

Nonlinear growth rate of a phase-space electron hole

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Based on the PhD work of **Alejandro Guillevic**

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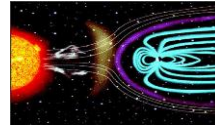
July 3rd, 2023



Motivation: essential role of phase-space structures

Vortices in phase-space are observed in experiments

- Space plasmas
- Laboratory linear plasmas
- Magnetic reconnection of toroidal plasmas
- Fusion plasmas
- Laser plasmas



Review: Eliasson & Shukla '06



Saeki '79

Fox '08

Kusama '99 ; Berk, Breizman & Pekker '96

Sarri '10

Deep implications for instabilities, turbulence, transport, heating

- Drive nonlinear instabilities
- Modify the magnitude of saturation, spectrum of turbulence
- Qualitative effect on transport
- Interact with large-scale flows
- Propagation of trapped turbulence

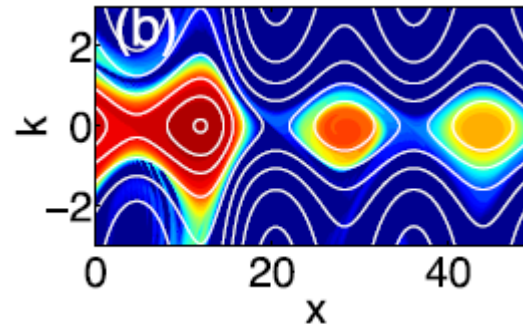
Dupree '82

Terry '90

Biglari '88

Kosuga '11

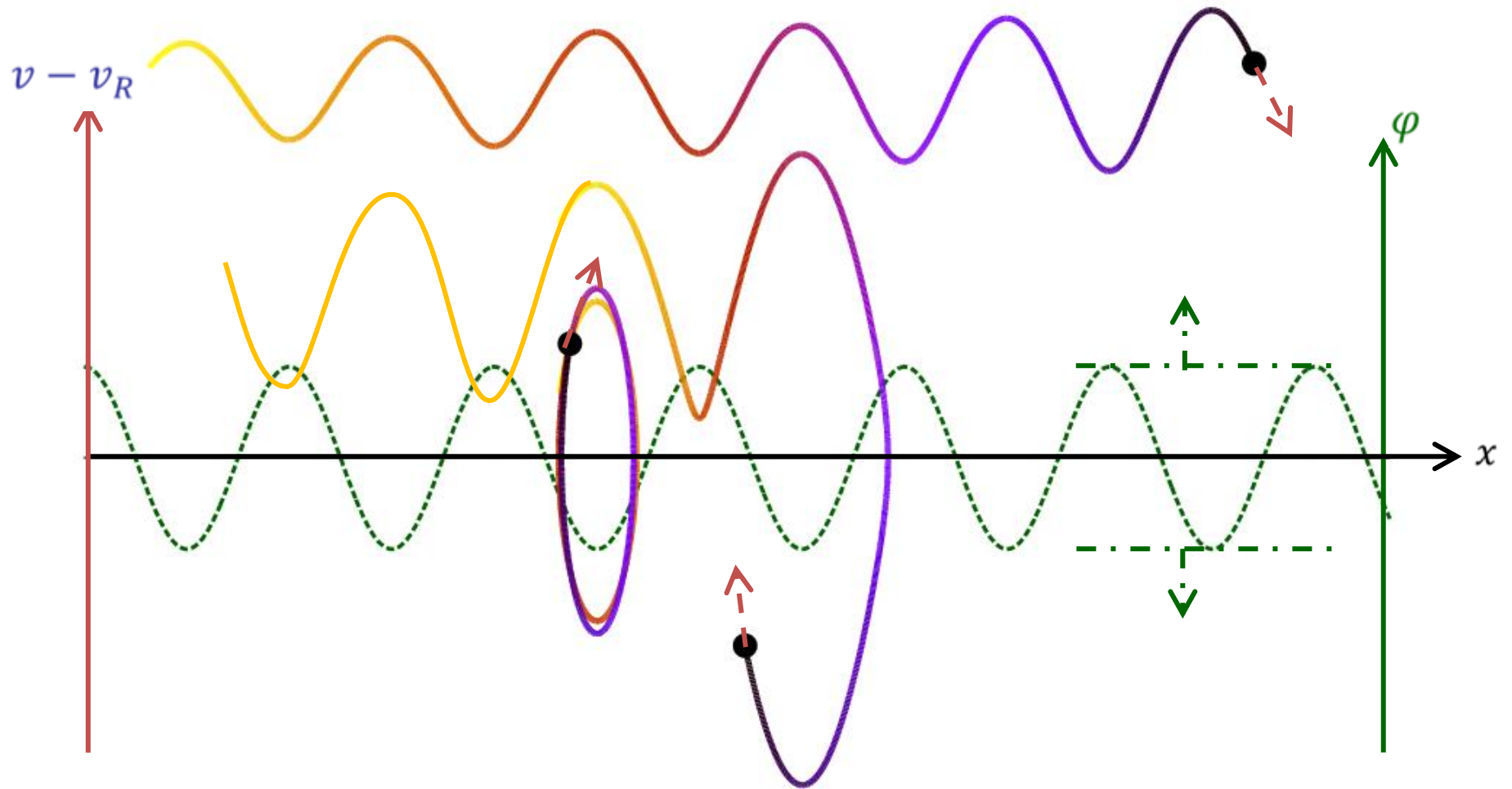
Sasaki '17



Many open questions remain

- Need for more detailed analysis in numerical simulations
- Need for more detailed measurements in fusion plasmas

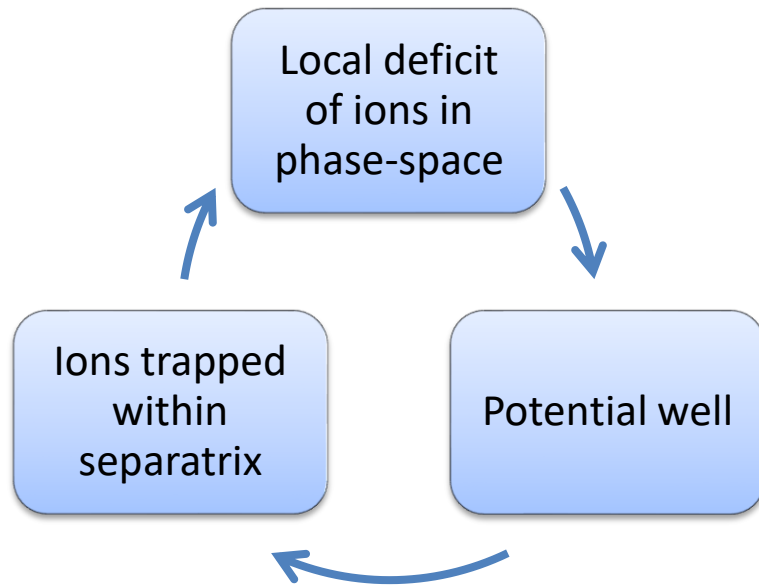
Electrostatic trapping



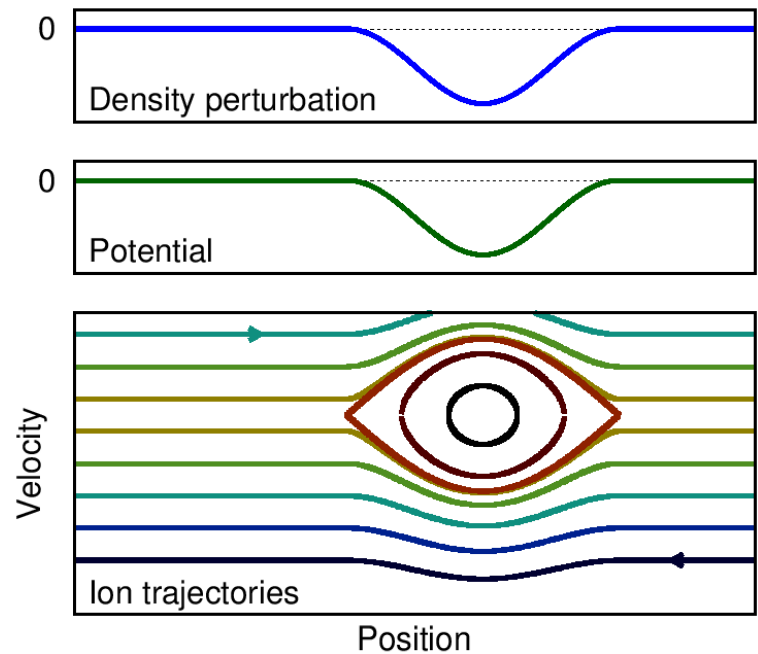
⇒ An electrostatic wave can grow by trapping “barely passing” particles

Electrostatic trapping yields phase-space vortex

Self-sustaining structure



Phase-space vortex formation is a fully nonlinear, kinetic process.



⇒ BGK mode or soliton

Bernstein & Green & Kruskal '57

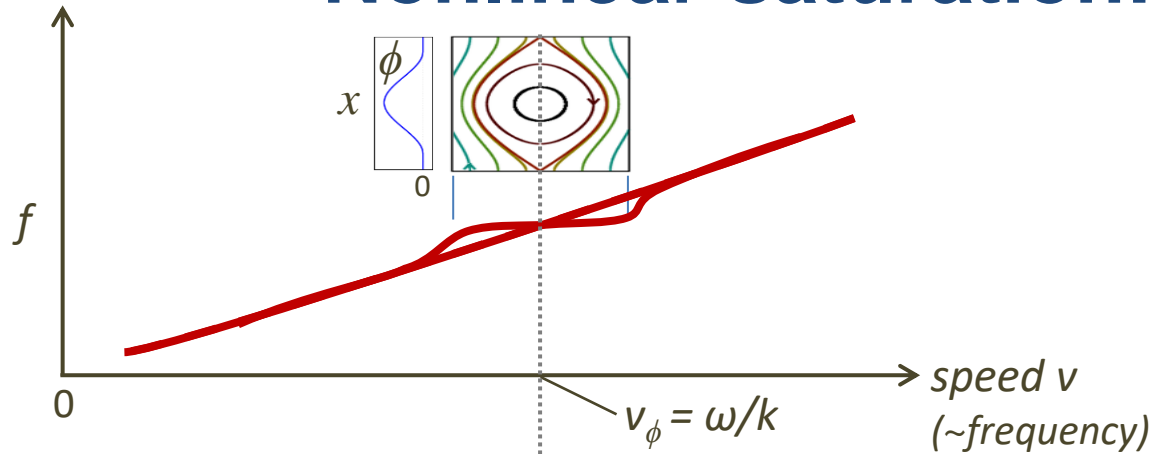
Dupree '83

Schamel '86

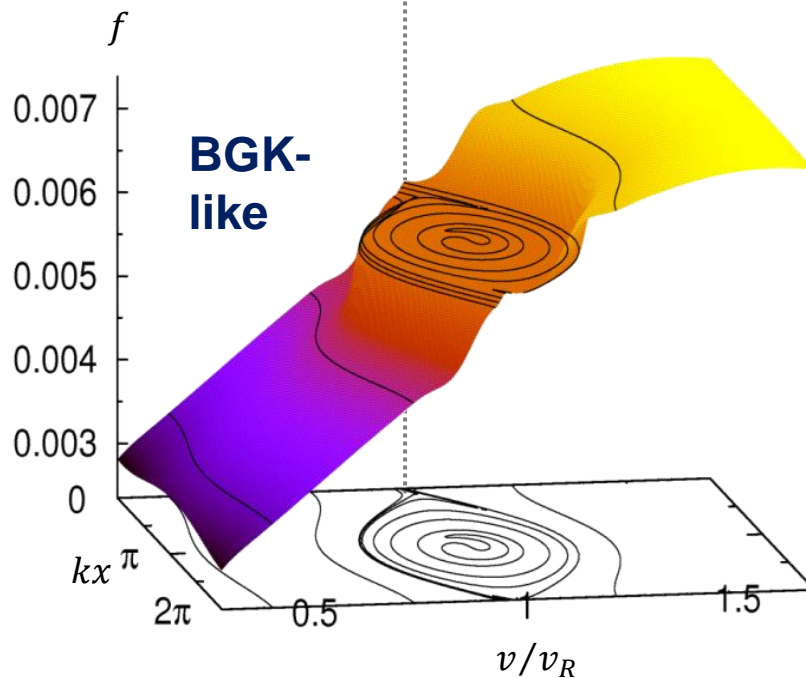
Berk & Breizman '99

⇒ **a PS vortex is not tied to a wave and can evolve independently**

Nonlinear saturation: BGK limit



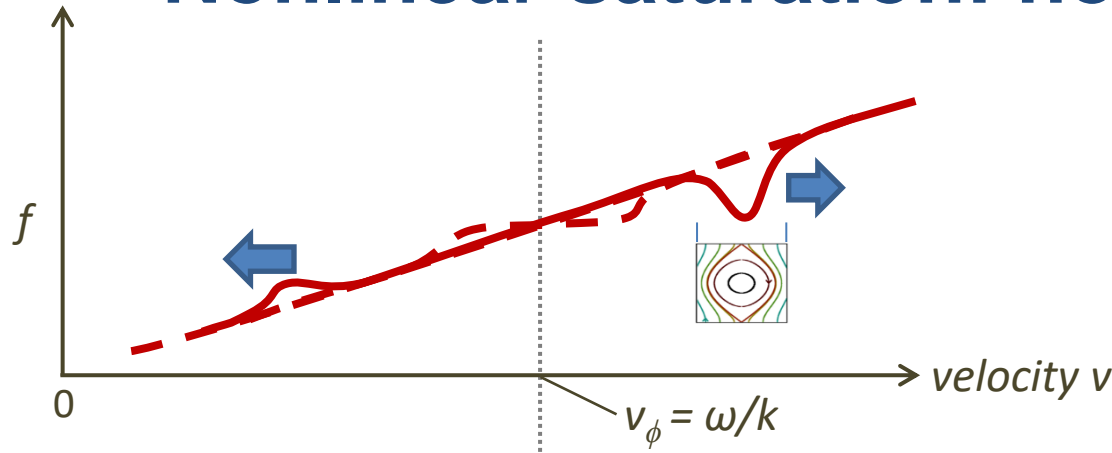
Case of a single mode, in a collisionless, closed system



Analytic theory: O'Neil '65

⇒ Island in phase-space (x,v) , consistent with a finite amplitude potential

Nonlinear saturation: holes and clumps

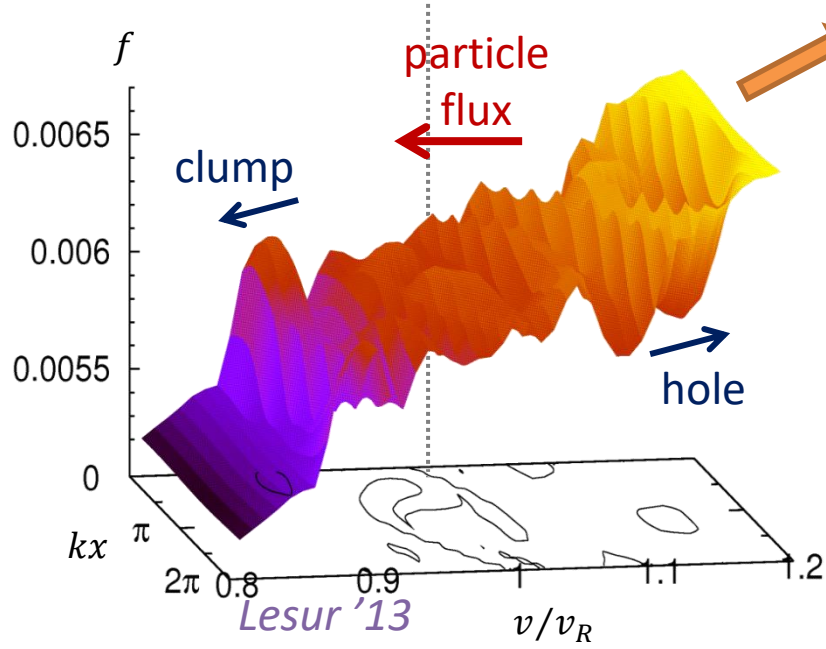


Case of a single mode, in a dissipative system, but weakly collisional

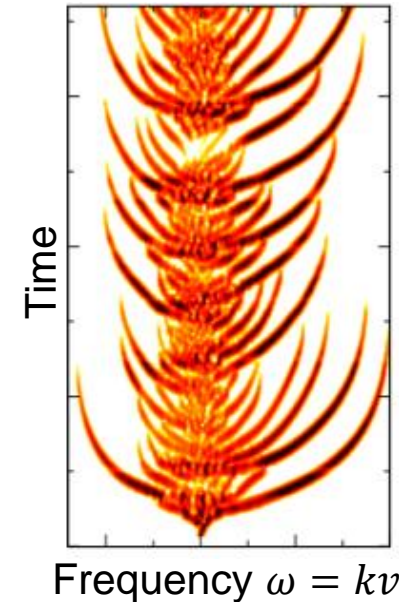
Schamel, '79

Dupree '82, Berman '82

Berk, Breizman, et al., '97



Nonlinear frequency sweeping (chirping)



\Rightarrow Chirping (fast frequency sweeping) = fast vortex dynamics

Current-driven ion-acoustic turbulence

1D model for ion-electron plasma with initial velocity drift

- 1D Vlasov for ions and electrons (collisionless)
+ Poisson equation

- Ion-acoustic modes

$$\omega \approx kc_s$$

- Linear instability threshold

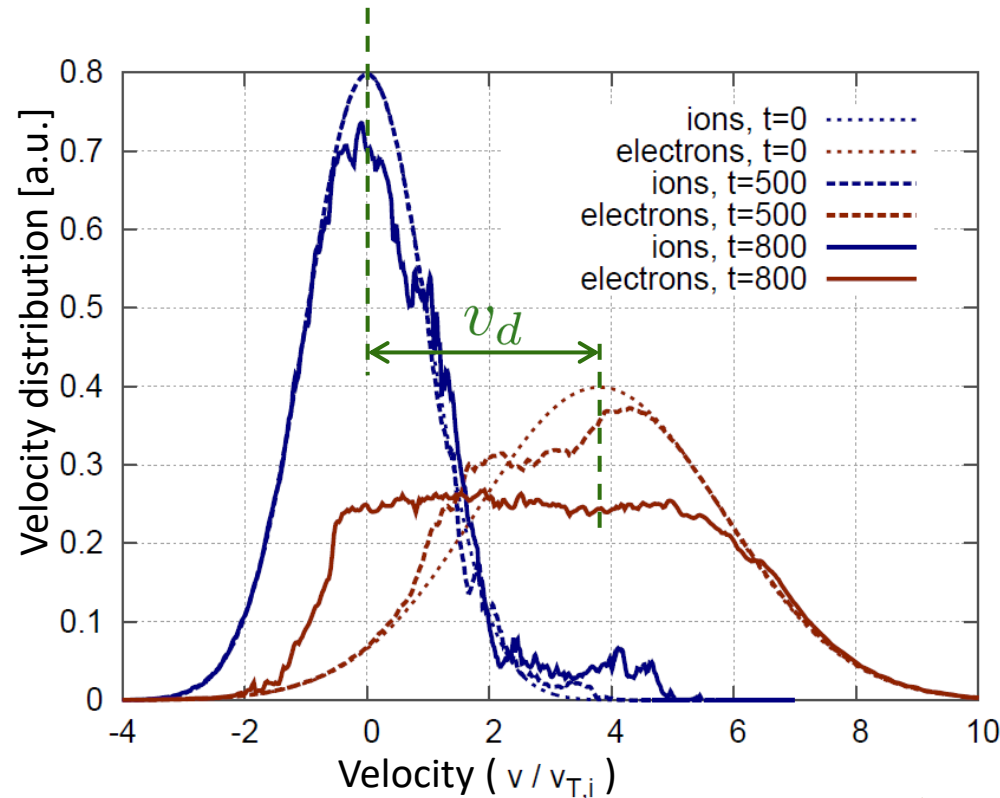
$$v_d > v_{d,cr}$$

Tonks & Langmuir '29

Revans '33

Velocity redistribution

- Anomalous resistivity
- Turbulent "heating"



Lesur '14

⇒ **Basic version of many types of fusion plasma turbulence**

Nonlinear growth of an electron hole

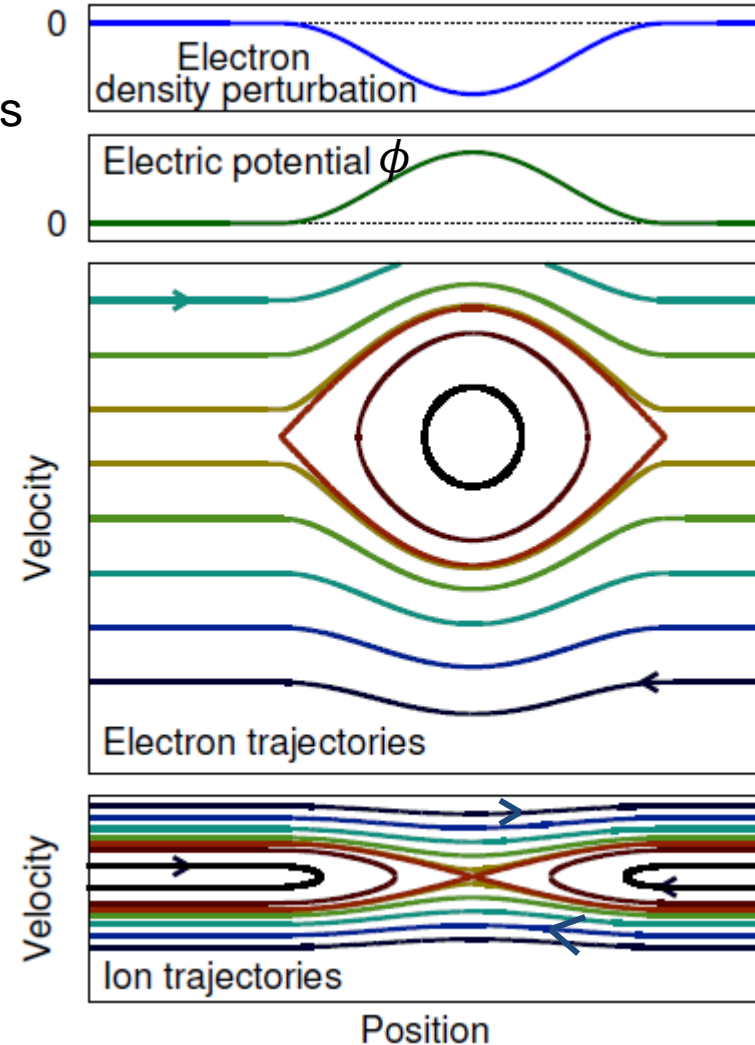
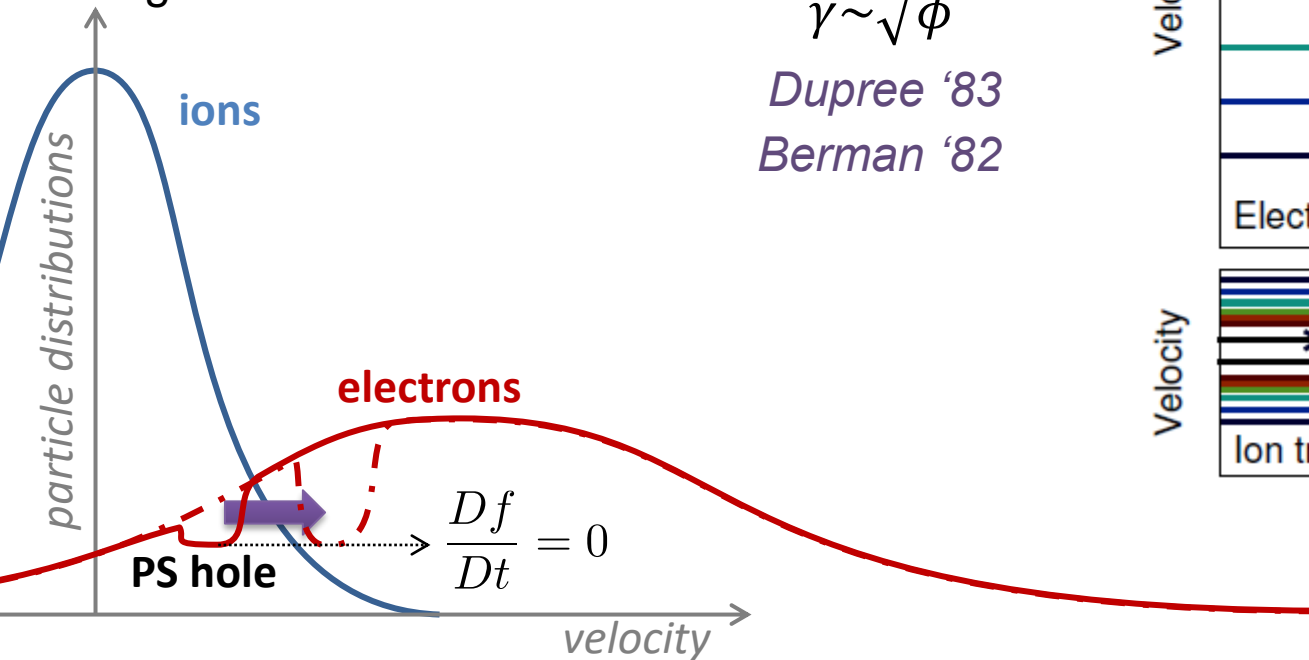
Growth mechanism

- Electron hole = positive charge \Rightarrow scatters ions
- Majority of ions encountering the structure **gain** momentum
- Mostly compensated by **lose of momentum of negative mass electron hole**
 \Rightarrow increase of structure velocity
- Lagrangian conservation of f yields Eulerian growth of δf

$$\gamma \sim \sqrt{\phi}$$

Dupree '83

Berman '82

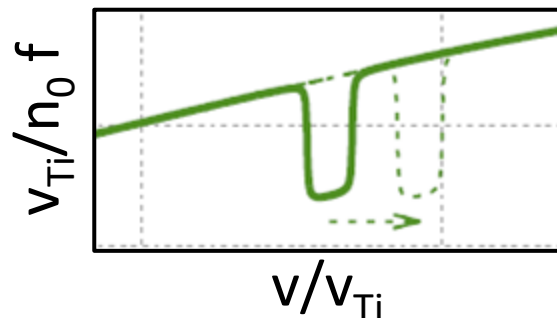


\Rightarrow **Nonlinear growth is due to dissipation (energy lost to ions)**

Analytic expression of nonlinear growthrate

Nonlinear growthrate of an isolated electron hole

- Based on simple model of “box hole” of width $\Delta x \times \Delta v$
- Relationship between growth and acceleration



$$\gamma^D \sim \partial_t \langle \tilde{f}^2 \rangle \sim \partial_t \phi \sim \partial_t |\delta f|_{\max} \sim a^D$$

- Lengthy calculation

Dupree '83, Tetreault '83

$$\Rightarrow \frac{\gamma^D}{\omega_{P,i}} = -2 \frac{\Delta v}{\Delta x} \frac{A}{(\lambda^{-2} + 4B(\partial_v f_{0,i}|_{v_h})^2)} \frac{\partial_v f_{0,e}|_{v_h}}{\partial_v f_{0,i}|_{v_h}} \sim \boxed{\sqrt{\phi_0} \partial_v f_e}$$

Screening length obtained from linear dispersion relation

\Rightarrow The goal here is to compare with numerical simulations

Numerical simulations of electron hole growth

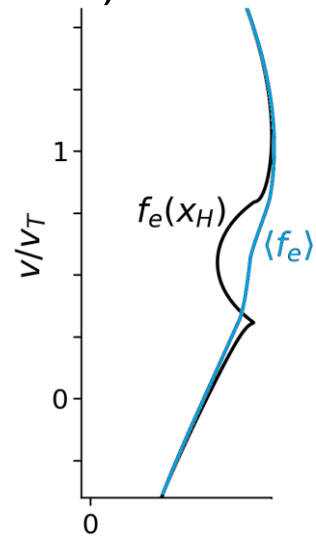
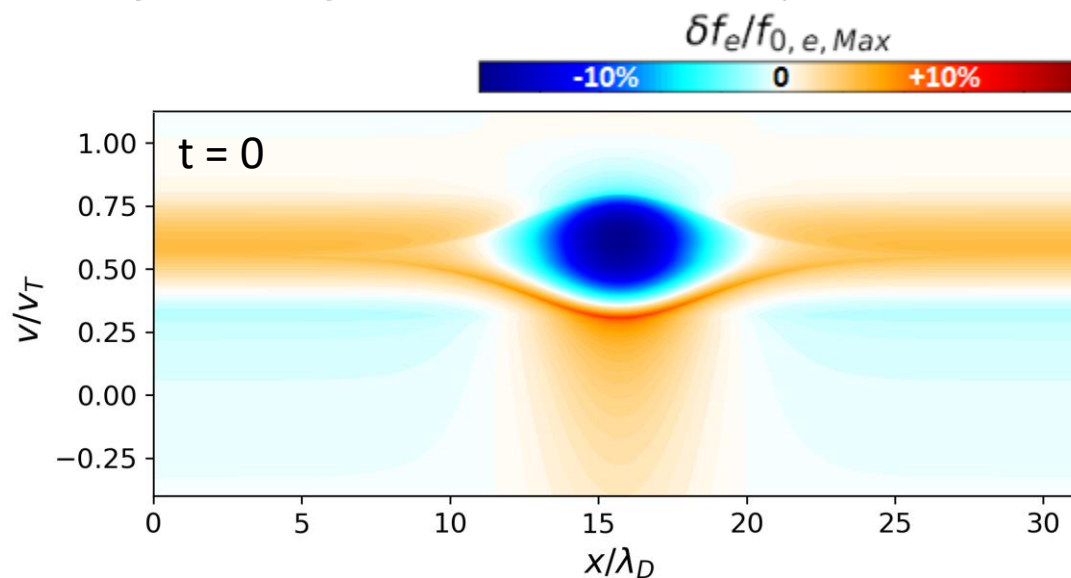
COBBLES code

Lesur '09

- 1D Vlasov for ions and electrons (collisionless) + Poisson equation
- Semi-Lagrangian
- Accurate treatment of phase-space filamentation (Liouville-theorem)

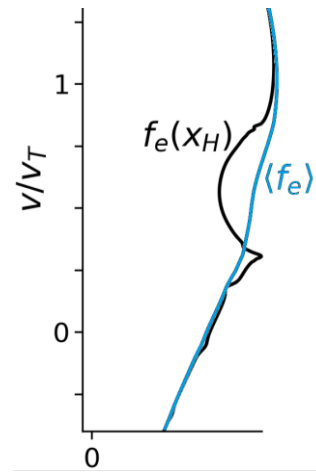
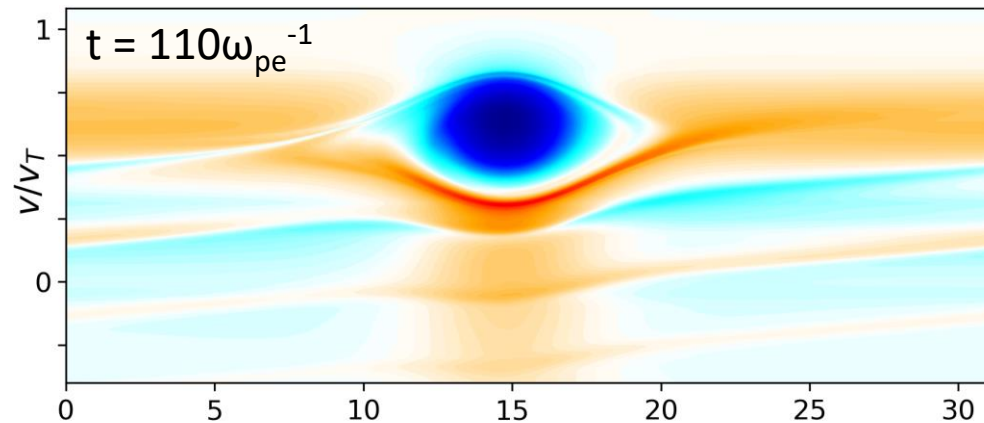
Initial setup

- Linearly stable conditions
- Prescribed initial electron hole



Time evolution

- Hole only slightly changing in shape
- Mainly accelerating



Prescribed initial electron hole

Special nonlinear solution

Schamel '71, Schamel '00

- Energy-dependent definition (trapping region & passing region)

$$f_{s,h}(x, v) = \frac{n_{s,0}}{v_{s,T} \sqrt{2\pi}} \exp \begin{cases} -\frac{1}{2} \left(\frac{\delta v_{s,h}}{v_{s,T}} \right)^2 - \beta_h \frac{E_s}{m_s v_{s,T}^2}, & \text{if } E_s \leq 0 \\ -\frac{1}{2v_{s,T}^2} \left(\sigma_v \sqrt{2 \frac{E_s}{m_s}} - \delta v_{s,h} \right)^2, & \text{if } E_s > 0 \end{cases}$$

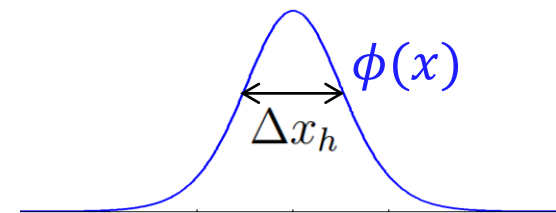
$$E_s(x, v) = \frac{1}{2} m_s \left(v - (v_{s,0} - \delta v_{s,h}) \right)^2 + q_s \phi(x - x_h)$$

$$\delta v_{s,h} = v_{s,0} - v_{h,0}$$

$$\sigma_v = \text{sgn} \left(v - \delta v_{s,h} \right)$$

- Consistent with

$$\phi(x) = \phi_0 \operatorname{sech}^4 \left(\frac{x - x_{h,0}}{\Delta x_h} \right)$$

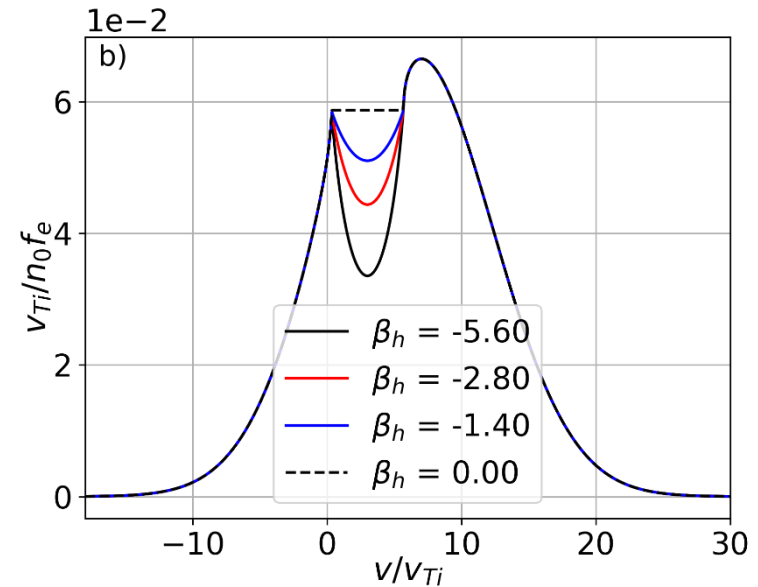


\Rightarrow 4 control parameters $(\phi_0, \beta_h, \delta v_{s,h}, \Delta x_h)$ - 2 independent

Two examples of hole parameters

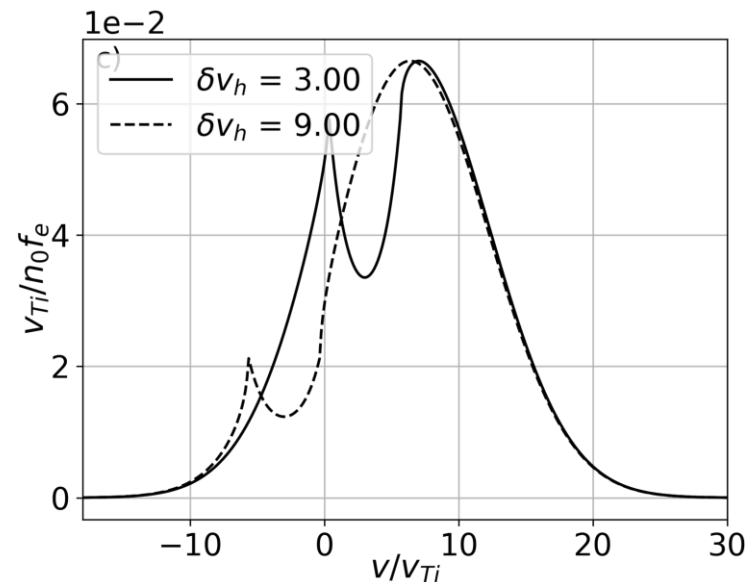
Trapping parameter (β_h)

- Negative for a hole
- Hole depth increases with increasing $|\beta_h|$ (all else being equal)



Hole shift (δv_h)

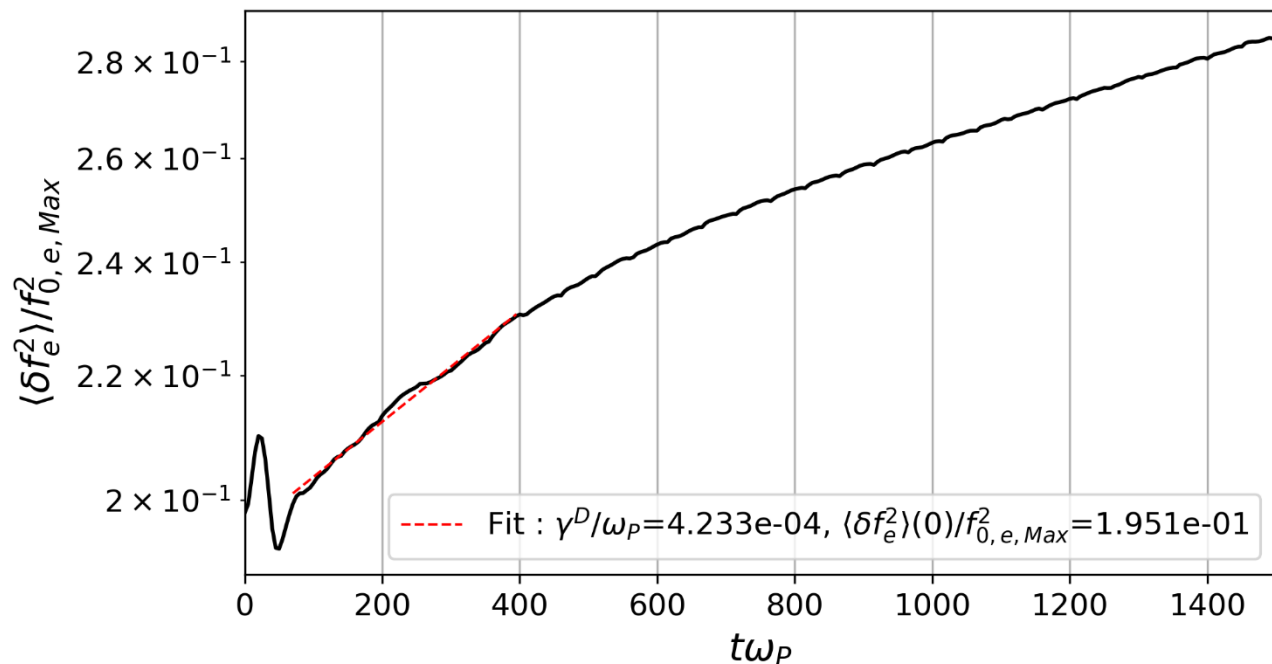
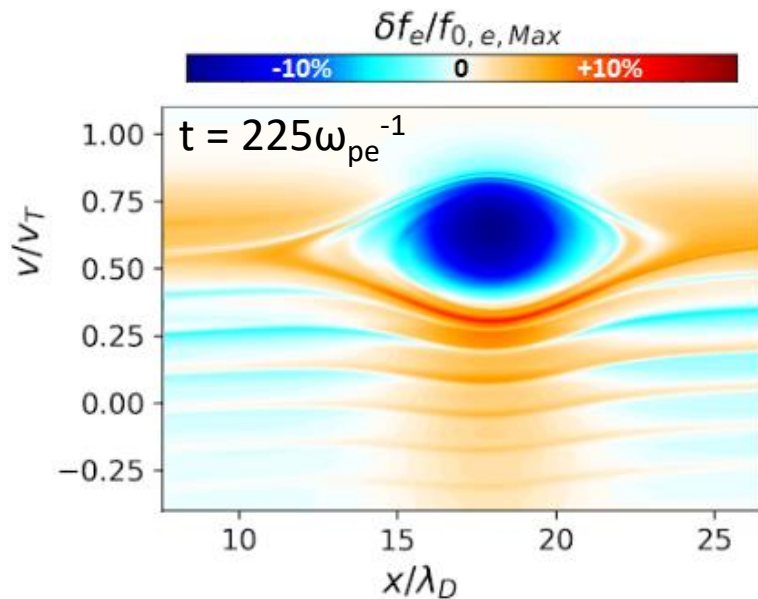
- Measures mean hole velocity, relative to electron frame-of-reference



Measuring the nonlinear growthrate

For each simulation:

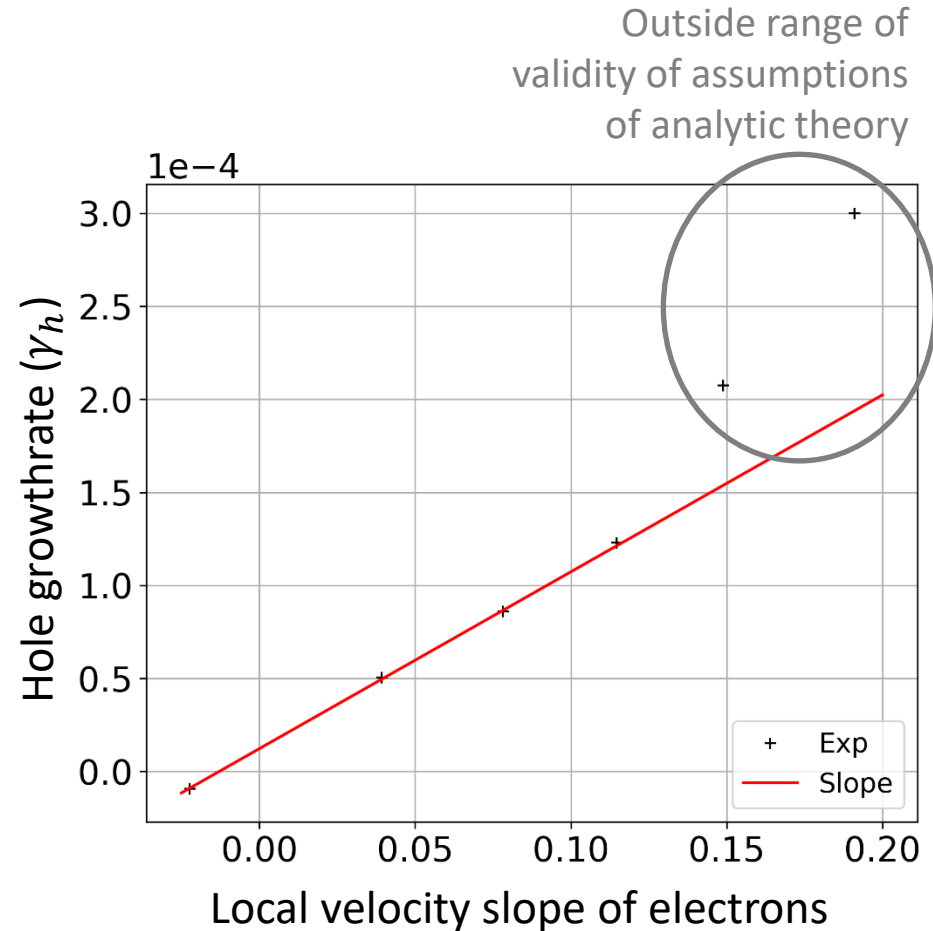
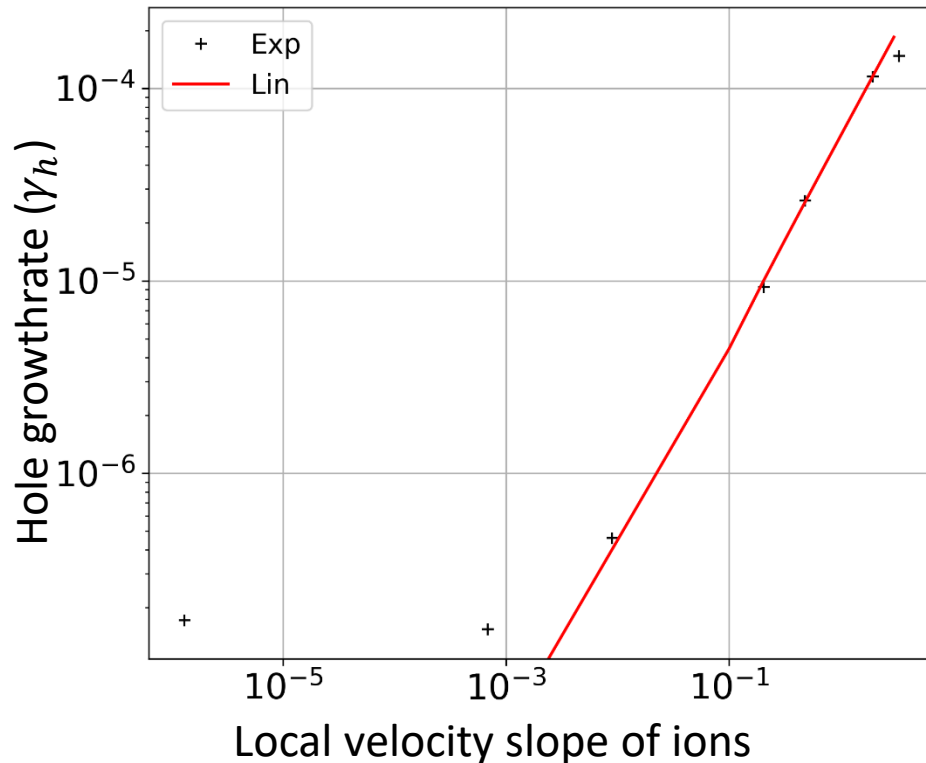
- Hole is tracked in numerical simulation
- Time evolution of hole parameters are extracted
- Fit on $\delta f^2(t)$ yields instantaneous value of nonlinear growthrate



Main dependencies of growthrate (1/2)

Against velocity slopes

- As expected, growthrate increases with both ion and electron velocity gradients
- Growthrate proportional to $\partial_v f_e$



⇒ **Qualitatively consistent with analytic theory**

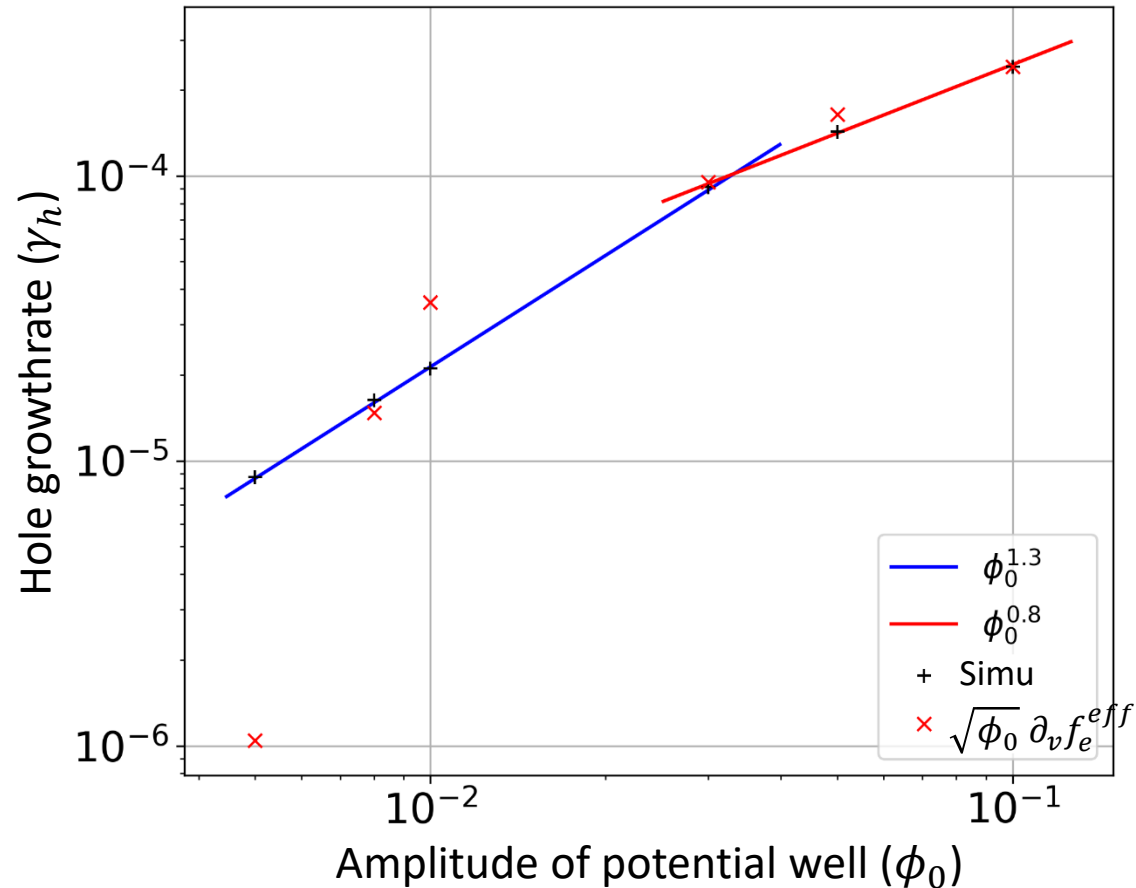
Main dependencies of growthrate (2/2)

Against amplitude of potential well

- Difficult to scan “all else being equal”
- We define an “effective” slope (spanned by the hole):

$$\partial_v f_e^{eff} = \frac{\iint \partial_v f_e(v) \delta f_e dx dv}{\iint \delta f_e dx dv}$$

- As amplitude is changed, effective slope varies significantly
- So the comparison is against the product $\sqrt{\phi_0} \partial_v f_e^{eff}$
- Another definition of $\partial_v f_e^{eff}$ may yield better agreement



⇒ Limited agreement with analytic theory, definition of $\partial_v f_e$ critical

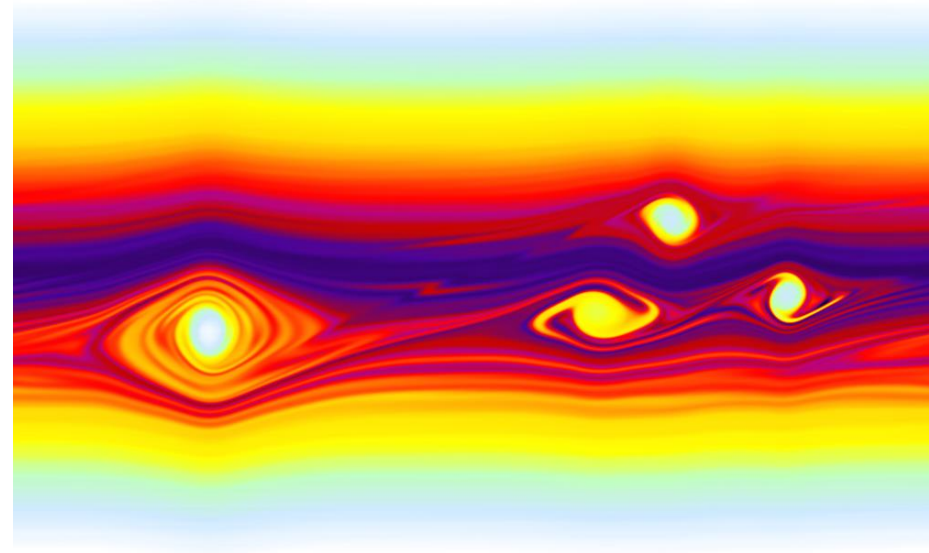
Summary & perspectives

Dynamics of an isolated electron hole in a collisionless plasma

- Nonlinear growth linked to hole acceleration
- Analytic theory predicts the nonlinear growth rate
- Qualitative agreement with numerical simulations, but correct definition of an effective local electron velocity slope is crucial

Perspectives: theoretical development of PS turbulence

- Dynamics of several PS vortices in interaction
- Statistical mechanics of many PS vortices



Acknowledgements



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Analogy with descriptions of turbulence in real space

Collection of waves

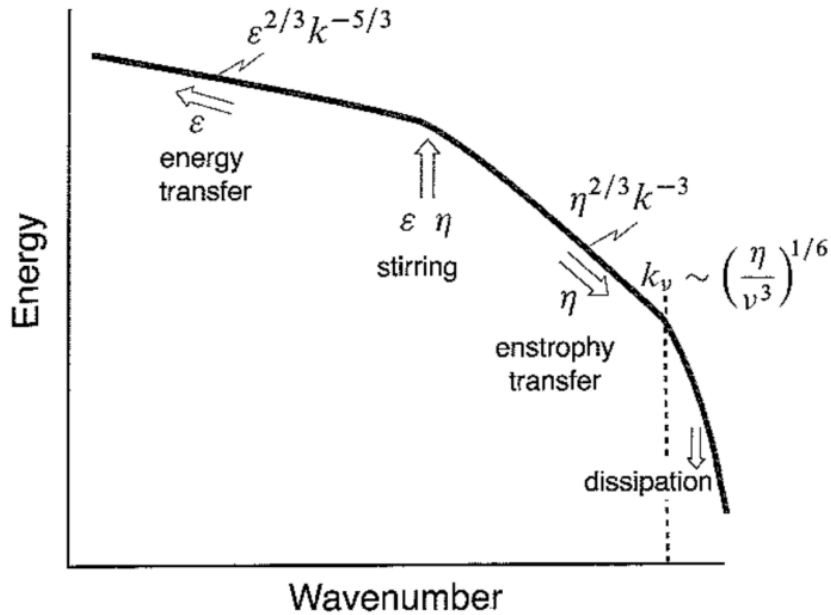
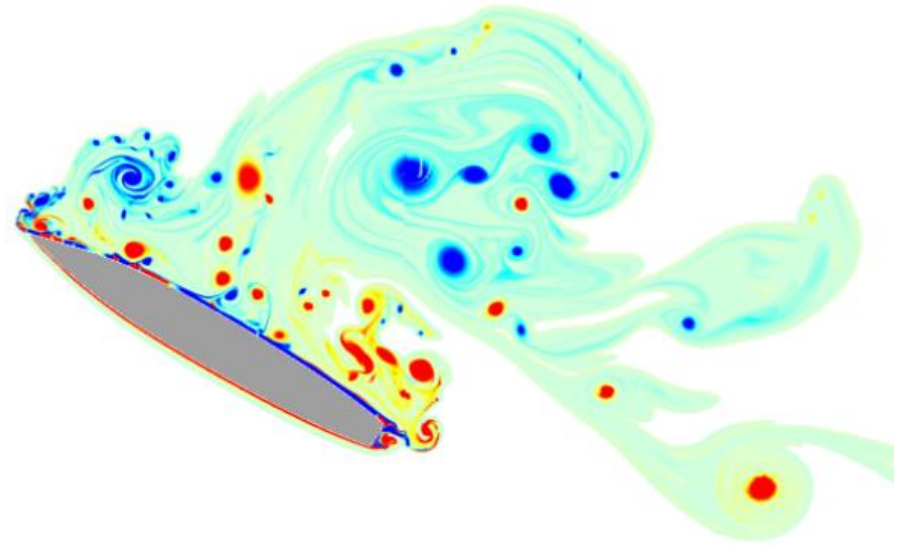


Illustration from Vallis '10

- Energy transfers?
- Phase dynamics?
- Intermittency?

⇒ Limited range
of applicability

Collection of vortices



Adapted from Rossi '14

⇒ **Reduction of
dimensionality**

- Reduced model for
smaller scales?