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On the road to Kubo > 1 :

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Alejandro Guillevic, Maxime Lesur, Xavier Garbet, A. Ghizzo, Etienne Gravier, et al.. On the road to Kubo > 1 : Quasi-linear theory in 1D plasma. Festival de Théorie 2022, Jul 2022, Aix-en-Provence, France. hal-04595326

HAL Id: hal-04595326

<https://hal.univ-lorraine.fr/hal-04595326>

Submitted on 31 May 2024

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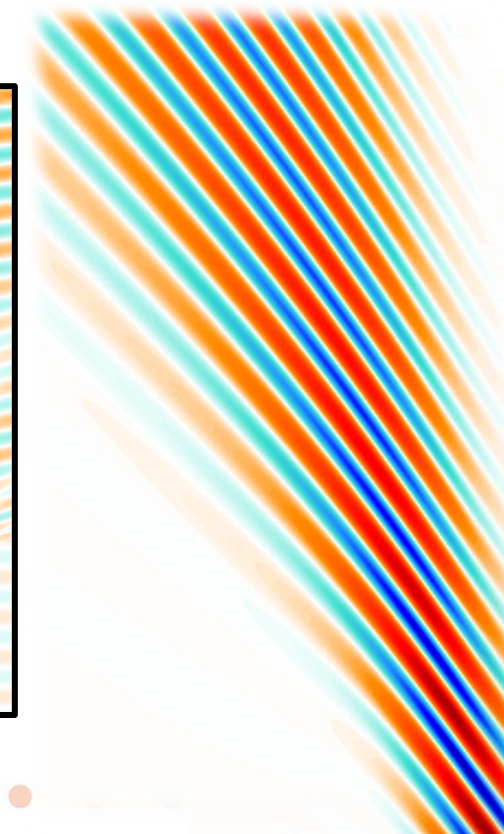
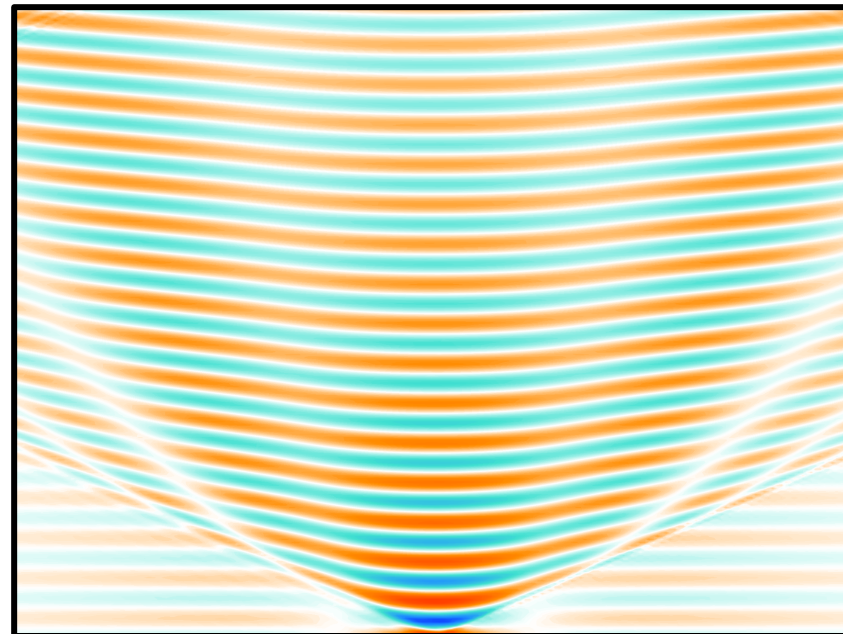
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On the road to Kubo > 1 : Quasi-linear theory in 1D plasma

Alejandro Guillevic

M. Lesur, X. Garbet, A. Ghizzo,
E. Gravier, G. Lo-Cascio,
T. Réveillé, D. Mandal,
and open to contributions

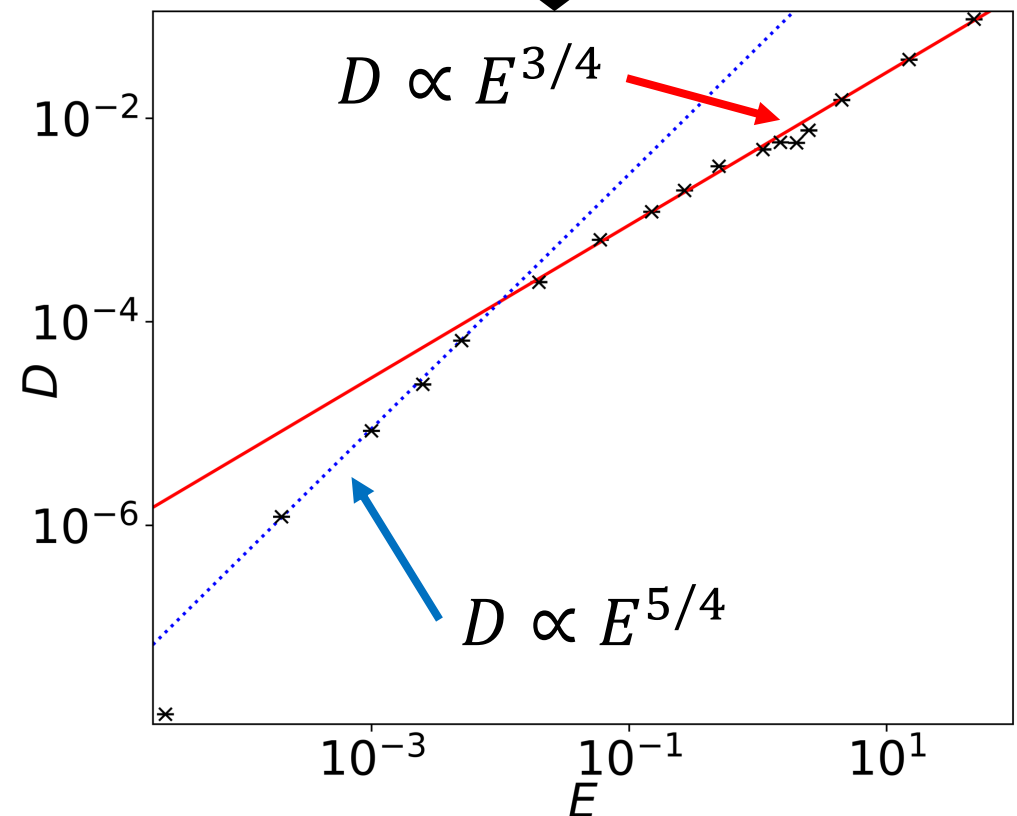
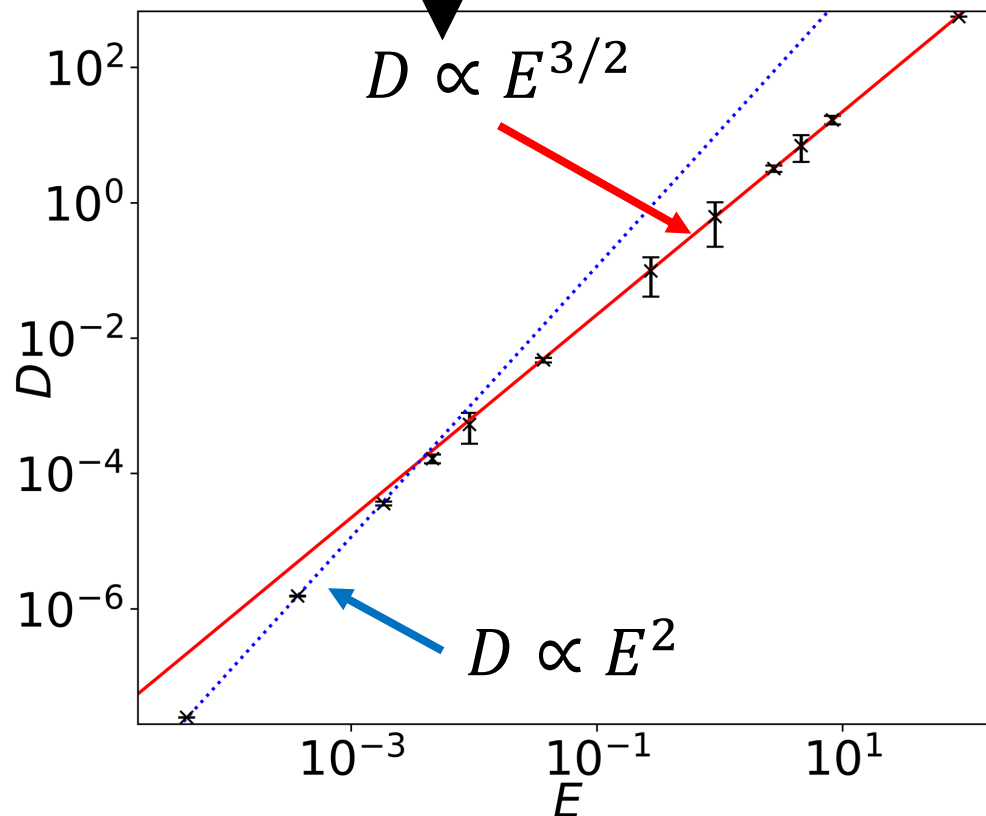
July 7, 2022



In a nutshell:

Diffusion scales as $E^{3/2}$ at high amplitude turbulence

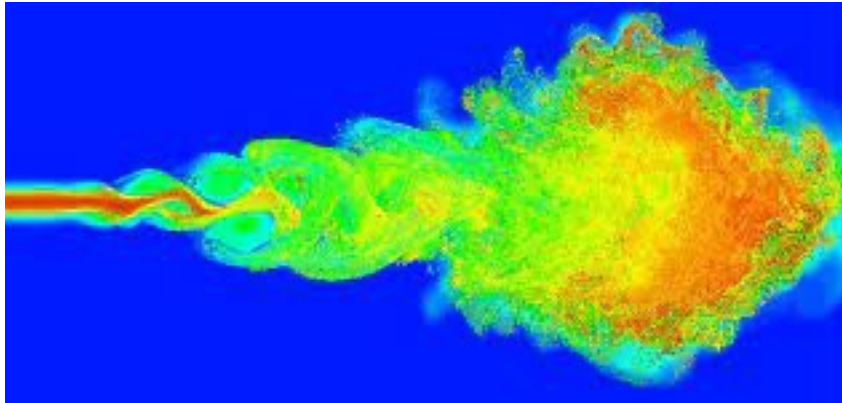
- Charged particles in an turbulent E field
 - Diffusion in v : $\dot{v} = E$
- At small E : $D \propto E^2$
- For larger Kubo and longer time scales



Context

Turbulence in nature

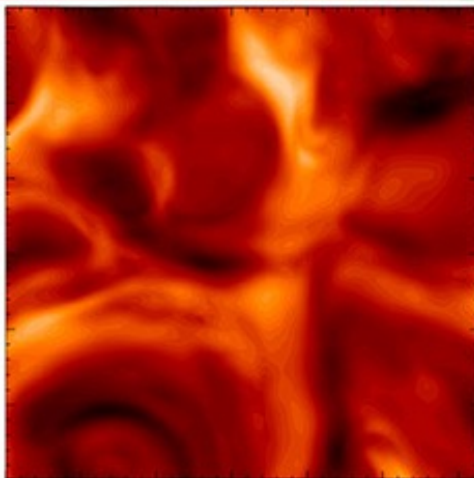
- Rapid fluctuation of physical parameters



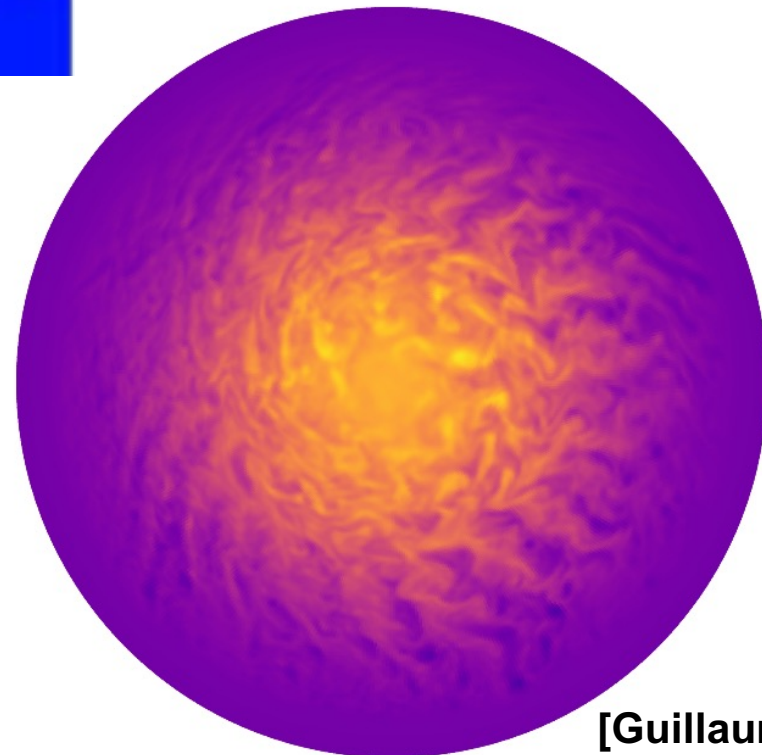
[Lecture, Yale univ., 2014]



- In plasmas:



[F. Califano, Frontiers, 2020]



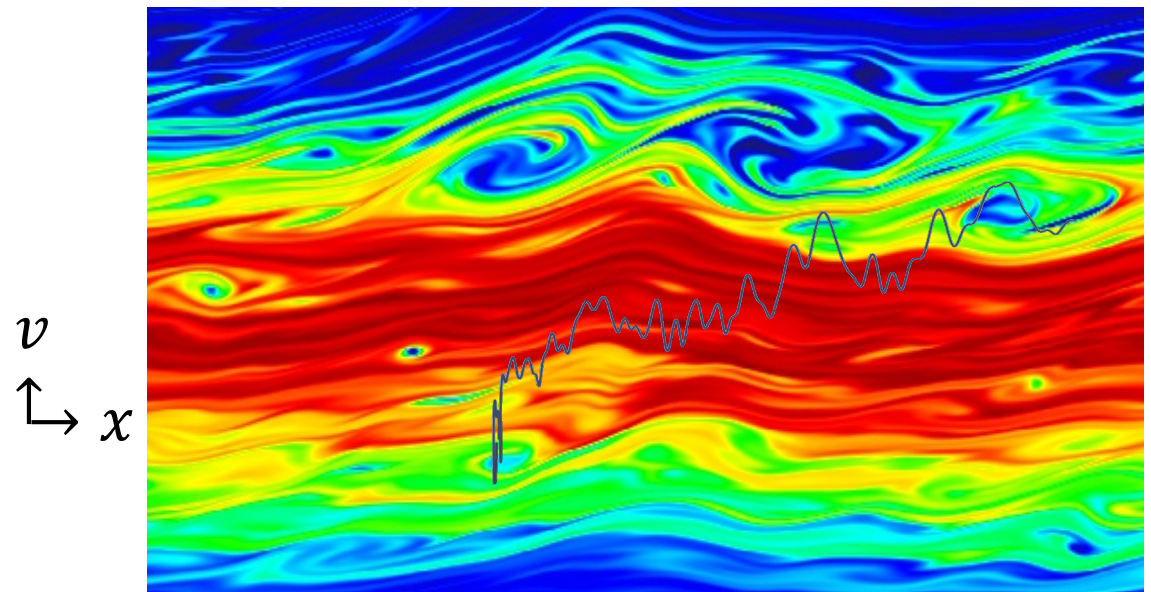
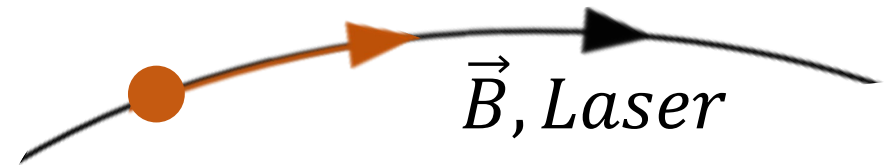
[Guillaume Lo-Cascio,
IJL, 2022]

Context

Collision-less 1D Plasma turbulence

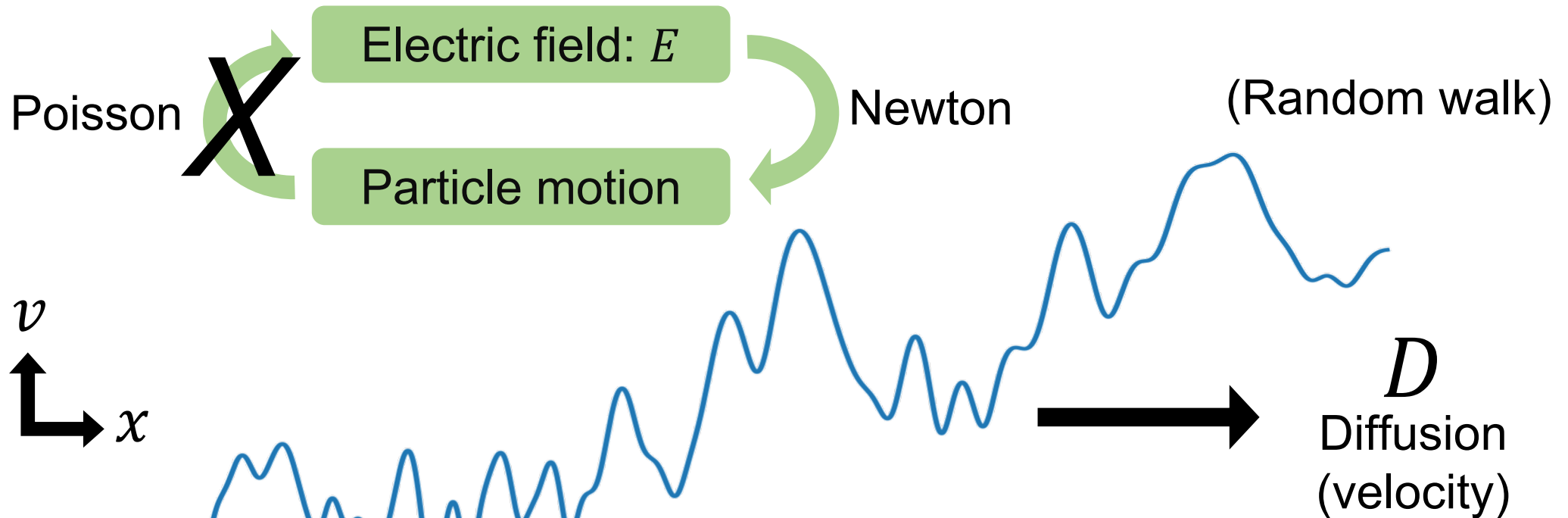
- Geometry:
 - Dynamics happen in 1D
 - Periodic limits
- In a 1D plasma
 - Diffusion in velocity v
 - Plasma heating
 - Instability triggering
- Types of 1D turbulence:
 - Ion-acoustic
 - Langmuir
 - $\omega = C^{const}$ (Artificial)

[F. Doveil & D. Gresillon, PoF, 1982]



Analytical model

1D random motion of charged particles



- Describe diffusion, in 3 regimes of increasing E :
 - $D \propto E^2$: Quasi-linear theory
 - $D \propto f(E)$: Resonance broadening
 - $D \propto E^{3/2}$?

Analytical model

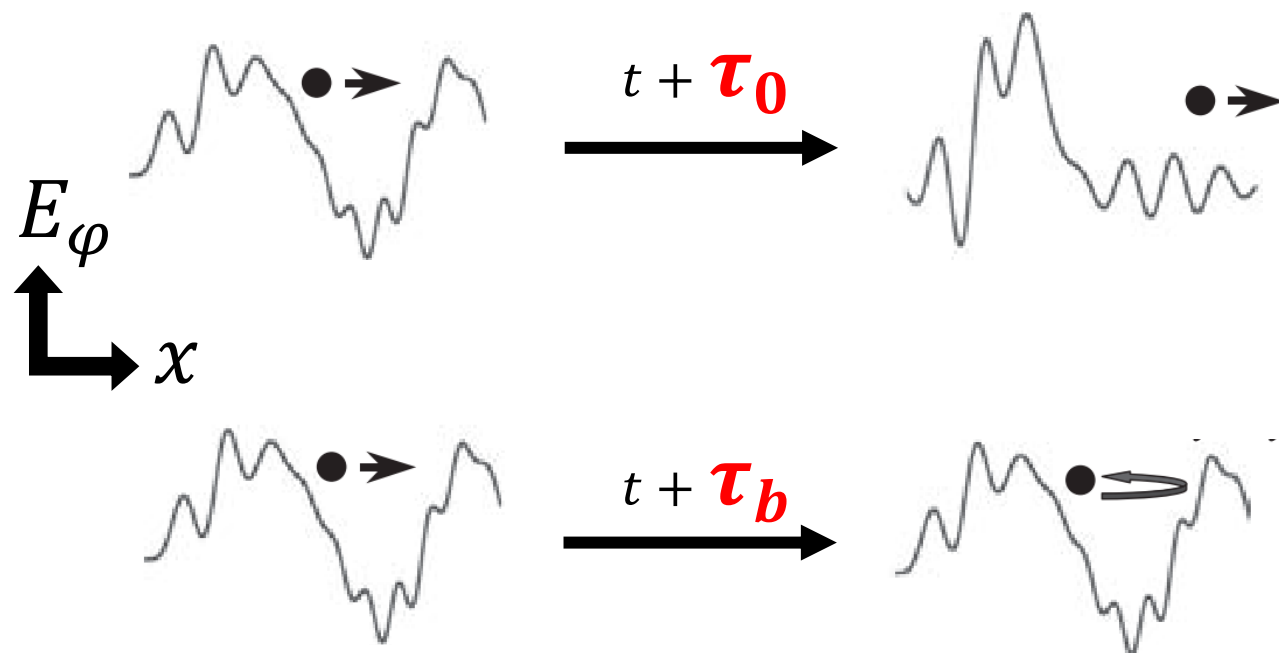
1D Turbulent electric field

Kubo number :

$$K = \frac{\tau_0}{\tau_b}$$

← Correlation time

← Trapped particle time



Particle experience
random E
 $K \ll 1$

Particle experience
potential wells
 $K \gg 1$

[P. H. Diamond, Modern Plasma Physics, 2010]

QL \longrightarrow $K \ll 1$

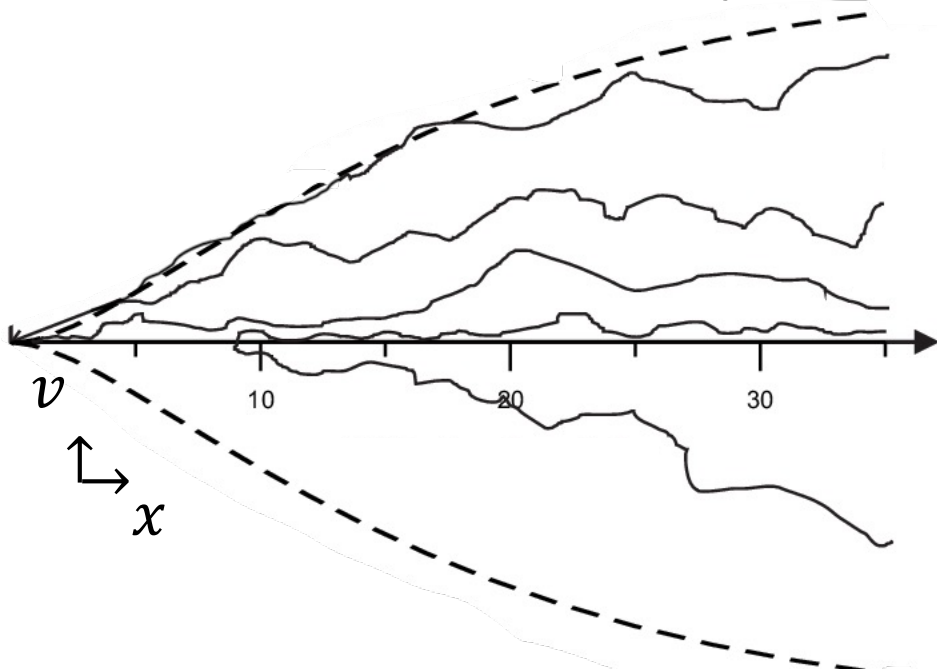
Analytical model

Quasi-linear diffusion: Particles dynamics

[A. Vedenov et al., 1962]

- Statistic on particle dynamics

$$\left\langle \frac{dv}{dt} = \frac{q}{m} E(x, t) \right\rangle$$



$$\langle \Delta v^2 \rangle = 2Dt$$

$$D = \frac{q^2}{m^2} \frac{1}{2t} \int_0^t \int_0^t dt' dt'' \langle E(x(t'), t') E(x(t''), t'') \rangle$$

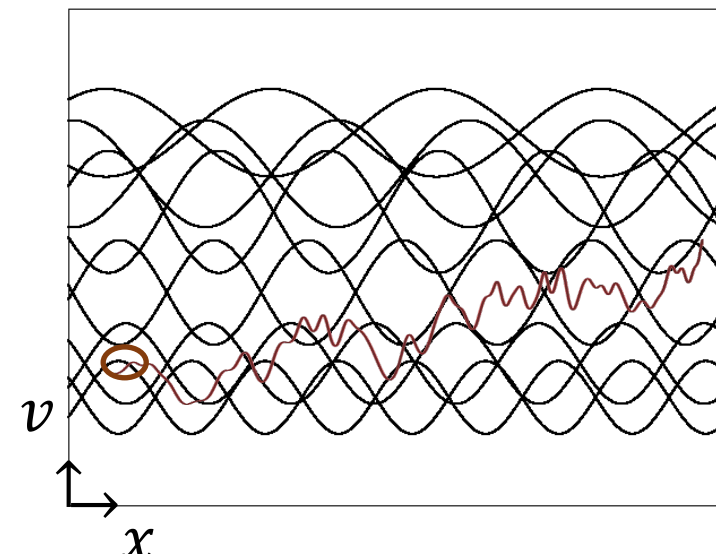
QL hypothèses: $x(t) \approx vt$

Analytical model

Quasi-linear diffusion $K \ll 1$

$$D_0(v) = \frac{q^2}{m^2} \int_0^{+\infty} \int_{-\infty}^{+\infty} dt dx \delta(x - vt) \langle E(0,0)E(x,t) \rangle$$

- Prescribe homogeneous steady state turbulence
- Sin waves with random initial phases



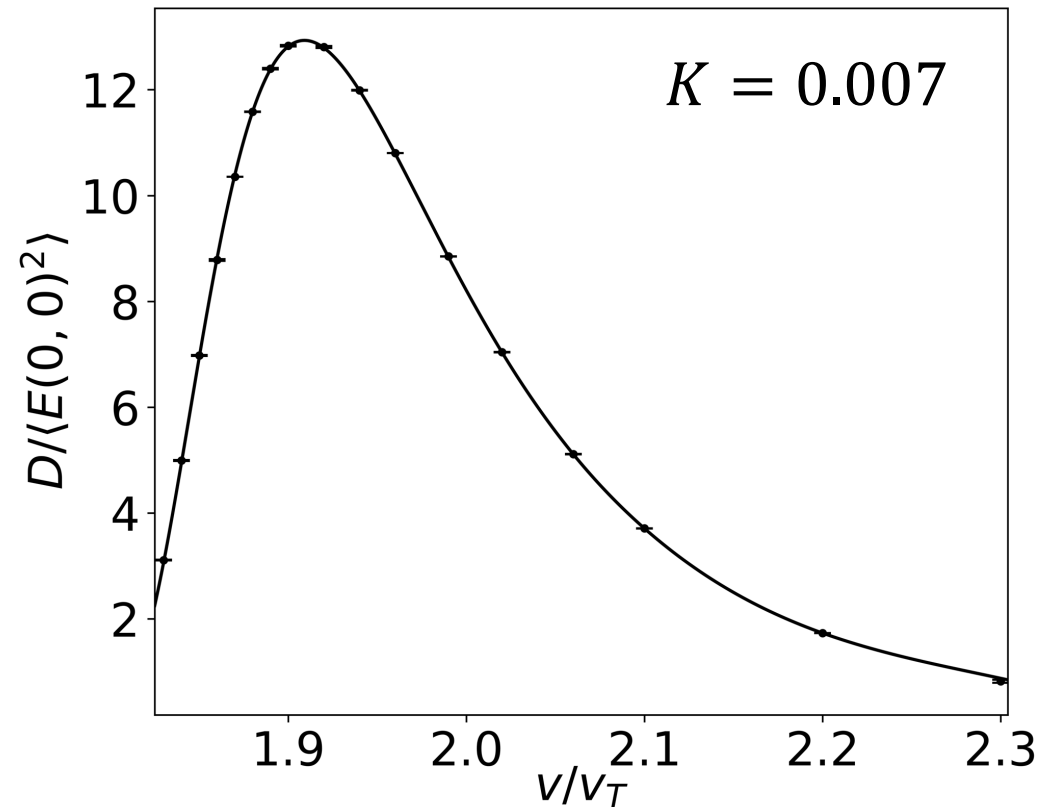
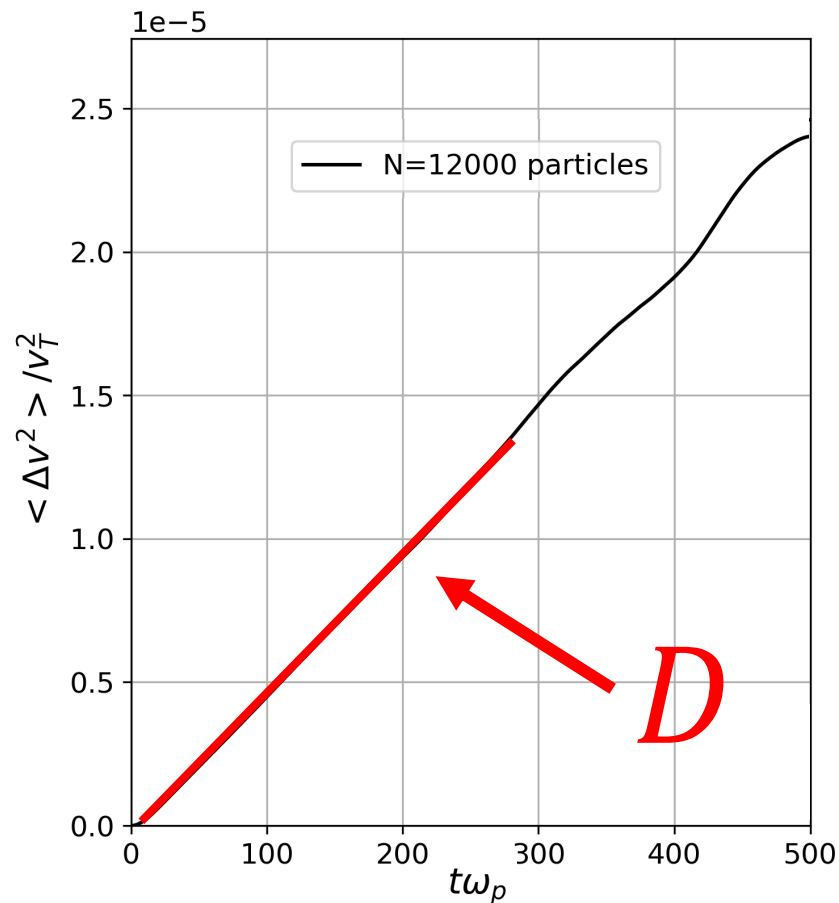
$$D_0(v) = \frac{q^2}{m^2} \int_0^{+\infty} dt \sum_k \frac{\hat{E}^2}{2} \cos[(kv - \omega)t]$$

For a prescribed turbulent E field

Numerical simulations

Statistics: Quasi-linear regime

- Quasi-linear regime: First diffusion
 - Th. & simulation agreement
 - $K < 0.01$



Analytical model

Limits on Quasi-linear theory: Resonance broadening

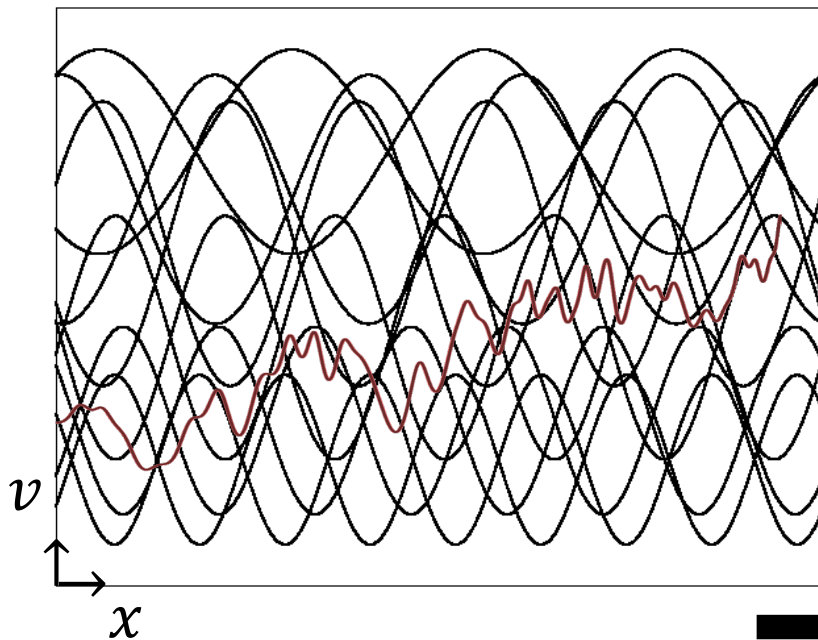
As $E \nearrow$:



$t + \tau_b$



$$K \simeq 0.2$$



- Considers:
 - Multi-mode resonances
- Parameters:
 - Distribution of modes
 - Underlying plasma wave

Resonance broadening

Analytical model

Resonance broadening

[T. H. Dupree, PoF, 1966]
[J. Weinstock, PoF, 1968]

- **D** : Iterative method
 - Distribution of particles around v

$$D_0(v) = \frac{q^2}{m^2} \int_0^{+\infty} \int_{-\infty}^{+\infty} dt dx \delta(x - vt) \langle E(0,0)E(x,t) \rangle$$

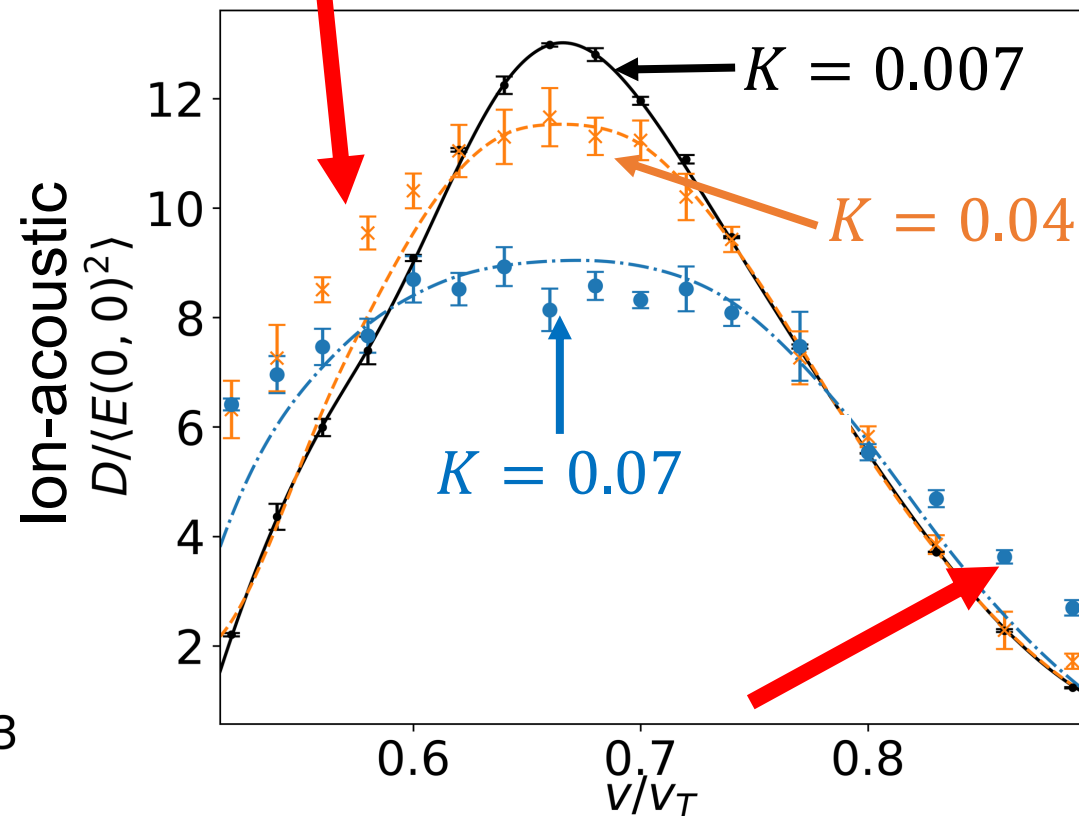
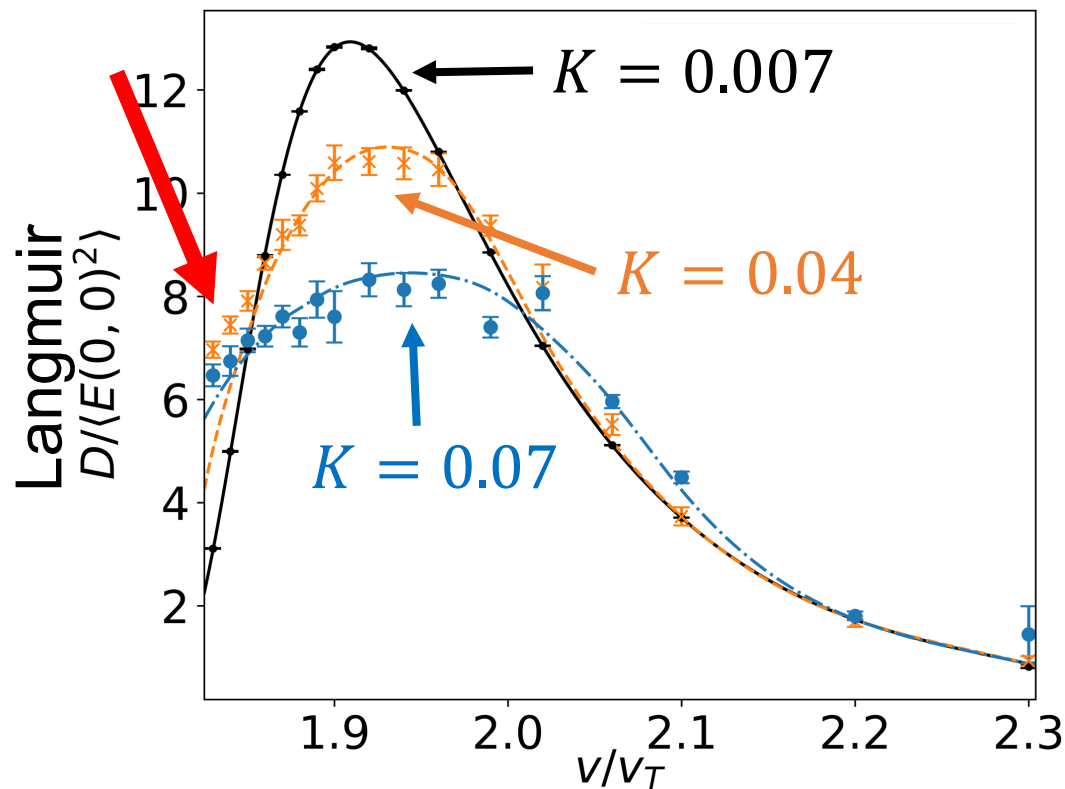
$$\delta(x - vt) \rightarrow A e^{-\frac{(x-vt)^2}{2\sigma_{x,n-1}^2}}$$

$$\longrightarrow D_n^{RB}(v) = \frac{q^2}{m^2} \int_0^{+\infty} dt \sum_k \frac{\hat{E}^2}{2} e^{-\frac{\sigma_{x,n-1}^2 k^2}{2}} \cos[(kv + \omega)t]$$

$$D_n^{RB}(v) \xrightarrow{n \rightarrow +\infty} D^{RB}(v)$$

Numerical simulations

Intermediate Kubo number

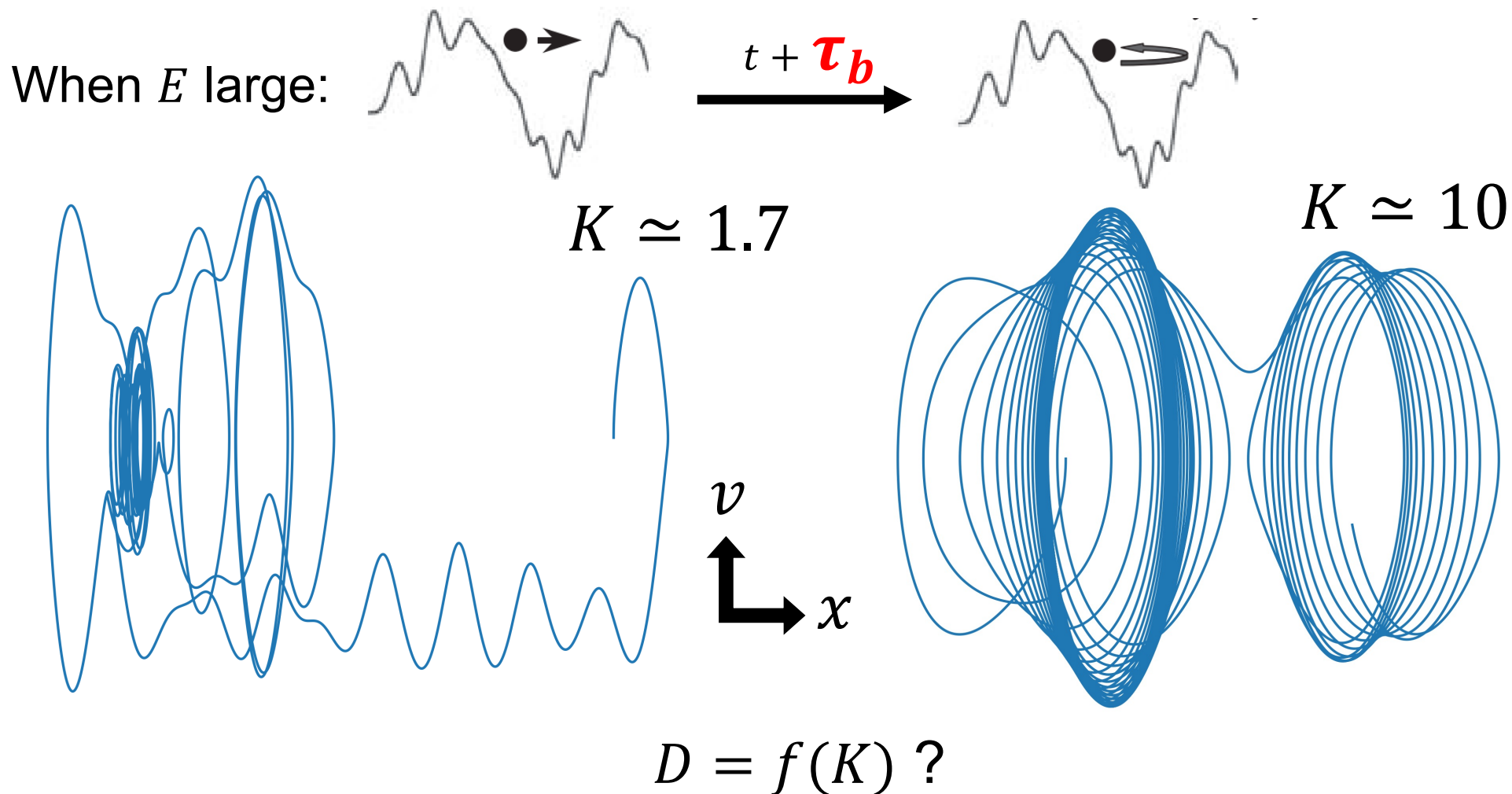


- Th. & Simulation Diff. agreement
- RB effects: K few percents
- D flattens $K \nearrow$

- Problems :
 - Discrepancy at large and small ν

Analytical model

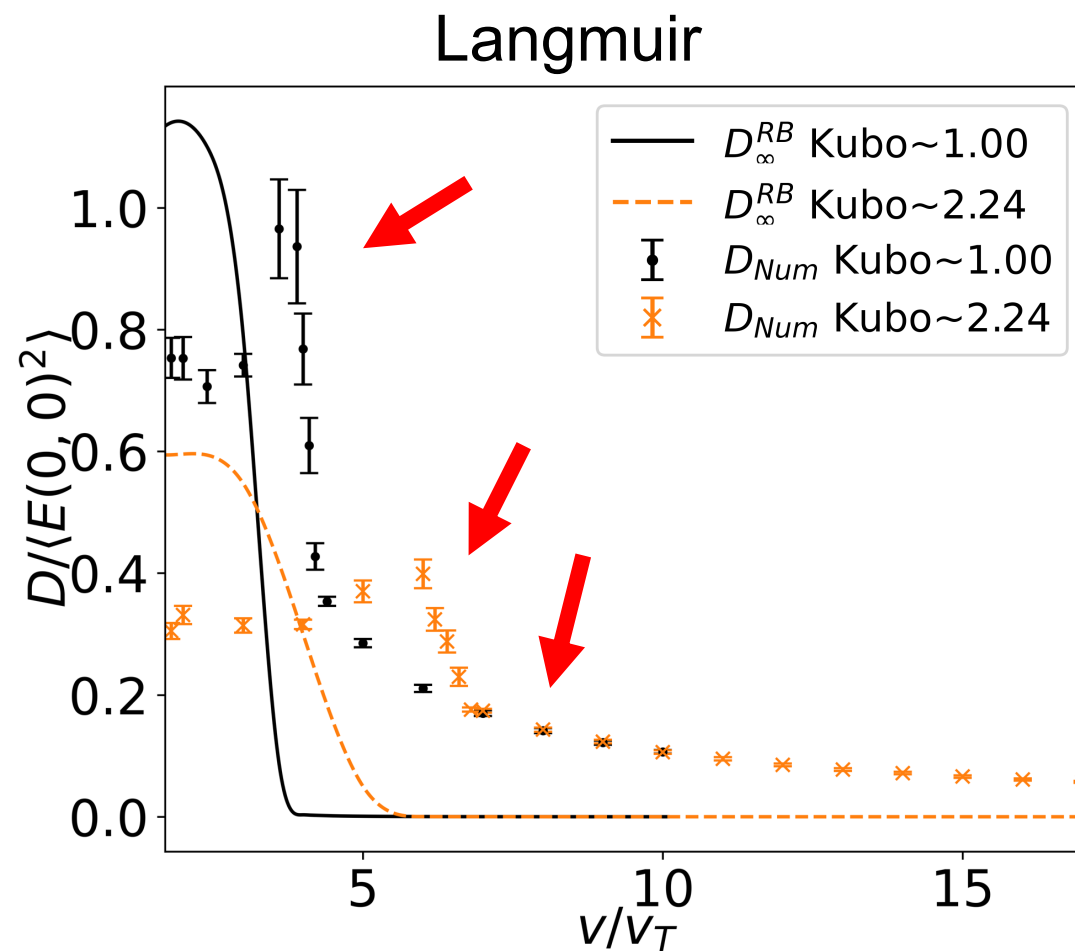
Limits on resonance broadening



Numerical simulations

Large Kubo number

- Study at fixed K
- QL & RB fails for large K
 - Large ν
- RB Th: Order of magnitude
- D converges $\forall K$
 - Finite periodic system



Numerical simulations

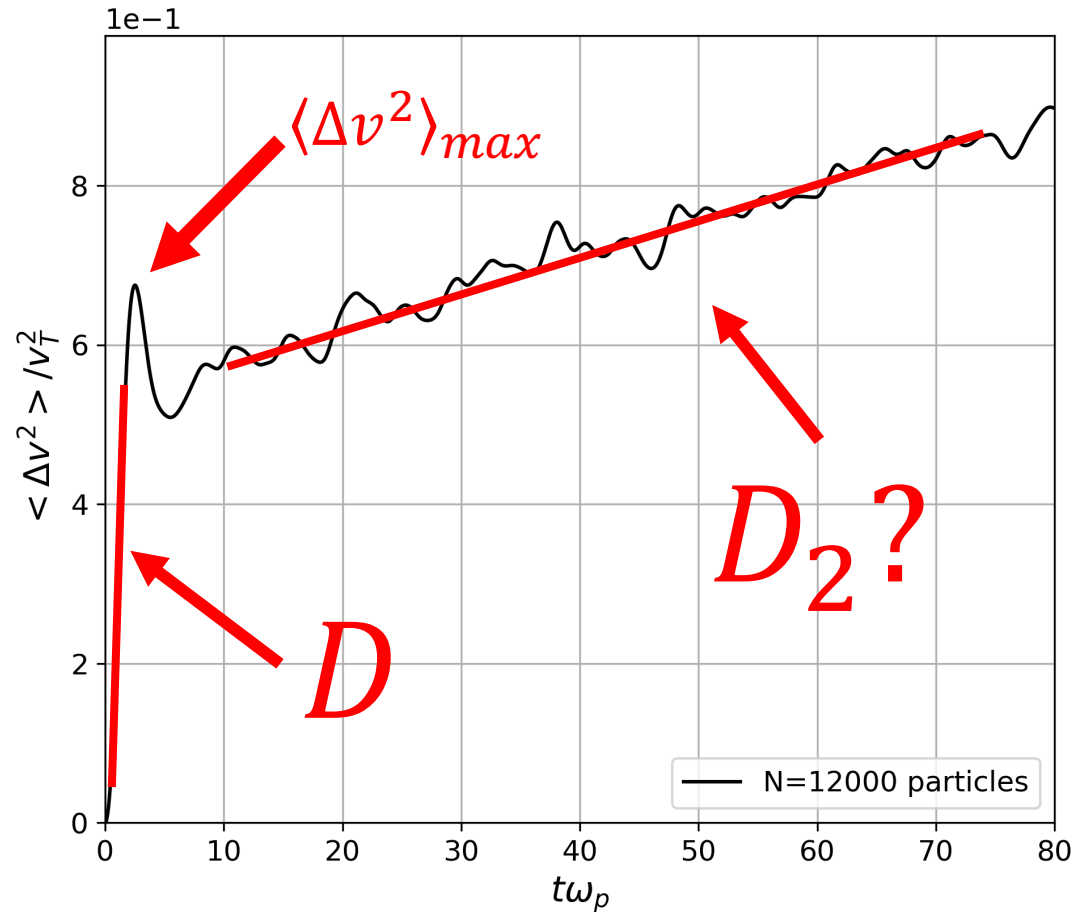
Remark: Two diffusions

- First diffusion regime:

- $t \ll \tau_{ac}$

- Second diffusion regime:

- $t \gtrsim \tau_{ac}$

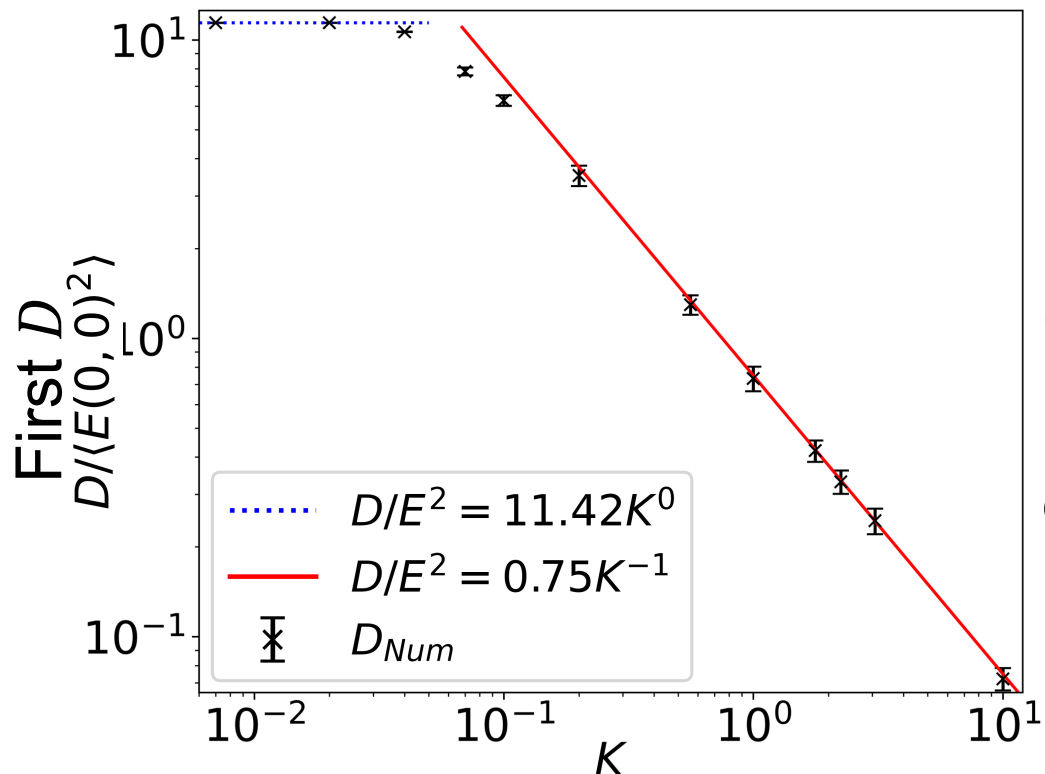


$$D_2 \sim \frac{\Delta v^2}{\Delta t} ?$$

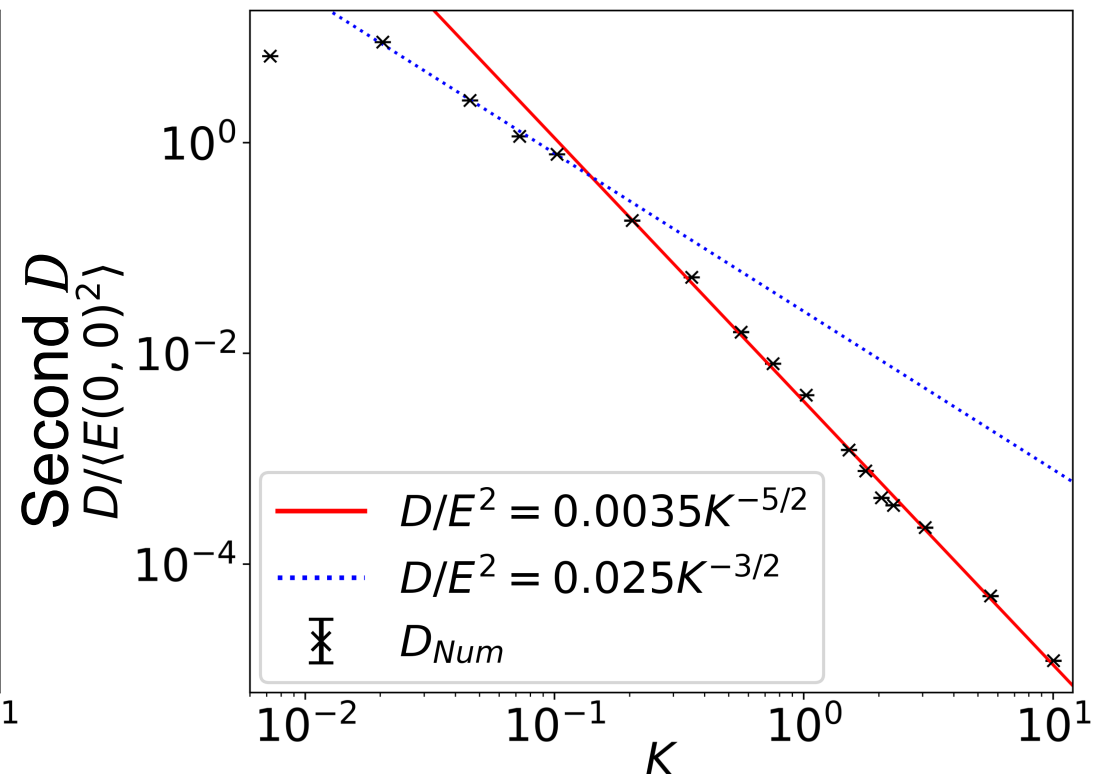
Numerical simulations

First and second diffusion

- Study at fixed v
- D evolves as a power Kubo: $K^\beta = E^{\beta/2}$



- For $K < 0.1$: $D \propto K^4 \propto E^2$
- For $K > 0.5$: $D \propto K^3 \propto E^{3/2}$



- For $K < 0.2$: $D \propto K^{5/2} \propto E^{5/4}$
- For $K > 0.2$: $D \propto K^{3/2} \propto E^{3/4}$

Future prospects

1D Plasma turbulence and transport

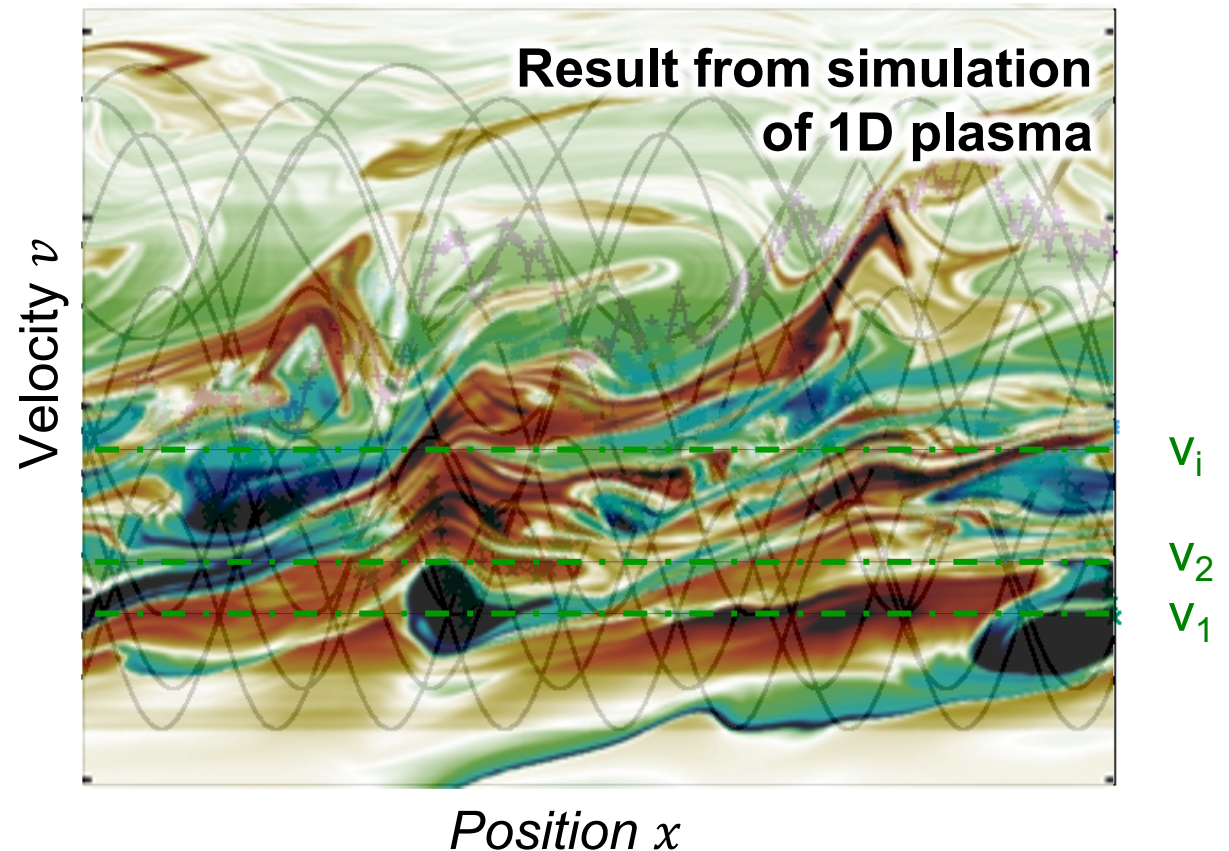
- PS structures generated by turbulence

➤ Modify turbulence

→ Non-linear effect

→ Transport & diffusion

Structures need to be taken
into account



[M. Lesur, P. H. Diamond, PPCF, 2014]

Future prospects

Conclusion

- We verified that QL and RB predict diffusion for small K , for realistic dispersion relation and spectrum.
- We observed the limits on RB for :
 - High and low velocities
 - High Kubo number
- We observed two diffusions at different time scales, where diffusion evolves as a power of Kubo number:

$$\text{➤ } \beta = 3, \quad \frac{5}{2}, \quad \frac{3}{2}$$