, “Ellipsoidal particles transport and deposition in turbulent channel flows,” *International Journal of Multiphase Flow* **27**, 971–1009 (2001).
- ⁶P. H. Mortensen, H. I. Andersson, J. J. J. Gillissen, and B. J. Boersma, “Dynamics of prolate ellipsoidal particles in a turbulent channel flow,” *Physics of Fluids* **20**, 093302 (2008).
- ⁷C. Marchioli, M. Fantoni, and A. Soldati, “Orientation, distribution, and deposition of elongated, inertial fibers in turbulent channel flow,” *Physics of fluids* **22**, 033301 (2010).
- ⁸L. Zhao, C. Marchioli, and H. I. Andersson, “Slip velocity of rigid fibers in turbulent channel flow,” *Physics of Fluids* **26**, 063302 (2014).
- ⁹N. R. Challabotla, L. Zhao, and H. I. Andersson, “Orientation and rotation of inertial disk particles in wall turbulence,” *Journal of Fluid Mechanics* **766** (2015).
- ¹⁰N. R. Challabotla, L. Zhao, and H. I. Andersson, “Gravity effects on fiber dynamics in wall turbulence,” *Flow, Turbulence and Combustion* **97**, 1095–1110 (2016).
- ¹¹N. R. Challabotla, L. Zhao, and H. I. Andersson, “On fiber behavior in turbulent vertical channel flow,” *Chemical Engineering Science* **153**, 75–86 (2016).

- ¹²X. Yuan, H. I. Andersson, L. Zhao, N. R. Challabotla, and J. Deng, “Dynamics of disk-like particles in turbulent vertical channel flow,” *International Journal of Multiphase Flow* **96**, 86–100 (2017).
- ¹³Y. Jie, L. Zhao, C. Xu, and H. I. Andersson, “Preferential orientation of tracer spheroids in turbulent channel flow,” *Theoretical and Applied Mechanics Letters* **9**, 212–214 (2019).
- ¹⁴W. Yuan, L. Zhao, H. I. Andersson, and J. Deng, “Three-dimensional voronoi analysis of preferential concentration of spheroidal particles in wall turbulence,” *Physics of Fluids* **30**, 063304 (2018).
- ¹⁵R. Ouchene, J. I. Polanco, I. Vinkovic, and S. Simoëns, “Acceleration statistics of prolate spheroidal particles in turbulent channel flow,” *Journal of Turbulence* **19**, 827–848 (2018).
- ¹⁶J. Happel and H. Brenner, *Low Reynolds number hydrodynamics: with special applications to particulate media*, Vol. 1 (2012).
- ¹⁷B. J. O’Donnell and B. Helenbrook, “Drag on ellipsoids at finite reynolds numbers,” *Atomization and Sprays* **15** (2005).
- ¹⁸A. Hölzer and M. Sommerfeld, “New simple correlation formula for the drag coefficient of non-spherical particles,” *Powder Technology* **184**, 361–365 (2008).
- ¹⁹A. Richter and P. A. Nikrityuk, “Drag forces and heat transfer coefficients for spherical, cuboidal and ellipsoidal particles in cross flow at sub-critical reynolds numbers,” *International Journal of Heat and Mass Transfer* **55**, 1343–1354 (2012).
- ²⁰R. Ouchene, M. Khalij, A. Tanière, and B. Arcen, “Drag, lift and torque coefficients for ellipsoidal particles: From low to moderate particle reynolds numbers,” *Computers & Fluids* **113**, 53–64 (2015).
- ²¹J. Militzer, J. M. Kan, F. Hamdullahpur, P. R. Amyotte, and A. A. Taweel, “Drag coefficient for axisymmetric flow around individual spheroidal particles,” *Powder technology* **57**, 193–195 (1989).
- ²²A. Tripathi, R. P. Chhabra, and T. Sundararajan, “Power law fluid flow over spheroidal particles,” *Industrial & engineering chemistry research* **33**, 403–410 (1994).
- ²³R. Chhabra, L. Agarwal, and N. K. Sinha, “Drag on non-spherical particles: an evaluation of available methods,” *Powder technology* **101**, 288–295 (1999).
- ²⁴G. H. Ganser, “A rational approach to drag prediction of spherical and nonspherical particles,” *Powder Technology* **77**, 143–152 (1993).

- ²⁵H. N. Yow, M. J. Pitt, and A. D. Salman, “Drag correlations for particles of regular shape,” *Advanced Powder Technology* **16**, 363–372 (2005).
- ²⁶R. Ouchene, A. Chadil, P. Fede, M. Khalij, A. Tanière, S. Vincent, J. L. Estivalezes, and B. Arcen, “Numerical simulation and modelling of the forces acting on single and multiple non-spherical particles,” in *ASME 2014 4th Joint US-European Fluids Engineering Division Summer Meeting collocated with the ASME 2014 12th International Conference on Nanochannels, Microchannels, and Minichannels* (American Society of Mechanical Engineers, 2014) pp. V002T34A001–V002T34A001.
- ²⁷S. F. Hoerner, “Fluid–dynamic drag, hoerner fluid dynamics, bakersfield, ca,” in *Library of Congress Catalog Card*, 64-19666 (1965).
- ²⁸L. Rosendahl, “Using a multi-parameter particle shape description to predict the motion of non-spherical particle shapes in swirling flow,” *Applied Mathematical Modelling* **24**, 11–25 (2000).
- ²⁹C. Yin, L. Rosendahl, S. S. Kær, and H. Sørensen, “Modelling the motion of cylindrical particles in a nonuniform flow,” *Chemical Engineering Science* **58**, 3489–3498 (2003).
- ³⁰M. Zastawny, G. Mallouppas, F. Zhao, and B. V. Wachem, “Derivation of drag and lift force and torque coefficients for non-spherical particles in flows,” *International Journal of Multiphase Flow* **39**, 227–239 (2012).
- ³¹R. Ouchene, M. Khalij, B. Arcen, and A. Tanière, “A new set of correlations of drag, lift and torque coefficients for non-spherical particles and large reynolds numbers,” *Powder Technology* **303**, 33–43 (2016).
- ³²S. K. Sanjeevi, J. Kuipers, and J. T. Padding, “Drag, lift and torque correlations for non-spherical particles from stokes limit to high reynolds numbers,” *International Journal of Multiphase Flow* **106**, 325–337 (2018).
- ³³Y. Cui, J. Ravnik, M. Hriberšek, and P. Steinmann, “A novel model for the lift force acting on a prolate spheroidal particle in an arbitrary non-uniform flow. part i. lift force due to the streamwise flow shear,” *International Journal of Multiphase Flow* **104**, 103–112 (2018).
- ³⁴Y. Cui, J. Ravnik, O. Verhnjak, M. Hriberšek, and P. Steinmann, “A novel model for the lift force acting on a prolate spheroidal particle in arbitrary non-uniform flow. part ii. lift force taking into account the non-streamwise flow shear,” *International Journal of Multiphase Flow* **111**, 232–240 (2019).

- ³⁵H. I. Andersson and F. Jiang, “Forces and torques on a prolate spheroid: Low-reynolds-number and attack angle effects,” *Acta Mechanica* **230**, 431–447 (2019).
- ³⁶Y. Rimon and H. J. Lugt, “Laminar flows past oblate spheroids of various thicknesses,” *The Physics of Fluids* **12**, 2465–2472 (1969).
- ³⁷J. H. Masliyah and N. Epstein, “Numerical study of steady flow past spheroids,” *Journal of Fluid Mechanics* **44**, 493–512 (1970).
- ³⁸J. Comer and C. Kleinstreuer, “A numerical investigation of laminar flow past nonspherical solids and droplets,” *J. Fluid Eng. Trans. ASME* **117**, 170–175 (1995).
- ³⁹R. L. Pitter, H. R. Pruppacher, and A. E. Hamielec, “A numerical study of viscous flow past a thin oblate spheroid at low and intermediate reynolds numbers,” *Journal of the Atmospheric Sciences* **30**, 125–134 (1973).
- ⁴⁰W. Li, Y. Yan, and J. Smith, “A numerical study of the interfacial transport characteristics outside spheroidal bubbles and solids,” *International journal of multiphase flow* **29**, 435–460 (2003).
- ⁴¹N. Kishore and S. Gu, “Momentum and heat transfer phenomena of spheroid particles at moderate reynolds and prandtl numbers,” *International Journal of Heat and Mass Transfer* **54**, 2595–2601 (2011).
- ⁴²C. Ke, S. Shu, H. Zhang, H. Yuan, and D. Yang, “On the drag coefficient and averaged nusselt number of an ellipsoidal particle in a fluid,” *Powder Technology* **325**, 134–144 (2018).
- ⁴³B. J. Lazaro and J. C. Lasheras, “Particle dispersion in a turbulent, plane, free shear layer,” *Physics of Fluids A: Fluid Dynamics* **1**, 1035–1044 (1989).
- ⁴⁴E. E. Michaelides, “Review—the transient equation of motion for particles, bubbles, and droplets,” *Journal of fluids engineering* **119**, 233 (1997).
- ⁴⁵S. V. Patankar and D. B. Spalding, “Mathematical-models of fluid-flow and heat-transfer in furnaces-review,” *Journal of the Institute of Fuel* **46**, 279–283 (1973).
- ⁴⁶L. Schiller and A. Naumann, “Fundamental calculations in gravitational processing,” *Zeitschrift Des Vereines Deutscher Ingenieure* **77**, 318–320 (1933).
- ⁴⁷G. E. Stringham, D. Simons, and H. P. Guy, *The behavior of large particles falling in quiescent liquids* (US Government Printing Office, 1969).
- ⁴⁸T. A. Johnson and V. C. Patel, “Flow past a sphere up to a reynolds number of 300,” *Journal of Fluid Mechanics* **378**, 19–70 (1999).

- ⁴⁹P. W. Holland and R. E. Welsch, “Robust regression using iteratively reweighted least-squares,” *Communications in Statistics-theory and Methods* **6**, 813–827 (1977).
- ⁵⁰A. Richter and P. A. Nikrityuk, “New correlations for heat and fluid flow past ellipsoidal and cubic particles at different angles of attack,” *Powder technology* **249**, 463–474 (2013).
- ⁵¹S. K. P. Sanjeevi and J. T. Padding, “On the orientational dependence of drag experienced by spheroids,” *Journal of Fluid Mechanics* **820** (2017).
- ⁵²R. Ouchene, S. Hamidouche, B. Arcen, M. Khalij, and A. Tanière, “Turbulent dispersion of ellipsoidal particles in a wall-bounded turbulent flow,” in *9th International Conference on Multiphase Flow* (2016).
- ⁵³B. Arcen, R. Ouchene, M. Khalij, and A. Tanière, “Prolate spheroidal particles’ behavior in a vertical wall-bounded turbulent flow,” *Physics of Fluids* **29**, 093301 (2017).

VIII. FIGURE CAPTIONS

Figure 1. The fluid velocity in frame of reference (x,y,z) and in and the particle frame (x'',y'',z'') . The angle of incidence ϕ is the angle between the fluid velocity vector and the revolution axis of the spheroid.

Figure 2. Geometrical configuration.

Figure 3. Computational mesh for flow past an oblate spheroid with an aspect ratio $\lambda = 0.2$.

Figure 4. Streamlines for the oblate spheroids at $\phi = 0^\circ$ and $Re_p = 100$. Aspect ratio from left to right are $\lambda = 0.2, 0.5$ and 0.8 .

Figure 5. Streamlines for the oblate spheroids at $\phi = 90^\circ$ and $Re_p = 100$. Aspect ratio from left to right are $\lambda = 0.2, 0.5$ and 0.8 .

Figure 6. Drag coefficient as a function of particle Reynolds numbers for $\phi = 0^\circ$ and $\phi = 90^\circ$. Symbols correspond to the results of simulations and corresponding solid lines in respective colors indicate the correlations (Eqs.18 and 19); the dashed line corresponds to Zastawny et al.³⁰.

Figure 7. Variation of the drag coefficient as a function of particle Reynolds number for an oblate spheroid with an aspect ratio $\lambda = 0.3$ and an incidence angle $\phi = 0^\circ$.

Figure 8. Normalized drag coefficient given by the theory (Eq.15) at low Reynolds numbers. (line) and results of simulations (symbols).

Figure 9. Lift coefficient as a function of incidence angle. Symbols correspond to the results of simulations and corresponding solid lines in respective colors indicate the correlations (Eq.23); the dashed line corresponds to Zastawny et al.³⁰.

Figure 10. Maximum lift coefficient as a function of particle Reynolds number. Symbols correspond to the results of simulations and corresponding solid lines in respective colors indicate the correlations (Eq.23); the dashed line corresponds to Zastawny et al.³⁰.

Figure 11. Pitching torque coefficient as a function of incidence angle. Symbols correspond to the results of simulations and corresponding solid lines in respective colors indicate the correlations (Eq.24); the dashed line corresponds to Zastawny et al.³⁰.

Figure 12. Maximum torque coefficient as a function of particle Reynolds number. Symbols correspond to the results of simulations and corresponding solid lines in respective colors indicate the correlations (Eq.24); the dashed line corresponds to Zastawny et al.³⁰.







































