

# Scaling of turbulent diffusion in the quasilinear resonance-broadening regime and beyond

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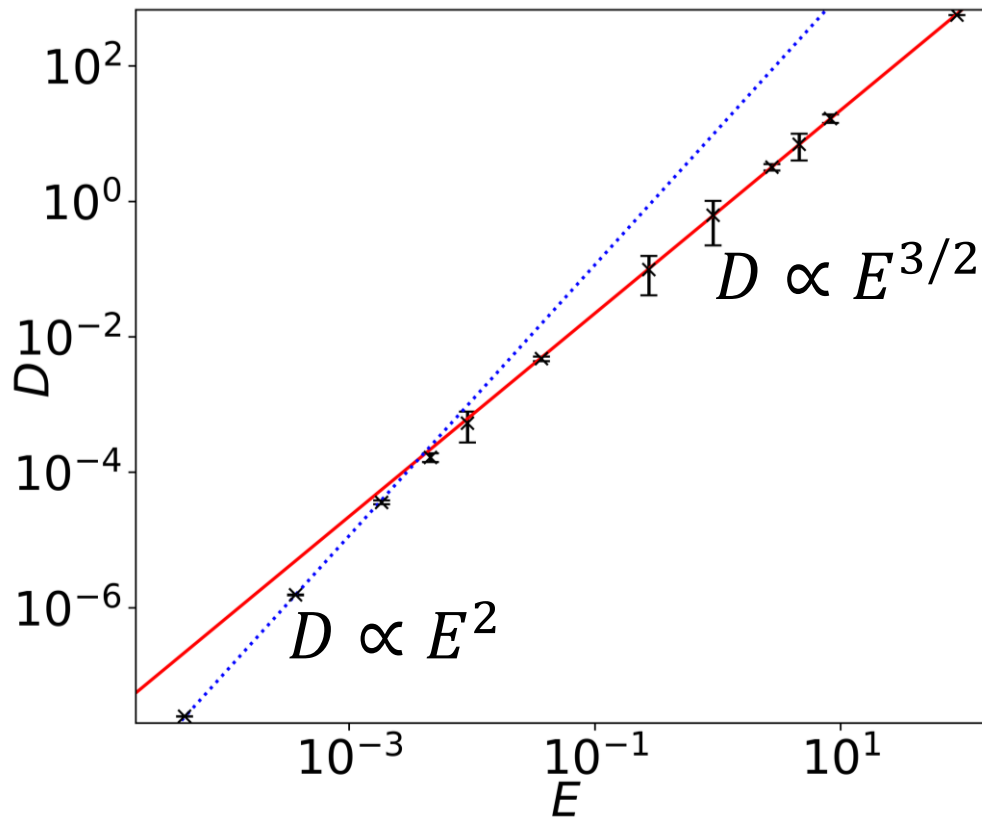
**AAPPS-DPP**

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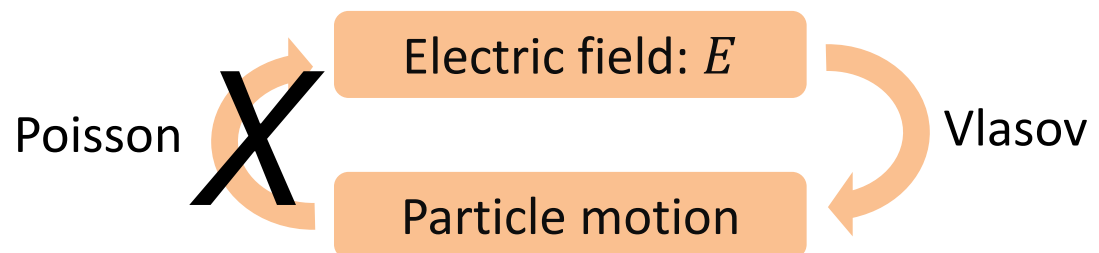
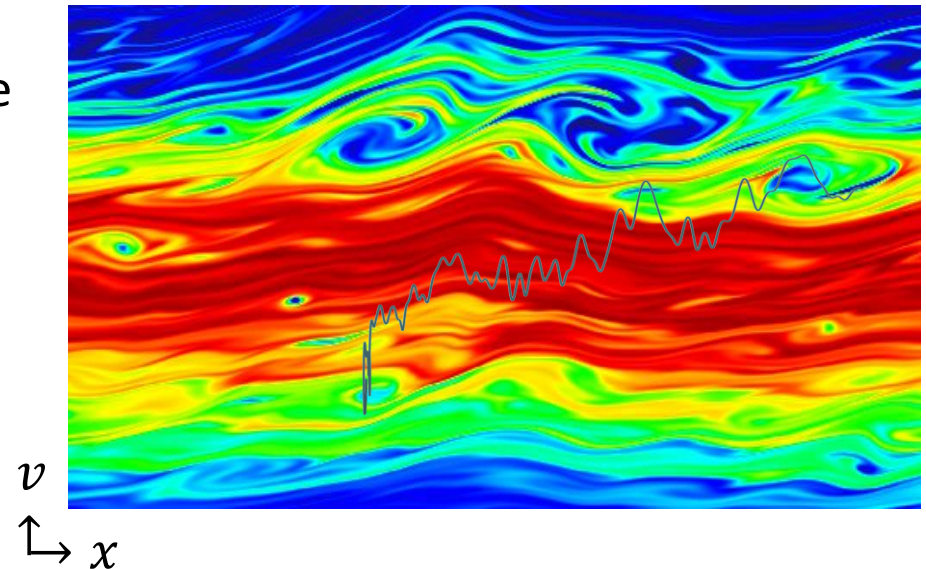
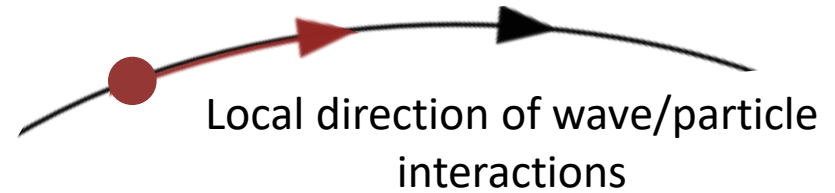
# High-amplitude turbulent diffusion scales as $E^{3/2}$

- Charged particles in an turbulent  $E$  field ( $\dot{v} = \frac{q}{m} E$ )
  - Diffusion in  $v$
- At small  $E$ :  $D \propto E^2$
- At large  $E$ :  $D \propto E^{3/2}$  **even for large Kubo!**



# Context: collisionless 1D plasma turbulence

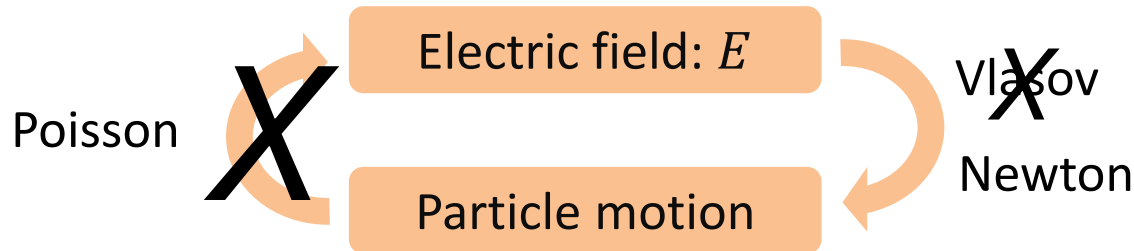
- Geometry:
  - 1D or 1D-reducible dynamics
  - Periodic boundary conditions
- In a homogeneous 1D plasma
  - Complex structures in phase-space
  - Transport in velocity  $v$ 
    - Plasma heating
    - Instability triggering
- Splitting the Vlasov-Poisson problem
  - Prescribed electric field
  - Describe particle transport
- Types of waves
  - $\omega = \text{const.}$  (artificial)  
*[Doveil & Gresillon, '82]*
  - Langmuir
  - ion-acoustic



*This work*

# Method: test particles in a prescribed turbulent electric field

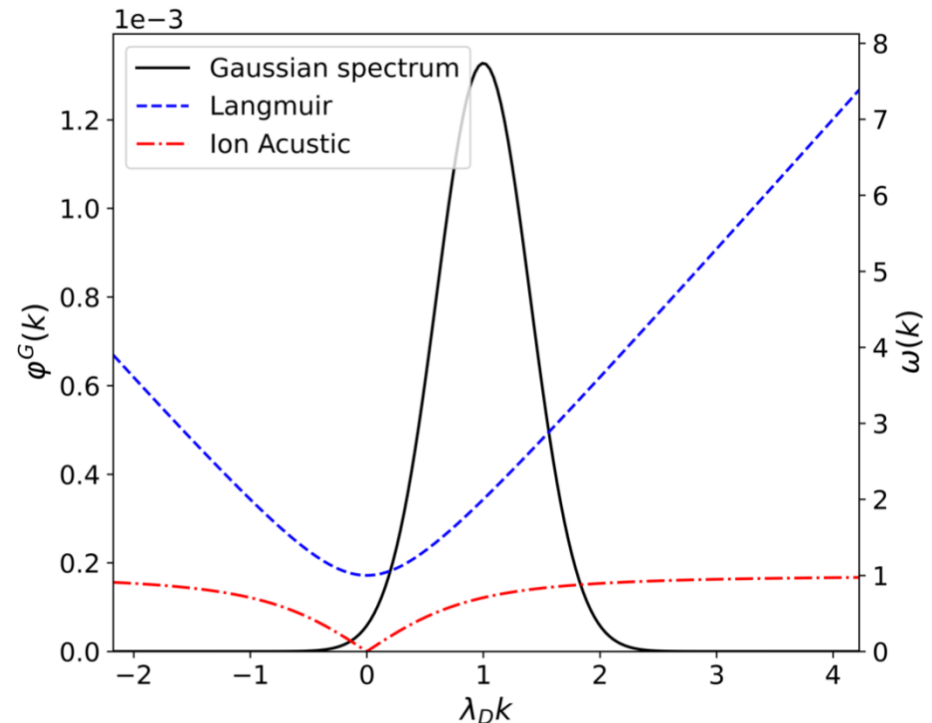
- Model for test particles in a prescribed electric field



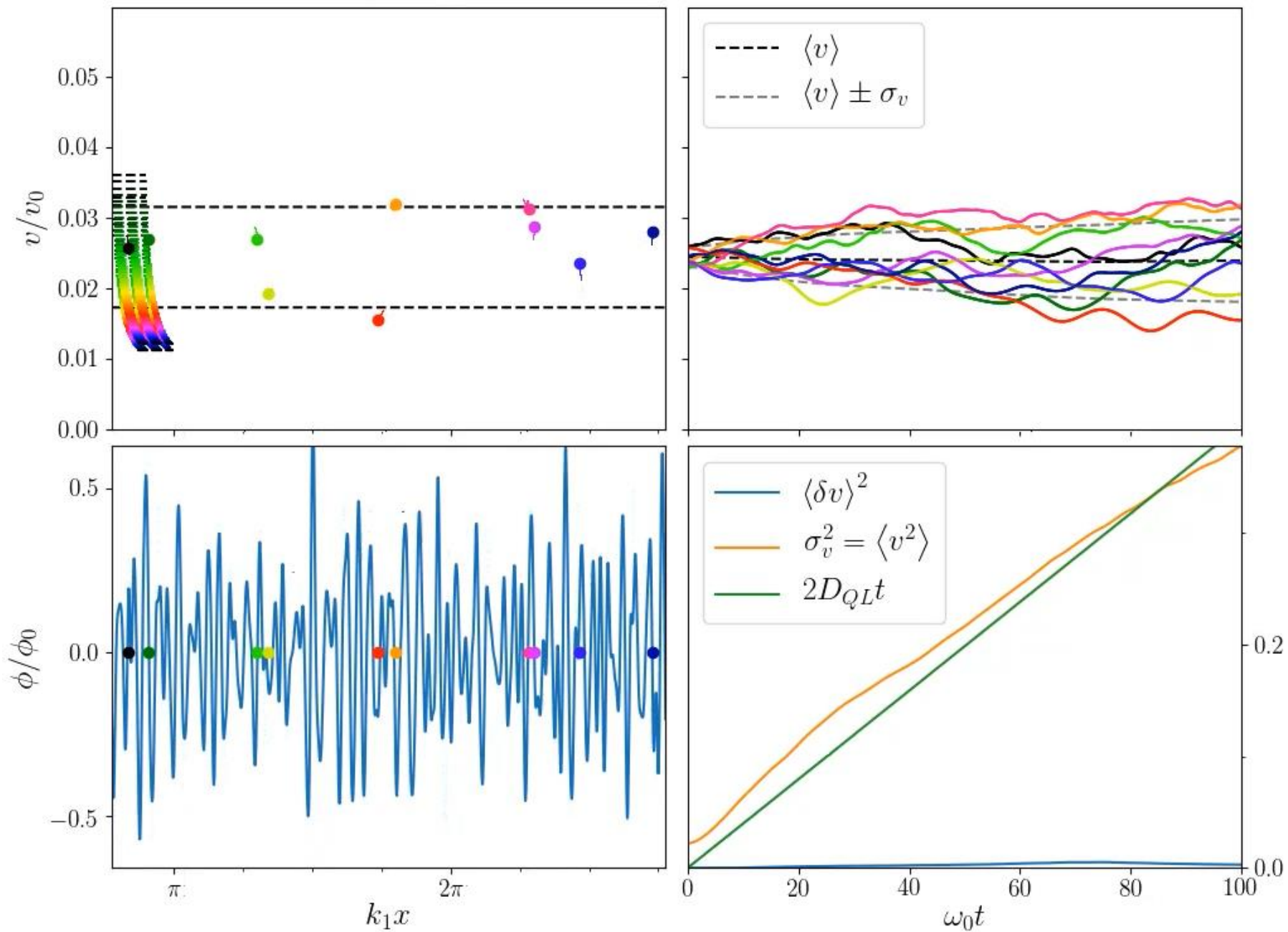
- Prescribed turbulent electric field

$$E(x, t) = \sum_k \hat{E}_k \sin[kx - \omega_k t + \beta_k]$$

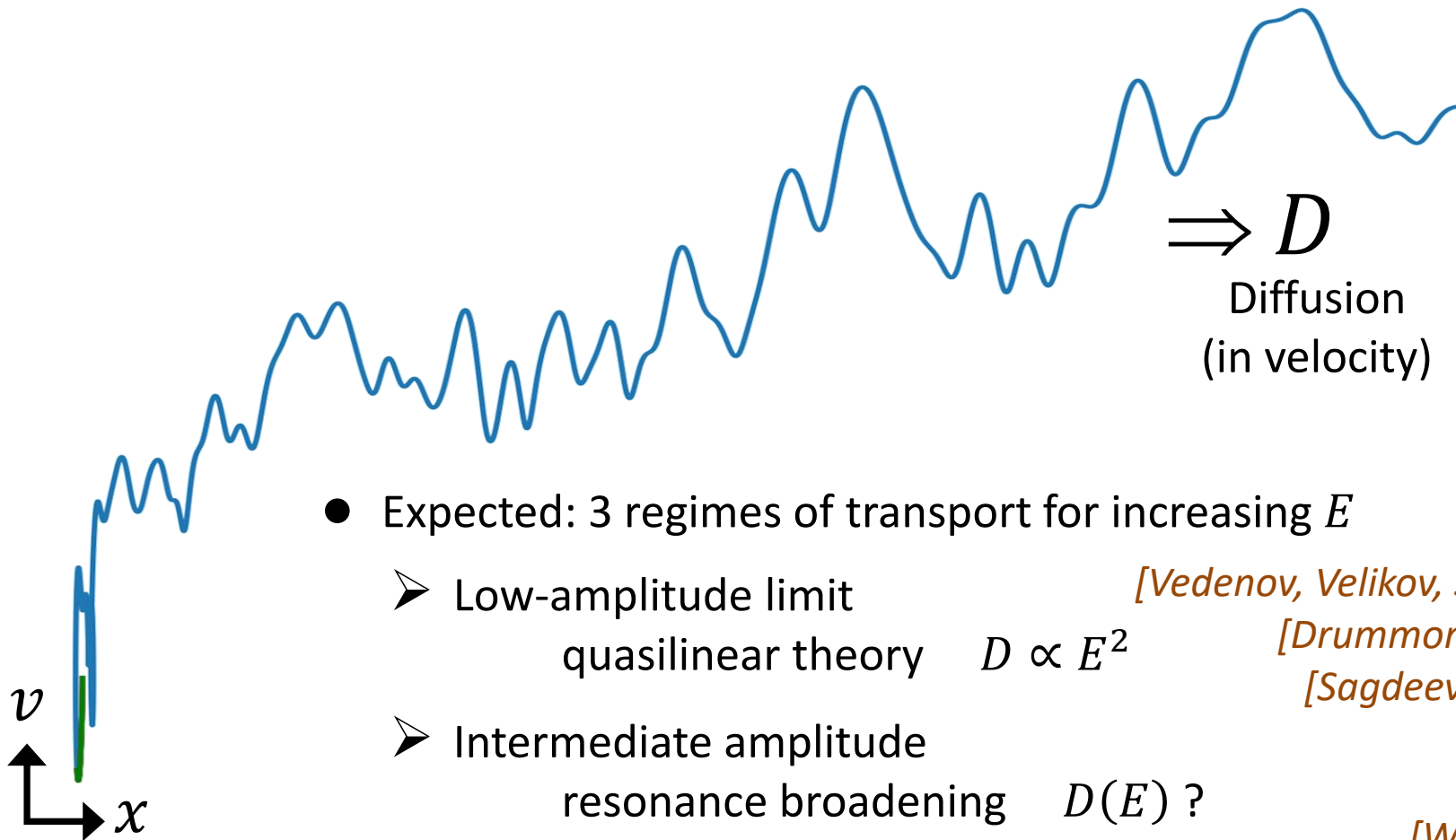
- Many modes (200)
- Steady state
- Random phases
- Realistic dispersion relation (Langmuir & ion-acoustic)
- Realistic spectrum (centered on finite  $k$ )



$\omega_0 t = 100.0$



# Objective: quantitative description of transport



- Expected: 3 regimes of transport for increasing  $E$

- Low-amplitude limit

quasilinear theory  $D \propto E^2$

*[Vedenov, Velikov, Sagdeev '61]*

*[Drummond, Pines '62]*

*[Sagdeev, Galeev '69]*

- Intermediate amplitude

resonance broadening  $D(E) ?$

*[Dupree '66]*

*[Weinstock '68]*

- Large-amplitude limit

strong trapping  $D(E) ?$

non-diffusive transport ?

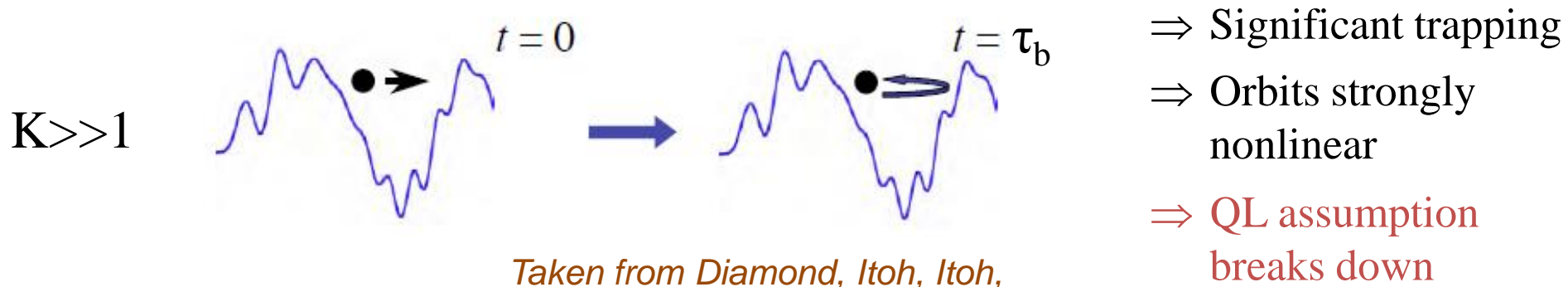
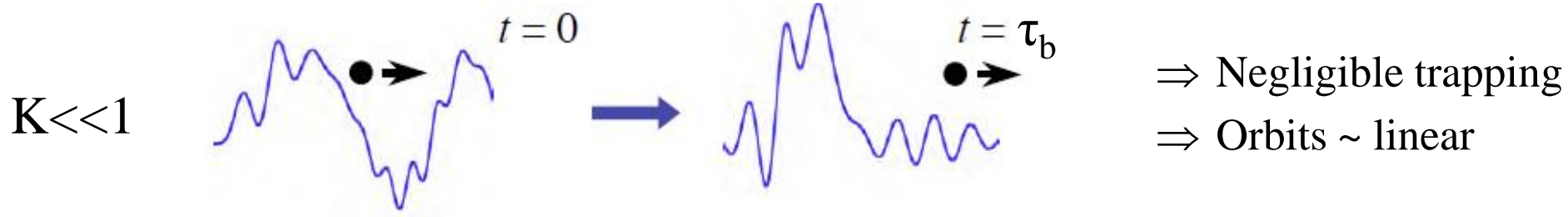
# The Kubo number measures nonlinear trapping

Kubo number:  $K = \frac{\tau_c}{\tau_b}$

$(K \propto \sqrt{E})$

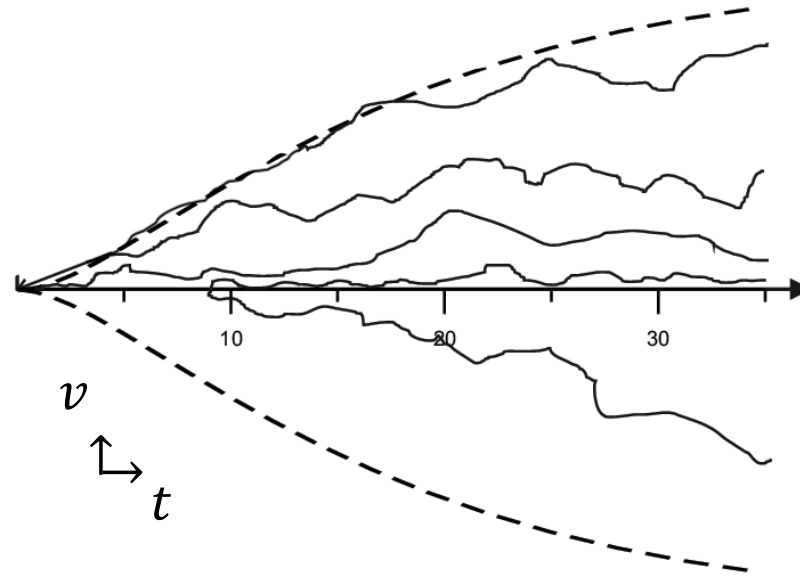
$\tau_c$ : autocorrelation time of turbulence  
 ~ effective duration of pattern of potential

$\tau_b$ : bounce time of a particle in the potential if it had a fixed pattern



*Taken from Diamond, Itoh, Itoh, Modern Plasma Physics*

# Variance of particle velocities involves the Lagrangian autocorrelation



$$\frac{dv}{dt} = \frac{q}{m} E(x, t)$$

$$\hookrightarrow \langle \Delta v^2 \rangle = \frac{q^2}{m^2} \int_0^t \int_0^t dt' dt'' \langle E(x(t'), t') E(x(t''), t'') \rangle$$



quasilinear hypothesis:  $x(t) = vt$

or resonance broadened hypothesis: statistical distribution of  $x(t)$

or trapped orbit  $x(t)$



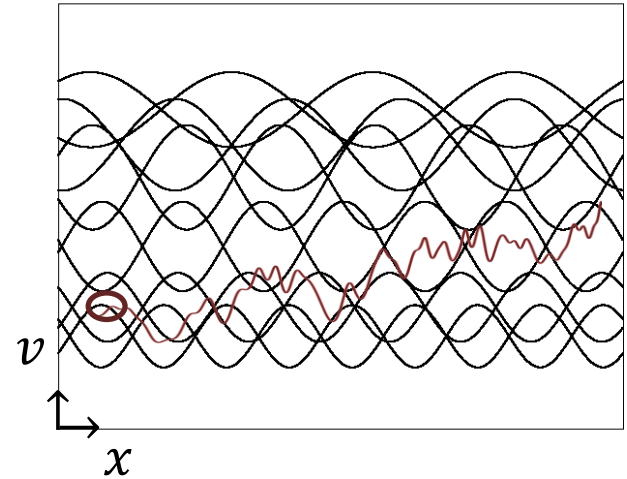
# Quasilinear diffusion

$K \ll 1$

$$x(t) = vt \longrightarrow \langle \Delta v^2 \rangle = 2D_0 t$$

$$D_0(v) = \frac{q^2}{m^2} \int_0^{+\infty} \int_{-\infty}^{+\infty} dt dx \delta(x - vt) \langle E(0,0) E(x,t) \rangle$$

$$E(x,t) = \sum_k \hat{E}_k \sin[kx - \omega_k t + \beta_k]$$

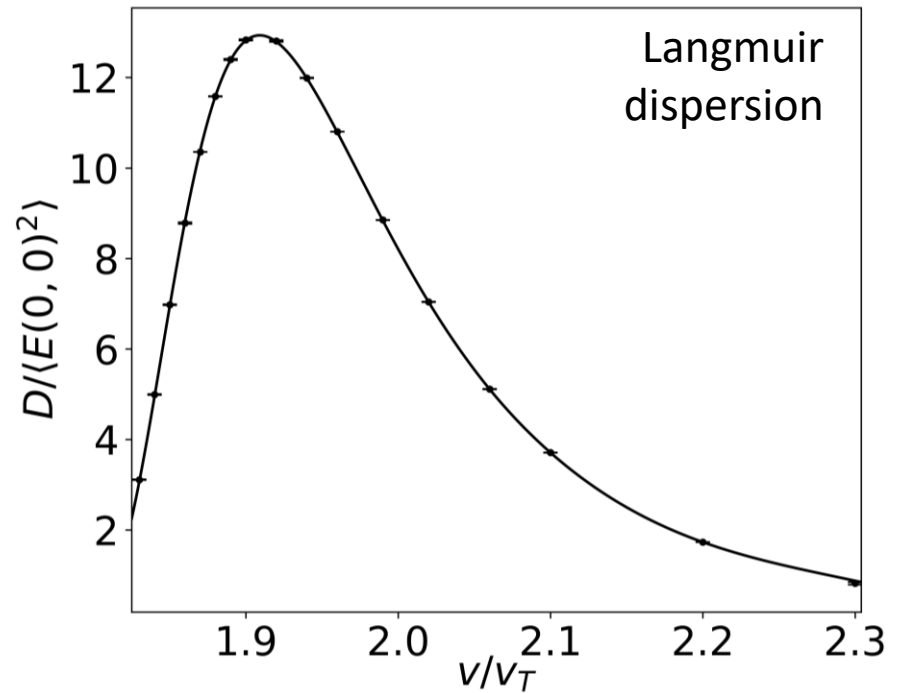
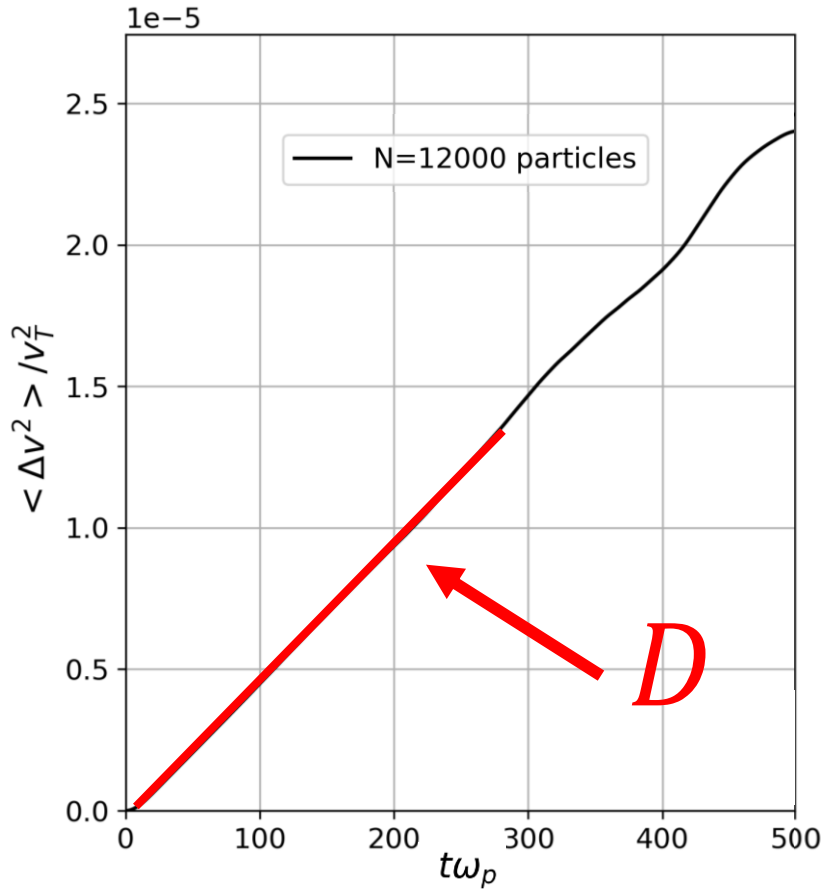


$$D_0(v) = \frac{q^2}{m^2} \int_0^{+\infty} dt \sum_k \frac{\hat{E}_k^2}{2} \cos[(kv - \omega)t] = \frac{q^2}{m^2} \tau_c(v) E^2$$

$\Rightarrow D_0$  proportional to  $E^2$

# Statistics for $K \ll 1$

$$K = 0.007$$



Similar curves and agreement with ion-acoustic dispersion

$\Rightarrow$  **Quasilinear theory very accurate in this regime**

# Resonance broadening (or orbit diffusion)

$$D_0(v) = \frac{q^2}{m^2} \int_0^{+\infty} \int_{-\infty}^{+\infty} dt dx \delta(x - vt) \langle E(0,0) E(x,t) \rangle$$

[Dupree, '66]

[Weinstock, '68]

$$P(x,t) \sim e^{-\frac{(x-vt)^2}{2\sigma_x^2}}$$

$$\longrightarrow D^{RB}(v) = \frac{q^2}{m^2} \int_0^{+\infty} dt \sum_k \frac{\hat{E}^2}{2} e^{-\frac{\sigma_x^2 k^2}{2}} \cos[(kv + \omega)t]$$

$$\text{with } \sigma_x^2 = \frac{2}{3} D^{RB} t^p$$

Iteratively accounting for orbit diffusion

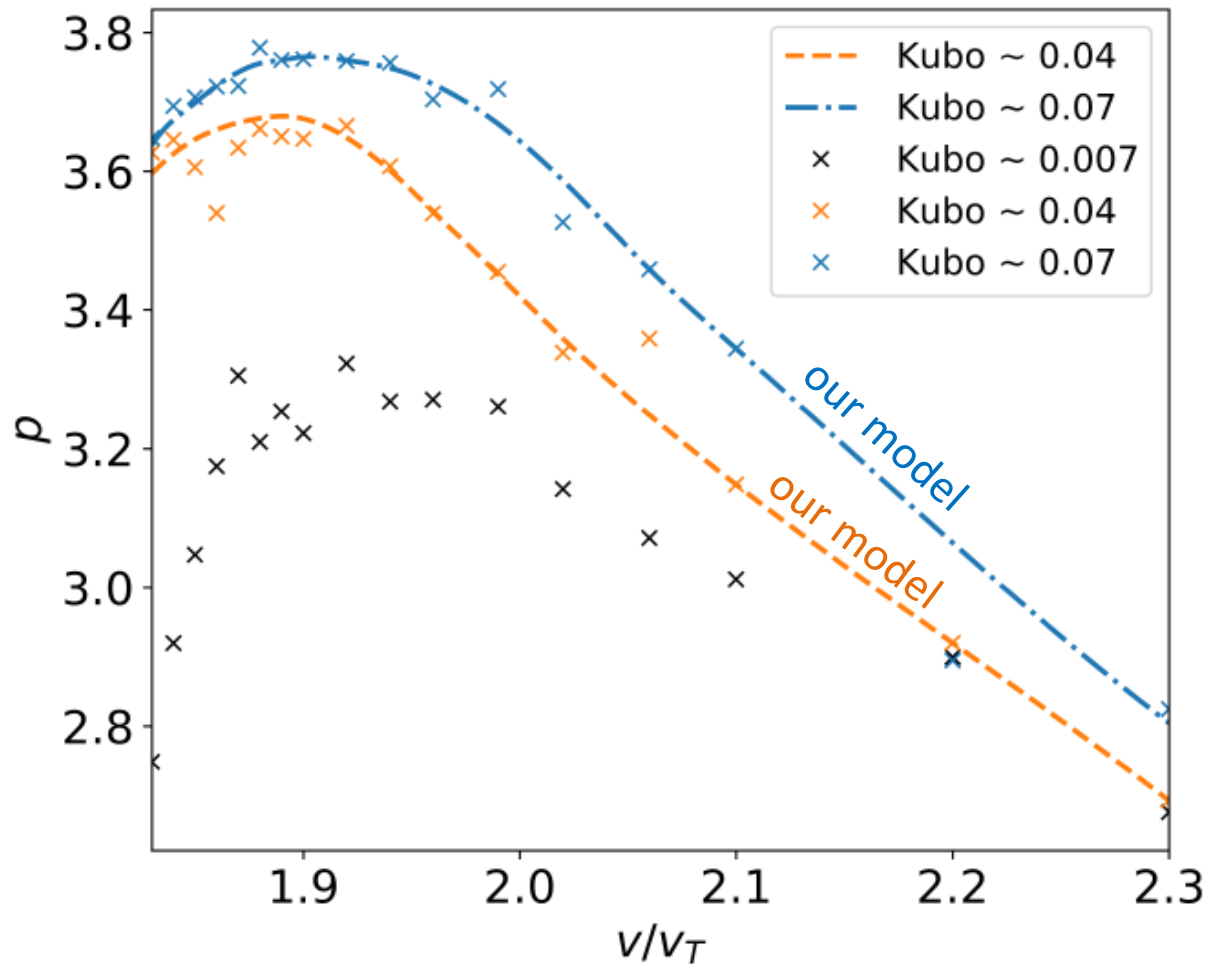
$$D_0 \longrightarrow \sigma_{x,n=0} \longrightarrow D_{n=1}^{RB} \longrightarrow \sigma_{x,n=1} \dashrightarrow \dots \dashrightarrow D_n^{RB} \xrightarrow[n \rightarrow +\infty]{} D^{RB}$$

$\Rightarrow$  requires a model for  $p$  in  $\sigma_x^2 = \frac{2}{3} D^{RB} t^p$

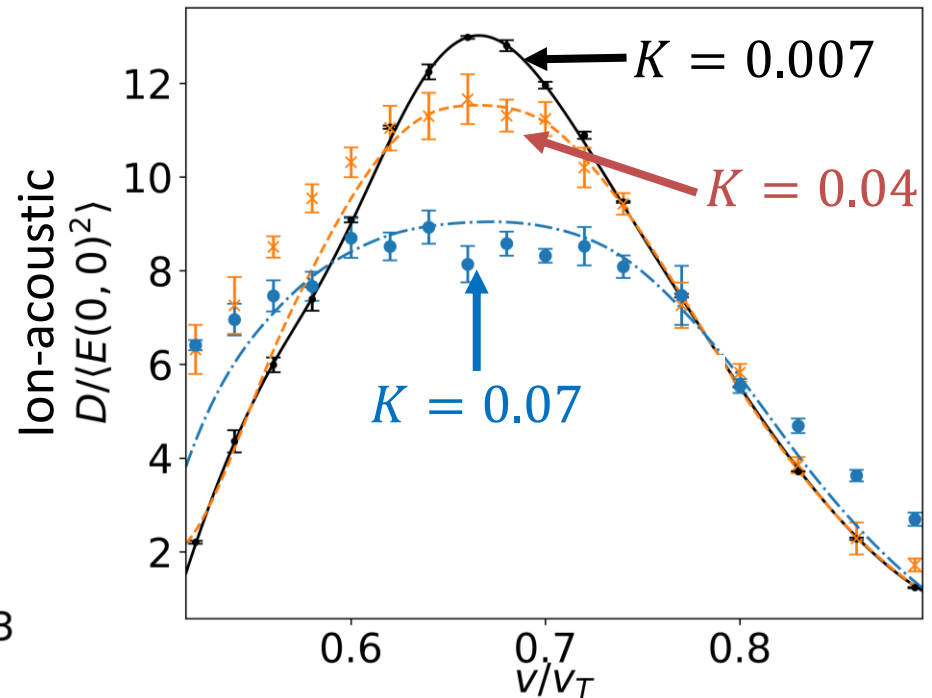
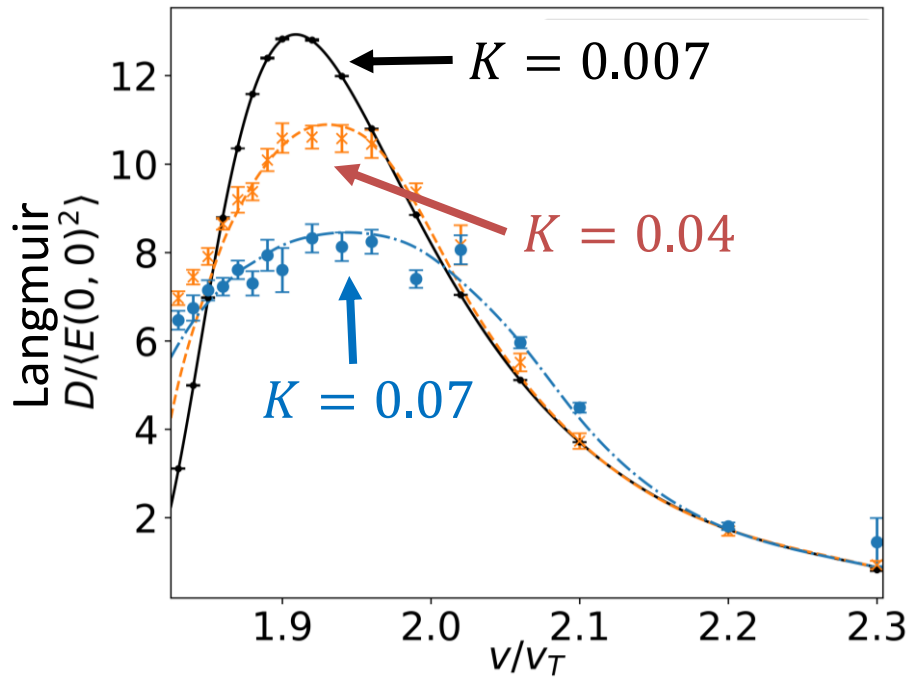
# Time dependency of the spatial variance

$$\sigma_x^2 = \frac{2}{3} D^{RB} t^p$$

$p$  converges to 3 for large enough times  
but for large electric fields, diffusion  
occurs on smaller timescale



# Intermediate Kubo number

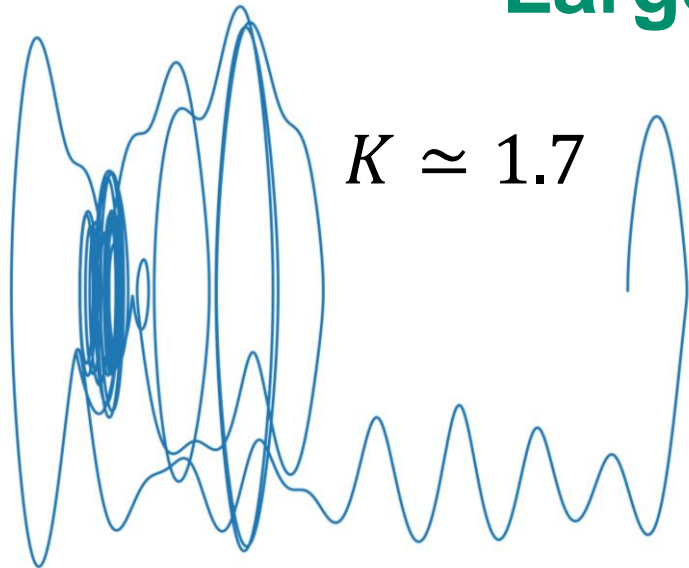


[Guillevic, in preparation]

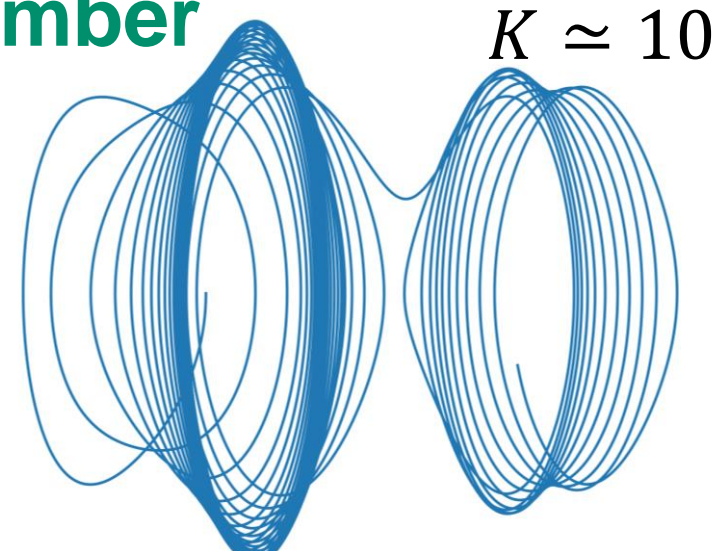
- Important resonance broadening for  $K$  of a few percents
- $D$  increases with  $K$ , but  $D/E$  decreases
- Discrepancies at small and large velocities not fully explained

⇒ Resonance broadening theory is precise (with correct  $p$ )

# Large Kubo number

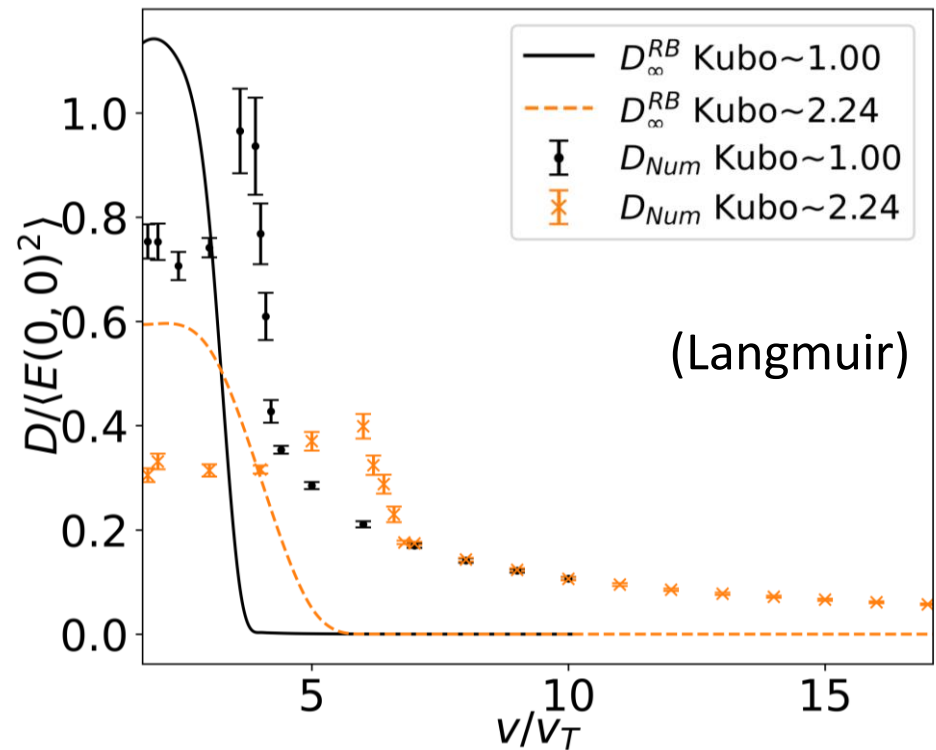


$K \approx 1.7$



$K \approx 10$

- QL & RB fails for large  $K$   
- especially at large  $v$
- However RB gives correct order of magnitude



⇒ **limit of resonance broadening theory reached at  $K \sim 1$**  14/28

# Scaling of diffusion against Kubo number

- Study at fixed  $v$

- For  $K < 0.1$ ,

$$D \propto K^4 \propto E^2$$

~ random walk

$$\Delta v \propto E \Delta t$$

$$\Delta t = \tau_c$$

- For  $K > 0.5$ ,

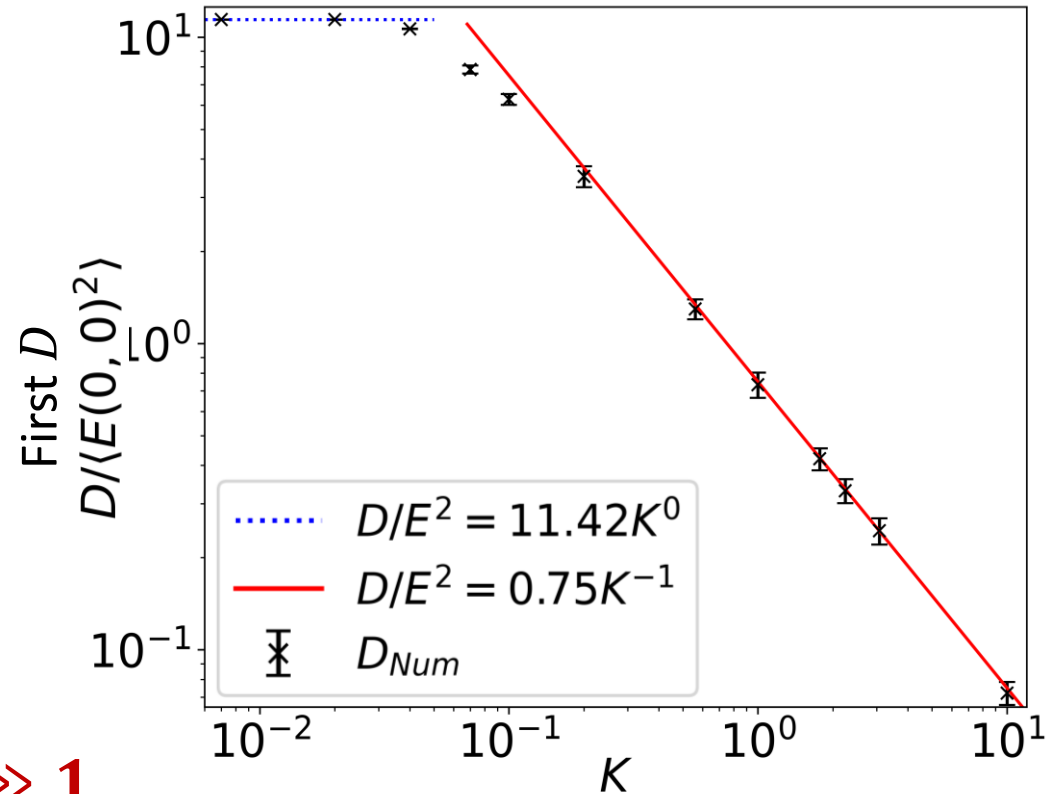
$$D \propto K^3 \propto E^{3/2}$$

~ random walk

$$\Delta v \propto D^{1/3} \text{ [Dupree, '66]}$$

$$\Delta t = \tau_b$$

**even for  $K \gg 1$**

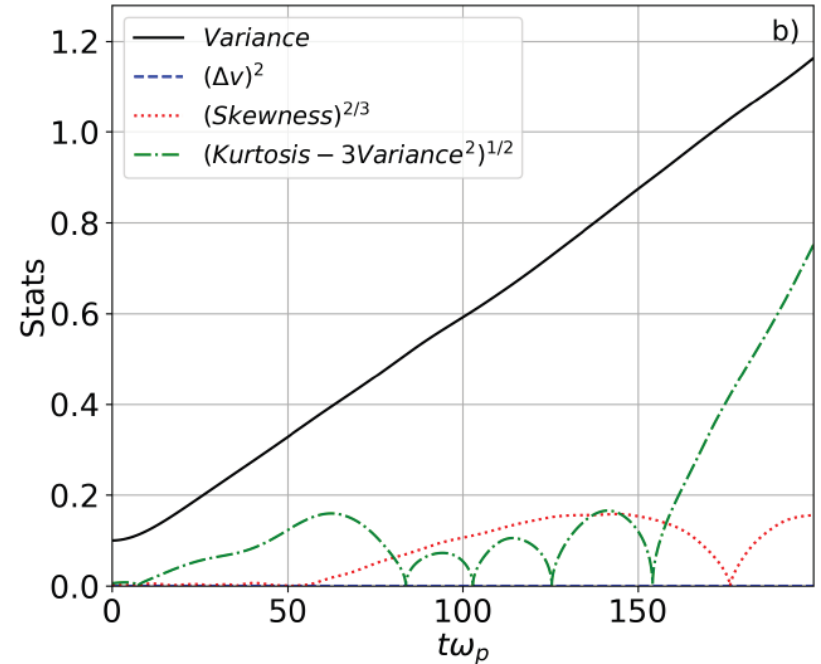
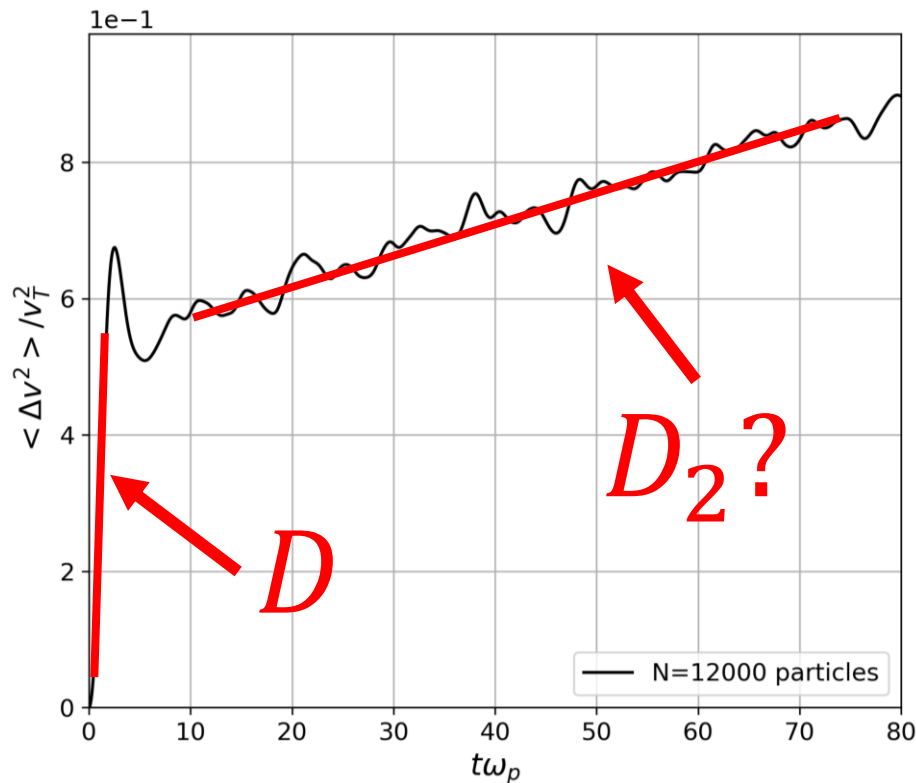


[Guillevic, in preparation]

⇒ In terms of scaling of diffusion with respect to turbulence intensity, resonance broadening works further than expected

# Later transport process includes diffusion

- First diffusion regime:
  - $t \sim \tau_{ac}$
- Second diffusion regime:
  - $t > \tau_{ac}$



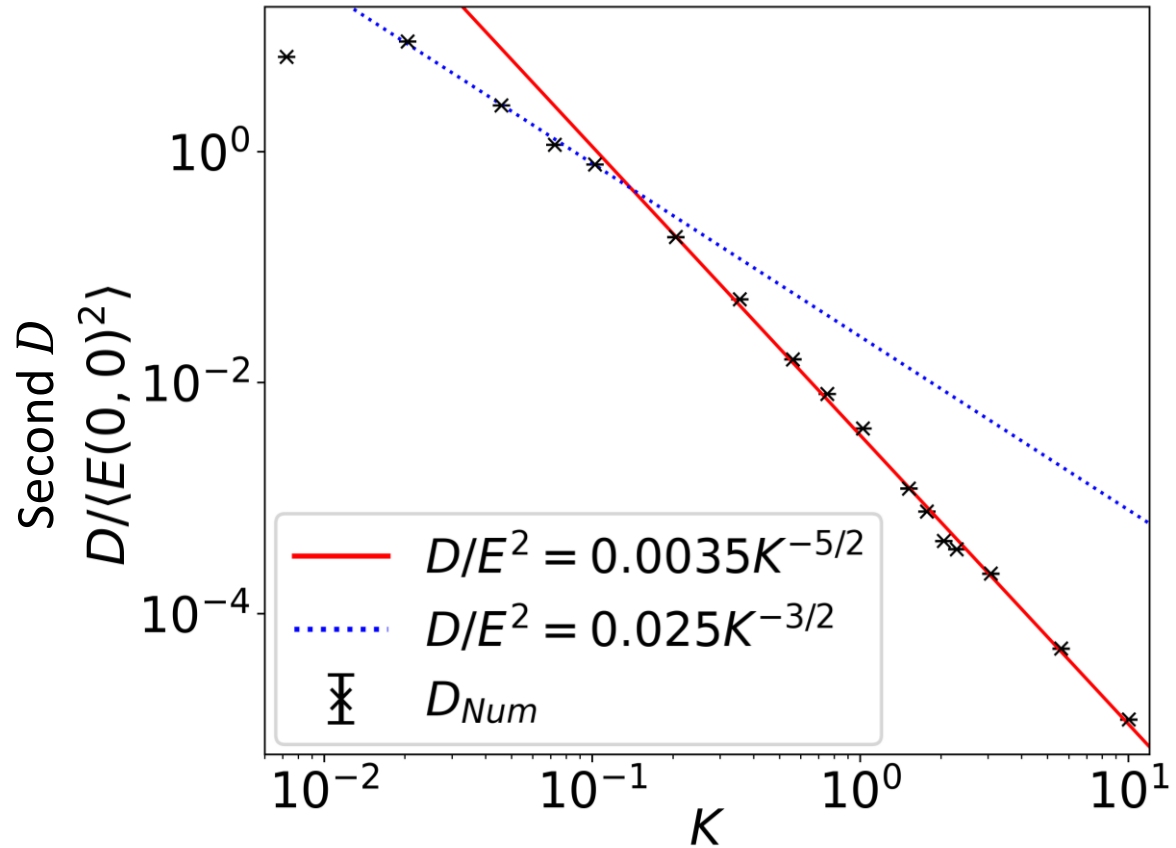
- Albeit not pure diffusion

⇒ Analysis of the second slope



# Scaling of the second diffusion

- Study at fixed  $v$
- For  $K < 0.1$ ,  
 $D \propto K^4 \propto E^2$
- For  $K > 0.5$ ,  
 $D \propto K^3 \propto E^{3/2}$
- Unsuccessful attempts to model as random walk

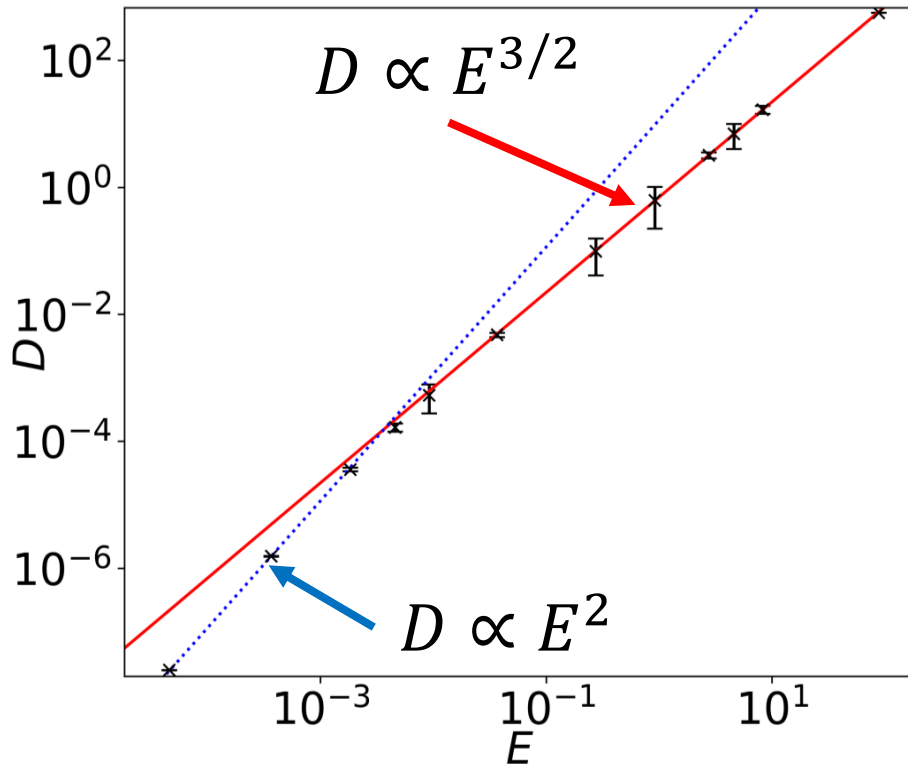


[Guillevic, in preparation]

⇒ Searching for a good framework of analysis

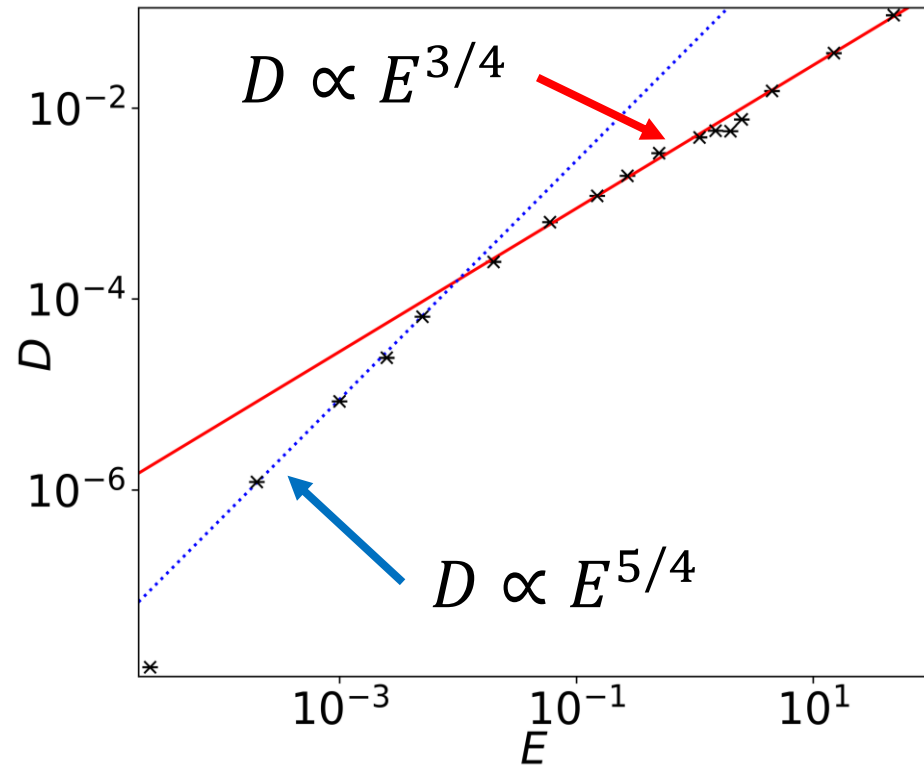
# Summary

"First diffusion"



- For  $K < 0.1$  :  $D \propto K^4 \propto E^2$
- For  $K > 0.5$  :  $D \propto K^3 \propto E^{3/2}$

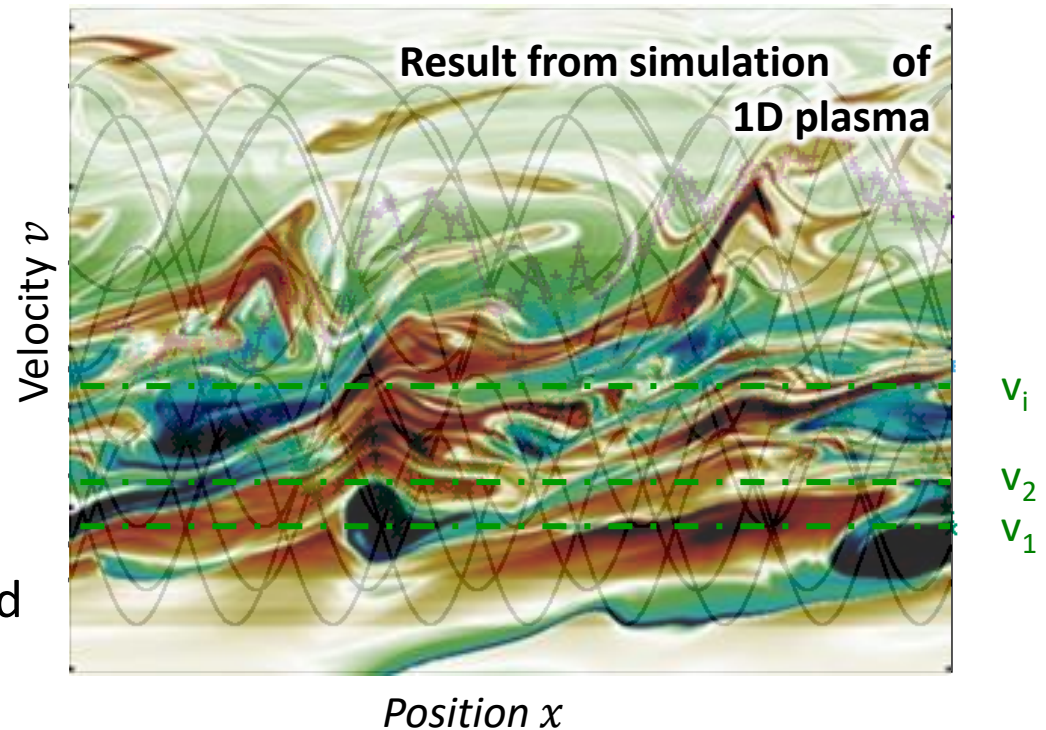
"Second diffusion"



- For  $K < 0.2$  :  $D \propto K^{5/2} \propto E^{5/4}$
- For  $K > 0.2$  :  $D \propto K^{3/2} \propto E^{3/4}$

# Prospects: including phase-space vortices

- Phase-space vortices spontaneously emerge in Vlasov turbulence
  - Loss of random-phases approximation
  - Expect strongly non-diffusive transport
- Preliminary results
  - PS structures may be accounted for simply by accounting for modified spectrum



[Lesur, '14]

## Acknowledgements

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