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THÈSE

en vue de l'obtention du titre de

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Spécialité INFORMATIQUE

présentée par

NGUYEN PHUONG ANH

Titre de la thèse :

LA PROGRAMMATION DC ET DCA POUR LA SÉCURITÉ DE
LA COUCHE PHYSIQUE DES RÉSEAUX SANS FIL

—
DC PROGRAMMING AND DCA FOR PHYSICAL LAYER
SECURITY IN WIRELESS NETWORK

soutenue le 30 avril 2020

Composition du Jury :

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THÈSE PRÉPARÉE AU SEIN DU LABORATOIRE D'INFORMATIQUE THÉORIQUE ET
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APPLICATIONS, LGIPM, UNIVERSITÉ DE LORRAINE, METZ, FRANCE

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- [2] Hoai An Le Thi, Phuong Anh Nguyen. *DC programming and DCA for Probabilistic Robust SWIPT Beamforming in Secure MISO Channel*. Submitted.

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- [1] Phuong Anh Nguyen, Hoai An Le Thi. *A DC Programming Approach for Worst-Case Secrecy Rate Maximization Problem*. In: Nguyen N., Pimenidis E., Khan Z., Trawinski B. (eds) *Computational Collective Intelligence. ICCCI 2018. Lecture Notes in Computer Science*, vol 11055, pp. 417–425, Springer, 2018.
- [2] Phuong Anh Nguyen, Hoai An Le Thi. *A DCA-Based Approach for Outage Constrained Robust Secure Power-Splitting SWIPT MISO System*. In: Le Thi H. A., Le H. M., Pham Dinh T. (eds) *Optimization of Complex Systems: Theory, Models, Algorithms and Applications. WCGO 2019. Advances in Intelligent Systems and Computing*, vol 991, pp. 289–298, Springer, 2019.

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- [1] Phuong Anh Nguyen, Hoai An Le Thi. *A DC Programming Approach for Worst-Case Secrecy Rate Maximization Problem*. Presentation in the 29th European Conference on Operational Research, Spain, July 8-11, 2018.

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Abbreviations and Notations

Throughout the dissertation, we use uppercase letters to denote matrices, and lowercase letters for vectors or scalars. Vectors are also regarded as matrices with one column. Some of the abbreviations and notations used in the dissertation are summarized as follows.

DC	Difference of Convex functions
DCA	DC Algorithm
SCA	Successive Convex Approximation
BS	Bisection search
1D	One-dimension line-search-based two-level optimization
AN	Artificial Noise
SINR	Signal-to-interference-plus-noise ratio
CSI	Channel State Information
WCSRM	Worst-case Secrecy rate Maximization
SWIPT	Simultaneous Wireless information and Power transfer
PLS	Physical layer Security
IR	Information receiver
ERs	Energy receivers
SRM	Secrecy rate Maximization
MISO	Multiple-input Single-output
\mathbb{R}	set of real numbers
\mathbb{R}^n	set of real column vectors of size n
$\overline{\mathbb{R}}$	set of extended real numbers, $\overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$
\mathbb{C}	set of complex numbers
\mathbb{C}^n	set of complex column vectors of size n
$\langle \cdot, \cdot \rangle$	scalar product, $\langle x, y \rangle = \sum_{i=1}^n x_i \cdot y_i$, $x, y \in \mathbb{R}^n$
$\chi_C(\cdot)$	indicator function of a set C , $\chi_C(x) = 0$ if $x \in C$, $+\infty$ otherwise
$\text{co}\{C\}$	convex hull of a set of points C
$\text{Proj}_C(x)$	projection of a vector x onto a set C
$\text{dom } f$	effective domain of a function f
$\nabla f(x)$	gradient of a function f at x
$\partial f(x)$	subdifferential of a function f at x
$\mathbb{H}^{N \times N}$	Hermitian matrix with size $N \times N$

\mathbf{A}^H	the Hermitian of matrix \mathbf{A}
$\text{Tr}(\mathbf{A})$	the trace of matrix \mathbf{A}
\mathbf{I}_K	the identity matrix of order K
$\ \mathbf{X}\ $	the Frobenius norm of matrix \mathbf{X}
$\mathbf{X} \otimes \mathbf{Y}$	the Kronecker product between matrices \mathbf{X} and \mathbf{Y}
$\text{vec}(\mathbf{X})$	the matrix vectorization of matrix \mathbf{X}
$\mathbf{X} \succeq \mathbf{Y}$	$\mathbf{X} - \mathbf{Y}$ is positive semi-definite matrix
$\log(\cdot)$	the logarithm function is taken to the base of 2
$x \sim \mathcal{CN}(\mu, \Omega)$	x is a random vector following a complex circular Gaussian distribution with mean μ and covariance Ω .
$\Re(x)$	the real part of complex number x
$[\mathbf{A}]_{(m,n)}$	the (m, n) th element of matrix \mathbf{A}
$\lambda(\mathbf{A})$	the minimum eigenvalue of matrix \mathbf{A} .

Résumé

La sécurité de la couche physique consiste à permettre la transmission des données confidentielles via un réseau sans fil en présence d'utilisateurs illégitimes, sans s'appuyer sur un cryptage de couche supérieure. L'essence de la sécurité de la couche physique est de maximiser le taux de secret, qui est le taux maximal d'informations sans interception par les espions. De plus, la conception de la sécurité de la couche physique prend en compte la minimisation de la puissance de transmission. Ces deux objectifs sont souvent en conflit l'un avec l'autre. Ainsi, la recherche sur les conceptions de sécurité de la couche physique se concentre souvent sur les deux principales classes de problèmes d'optimisation: maximiser le taux de secret sous contrainte de puissance de transmission et minimiser la puissance de transmission sous contrainte de taux de secret. Ces problèmes sont non convexes, donc difficiles à résoudre. Dans cette thèse, nous nous concentrons sur le développement des méthodes d'optimisation pour résoudre ces deux classes de problèmes d'optimisation. Nos méthodes sont basées sur la programmation DC (Difference of Convex functions) et DCA (DC Algorithm) étant reconnues comme des outils puissants d'optimisation non convexe.

Dans la première partie, nous considérons trois classes de problèmes de maximisation du taux de secret (chapitre 2, 3, 4). Le chapitre 2 étudie la transmission sécurisée des informations dans un système de relais MISO. Le relais utilise une combinaison de technique de formation de faisceau et de technique de bruit artificiel sous les modèles déterministes des canaux d'incertitude. Sans utiliser de relais, le chapitre 3 étudie le problème du transfert simultané d'information sans fil et de énergie dans un système sécurisé sous parfaite connaissance des canaux et imparfaite connaissance des canaux. Deux stratégies de transmission: formation de faisceaux avec bruit artificiel et sans bruit artificiel sont étudiés. Avec l'hypothèse du canal statistique des espions, chapitre 4 aborde le problème de maximisation du taux de secret sous la contrainte en probabilité dans un système SWIPT ("simultaneous wireless information and power transfer (SWIPT)" en anglais) multi-utilisateur. L'approche unifiée basée sur la programmation DC et DCA est proposée pour résoudre trois classes de problèmes d'optimisation. Le problème d'optimisation du chapitre 2 est reformulé comme deux programmes DC généraux. Les schémas DCA généraux sont proposés pour résoudre ces deux programmes DC. Dans le chapitre 3, nous considérons quatre problèmes d'optimisation conformément à quatre scénarios. Nous exploitons la structure particulière des problèmes considérés de les reformuler comme programmes DC généraux. Les schémas DCA généraux correspondants sont développés pour les

résoudre. Dans le chapitre 4, nous transformons d'abord le problème considéré en une forme traitable. Nous développons ensuite un algorithme alternatif pour résoudre le problème transformé. Deux programmes DC généraux sont proposés à chaque itération du schéma alternatif. Pour résoudre ces programmes DC, nous étudions une variante de DCA général, à savoir le schéma DCA_ρ . La convergence de l'algorithme proposé est rigoureusement prouvée.

La deuxième partie étudie le problème de minimisation de la puissance de transmission sous les contraintes de probabilité du taux de secret et de récolte l'énergie dans le réseau SWIPT (chapitre 5). Nous reformulons le problème d'origine comme trois programmes DC généraux pour lequel nous développons trois schémas DCA généraux correspondants. Les résultats numériques démontrent l'efficacité de les algorithmes proposés.

Abstract

Physical layer security is to enable confidential data transmission through wireless networks in the presence of illegitimate users, without basing on higher-layer encryption. The essence of physical layer security is to maximize the secrecy rate, that is the maximum rate of information without intercepted by the eavesdroppers. Besides, the design of physical layer security considers the transmit power minimization. These two objectives conflict with each other. Consequently, the research on physical layer security designs often focuses on the two main classes of optimization problems: maximizing secrecy rate under the transmit power constraint and minimizing power consumption while guaranteeing the secrecy rate constraint. These problems are nonconvex, thus, hard to solve. In this thesis, we focus on developing optimization approaches to solve these two optimization problem classes. Our methods are based on DC (Difference of Convex functions) programming and DCA (DC Algorithm) which well-known as one of the most powerful approaches in optimization.

In the first part, we consider three classes of secrecy rate maximization problems (chapters 2, 3, 4). In particular, chapter 2 studies the secure information transmission in a multiple-input single-output (MISO) relay system by using joint beamforming and artificial noise strategy under the deterministic uncertainty channel models of all links. Without using a relay, chapter 3 addresses the problem of transfer wireless information and power simultaneously in MISO secure system where scenarios of perfect channel state information and deterministic uncertainty channel models are concerned. Transmit beamforming without artificial noise and that with artificial noise are investigated. Under the assumption of statistical channel state information to eavesdroppers, chapter 4 studies the probability constrained secrecy rate maximization problem in multiuser MISO simultaneous wireless information and power transfer (SWIPT) system. The unified approach based on DC programming and DCA is proposed to solve three classes of optimization problems. The optimization problem in chapter 2 is recast as two general DC programs. The general DCA schemes are proposed to solve these two DC programs. In chapter 3, we consider four optimization problems in accordance with four scenarios. Exploiting the special structures of these original optimization problems, we transform it into four general DC programs for which the corresponding general DCA based algorithms are developed. In chapter 4, we first transform the considered problem into a tractable form. We then develop an alternating scheme to solve the transformed problem. Two general DC programs are proposed in each step of the alternating scheme. For solving these DC programs, we study a variant of general DCA, namely, $DCA_{-\rho}$ scheme. The convergence of alternating general $DCA_{-\rho}$ scheme is proven.

The second part studies the transmit power optimization problem under the probability constraints of secrecy rate and harvested energy in a MISO SWIPT system (chapter 5). We reformulate the original problem as three general DC programs for which the corresponding general DCA-based algorithms are investigated. Numerical results demonstrate the efficiency of the proposed algorithms.

Introduction générale

Cadre général et motivations

En raison du développement rapide de la technologie de communication sans fil et des équipements sans fil, les réseaux sans fil deviennent indispensables dans la vie quotidienne et professionnelle. Cependant, la nature du support de diffusion sans fil rend l'échange d'informations dans ces réseaux vulnérable aux attaques d'espionnage. Ainsi, la sécurité des réseaux sans fil est l'une des principales préoccupations durant ces dernières années. Dans le développement des réseaux de communication classiques, la confidentialité repose sur des techniques cryptographiques. Ces techniques sont basées sur les problèmes NP-Difficiles dont la résolution nécessite un effort considérable en calcul. L'augmentation rapide de la puissance de calcul et l'arrivée des ordinateurs quantiques sont des menaces pour les systèmes de cryptographie. Par exemple, le cryptogramme DES ("Data Encryption Standard" en anglais) a été brisé en seulement 56 heures en 1998 [57].

La sécurité de la couche physique ("physical layer security" en anglais) est un nouveau paradigme visant à sécuriser les communications entre les parties légitimes au niveau de la couche physique. Contrairement au paradigme cryptographique, la sécurité de la couche physique ne délimite pas une puissance de calcul spécifique des espions. La sécurité de la couche physique s'appuie sur les résultats novateurs développés par Wyner. H [90]. La performance de la sécurité de la couche physique est mesurée par le taux de secret ("secrecy rate" en anglais). Le taux de secret est défini comme la différence entre le débit de données du canal légitime et du canal d'espion [2]. Afin d'améliorer la sécurité de la transmission, il est important de maximiser le taux de secret. Cependant, le design des techniques de sécurité de la couche physique n'est pas simple, parce que la maximisation du taux de secret et la minimisation de la puissance de transmission ("transmit power" en anglais) ne sont pas compatibles et ne peuvent pas être satisfaits simultanément [5]. Il est donc impératif de trouver un bon équilibre entre la sécurité de la transmission de l'information et la puissance de transmission.

Le compromis entre la sécurité de la transmission de l'information et la puissance de transmission est formulé en deux classes de problèmes : dans la première, on maximise le taux de secret sous contraintes de la puissance de transmission tandis que dans la deuxième, on minimise la puissance de transmission sous contrainte de taux de secret. Dans cette thèse, nous nous concentrons sur l'utilisation de techniques d'optimisation

pour résoudre efficacement ces deux classes de problèmes. Pour la première, nous traitons les problèmes de maximisation du taux de secret sous contraintes de puissance de transmission pour plusieurs architectures de réseaux de communication sans fil. Tout au long de cette thèse, nous considérons que la communication peut être directe entre source et destination, ou passer par un relais de communication. Des modèles déterministes et statistiques de l'incertitude du canal sont considérés. Pour la deuxième classe de problèmes, nous étudions la minimisation de la puissance de transmission avec les contraintes de probabilités de panne du taux secret et de récolte d'énergie dans un système SWIPT ("simultaneous wireless information and power transfer (SWIPT)" en anglais). Ces problèmes sont non convexes, donc difficiles à résoudre. Ils appartiennent à une grande classe des problèmes nommés DC (Difference of Convex functions) généraux qui consistent à minimiser une fonction DC sous des contraintes DC.

La thèse a proposé une approche unifiée, fondée sur la programmation DC (Difference of Convex functions) et DCA (DC Algorithm) qui sont des outils puissants d'optimisation non convexe. Ces outils connaissent un grand succès, au cours des deux dernières décennies, dans la résolution de nombreux problèmes d'application dans divers domaines de sciences appliquées en général (voir par exemple [35, 34, 45, 46, 70, 71, 72, 41, 37, 43, 28, 23] et les références dans [41]), et des systèmes de communication en particulier (voir par exemple [86, 103, 85, 15, 1, 36, 39, 77, 78, 79, 40, 81, 44, 83, 82, 66, 65] et la liste des références dans [25]). De nombreuses expérimentations numériques réalisées dans cette thèse ont prouvé l'efficacité, la scalabilité, la rapidité des algorithmes proposés et leur supériorité par rapports aux méthodes standards.

La forme standard de la programmation DC s'écrit comme.

$$\alpha = \inf\{f(x) := g(x) - h(x) : x \in \mathbb{R}^n\} \quad (P_{dc}),$$

où g et h sont des fonctions convexes définies sur \mathbb{R}^n et à valeurs dans $\mathbb{R} \cup \{+\infty\}$, semi-continues inférieurement et propres. La fonction f est appelée fonction DC avec les composantes DC g et h , et $g - h$ est une décomposition DC de f . DCA est basé sur la dualité DC et des conditions d'optimalité locale. La construction de DCA implique les composantes DC g et h et non la fonction DC f elle-même. Chaque fonction DC admet une infinité des décompositions DC qui influencent considérablement sur la qualité (la rapidité, l'efficacité, la globalité de la solution obtenue ...) de DCA. Ainsi, au point de vue algorithmique, la recherche d'une "bonne" décomposition DC et d'un "bon" point initial est très importante dans le développement de DCA pour la résolution d'un programme DC.

L'utilisation de la programmation DC et DCA dans cette thèse est justifiée par de multiple arguments [72]:

- La programmation DC et DCA fournissent un cadre très riche pour les problèmes d'apprentissage automatique et fouille de données: l'apprentissage automatique et fouille de données constituent *une mine des programmes DC* dont la résolution appropriée devrait recourir à la programmation DC et DCA. En effet la liste

- indicative (non exhaustive) des références dans [25] témoigne de la vitalité la puissance et la percée de cette approche dans la communauté d'apprentissage automatique et fouille de données.
- DCA est une philosophie plutôt qu'un algorithme. Pour chaque problème, nous pouvons concevoir une famille d'algorithmes basés sur DCA. La flexibilité de DCA sur le choix des décomposition DC peut offrir des schémas DCA plus performants que des méthodes standard.
 - L'analyse convexe fournit des outils puissants pour prouver la convergence de DCA dans un cadre général. Ainsi tous les algorithmes basés sur DCA bénéficient (au moins) des propriétés de convergence générales du schéma DCA générique qui ont été démontrées.
 - DCA est une méthode efficace, rapide et scalable pour la programmation non convexe. A notre connaissance, DCA est l'un des rares algorithmes de la programmation non convexe, non différentiable qui peut résoudre des programmes DC de très grande dimension.

Il est important de noter qu'avec les techniques de reformulation en programmation DC et les décompositions DC appropriées, on peut retrouver la plupart des algorithmes existants en programmation convexe/non convexe comme cas particuliers de DCA.

Nos contributions

Les principales contributions de la thèse consistent à développer les techniques d'optimisation non convexe pour résoudre deux classes de problèmes en sécurité de la couche physique des réseaux sans fil: maximiser le taux de secret et minimiser la puissance de transmission. Tous ces problèmes peuvent être formulés comme la minimisation d'une fonction DC sous des contraintes DC (problème DC général). Nous étudions la programmation DC et DCA pour leur résolution. Sur l'idée de base de DCA, le schéma DCA pour un problème DC général [28, 41] consiste à approcher, à chaque itération, la fonction objectif DC (resp. des contraintes DC) par une fonction convexe (resp. des contraintes convexes) puis résoudre le problème convexe résultant.

Les difficultés rencontrées sont multiples dans cette étude. Tout d'abord, intuitivement, un problème DC général est beaucoup plus difficile qu'un problème DC standard car la non convexité représente à la fois en fonction objectif et contraintes. Certaines contraintes de ces problèmes sont fractionnaires. Par ailleurs, quelques problèmes contiennent des modèles de canaux d'incertitude ou des contraintes de probabilité. Ces types de contraintes sont très difficiles à traiter. De plus, l'approximation une contrainte DC par une contrainte convexe peut impliquer l'infaisabilité du sous-problème convexe résultant. Tenant compte de la structure spécifique du problème, nous reformulons les problèmes considérés comme des programmes DC et développons des algorithmes basés sur DCA pour leur résolution. De plus, une variante de DCA (ADCA- ρ) est développée au chapitre 4. Tout au long de la thèse les questions suivantes sont soigneusement étudiées : i) quelle est la formulation convenable pour rendre le problème moins difficile? ii) quelle est la "bonne" décomposition DC (celle qui correspond à un schéma DCA simple et efficace dont la résolution du sous-problème convexe

est non coûteuse)? iii) comment aborder l'infaisabilité du sous-problème convexe suite à l'approximation des contraintes DC? Nous donnons ci-après une description détaillée des nos contributions.

Dans le premier temps, nous étudions le problème de maximisation du taux de secret avec la contrainte de la puissance de transmission avec relais (chapitre 2). Le relais utilise une combinaison de technique de formation de faisceau ("beamforming" en anglais) et de technique de bruit artificiel ("artificial noise" en anglais). Ce problème a été formulé dans [95] (appelé problème WCSRM) et très difficile à résoudre directement. Zhang et al. [95] a transformé le problème WCSRM en une formulation traitable. Le problème transformé a $M + 1$ (M - le nombre des espions) contraintes DC fractionnaires. Les auteurs ont converti ces contraintes fractionnaires en $2M + 2$ contraintes DC exponentielles, où les termes exponentiels représentent dans les deux composants DC (g et h). Le schéma SCA ("successive convex approximation (SCA)" en anglais) a été étudié pour résoudre ce programme DC. Dans le premier temps, nous proposons un nouveau programme DC pour le problème WCSRM transformé dans [95]. Nous reformulons les $M + 1$ contraintes DC fractionnaires en $M + 1$ contraintes DC, où la première composante DC (fonction g) est quadratique et la deuxième composante DC (fonction h) est exponentielle. Le schéma DCA₁ est développé pour résoudre le premier programme DC. L'avantage de ce programme DC est qu'il réduit à la moitié du nombre des contraintes exponentielles. De plus, les sous-problèmes de DCA₁ contiennent des contraintes quadratiques tandis que celui de SCA contient des contraintes exponentielles. Par conséquent, DCA₁ devrait surpasser SCA. Ensuite, nous proposons une autre formulation DC du problème WCSRM, résulte en DCA₂. La S-procédure [95] est appliquée pour contourner les modèles de canaux d'incertitude. L'avantage de cette formulations est qu'il n'y a que des contraintes quadratiques dans cette formulation DC. Les résultats d'expérimentations montrent que DCA₂ est beaucoup plus efficace que DCA₁ et nos approches surpassent SCA à la fois sur le temps de calcul et le taux de secret.

Dans le chapitre 3, nous traitons les problèmes de maximisation du taux de secret dans les réseaux SWIPT sans utilisation de relais. Les quatre scénarios suivants sont étudiés: la technique de formation de faisceau sans bruit artificiel et avec bruit artificiel sous CSI ("channel state information (CSI)" en anglais) parfait et imparfait. Ces scénarios conduisent aux quatre problèmes d'optimisation qui maximisent le taux de secret avec les contraintes de la puissance de transmission et de récolte d'énergie ("harvested energy" en anglais). Les méthodes de résolution existantes sont basées sur 1D ("one dimensional line search based two level optimization (1D)" en anglais) et SCA. Les approches 1D décomposent les problèmes en deux niveaux: les problèmes de niveau extérieur sont traités par la recherche linéaire tandis que les problèmes de niveau interne sont résolus par la technique de relaxation semidéfinie ("semidefinite relaxation" en anglais). Puisque les approches 1D sont compliquées avec deux niveaux, les auteurs ont proposé des approches SCA pour résoudre les problèmes considérés. Cependant, les sous-problèmes des schémas SCA contiennent des contraintes exponentielles. Nous exploitons d'abord la structure particulière des problèmes considérés pour les reformuler comme programmes DC généraux mais sans les contraintes DC exponentielles. La S-procédure est appliquée pour contourner les modèles de canaux

d'incertitude. Ensuite, les schémas DCA généraux sont développés pour les résoudre. Ces schémas amènent à des sous-problèmes simples avec des contraintes quadratiques. Nous étudions la technique de relaxation en ajoutant une variable pour contourner l'infaisabilité des sous-problèmes convexes dans les schémas DCA, qui est causée par la relaxation convexe des contraintes DC. Les expérimentations montrent que les taux de secret et les temps de calcul obtenus par les algorithmes DCA sont considérablement meilleurs que ceux obtenus par les méthodes existantes.

Dans le chapitre 4, nous étudions le problème de maximisation du taux de secret sous contrainte de probabilité dans un système multi-utilisateur avec l'hypothèse du canal statistique d'espions. L'objectif est de maximiser le taux de secret avec les contraintes de la puissance de transmission, de récolte d'énergie, et la contrainte de probabilité du taux de secret. Le problème est non convexe et intraitable en raison de l'incertitude du canal et de la contrainte de probabilité. Nous reformulons d'abord le problème considéré par techniques de relaxation semi-définie et de S-procédure. Le problème résultant reste difficile si on optimise simultanément toutes les variables. Une idée naturelle est de découpler les variables en deux sous-ensembles. Nous développons un algorithme alternatif: fixer un sous-ensemble des variables à chaque itération alternativement, la solution optimale du problème par rapport à l'autre sous-ensemble variable est calculée par DCA- ρ général. DCA- ρ général est une extension de DCA général. Généralement, la formulation DC nécessite le calcul d'un paramètre ρ pour garantir les composants DC convexes (g et h). En pratique, ρ est généralement estimée par une valeur assez élevée qui pourrait rendre le DCA général inefficace. Ainsi, nous proposons une variante de DCA général (DCA- ρ général), dans laquelle une mise à jour de ρ est effectuée à chaque itération et la convexité de H (la seconde fonction dans la décomposition) n'est pas nécessaire. Nous prouvons que l'algorithme proposé converge vers un point KKT (The Karush-Kuhn-Tucker). Les performances des algorithmes proposés sont soigneusement examinées en les comparant avec la méthode existante (SCA).

Dans le chapitre 5, nous formulons le problème de minimisation de la puissance de transmission avec les hypothèses de canaux stochastiques dans le réseau SWIPT. Nous étudions un scénario pratique dans lequel les espions ne sont pas des utilisateurs du système. Plus précisément, l'objectif est de minimiser la puissance de transmission avec les contraintes de probabilité du taux secret et de récolte d'énergie. Nous transformons les contraintes probabilistes en contraintes déterministes par appliquant les inéquations de Bernstein [92, 13]. La reformulation reste difficile en raison de la non convexité. Nous reformulons le problème transformé déterministe comme un DC général et proposons DCA₁ pour sa résolution. Ce premier problème DC contient des contraintes DC exponentielles qui devraient être remplacées par des contraintes plus simples. C'est ainsi que nous proposons une autre formulation DC pour le problème transformé déterministe, qui contient des contraintes quadratiques. Le schéma DCA₂ est développé pour résoudre ce deuxième problème DC. Pour exploiter l'effet de décomposition DC, nous étudions une autre décomposition DC pour ce dernier et développons le schéma DCA₃ pour sa résolution. Comme dans le chapitre précédent, la technique de relaxation est étudiée pour contourner l'infaisabilité des sous-problèmes dans les schémas DCA. Les résultats numériques indiquent que nos approches sont prometteuses. En outre,

DCA_2 et DCA_3 sont beaucoup plus efficaces que DCA_1 en le temps de calcul et la qualité tandis que DCA_3 est meilleur que DCA_2 sur l'optimisation de la puissance de transmission.

Organisation de la Thèse

La thèse est composée de cinq chapitres. Le chapitre 1 présente les concepts et les résultats fondamentaux en analyse convexe ainsi que la programmation DC et DCA, qui sont la base théorique et algorithmique pour les autres chapitres. Les quatre chapitres suivants sont divisés en deux parties : la première partie (chapitres 2, 3 et 4) aborde le problème de maximisation du taux de secret dans trois diverses architectures du réseau sans fil. Plus précisément, dans le chapitre 2, nous traitons le problème de maximisation du taux de secret en utilisant la technique de relais et la technique de formation de faisceaux avec bruit artificiel sous l'erreur d'estimation déterministe du canal. Le chapitre 3 concerne les problèmes de maximisation du taux de secret du système SWIPT sans relais. Considérant l'erreur d'estimation du canal aléatoire, le chapitre 4 étudie le problème de maximisation du taux de secret sous contraintes de probabilité. Ensuite, dans la deuxième partie (chapitre 5), nous étudions le problème de minimisation de la puissance de transmission sous les contraintes de probabilité du taux de secret et énergie récoltée.

Chapter 1

Methodology

DC programming and DCA, which constitute the backbone of nonconvex programming and global optimization, were introduced by Pham Dinh Tao in their preliminary form in 1985 [69]. Important developments and improvements on both theoretical and computational aspects have been completed since 1994 throughout the joint works of Le Thi Hoai An and Pham Dinh Tao. The readers are referred to the seminal paper [41] for the thirty years of developments of DCA.

In this chapter, we present some basic properties of convex analysis, DC optimization, and DC Algorithms that computational methods of this dissertation are based on. The materials of this chapter are extracted from [38, 70, 24, 23].

1.1 Fundamental convex analysis

We first recall some notions and results in convex analysis related to the dissertation (refer to the references [3, 70, 75] for more details). Let denote X the Euclidean space \mathbb{R}^n and $\overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$ is the set of extended real numbers.

A subset C of X is said to be *convex* if $(1 - \lambda)x + \lambda y \in C$ for any $x, y \in C$ and any $\lambda \in [0, 1]$.

The *convex hull* of a set C , denoted by $\text{conv}C$ is the set of all convex combinations of points in C .

Let C be a convex set. A function $f : C \rightarrow (-\infty, +\infty]$ is said to be *convex* on C if

$$f((1 - \lambda)x + \lambda y) \leq (1 - \lambda)f(x) + \lambda f(y), \quad \forall x, y \in C, \forall \lambda \in [0, 1].$$

A real-value function f on a convex set C is said to be *strictly convex* on C if the inequality above holds strictly whenever $x \neq y$ and $0 < \lambda < 1$.

The *effective domain* of a convex function f on C , denoted by $\text{dom}f$, is the set

$$\text{dom}f = \{x \in X : f(x) < +\infty\}$$

Obviously, $\text{dom } f$ is a convex set in X .

The convex function f is called *proper* if $\text{dom } f \neq \emptyset$ and $f(x) > -\infty$ for all x .

The function $f : C \rightarrow (-\infty, +\infty]$ is said to be *lower semi-continuous* at a point x of C if

$$f(x) \leq \liminf_{y \rightarrow x} f(y).$$

Denote by $\Gamma_0(X)$ the set of all proper lower semi-continuous convex functions on X .

Let ρ be a nonnegative number and C be a convex subset of X . A function $\theta : C \rightarrow (-\infty, +\infty]$ is ρ -convex if

$$\theta[\lambda x + (1 - \lambda)y] \leq \lambda\theta(x) + (1 - \lambda)\theta(y) - \frac{\lambda(1 - \lambda)}{2}\rho\|x - y\|^2$$

for all $x, y \in C$ and $\lambda \in (0, 1)$.

It is easy to see that $\theta - (\rho/2)\|\cdot\|^2$ is convex on C .

The modulus of strong convexity of θ on C , denoted by $\rho(\theta, C)$ or $\rho(\theta)$ if $C = X$, is given by

$$\rho(\theta, C) = \sup\{\rho \geq 0 : \theta - (\rho/2)\|\cdot\|^2 \text{ is convex on } C\}.$$

θ is *strongly convex* on C if $\rho(\theta, C) > 0$.

A vector y is said to be a *subgradient* of a convex function f at a point x^0 if

$$f(x) \geq f(x^0) + \langle x - x^0, y \rangle, \quad \forall x \in X.$$

The set of all subgradients of f at x^0 is called the *subdifferential* of f at x^0 and is denoted by $\partial f(x^0)$. If $\partial f(x)$ is not empty, f is said to be *subdifferentiable* at x .

For $\varepsilon > 0$, a vector y is said to be an ε -*subgradient* of a convex function f at a point x^0 if

$$f(x) \geq (f(x^0) - \varepsilon) + \langle x - x^0, y \rangle, \quad \forall x \in X.$$

The set of all ε -subgradients of f at x^0 is called the ε -*subdifferential* of f at x^0 and is denoted by $\partial_\varepsilon f(x^0)$.

Let us describe two basic notations as follows.

$$\text{dom } \partial f = \{x \in X : \partial f(x) \neq \emptyset\} \quad \text{and} \quad \text{range } \partial f = \cup\{\partial f(x) : x \in \text{dom } \partial f\}.$$

Proposition 1.1. *Let f be a proper convex function. Then*

1. $\partial_\varepsilon f(x)$ is a closed convex set, for any $x \in X$ and $\varepsilon \geq 0$.
2. $\text{ri}(\text{dom } f) \subset \text{dom } \partial f \subset \text{dom } f$
where $\text{ri}(\text{dom } f)$ stands for the relative interior of $\text{dom } f$.
3. If f is differentiable at $x \in \text{dom } f$, then $\partial f(x) = \{\nabla f(x)\}$.
4. $x_0 \in \text{argmin}\{f(x) : x \in X\}$ if and only if $0 \in \partial f(x_0)$.

Let C be a nonempty convex subset of \mathbb{R}^n . The *indicator function* of C , denoted by χ_C , is the function

$$\chi_C(x) = \begin{cases} 0 & \text{if } x \in C, \\ +\infty & \text{otherwise} \end{cases}$$

The *normal cone* of C at $x \in C$, denoted $N_C(x)$, is given by

$$N_C(x) = \partial\chi_C(x) = \{u \in \mathbb{R}^n : \langle u, y - x \rangle \leq 0 \forall y \in C\}.$$

A function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be *Lipschitz continuous* on C if there exists a real number $\lambda \geq 0$ such that

$$\|f(x_1) - f(x_2)\| \leq \lambda \|x_1 - x_2\| \quad \forall x_1, x_2 \in C.$$

Such a number λ is called a Lipschitz constant of f on C .

A function $f : \mathbb{R}^n \rightarrow (-\infty, +\infty]$ is said to be *locally Lipschitz* at $x \in \mathbb{R}^n$ if there exists a neighborhood U_x of x such that f is Lipschitz continuous on U_x .

Let $f : \mathbb{R}^n \rightarrow (-\infty, +\infty]$ be a locally Lipschitz function at a given $x \in \mathbb{R}^n$. The *Clarke directional derivative* and the *Clarke subdifferential* of f at x is given by the following formulas.

$$f^\uparrow(x, v) = \limsup_{(t, y) \rightarrow (0^+, x)} \frac{f(y + tv) - f(y)}{t},$$

$$\partial^\uparrow f(x) = \{x^* \in \mathbb{R}^n : \langle x^*, v \rangle \leq f^\uparrow(x, v) \quad \forall v \in \mathbb{R}^n\}.$$

If f is continuously differentiable at x then $\partial^\uparrow f(x) = \nabla f(x)$. When f is a convex function, then $\partial^\uparrow f(x)$ coincides with the subdifferential $\partial f(x)$.

Conjugates of convex functions

The *conjugate* of a function $f : X \rightarrow \overline{\mathbb{R}}$ is the function $f^* : X \rightarrow \overline{\mathbb{R}}$, defined by

$$f^*(y) = \sup_{x \in X} \{\langle x, y \rangle - f(x)\}.$$

Proposition 1.2. *Let $f \in \Gamma_0(X)$. Then we have*

1. $f^* \in \Gamma_0(X)$ and $f^{**} = f$.
2. $f(x) + f^*(y) \geq \langle x, y \rangle$, for any $x, y \in X$.
3. $f(x) + f^*(y) = \langle x, y \rangle \Leftrightarrow y \in \partial f(x) \Leftrightarrow x \in \partial f^*(y)$.

Polyhedral Functions

A *polyhedral* set is a closed convex set that is of form

$$C = \{x \in X : \langle b_i, x \rangle \leq \beta_i, \quad \forall i = 1, \dots, m\}$$

where $b_i \in X$ and $\beta_i \in \mathbb{R}$ for all $i = 1, \dots, m$.

A function $f \in \Gamma_0(X)$ is said to be *polyhedral* if

$$f(x) = \max\{\langle a_i, x \rangle - \alpha_i : i = 1, \dots, k\} + \chi_C(x), \quad \forall x \in X \quad (1.1)$$

where $a_i \in X$, $\alpha_i \in \mathbb{R}$ for all $i = 1, \dots, k$ and C is a nonempty polyhedral set. It is clear that $\text{dom } f = C$.

Proposition 1.3. [12] *Let f be a polyhedral convex function, and $x \in \text{dom } f$. Then we have*

1. *f is subdifferentiable at x , and $\partial f(x)$ is a polyhedral convex set. In particular, if f is defined by (1.1) with $C = X$ then*

$$\partial f(x) = \text{co}\{a_i : i \in I(x)\}$$

where $I(x) = \{i \in \{1, \dots, k\} : \langle a_i, x \rangle - \alpha_i = f(x)\}$.

2. *The conjugate f^* is a polyhedral convex function. Moreover, if $C = X$ then*

$$\begin{aligned} \text{dom } f^* &= \text{co}\{a_i : i = 1, \dots, k\}, \\ f^*(y) &= \inf \left\{ \sum_{i=1}^k \lambda_i \alpha_i : \sum_{i=1}^k \lambda_i a_i = y, \sum_{i=1}^k \lambda_i = 1, \lambda_i \geq 0, \forall i = 1, \dots, k \right\}. \end{aligned}$$

In particular,

$$f^*(a_i) = \alpha_i, \quad \forall i = 1, \dots, k.$$

DC functions

A function f is called *DC function* on X if it is of the form

$$f(x) = g(x) - h(x), \quad x \in X$$

where g and h belong to $\Gamma_0(X)$. One says that $g - h$ is a *DC decomposition* of f and the functions g , h are its *DC components*. If, in addition, g and h are finite at all points of X then f is said to be finite DC function on X . The set of DC functions (resp. finite DC functions) on X is denoted by $\mathcal{DC}(X)$ (resp. $\mathcal{DC}_f(X)$).

Remark 1.1. *If f is a DC function with DC decomposition $f = g - h$ then for every $\theta \in \Gamma_0(X)$ finite on X , $f = (g + \theta) - (h + \theta)$ is another DC decomposition of f . Thus, a DC function f has infinitely many DC decompositions.*

1.2 DC Programming and DCA

In this section, we briefly point out the main results in the standard and general DC optimization.

1.2.1 Standard DC program and Standard DCA

Standard DC program

In the sequel, we use the convention $+\infty - (+\infty) = +\infty$.

For $g, h \in \Gamma_0(X)$, a standard *DC program* is of the form

$$(P) \quad \alpha = \inf\{f(x) = g(x) - h(x) : x \in X\}.$$

Remark 1.2. *Let C be a nonempty closed convex set. Then, the constrained problem*

$$\inf\{f(x) = g(x) - h(x) : x \in C\}$$

can be transformed into an unconstrained DC program by adding the indicator function χ_C of C to the first DC component, i.e.

$$\inf\{f(x) = g(x) + \chi_C(x) - h(x) : x \in X\}$$

The dual program of (P) is also a DC program with the same optimal value and defined by

$$(D) \quad \alpha = \inf\{h^*(y) - g^*(y) : y \in X\}.$$

It is noted that there is a perfect symmetry between primal and dual programs (P) and (D) : the dual program of (D) is exactly (P) .

We will always keep the following assumption that is deduced from the finiteness of α

$$\text{dom } g \subset \text{dom } h \quad \text{and} \quad \text{dom } h^* \subset \text{dom } g^*. \quad (1.2)$$

Polyhedral DC program

In the problem (P) , if one of the DC components g and h is polyhedral function, we call (P) a *polyhedral DC program*.

Optimality conditions for DC optimization

A point x^* is said to be a *local minimizer* of $g - h$ if $x^* \in \text{dom } g \cap \text{dom } h$ and there is a neighborhood U of x^* such that

$$g(x) - h(x) \geq g(x^*) - h(x^*), \quad \forall x \in U. \quad (1.3)$$

A point x^* is said to be a *critical point* of $g - h$ if

$$\partial g(x^*) \cap \partial h(x^*) \neq \emptyset. \quad (1.4)$$

Optimality conditions for DC standard programs are shown in the following theorem (see [37]).

Theorem 1.1. i) *Global optimality condition: x^* is an optimal solution of the problem (P) if and only if*

$$\partial_\varepsilon h(x) \subset \partial_\varepsilon g(x), \quad \forall \varepsilon > 0.$$

ii) *Necessary condition for local optimality: if x^* is a local minimizer of $g - h$, then*

$$\partial h(x^*) \subset \partial g(x^*).$$

iii) *Sufficient condition for local optimality: Let x^* be a critical point of $g - h$ and $y^* \in \partial g(x^*) \cap \partial h(x^*)$. Let U be a neighborhood of x^* such that $(U \cap \text{dom } g) \subset \text{dom } \partial h$. If for any $x \in U \cap \text{dom } g$, there is $y \in \partial h(x)$ such that $h^*(y) - g^*(y) \geq h^*(y^*) - g^*(y^*)$, then x^* is a local minimizer of $g - h$. More precisely,*

$$g(x) - h(x) \geq g(x^*) - h(x^*), \quad \forall x \in U \cap \text{dom } g.$$

Remark 1.3. a) *By the symmetry of the DC duality, Theorem 1.1 has its corresponding dual part.*

b) *The necessary local optimality condition $\partial h^*(x^*) \subset \partial g^*(x^*)$ is also sufficient for many important classes programs, for example, if h is polyhedral convex, or when f is locally convex at x^* , i.e. there exists a convex neighborhood U of x^* such that f is finite and convex on U . We know that a polyhedral convex function is differentiable everywhere except on a set of measure zero. Thus, if h is a polyhedral convex function, then a critical point of $g - h$ is almost always a local solution to (P).*

c) *If $f = g - h$ is actually convex on X , we call (P) a “false” DC program. In addition, if $\text{ri}(\text{dom } g) \cap \text{ri}(\text{dom } h) \neq \emptyset$ and $x^* \in \text{dom } g$ such that g is continuous at x^* , then $0 \in \partial f(x^*) \Leftrightarrow \partial h(x^*) \subset \partial g(x^*)$. Thus, in this case, the local optimality is also sufficient for the global optimality. If, in addition, h is differentiable, a critical point is also a global solution.*

Standard DCA

The idea of DCA for solving the problem (P) is that each iteration k of DCA approximates the concave part $-h$ by its affine majorization (that corresponds to taking $(y^k \in \partial h(x^k))$) and minimizes the resulting convex function. This algorithm can be summarized as follows.

Algorithm 1.1. Standard DCA

Initialization. Choose an initial point $x^0 \in X$, set $k := 0$.

Repeat

1. Compute $y^k \in \partial h(x^k)$.
2. Compute $x^{k+1} \in \arg \min \{g(x) - h(x^k) - \langle x - x^k, y^k \rangle : x \in X\}$.
3. Set $k := k + 1$.

Until convergence of $\{x^k\}$.

The convergence properties of DCA was completely investigated in [70]. The following theorem indicates some important results.

Theorem 1.2. *Suppose that the sequences $\{x^k\}$ and $\{y^k\}$ are generated by Algorithm 1.1. Then the following statements hold*

- i) *The sequences $\{g(x^k) - h(x^k)\}$ and $\{h^*(y^k) - g^*(y^k)\}$ are decreasing.*
- ii) *If $g(x^{k+1}) - h(x^{k+1}) = g(x^k) - h(x^k)$ then x^k, x^{k+1} are the critical points of $g - h$. In this case, DCA terminates after a finite number of iterations.*
- iii) *If $\rho(h) + \rho(g) > 0$ (resp. $\rho(h^*) + \rho(g^*) > 0$), then the sequence $\{\|x^{k+1} - x^k\|^2\}$ (resp. $\{\|y^{k+1} - y^k\|^2\}$) converges.*
- iv) *If the optimal value α is finite and the sequences $\{x^k\}$ and $\{y^k\}$ are bounded, then every limit point x^* (resp. y^*) of the sequence $\{x^k\}$ (resp. $\{y^k\}$) is a critical point of $g - h$ (resp. $h^* - g^*$).*
- v) *For polyhedral DC programs, the sequences $\{x^k\}$ and $\{y^k\}$ contain finitely many elements and DCA has a finite convergence. Especially, if h is differentiable at x^* , then x^* is a local minimizer to the problem (P).*

Remark 1.4. a) *When h is a polyhedral function, the calculation of the subdifferential is explicit by Proposition 1.3. With a fixed choice of subgradients of h , the sequence $\{y^k\}$ has only finitely many different elements. This leads to finite convergence of DCA.*

- b) *DCA's distinctive feature relies on upon the fact that DCA deals with the convex DC components g and h but not with the DC function f itself. Moreover, a DC function f has infinitely many DC decompositions which have crucial implications for the qualities (e.g. convergence speed, robustness, efficiency, globality of computed solutions) of DCA. For a given DC program, the choice of optimal DC decompositions is still open. Of course, this depends strongly on the very specific structure of the problem being considered.*

Recently, a novel generation of DCA has been developed in order to improve the standard DCA. For example: DCA with successive DC decomposition [41]; approximate DCA [41]; accelerated DCA [73]; DCA-Like and its accelerated version [33]; online DCA [27]; collaborative DCA [26]; stochastic DCA [32, 31, 42, 30].

1.2.2 General DC program and General DCA

General DC program

A general DC program is of the form

$$\begin{aligned} & \min_x f_0(x), \\ \text{s. t. } & f_i(x) \leq 0, \quad i = 1, \dots, m, \\ & x \in C. \end{aligned} \tag{1.5}$$

where C is a nonempty closed convex set in \mathbb{R}^n ; $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$ ($i = 0, \dots, m$) are DC functions.

This class of nonconvex programs is quite large in DC programming and more difficult than standard DC programs because of the nonconvexity of the constraints. Two approaches for the problem (1.5) were proposed in [28]. The first one employs a penalty technique in DC programming to reformulate the problem (1.5) as a standard DC program. The second one linearizes concave parts in DC constraints to build convex inner approximations of the feasible set. Before presenting these two approaches, we recall some definitions.

Let F be the feasible set of (1.5). A point $x^* \in F$ is a Karush-Kuhn-Tucker (KKT) point for the problem (1.5) if there exist nonnegative scalars λ_i , $i = 1, \dots, m$ such that

$$\begin{cases} 0 \in \partial^\dagger f_0(x^*) + \sum_{i=1}^m \lambda_i \partial^\dagger f_i(x^*) + N(C, x^*), \\ \lambda_i f_i(x^*) = 0, \quad i = 1, \dots, m. \end{cases}$$

Denote

$$\begin{aligned} p(x) &= \max\{f_1(x), f_2(x), \dots, f_m(x)\}, \\ I(x) &= \{i \in \{1, \dots, m\} : f_i(x) = p(x)\}; p^+(x) = \max\{p(x), 0\}. \end{aligned}$$

One says that the *extended Mangasarian-Fromowitz constraint qualification (EMFCQ)* is satisfied at $x^* \in F$ with $I(x^*) \neq \emptyset$ if there is a vector $d \in \text{textcone}(C - \{x^*\})$ (the cone hull of $C - \{x^*\}$) such that

$$f_i^\uparrow(x^*, d) < 0 \quad \forall i \in I(x^*).$$

When f_i 's are continuously differentiable, then $f_i^\uparrow(x^*, d) = \langle \nabla f_i(x^*), d \rangle$. Therefore, the EMFCQ becomes the well-known Mangasarian-Fromowitz constraint qualification.

General DCA using l_∞ -penalty function with updated penalty parameter

Consider the following penalized problem

$$\begin{aligned} & \min_x \phi_k(x) = f_0(x) + \beta_k p^+(x), \\ \text{s. t. } & x \in C, \end{aligned} \tag{1.6}$$

where β_k are penalty parameters. Since $f_i(x)$ ($i = 0, 1, \dots, m$) are DC functions, so is p^+ . Suppose that f_0 and p^+ are decomposed into the difference of two convex functions as follows

$$f_0(x) = g_0(x) - h_0(x), \quad p^+(x) = p_1(x) - p_2(x)$$

where g_0, h_0, p_1, p_2 are convex functions defined on the whole space. Then a DC decomposition of ϕ_k can be chosen to be

$$\phi_k(x) = g_k(x) - h_k(x)$$

where

$$g_k(x) = g_0(x) + \beta_k p_1(x), \quad h_k(x) = h_0(x) + \beta_k p_2(x).$$

DCA with updated penalty parameter is described in the following algorithm.

Algorithm 1.2. General DCA using l_∞ -penalty function

Initialization: Take an initial point $x^1 \in C$; $\delta > 0$, an initial penalty parameter $\beta_1 > 0$. Set $k := 1$.

1. Compute $y^k \in \partial h_k(x^k)$.
2. Compute x^{k+1} by solving the convex program

$$\min\{g_k(x) - \langle x, y^k \rangle : x \in C\}.$$

3. Stopping test
Stop if $x^{k+1} = x^k$ and $p(x^k) \leq 0$.
4. Penalty parameter update.
Compute $r_k = \min\{p(x^k), p(x^{k+1})\}$ and set

$$\beta_{k+1} = \begin{cases} \beta_k & \text{if either } \beta_k \geq \|x^{k+1} - x^k\|^{-1} \text{ or } r_k \leq 0, \\ \beta_k + \delta & \text{if } \beta_k < \|x^{k+1} - x^k\|^{-1} \text{ and } r_k > 0. \end{cases}$$

5. Set $k := k + 1$ and go to Step 1.
-

In the global convergence theorem, the authors use the following assumptions.

Assumption 1.1. $f_i (i = 0, 1, \dots, m)$ are locally Lipschitz functions at every point of C .

Assumption 1.2. Either g_k or h_k is differentiable on C , and $\rho(g_0) + \rho(h_0) + \rho(p_1) + \rho(p_2) > 0$.

Assumption 1.3. The EMFCQ is satisfied at any $x \in \mathbb{R}^n$ with $p(x) \geq 0$.

Theorem 1.3. Suppose that C is a nonempty closed convex set in \mathbb{R}^n and $f_i, i = 0, 1, \dots, m$ are DC functions on C . Suppose further that Assumptions 1.1 – 1.3 are verified. Let $\delta > 0, \beta_1 > 0$ be given and $\{x^k\}$ be a sequence generated by Algorithm 1.2. Then Algorithm 1.2 either stops, after finitely many iterations, at a KKT point x^k for the problem (1.5) or generates an infinite sequence $\{x^k\}$ of iterates such that $\lim_{k \rightarrow \infty} \|x^{k+1} - x^k\| = 0$ and every limit point x^∞ of the sequence $\{x^k\}$ is a KKT point of the problem (1.5).

This theorem is proved in detail in [28].

General DCA using slack variables with updated parameter

Since f_i ($i = 0, 1, \dots, m$) are DC functions, they can be decomposed into the difference of two convex functions as follows $f_i(x) = g_i(x) - h_i(x)$, $x \in \mathbb{R}^n$, $i = 0, \dots, m$.

In this second approach, at each iteration, one solves the following convex subproblem, which is obtained by replacing the concave parts of the DC structure with their affine majorization.

$$\begin{aligned} \min_x \quad & g_0(x) - \langle y_0^k, x \rangle, \\ \text{s. t.} \quad & g_i(x) - h_i(x^k) - \langle y_i^k, x - x^k \rangle \leq 0, \quad \forall i = 1, \dots, m, \\ & x \in C. \end{aligned} \tag{1.7}$$

where $x^k \in \mathbb{R}^n$ is the current iterate, $y_i^k \in \partial h_i(x^k)$ for $i = 0, 1, \dots, m$. However, the inner convex approximation of the feasible set of the problem (1.5) is quite often poor and can lead to infeasibility of the convex subproblem (1.7). To deal with the infeasibility of subproblems, a relaxation technique was proposed. Instead of (1.7), the authors consider the subproblem

$$\min_x \quad g_0(x) - \langle y_0^k, x \rangle + t_k s, \tag{1.8}$$

$$\begin{aligned} \text{s. t.} \quad & g_i(x) - h_i(x^k) - \langle y_i^k, x - x^k \rangle \leq s, \quad \forall i = 1, \dots, m, \\ & x \in C, \quad s \geq 0. \end{aligned} \tag{1.9}$$

where $t_k > 0$ is a penalty parameter. Clearly, (1.8) is a convex problem that is always feasible. Moreover, the Slater constraint qualification is satisfied for the constraints of (1.8), thus the Karush-Kuhn-Tucker (KKT) optimality condition holds for some solution (x^{k+1}, s^{k+1}) of (1.8). The algorithm is summarized as follows.

Algorithm 1.3. General DCA using slack variables

Initialization: Choose an initial point $x^1 \in C$; $\delta_1, \delta_2 > 0$, an initial penalty parameter $t_1 > 0$. Set $k := 1$.

1. Compute $y_i^k \in \partial h_i(x^k)$, $i = 0, 1, \dots, m$.
2. Compute (x^{k+1}, s^{k+1}) as a solution to the convex problem (1.8) and the Lagrange multipliers λ_i^{k+1} associated with the constraints (1.9).
3. Stopping test
If $x^{k+1} = x^k$ and $s^{k+1} = 0$, then stop, otherwise go to Step 4.
4. Penalty parameter update. Compute

$$r_k = \min \left(\|x^{k+1} - x^k\|^{-1}, \sum_{i=1}^m |\lambda_i^{k+1}| + \delta_1 \right)$$

and set

$$t_{k+1} = \begin{cases} t_k & \text{if } t_k \geq r_k, \\ t_k + \delta_2 & \text{if } t_k < r_k. \end{cases}$$

5. Set $k := k + 1$ and go to Step 1.

The global convergence of the above algorithm is shown in the theorem below.

Theorem 1.4. *Suppose that C is a nonempty closed convex set in \mathbb{R}^n and f_i , $i = 0, 1, \dots, m$ are DC functions on C such that Assumptions 1.1 and 1.3 are verified. Suppose further that for each $i = 0, 1, \dots, m$ either g_i or h_i is differentiable on C and that*

$$\rho = \rho(g_0) + \rho(h_0) + \min\{\rho(g_i) : i = 0, 1, \dots, m\} > 0.$$

Let $\delta_1, \delta_2 > 0$, $t_1 > 0$ be given and $\{x^k\}$ be a sequence generated by Algorithm 1.3. Then Algorithm 1.3 either stops, after finitely many iterations, at a KKT point x^k for the problem (1.5) or generates an infinite sequence $\{x^k\}$ of iterates such that $\lim_{k \rightarrow \infty} \|x^{k+1} - x^k\| = 0$ and every limit point x^∞ of the sequence $\{x^k\}$ is a KKT point of the problem (1.5) .

The proof of this theorem is presented in [43].

Part I

Secrecy rate maximization

Chapter 2

DC Programming and DCA for Worst-Case Secrecy Rate Maximization Problem

Abstract: This chapter is concerned with the problem of secure transmission for amplify-and-forward multi-antenna relay system in the presence of multiple eavesdroppers. Specifically, joint beamforming and artificial noise are chosen as the strategy for secure transmission against imperfect channel state information of the intended receiver and the eavesdroppers. In such a scenario, the objective is to maximize the worst-case secrecy rate while guaranteeing the transmit power constraint and the norm-bounded channel uncertainty at the relay. The resulting optimization problem is nonsmooth, nonconvex, and very hard to solve. We first reformulate the problem as two general DC programs with DC constraints and then develop two efficient general DCA based algorithms for solving them. Numerical results illustrate the effectiveness of the proposed algorithms and their superiority versus the existing approach.

1. The material of this chapter is developed from the following work:
[1]. Phuong Anh Nguyen, Hoai An Le Thi. *A DC Programming Approach for Worst-Case Secrecy Rate Maximization Problem*. Proceedings of the 10th International Conference on Computational Collective Intelligence ICCCI 2018, vol 11055, pp. 417–425, Springer, 2018.

2.1 Introduction

Wireless networks have kept evolved with the expansion of features and capacity. However, wireless networks are more vulnerable than wired networks due to their broadcast nature. Traditionally, information transmission is secured by encryption techniques on the application layer via computational complexity. This solution has disadvantages and could be completely broken with the rapid development of computational devices [21]. In recent years, physical layer security has emerged as an alternative or a complement to cryptosystems [53].

The concept of physical layer security was originally developed by Wyner for discrete memoryless channels [90]. In 1978, Cheong and Hellman extended Wyner's results to Gaussian wiretap channels [48], where the secrecy capacity was shown to be the difference between the capacity of the main channel and that of the wiretap channels. If the secrecy capacity falls below zero, wireless communications become insecure [106]. Hence, considerable research efforts have been devoted to the secrecy performance improvement either by enhancing the main channel (known as beamforming) or exploiting artificial noise to degrade the wiretap channels.

The principle of artificial noise paradigm was first proposed in [61], and received notable efforts to advance the first scheme [104, 50, 9, 94]. It was proven that instead of increasing the signal strength, generating artificial noise results in a higher secrecy rate [104]. On the other hand, beamforming is used to boost the reception quality at intended receiver and become a major concept in physical layer security as discussed in [52, 74, 56, 98, 14, 60]. Especially, it is worth mentioning that beamforming techniques can be capable of gaining a better secrecy capacity than artificial noise approaches [60].

Naturally, beamforming and artificial noise may also be joint to further boost the secrecy capacity. In [55], a joint beamforming and artificial noise design was studied in the presence of multiple eavesdroppers. Based on total transmit power minimization, such joint design problem under a target signal-to-interference-plus-noise ratio is mathematically nonconvex. This problem was solved by using a convex approximation method called semidefinite program relaxation. It was pointed out that the power performance in the case of artificial noise-aided design can be improved as compared to the no-artificial noise design under the same signal-to-interference-plus-noise ratio. Here, the channel state information (CSI) at both receiver and all eavesdroppers is assumed to be perfectly known.

In practical applications, CSI uncertainty is unavoidable due to non-ideal factors such as estimation error, quantization error and feedback error [68]. As a result, the robust transmission designs against imperfect CSI recently become a trend [102, 54, 11, 76, 91]. The optimization problems in secure transmission with CSI uncertainty are more challenging owing to nonsmooth, nonconvexity. In [54], Li et al. considered a secrecy rate maximization problem toward the eavesdropper's channel uncertainties. A two-level optimization approach was proposed to solve the problem where the semi-definite relaxation based solution was applied for the inner problem and the outer one was solved by one-dimensional search. In [91], for tackling the nonconvex signal-to-interference-

plus-noise ratio maximization problem against imperfect CSI of eavesdroppers, DCA was applied which is especially designed for facing nonconvex, nonsmooth problems. These works are restricted by perfect CSI assumption at legitimate receiver which may lead to information leakage at experienced eavesdroppers.

As the aforementioned analysis, joint beamforming and artificial noise strategy against CSI uncertainty at intended receiver and eavesdroppers is the most effective way to improve the secrecy performance and reflect the most fact. In such a scenario, Zhang et al. investigated the worst-case secrecy rate maximization (WCSRM) problem in [95]. This problem is very difficult to solve in original form due to its nonconvexity and the infinitely many constraints on the CSI error. The authors applied S-procedure to transform the problem into a tractable form. The transformed problem has $M + 1$ (M - the number of eavesdroppers) fractional DC constraints. The authors converted these fractional constraints as $2M + 2$ exponential DC constraints, for which SCA scheme (successive convex approximation) was developed to solve the considered problem in [95].

It is well known that (see e.g. [41]) SCA is a special version of DCA, a power approach in nonconvex programming framework. The construction of DCA is based on the convex DC components but not the DC function f itself. There are as many DCA as there are DC decompositions which decide the efficiency of resulting DCA (the speed of convergence, robustness, globality of optimal solutions). Hence, finding a suitable DC decomposition which is vital but also difficult, depends on special structure of the corresponding problem.

Since the main feature of DCA is its flexibility in the choice of DC decomposition/DC formulation, we investigate DC programming and DCA to solve the WCSRM problem in [95].

Our contributions are twofold.

Firstly, we develop a new DC program for the transformed WCSRM problem in [95]. We reformulate the $M + 1$ fractional DC constraints of the transformed WCSRM problem as $M + 1$ DC constraints, where the first DC components (g function) are quadratic and the second DC components (h function) are exponential. The number of DC exponential constraints in our proposed DC program is half less than that in [95]. In addition, exponential terms appear at both DC components in the existing DC programs [95]. We propose DCA₁ scheme to solve this DC program. The subproblems of DCA₁ contain quadratic constraints while that of SCA contain exponential ones.

Secondly, we propose another DC formulation for the WCSRM problem, for which results in DCA₂. Here, we replace the DC exponential constraints in the first DC program by quadratic ones. In DCA scheme, the linearization of a quadratic DC function is expected more efficiently than that of exponential DC function. As a result, DCA₂ is expected superior to DCA₁. Numerical experiments confirm that DCA₁ and DCA₂ outperform than SCA while DCA₂ is better than DCA₁ on both quality and rapidity.

The rest of this chapter is organized as follows. The considered secrecy rate maximization problem is stated in Section 2.2 while the solution methods based on DC programming and DCA are presented in Section 2.3. Numerical experiments are reported in Section 2.4, and finally, Section 2.5 concludes the chapter.

2.2 System model

In this section, let us briefly restate the problem formulated in [95].

A relay network comprises a transmitter, a receiver in presence of M eavesdroppers through a K -antenna relay. Especially, the CSI knowledge at the receiver and the eavesdroppers are imperfect.

The system consists of two hops.

- *Hop 1*: In the first hop, information is transmitted with the power P_s . The channel coefficient from source to relay is denoted by $\mathbf{f} \in \mathbb{C}^{K \times 1}$. Let $\mathbf{x}_r \in \mathbb{C}^{K \times 1}$ represents the signal vector to be received at the relay,

$$\mathbf{x}_r = \sqrt{P_s} \mathbf{f} s + \mathbf{n}_r,$$

where $\mathbf{n}_r \sim \mathcal{CN}(0, \mathbf{I}_K)$ is the additive white Gaussian noise vector received at the relay.

- *Hop 2*: In the second hop, the relay forwards the source information to receiver by a beamforming matrix $\mathbf{W} \in \mathbb{C}^{K \times K}$ and then superimposes the artificial noise signal $\mathbf{v}_r \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Phi})$ with $\mathbf{\Phi} \succ \mathbf{0}$. Hence, the signal vector to be transmitted from the relay \mathbf{x}_r is

$$\bar{\mathbf{x}}_r = \mathbf{W} \mathbf{x}_r + \mathbf{v}_r,$$

Let $y_b, y_{e,i}$ denote the received signal of legitimate receiver and i th eavesdropper, respectively, as

$$\begin{aligned} y_b &= \mathbf{h}^H \bar{\mathbf{x}}_r + n_b, \\ y_{e,i} &= \mathbf{g}_i^H \bar{\mathbf{x}}_r + n_{e,i}, \quad i = 1, \dots, M, \end{aligned}$$

and $\mathbf{h} \in \mathbb{C}^{K \times 1}$ and $\mathbf{g}_i \in \mathbb{C}^{K \times 1}$ be the channel coefficients from the relay to receiver and the i th eavesdropper, respectively; $n_b \sim \mathcal{CN}(0, 1)$ and $n_{e,i} \sim \mathcal{CN}(0, 1)$ are the additive zero-mean Gaussian noise components at receiver and i^{th} eavesdropper, respectively.

The relay is assumed to have perfect CSI from source to the relay, but only knows imperfect CSI of legitimate receiver and eavesdroppers. The actual channel vectors of intended receiver and eavesdroppers take the following forms:

$$\mathbf{h} = \bar{\mathbf{h}} + \Delta \mathbf{h}, \quad \mathbf{g}_i = \bar{\mathbf{g}}_i + \Delta \mathbf{g}_i, \quad i = 1, \dots, M,$$

where $\bar{\mathbf{h}}$ and $\bar{\mathbf{g}}_i$ are the estimated channels of intended receiver and the i th eavesdropper at relay; $\Delta \mathbf{h}$ and $\Delta \mathbf{g}_i$ represent the channel errors which are bounded

within spheres:

$$\|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}, \quad \|\Delta \mathbf{g}_i\| \leq \epsilon_{\mathbf{g}_i}, \quad i = 1, \dots, M,$$

where $\epsilon_{\mathbf{h}}$ and $\epsilon_{\mathbf{g}_i}$ are called coefficient errors.

The achievable secrecy rate can be expressed as

$$\begin{aligned} R_s &= \{R_d - R_e\}^+, \quad \text{where} \\ R_d &= \min_{\Delta \mathbf{h}} \frac{1}{2} \log \left(1 + \frac{P_s \mathbf{h}^H \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H \mathbf{h}}{1 + \mathbf{h}^H \Phi \mathbf{h} + \mathbf{h}^H \mathbf{W} \mathbf{W}^H \mathbf{h}} \right), \\ R_e &= \max_i \max_{\Delta \mathbf{g}_i} \frac{1}{2} \log \left(1 + \frac{P_s \mathbf{g}_i^H \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H \mathbf{g}_i}{1 + \mathbf{g}_i^H \Phi \mathbf{g}_i + \mathbf{g}_i^H \mathbf{W} \mathbf{W}^H \mathbf{g}_i} \right). \end{aligned}$$

The transmit power of all antennas at the relay is calculated by

$$P_r = \text{Tr}(P_s \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H) + \text{Tr}(\mathbf{W} \mathbf{W}^H + \Phi).$$

A widely used secrecy design formulation is to maximize the worst-case secrecy rate under the relay power constraint. The problem can be formulated as

$$\begin{aligned} &\max_{R_d, R_e, \mathbf{W}, \Phi} R_d - R_e, \\ \text{s. t. } &\text{Tr}(P_s \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H) + \text{Tr}(\mathbf{W} \mathbf{W}^H + \Phi) \leq P, \\ &\Phi \succeq \mathbf{0}. \end{aligned}$$

In [95], by using slack variables \mathbf{X} , \mathbf{F} , \mathbf{E} ($\mathbf{X} = \text{vec}(\mathbf{W})[\text{vec}(\mathbf{W})]^H$, $\mathbf{F} = P_s \mathbf{f}^T \otimes \mathbf{I}_K \mathbf{X} (\mathbf{f}^T \otimes \mathbf{I}_K)^H$, $\mathbf{E} = \sum_{i=1}^k \mathbf{E}_i \mathbf{X} \mathbf{E}_i^H + \Phi$ where $\mathbf{E}_i = \mathbf{e}_i^T \otimes \mathbf{I}_M$ (\mathbf{e}_i is a unit vector with i th entry being one), the authors reformulated R_d , R_e , P_r as

$$\begin{aligned} R_d &= \min_{\Delta \mathbf{h}} \frac{1}{2} \log \left(1 + \frac{\mathbf{h}^H \mathbf{F} \mathbf{h}}{1 + \mathbf{h}^H \mathbf{E} \mathbf{h}} \right), \\ R_e &= \max_i \max_{\Delta \mathbf{g}_i} \frac{1}{2} \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{F} \mathbf{g}_i}{1 + \mathbf{g}_i^H \mathbf{E} \mathbf{g}_i} \right), \\ P_r &= \text{Tr}(\mathbf{F}) + \text{Tr}(\mathbf{X}) + \text{Tr}(\Phi). \end{aligned}$$

Then, the WCSRM problem was considered in [95] as following formulation

$$\max_{R_d, R_e, \Phi, \mathbf{F}, \mathbf{E}, \mathbf{X}} R_d - R_e, \quad (2.1a)$$

$$\text{s. t. } \text{Tr}(\mathbf{F}) + \text{Tr}(\mathbf{X}) + \text{Tr}(\Phi) \leq P, \quad (2.1b)$$

$$\mathbf{X} \succeq \mathbf{0}, \quad \Phi \succeq \mathbf{0}, \quad R_d > 0, \quad R_e \geq 0, \quad (2.1c)$$

$$\mathbf{F} = P_s \mathbf{f}^T \otimes \mathbf{I}_K \mathbf{X} (\mathbf{f}^T \otimes \mathbf{I}_K)^H, \quad (2.1d)$$

$$\mathbf{E} = \sum_{i=1}^k \mathbf{E}_i \mathbf{X} \mathbf{E}_i^H + \Phi, \quad (2.1e)$$

$$R_d = \min_{\Delta \mathbf{h}} \frac{1}{2} \log \left(1 + \frac{\mathbf{h}^H \mathbf{F} \mathbf{h}}{1 + \mathbf{h}^H \mathbf{E} \mathbf{h}} \right), \quad (2.1f)$$

$$R_e = \max_{\Delta \mathbf{g}_i} \frac{1}{2} \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{F} \mathbf{g}_i}{1 + \mathbf{g}_i^H \mathbf{E} \mathbf{g}_i} \right), \quad i = 1, \dots, M. \quad (2.1g)$$

The problem is still challenging due to the nonconvex constraints (2.1f) and (2.1g). We focus on developing efficient algorithms based on DC programming and DCA to solve the problem (2.1).

2.3 Solution method based on DC programming and DCA

2.3.1 The first DCA scheme for solving the problem (2.1)

The problem (2.1) is intractable to solve in the original form. In [95], Zhang et al. reformulated this problem as following

$$\max_{\mathbf{F}, \mathbf{E}, \mathbf{X}, \Phi, R_d, R_e, \{x_i\}, \{y_i\}, \{\gamma_i^l\}, \{\gamma_i^u\}, i=0, \dots, M} R_d - R_e, \quad (2.2a)$$

$$\text{s. t. (2.1b) - (2.1e),} \quad (2.2b)$$

$$\begin{bmatrix} -\gamma_0^u \mathbf{I}_K - \mathbf{F} & -\mathbf{F}\bar{\mathbf{h}} \\ -\bar{\mathbf{h}}^H \mathbf{F} & -\bar{\mathbf{h}}^H \mathbf{F}\bar{\mathbf{h}} + x_0 + \gamma_0^u \epsilon_{\mathbf{h}}^2 \end{bmatrix} \preceq \mathbf{0}, \quad (2.2c)$$

$$\begin{bmatrix} -\gamma_0^l \mathbf{I}_K + \mathbf{E} & \mathbf{E}\bar{\mathbf{h}} \\ -\bar{\mathbf{h}}^H \mathbf{E} & 1 + \bar{\mathbf{h}}^H \mathbf{E}\bar{\mathbf{h}} - y_0 + \gamma_0^l \epsilon_{\mathbf{h}}^2 \end{bmatrix} \preceq \mathbf{0}, \quad (2.2d)$$

$$\begin{bmatrix} -\gamma_i^u \mathbf{I}_K + \mathbf{F} & \mathbf{F}\bar{\mathbf{g}}_i \\ \bar{\mathbf{g}}_i^H \mathbf{F} & \bar{\mathbf{g}}_i^H \mathbf{F}\bar{\mathbf{g}}_i - x_i + \gamma_i^u \epsilon_{\mathbf{g}_i}^2 \end{bmatrix} \preceq \mathbf{0}, \quad (2.2e)$$

$$\begin{bmatrix} -\gamma_i^l \mathbf{I}_K - \mathbf{E} & -\mathbf{E}\bar{\mathbf{g}}_i \\ -\bar{\mathbf{g}}_i^H \mathbf{E} & -1 - \bar{\mathbf{g}}_i^H \mathbf{E}\bar{\mathbf{g}}_i + y_i + \gamma_i^l \epsilon_{\mathbf{g}_i}^2 \end{bmatrix} \preceq \mathbf{0}, \quad (2.2f)$$

$$\gamma_i^l \geq 0, \gamma_i^u \geq 0, \quad i = 0, \dots, M. \quad (2.2g)$$

$$\frac{x_0}{y_0} \geq 2^{2R_d} - 1, \quad (2.2h)$$

$$\frac{x_i}{y_i} \leq 2^{2R_e} - 1, \quad i = 1, \dots, M. \quad (2.2i)$$

The transformed problem is nonconvex due to the nonconvex constraints (2.2h) and (2.2i).

It is easy to see that the constraints (10d), (10f) in [95] hold with equality at the optimal solution $(\mathbf{W}^*, \Phi^*, y_i^*)$. Thus,

$$\max_{\|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}} 1 + (\bar{\mathbf{h}} + \Delta \mathbf{h})^H \mathbf{W}^* (\mathbf{W}^*)^H (\bar{\mathbf{h}} + \Delta \mathbf{h}) + (\bar{\mathbf{h}} + \Delta \mathbf{h})^H \Phi^* (\bar{\mathbf{h}} + \Delta \mathbf{h}) = y_0^*, \quad (2.3)$$

$$\min_{\|\Delta \mathbf{g}_i\| \leq \epsilon_{\mathbf{g}_i}} 1 + (\bar{\mathbf{g}}_i + \Delta \mathbf{g}_i)^H \mathbf{W}^* (\mathbf{W}^*)^H (\bar{\mathbf{g}}_i + \Delta \mathbf{g}_i) + (\bar{\mathbf{g}}_i + \Delta \mathbf{g}_i)^H \Phi^* (\bar{\mathbf{g}}_i + \Delta \mathbf{g}_i) = y_i^*, \quad i = 1, \dots, M. \quad (2.4)$$

Since the left side of (2.3), (2.4) are positive, we add to the problem (2.2) the constraints $y_i \in \mathbb{R}^+$, $i = 0, \dots, M$.

For convenience, we denote $\mathbf{z}_{01} = (x_0, y_0, R_d)$, $\mathbf{z}_{i1} = (x_i, y_i, R_e)$ with $i = 1, \dots, M$.

We first transform the nonconvex constraints (2.2h) and (2.2i) into DC constraints.

a) The nonconvex constraint (2.2h) is expressed as a DC constraint

$$\begin{aligned} F_{01}(\mathbf{z}_{01}) &= G_{01}(\mathbf{z}_0) - H_{01}(\mathbf{z}_{01}) \leq 0, \text{ where} \\ G_{01}(\mathbf{z}_{01}) &= \frac{1}{2}\rho_0(x_0^2 + y_0^2 + R_d^2), \\ H_{01}(\mathbf{z}_{01}) &= \frac{1}{2}\rho_0(x_0^2 + y_0^2 + R_d^2) + x_0 + y_0 - y_0 2^{2R_d} \end{aligned}$$

are convex functions for ρ_0 being determined in Proposition 2.1.

b) The nonconvex constraints (2.2i) are reformulated as DC constraints

$$\begin{aligned} F_{i1}(\mathbf{z}_i) &= G_{i1}(\mathbf{z}_{i1}) - H_{i1}(\mathbf{z}_{i1}) \leq 0, \quad i = 1, \dots, M, \text{ where} \\ G_{i1}(\mathbf{z}_{i1}) &= \frac{1}{2}\rho_i(x_i^2 + y_i^2 + R_e^2), \\ H_{i1}(\mathbf{z}_{i1}) &= \frac{1}{2}\rho_i(x_i^2 + y_i^2 + R_e^2) - x_i - y_i + y_i 2^{2R_e} \end{aligned}$$

are convex functions for ρ_i being determined in Proposition 2.1.

Proposition 2.1. Denote $D_0 = P(\|\bar{\mathbf{h}}\| + \epsilon_h)^2 + 1$ and $D_i = P(\|\bar{\mathbf{g}}_i\| + \epsilon_{g_i})^2 + 1$ for $i = 1, \dots, M$. The following statements hold

- a) If $\rho_0 \geq 2(\ln 2)D_0 + 2(\ln 2)^2 D_0^2$, then both $G_{01}(\mathbf{z}_{01}), H_{01}(\mathbf{z}_{01})$ are convex.
b) If $\rho_i \geq 2(\ln 2)D_0 + 2(\ln 2)^2 D_0 D_i$, then both $G_{i1}(\mathbf{z}_{i1}), H_{i1}(\mathbf{z}_{i1})$ are convex for $i = 1, \dots, M$.

Proof. a) First, we estimate the feasible region for R_d . From the constraint

$$\text{Tr}(P_s \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H) + \text{Tr}(\mathbf{W} \mathbf{W}^H + \mathbf{\Phi}) \leq P$$

and positive definite hermitian matrix $\mathbf{\Phi} + \mathbf{W} \mathbf{W}^H$, we achieve

$$\begin{aligned} \|\mathbf{\Phi} + \mathbf{W} \mathbf{W}^H\| &\leq \sqrt{[\text{Tr}(\mathbf{\Phi} + \mathbf{W} \mathbf{W}^H)]^2} \leq P, \\ \text{Tr}(P_s \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H) &= P_s \|\mathbf{W} \mathbf{f}\|^2 \leq P. \end{aligned}$$

We rewrite

$$R_d = \min_{\Delta \mathbf{h}} \frac{1}{2} \log \left(1 + \frac{P_s \mathbf{h}^H \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H \mathbf{h}}{1 + \mathbf{h}^H \mathbf{\Phi} \mathbf{h} + \mathbf{h}^H \mathbf{W} \mathbf{W}^H \mathbf{h}} \right)$$

as

$$\frac{P_s \mathbf{h}^H \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H \mathbf{h}}{1 + \mathbf{h}^H \mathbf{\Phi} \mathbf{h} + \mathbf{h}^H \mathbf{W} \mathbf{W}^H \mathbf{h}} \geq 2^{2R_d} - 1 \text{ with } \forall \Delta \mathbf{h} : \|\Delta \mathbf{h}\| \leq \epsilon_h.$$

Thus,

$$\begin{aligned} 2^{2R_d} - 1 &\leq P_s \mathbf{h}^H \mathbf{W} \mathbf{f} \mathbf{f}^H \mathbf{W}^H \mathbf{h}. \text{ Then, apply the Cauchy- Schwarz inequality,} \\ 2^{2R_d} &\leq P_s \|\mathbf{h}\|^2 \|\mathbf{W} \mathbf{f}\|^2 + 1 \leq P(\|\bar{\mathbf{h}}\| + \|\Delta \mathbf{h}\|)^2 + 1 \leq P(\|\bar{\mathbf{h}}\| + \epsilon_h)^2 + 1 = D_0. \end{aligned}$$

Next, we estimate the feasible region for y_0 . Similar as above, we gain

$$1 + \mathbf{h}^H \Phi \mathbf{h} + \mathbf{h}^H \mathbf{W} \mathbf{W}^H \mathbf{h} \leq P(\|\bar{\mathbf{h}}\| + \epsilon_{\mathbf{h}})^2 + 1 = D_0.$$

The constraint (10d) in [95]

$$\max_{\|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}} 1 + \mathbf{h}^H \mathbf{W} \mathbf{W}^H \mathbf{h} + \mathbf{h}^H \Phi \mathbf{h} \leq y_0$$

holds with equality at the optimal solution so that we constraint $y_0 \leq D_0$.

It is noted that for any $\rho_0 > 0$, the function $G_{01}(\mathbf{z}_{01})$ is convex. We aim to determine $\rho_0 \geq 2(\ln 2)D_0 + 2(\ln 2)^2 D_0^2$ to guarantee the convexity of H function. Indeed,

$$\begin{aligned} \nabla^2 F_0 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2(\ln 2)2^{2R_d} \\ 0 & 2(\ln 2)2^{2R_d} & y_0(2 \ln 2)^2 2^{2R_d} \end{bmatrix}, \\ \|\nabla^2 F_0\|_1 &= 2(\ln 2)2^{2R_d} + y_0(2 \ln 2)^2 2^{2R_d}, \\ \nabla^2 H_0 &= \rho_0 \mathbf{I}_3 - \nabla^2 F_0, \text{ thus, } \nabla^2 H_0 > 0 \text{ if } \rho_0 \geq 2(\ln 2)D_0 + 2(\ln 2)^2 D_0^2. \end{aligned}$$

b) Proof is similar to a)

□

From the Proposition 2.1, we can choose $\rho_0 = 2(\ln 2)D_0 + 2(\ln 2)^2 D_0^2$ and $\rho_i = 2(\ln 2)D_0 + 2(\ln 2)^2 D_0 D_i$, $i = 1, \dots, M$. Hence, we obtain the following DC formulation of the problem (2.2)

$$\begin{aligned} & \min_{\substack{\mathbf{F}, \mathbf{E}, \mathbf{X}, \Phi, R_d, R_e, \\ \{x_i\}, \{y_i\}, \{\gamma_i^l\}, \{\gamma_i^u\}, i=0, \dots, M}} R_e - R_d, \tag{2.5} \\ \text{s. t. } & (2.2\text{b}) - (2.2\text{g}), \\ & x_i \geq 0, y_i > 0, i = 0, \dots, M, \\ & G_{i1}(\mathbf{z}_{i1}) - H_{i1}(\mathbf{z}_{i1}) \leq 0, i = 0, \dots, M. \end{aligned}$$

Following the general DCA scheme, a DCA based algorithm for solving the problem (2.5) can be described in Algorithm 2.1.

Algorithm 2.1. DCA based algorithm for solving the problem (2.5)

Initialization: Let $(x_i^0, y_i^0, R_e^0, R_d^0)$ be an initial point, $i = 0, \dots, M$, $k \leftarrow 0$.
repeat

1. For $i = 1, \dots, M$, compute

$$\begin{aligned} \nabla H_{01}(\mathbf{z}_{01}^k) &= (\rho_0 x_0^k + 1, \rho_0 y_0^k + 1 - 2^{2R_d^k}, \rho_0 R_d^k - 2 \ln 2 y_0^k 2^{2R_d^k}), \\ \nabla H_{i1}(\mathbf{z}_{i1}^k) &= (\rho_i x_i^k - 1, \rho_i y_i^k - 1 + 2^{2R_e^k}, \rho_i R_e^k + 2 \ln 2 y_i^k 2^{2R_e^k}), \end{aligned}$$

2. Compute $(x_i^{k+1}, y_i^{k+1}, R_e^{k+1}, R_d^{k+1})$, $i = 0, \dots, M$ by solving the problem

$$\begin{aligned} & \min_{\mathbf{F}, \mathbf{E}, \mathbf{X}, \Phi, R_d, R_e, \{x_i\}, \{y_i\}, \{\gamma_i^l\}, \{\gamma_i^u\}, i=0, \dots, M} R_e - R_d, \\ & \text{s. t. (2.2b) - (2.2g),} \\ & x_i \geq 0, y_i > 0, i = 0, \dots, M, \\ & G_{i1}(\mathbf{z}_{i1}) - H_{i1}(\mathbf{z}_{i1}^k) - \langle \nabla H_{i1}(\mathbf{z}_{i1}^k), \mathbf{z}_{i1} - \mathbf{z}_{i1}^k \rangle \leq 0, i = 0, \dots, M. \end{aligned} \quad (2.6)$$

3. $k \leftarrow k + 1$.

until stopping criterion.

This first DCA scheme leads to $M + 1$ quadratic constraints at each subproblems. This algorithm has drawbacks with the large number of eavesdroppers. It motivates us to deal with this disadvantage as the following section.

2.3.2 The second DCA scheme for solving the problem (2.1)

We first transform the problem (2.1) into a tractable form. Since the binary logarithm function is increasing function,

$$\min_x \log(f(x)) = \log \min_x f(x), \quad \max_x \log(f(x)) = \log \max_x f(x).$$

Thus, we can rewrite the problem (2.1) as

$$\begin{aligned} & \max_{R_d, R_e, \Phi, \mathbf{F}, \mathbf{E}, \mathbf{X}} R_d - R_e, \\ & \text{s. t. (2.1b) - (2.1e),} \\ & R_d = \frac{1}{2} \log \min_{\Delta \mathbf{h}} \left(1 + \frac{\mathbf{h}^H \mathbf{F} \mathbf{h}}{1 + \mathbf{h}^H \mathbf{E} \mathbf{h}} \right), \\ & R_e = \frac{1}{2} \log \max_{\Delta \mathbf{g}_i} \left(1 + \frac{\mathbf{g}_i^H \mathbf{F} \mathbf{g}_i}{1 + \mathbf{g}_i^H \mathbf{E} \mathbf{g}_i} \right), i = 1, \dots, M. \end{aligned}$$

We introduce two slack variables u , v .

$$\begin{aligned} v &= \min_{\Delta \mathbf{h}} \frac{1 + \mathbf{h}^H (\mathbf{F} + \mathbf{E}) \mathbf{h}}{1 + \mathbf{h}^H \mathbf{E} \mathbf{h}}, \quad v \geq 1, \\ u &= \max_{\Delta \mathbf{g}_i} 1 + \frac{\mathbf{g}_i^H \mathbf{F} \mathbf{g}_i}{1 + \mathbf{g}_i^H \mathbf{E} \mathbf{g}_i}, \quad u \geq 1. \end{aligned}$$

Substituting variables u and v into variables R_d and R_e , the WCSRM problem (2.1) is transformed into

$$\min_{\mathbf{F}, \mathbf{E}, \mathbf{X}, \Phi, u, v} \frac{1}{2} \log u - \frac{1}{2} \log v, \quad (2.7a)$$

$$\text{s. t. (2.1b) - (2.1e),} \quad (2.7b)$$

$$v \geq 1, \quad u \geq 1, \quad (2.7c)$$

$$\min_{\Delta \mathbf{h}} \frac{1 + \mathbf{h}^H (\mathbf{F} + \mathbf{E}) \mathbf{h}}{1 + \mathbf{h}^H \mathbf{E} \mathbf{h}} = v, \quad (2.7d)$$

$$\max_{\Delta \mathbf{g}_i} 1 + \frac{\mathbf{g}_i^H \mathbf{F} \mathbf{g}_i}{1 + \mathbf{g}_i^H \mathbf{E} \mathbf{g}_i} = u, \quad i = 1, \dots, M. \quad (2.7e)$$

This converted problem is nonconvex due to nonconvex objective function (2.7a) and nonconvex constraints (2.7d)-(2.7e).

For convenience, we denote $\mathbf{z}_{02} = (u, v)$, $\mathbf{z}_{12} = (x, y, v)$. We carefully study DC decomposition for (2.7a) and DC constraints for (2.7d)- (2.7e) as following.

a) A DC decomposition for (2.7a):

$$\begin{aligned} F_{02}(\mathbf{z}_{02}) &= G_{02}(\mathbf{z}_{02}) - H_{02}(\mathbf{z}_{02}), \text{ where} \\ G_{02}(\mathbf{z}_{02}) &= -\frac{1}{2} \log v, \quad H_{02}(\mathbf{z}_{02}) = \frac{1}{2} \log u. \end{aligned}$$

b) The constraint (2.7d) can be transformed as

$$\frac{1 + \mathbf{h}^H (\mathbf{F} + \mathbf{E}) \mathbf{h}}{1 + \mathbf{h}^H \mathbf{E} \mathbf{h}} \geq v \text{ with } \forall \Delta \mathbf{h} : \|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}, \quad (2.8)$$

It can be seen that (2.8) must hold with equality at the optimal solution, if not, we can increase v then the objective value is reduced. We rewrite (2.8) as

$$\left\{ \begin{array}{l} 1 + \mathbf{h}^H (\mathbf{F} + \mathbf{E}) \mathbf{h} \geq x \text{ with } \forall \Delta \mathbf{h} : \|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}, \\ 1 + \mathbf{h}^H \mathbf{E} \mathbf{h} \leq y \text{ with } \forall \Delta \mathbf{h} : \|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}, \\ \frac{x}{y} \geq v, \\ x \geq 0, \quad y \geq 1. \end{array} \right. \quad (2.9)$$

(2.8) can be considered equivalent to (2.9) because the inequality constraints of (2.9) hold with equalities at the optimal solution. In otherwise, we increase x and reduce y . Then v is increased, which leads to decreased objective value. (2.9) can be expressed as

$$1 + \mathbf{h}^H (\mathbf{F} + \mathbf{E}) \mathbf{h} \geq x \quad \forall \Delta \mathbf{h} : \|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}, \quad (2.10a)$$

$$1 + \mathbf{h}^H \mathbf{E} \mathbf{h} \leq y \quad \forall \Delta \mathbf{h} : \|\Delta \mathbf{h}\| \leq \epsilon_{\mathbf{h}}, \quad (2.10b)$$

$$-x + yv \leq 0, \quad (2.10c)$$

$$x \geq 0, \quad y \geq 1. \quad (2.10d)$$

The channel error models can be coped with S-Procedure as following Lemma

Lemma 2.1. (*[95]*) Define $f_i(x) = x^H A_i x + 2\Re.(b_i^H x) + c_i$, where $A_i \in \mathbb{C}^{n \times n}$ is Hermitian, $b_i \in \mathbb{C}^{n \times 1}$ and $x \in \mathbb{C}^{n \times 1}$, $i = 1, 2$. The implication $f_1(x) \leq 0 \Rightarrow f_2(x) \leq 0$ holds if and only if there exists $\lambda \geq 0$ such that

$$\lambda \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{bmatrix} \succeq \begin{bmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2^H & c_2 \end{bmatrix}.$$

Applying S-procedure for (2.10a), (2.10b), we gain

$$\begin{bmatrix} \beta \mathbf{I}_K + (\mathbf{F} + \mathbf{E}) & (\mathbf{F} + \mathbf{E}) \bar{\mathbf{h}}_s \\ \bar{\mathbf{h}}_s^H (\mathbf{F} + \mathbf{E}) & \bar{\mathbf{h}}_s^H (\mathbf{F} + \mathbf{E}) \bar{\mathbf{h}}_s + 1 - x - \beta \epsilon_s^2 \end{bmatrix} \succeq \mathbf{0}, \quad \beta \geq 0, \quad (2.11a)$$

$$\begin{bmatrix} \alpha \mathbf{I}_K - \mathbf{E} & -\mathbf{E} \bar{\mathbf{h}}_s \\ -\bar{\mathbf{h}}_s^H \mathbf{E} & -\bar{\mathbf{h}}_s^H \mathbf{E} \bar{\mathbf{h}}_s + y - 1 - \alpha \epsilon_s^2 \end{bmatrix} \succeq \mathbf{0}, \quad \alpha \geq 0. \quad (2.11b)$$

The constraint (2.10c) can be expressed as a DC constraint:

$$\begin{aligned} F_{12}(\mathbf{z}_{12}) &= G_{12}(\mathbf{z}_{12}) - H_{12}(\mathbf{z}_{12}) \leq 0, \text{ where} \\ G_{12}(\mathbf{z}_{12}) &= \frac{1}{2} \rho(x^2 + y^2 + v^2), \\ H_{12}(\mathbf{z}_{12}) &= \frac{1}{2} \rho(x^2 + y^2 + v^2) + x - yv \end{aligned}$$

are convex functions with $\rho \geq 1$.

c) (2.7e) can be reformulated as

$$1 + \frac{\mathbf{g}_i^H \mathbf{F} \mathbf{g}_i}{1 + \mathbf{g}_i^H \mathbf{E} \mathbf{g}_i} \leq u \text{ with } \forall \Delta \mathbf{g}_i : \|\Delta \mathbf{g}_i\| \leq \epsilon_{\mathbf{g}_i}, \quad (2.12)$$

It is easy to see that (2.12) must hold with equality at the optimal solution. Assume the contrast, we can decrease u such that the objective value can be further reduced. (2.12) is reformulated as

$$\mathbf{g}_i^H (\mathbf{F} - \mathbf{V}) \mathbf{g}_i - u \leq 0 \text{ with } \forall \Delta \mathbf{g}_i : \|\Delta \mathbf{g}_i\| \leq \epsilon_{\mathbf{g}_i}, \quad (2.13a)$$

$$\mathbf{V} = (u - 1) \mathbf{E}. \quad (2.13b)$$

Lemma 2.2. (*[74]*) The equality $\mathbf{X}_2 = \mathbf{X}_0 \mathbf{X}_1$ is equivalent to the following constraints in additional matrix variables \mathbf{Y}_1 and \mathbf{Y}_2 :

$$\begin{cases} \begin{bmatrix} \mathbf{Y}_1 & \mathbf{X}_2 & \mathbf{X}_0 \\ \mathbf{X}_2^H & \mathbf{Y}_2 & \mathbf{X}_1^H \\ \mathbf{X}_0^H & \mathbf{X}_1 & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \\ \text{Tr}(\mathbf{Y}_1) - \text{Tr}(\mathbf{X}_0 \mathbf{X}_0^H) \leq 0. \end{cases}$$

By exploiting S-procedure for (2.13a) and Lemma 2.2 for (2.13b), respectively, they are transformed into

$$\begin{bmatrix} \lambda_i \mathbf{I}_K + (\mathbf{F} - \mathbf{V}) & (\mathbf{F} - \mathbf{V}) \bar{\mathbf{g}}_i \\ \bar{\mathbf{g}}_i^H (\mathbf{F} - \mathbf{V}) & \bar{\mathbf{g}}_i^H (\mathbf{F} - \mathbf{V}) \bar{\mathbf{g}}_i + u - 1 - \lambda_i \epsilon_i^2 \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_i \geq 0, \quad (2.14a)$$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{V} & (u-1)\mathbf{I}_K \\ \mathbf{V}^H & \mathbf{A}_2 & \mathbf{E}^H \\ (u-1)\mathbf{I}_K & \mathbf{E} & \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \quad (2.14b)$$

$$-\text{Tr}(\mathbf{A}\mathbf{1}) - K(u-1)^2 \leq 0. \quad (2.14c)$$

Here, the difficulty is the nonconvex constraint (2.14c). We propose a DC decomposition for the left side of (2.14c) as

$$F_{22}(\mathbf{z}_{22}) = G_{22}(\mathbf{z}_{22}) - H_{22}(\mathbf{z}_{22}), \quad \mathbf{z}_{22} = ([\mathbf{A}_1]_{(i,i)}(i = 1, \dots, K), u), \quad \text{where} \\ G_{22}(\mathbf{z}_{22}) = -\text{Tr}(\mathbf{A}\mathbf{1}), \quad H_{22}(\mathbf{z}_{22}) = K(u-1)^2.$$

From above discussion, we gain the DC formulation of the problem (2.1) as following

$$\begin{aligned} & \min_{\substack{\mathbf{F}, \mathbf{E}, \mathbf{X}, \Phi, \mathbf{A}\mathbf{1}, \mathbf{A}\mathbf{2}, \\ u, v, x, y, \alpha, \beta, \{\lambda_i\}, i=1, \dots, M}} G_{02}(\mathbf{z}_{02}) - H_{02}(\mathbf{z}_{02}), & (2.15) \\ \text{s. t. } & (2.1b) - (2.1e), (2.11a) - (2.11b), (2.14a) - (2.14b), \\ & v \geq 0, \quad u \geq 1, \quad x \geq 0, \quad y \geq 1, \\ & G_{i2}(\mathbf{z}_{i2}) - H_{i2}(\mathbf{z}_{i2}) \leq 0, \quad i = 1, 2. \end{aligned}$$

A DCA based algorithm to solve the problem (2.15) is given in Algorithm 2.2.

Algorithm 2.2. DCA based algorithm for solving the problem (2.15)

Initialization: Let (u^0, v^0, x^0, y^0) be an initial point, $k \leftarrow 0$.

repeat

1. Compute

$$\begin{aligned} \nabla H_{02}(\mathbf{z}_{02}^k) &= \frac{1}{2} \frac{1}{u^k \ln 2}, \\ \nabla H_{12}(\mathbf{z}_{12}^k) &= (\rho x^k + 1, \quad \rho y^k - v^k, \quad \rho v^k - y^k), \\ \nabla H_{22}(\mathbf{z}_{22}^k) &= 2K(u^k - 1). \end{aligned}$$

2. Compute $(u^{k+1}, v^{k+1}, x^{k+1}, y^{k+1})$ by solving the problem

$$\begin{aligned} & \min_{\substack{\mathbf{F}, \mathbf{E}, \mathbf{X}, \Phi, \mathbf{A}\mathbf{1}, \mathbf{A}\mathbf{2}, \\ u, v, x, y, \alpha, \beta, \{\lambda_i\}, i=1, \dots, M}} -\frac{1}{2} \log v + \frac{1}{2} \log u^k + \frac{1}{2} \frac{u - u^k}{u^k \ln 2}, & (2.16) \\ \text{s. t. } & (2.1b) - (2.1e), (2.11a) - (2.11b), (2.14a) - (2.14b), \\ & v \geq 1, \quad u \geq 1, \quad x \geq 0, \quad y \geq 1, \\ & G_{i2}(\mathbf{z}_{i2}) - H_{i2}(\mathbf{z}_{i2}^k) - \langle \nabla H_{i2}(\mathbf{z}_{i2}^k), \mathbf{z}_{i2} - \mathbf{z}_{i2}^k \rangle \leq 0, \quad i = 1, 2. \end{aligned}$$

3. $k \leftarrow k + 1$,

until stopping criterion.

2.4 Numerical experiments

Experiment settings. All algorithms were implemented in Matlab R2017b and executed on a PC Intel(R) Core(TM)i5-2450M CPU 2.50 GHz of 4GB RAM. The CVX toolbox [10] (based on sedumi solver) is used to solve subproblems of all algorithms. All channel coefficients are generated by complex Gaussian distribution with zero mean and unit variance. The number of antennas at relay is 4. The transmit power at the source is $P_s = 20$ dB. Without loss of generality, we assume the coefficient error $\epsilon = \epsilon_{\mathbf{h}} = \epsilon_{\mathbf{g}_i}$, $i = 1, \dots, M$. The stopping criterion for all algorithms is $|R_d^k - R_e^k - (R_d^{k-1} - R_e^{k-1})| < 10^{-3}$.

The purpose of our experiments are twofold. The first is to evaluate the efficiency of the proposed algorithms DCA_1 , DCA_2 in comparing with the existing method SCA. Our second purpose is to study the performance of the two different DC formulations in comparing DCA_1 to DCA_2 .

For the two mentioned purposes, we are interested in the effect of the number of eavesdroppers, of the relay power and of the coefficient error. All the algorithms are tested on 10 independent channel realizations. For each value of these parameters, the average values of the secrecy rate (SSR) and CPU time (CPU(s)) are reported in Table 2.1 (when $P = 20$ dB, $\epsilon = 0.1$, with different number of eavesdroppers), Table 2.2 (when $M = 2$, $\epsilon = 0.1$ and the relay power varies) and Table 2.3 (when $M = 2$, $P = 20$ dB and the coefficient error varies).

Comments on numerical results. It is seen from Table 2.1 that the larger the number of eavesdroppers is, the smaller the worst-case secrecy rate is. In Table 2.2, as we expected, increasing the relay power can yield a higher secrecy rate. Table 2.3 shows the sensitivity of the secrecy rate to the CSI error. Moreover, as justification for the performance of our proposed algorithms, we provide some observations as followings.

- DCAs versus SCA

In terms of secrecy rate, we observe from Table 2.1 – 2.3 that DCA_1 and DCA_2 are more efficient than SCA on both SSR and CPU(s). Specifically, DCA_2 is the best - the gain varies from 67.4% to 134.8% between DCA_2 and SCA on SSR (Table 2.3). The second is DCA_1 which outperforms SCA - the gain varies from 25.7% to 32.4% (Table 2.3).

In terms of CPU time, it can be seen that SCA scales very fast with rise in the number of eavesdroppers while DCAs increase more slowly as compared to the former. Specifically, DCA_2 is faster than SCA from 2.5 to 3.0 times (Table 2.1). DCA_1 and SCA solve the same reformulated problem of the original problem but different DC formulation. DCA_1 is faster than SCA from 1.6 to 2.0 times (Table 2.1). This fact is because at each iteration, DCA_1 and DCA_2 involve quadratic constraints while the subproblem of SCA contains exponential constraints.

- DCA_1 versus DCA_2

DCA_1 and DCA_2 correspond to two different DC reformulations of the original SRM problem. Regarding the secrecy rate, DCA_2 is much more efficient than DCA_1 . The gain on SSR varies from 28.7% to 86.7% (Table 2.3). In terms of

CPU times, DCA₂ is more cost-effective than DCA₁ in almost cases.

Table 2.1 – System secrecy rate versus the number of eavesdroppers

M	DCA ₂		DCA ₁		SCA	
	SSR(bit/s/Hz)	CPU(s)	SSR(bit/s/Hz)	CPU(s)	SSR(bit/s/Hz)	CPU(s)
2	3.8887	11.1444	3.0757	13.9171	2.3224	27.3869
3	3.5956	13.5399	2.9263	20.3791	2.1325	32.9894
4	3.4841	14.2911	2.6477	21.2082	2.0254	38.0060
5	3.3873	14.8600	2.4873	21.9338	1.9671	44.6234
6	3.3022	16.2305	2.2283	32.5916	1.8969	51.7861

Table 2.2 – System secrecy rate versus the relay power

P (dB)	DCA ₂		DCA ₁		SCA	
	SSR(bit/s/Hz)	CPU(s)	SSR(bit/s/Hz)	CPU(s)	SSR(bit/s/Hz)	CPU(s)
10	3.6220	12.0521	2.2831	14.4632	2.0070	29.1921
15	3.7865	11.8074	2.4004	16.1351	2.1970	28.7600
20	3.8887	11.1444	3.0757	13.9171	2.3224	27.3869
25	3.9595	10.9237	3.2613	13.8592	2.4136	25.6296

Table 2.3 – System secrecy rate versus the coefficient error

ϵ	DCA ₂		DCA ₁		SCA	
	SSR(bit/s/Hz)	CPU(s)	SSR(bit/s/Hz)	CPU(s)	SSR(bit/s/Hz)	CPU(s)
0.1	3.8887	11.1444	3.0757	13.9171	2.3224	27.3869
0.2	3.7902	12.0513	2.7286	10.7409	2.1051	25.4291
0.3	3.6882	11.5598	2.1537	15.0775	1.8351	25.1544
0.4	3.5813	11.3708	1.9178	21.2422	1.5251	27.7326

2.5 Conclusion

We have intensively investigated DC program and DCA for solving the WCSRM problem in the presence of imperfect CSI at both legitimate receiver and eavesdroppers. First, using the existing reformulation of the WCSRM problem in [95], we have devoted to propose new DC formulation which results in the corresponding general DCA scheme (named DCA₁). Then, we have transformed the original WCSRM problem into a new reformulation. We have thoroughly studied the special structure of the resulting reformulation in a suitable way to propose a DC program for which a DCA based algorithm (named DCA₂) has been developed. Numerical results have confirmed the superiority of DCAs as compared with the existing approach SCA on both quality and rapidity. In addition, numerical results have shown that DCA₂ is superior to DCA₁.

Chapter 3

Efficient DCA based Algorithms for Simultaneous Wireless Information and Power Transfer in MISO Secrecy Channel

Abstract: This chapter addresses the emerging practical requirement that wireless information and power transfer are employed simultaneously in a multiple-input single-output (MISO) secrecy channel. Transmit beamforming without artificial noise and that with artificial noise are considered. In addition, perfect and imperfect channel state information of both the legitimate receivers and eavesdroppers are investigated. These four scenarios correspond to four optimization problems. The considered problems aim to maximize the secrecy rate with constraints on the transmit power and the harvested energy at each receiver. Since these optimization problems are nonconvex, we propose the efficient algorithms based on DC programming and DCA - an innovative approach in nonconvex optimization. Exploiting the special structure of these problems, we utilize the semidefinite relaxation technique and S-procedure to reformulate them as general DC programs. The efficient algorithms based on general DCA are developed to solve the corresponding general DC programs. Numerical results are presented which illustrate the effectiveness of our proposed algorithms, they outperform the existing approaches on both quality and rapidity.

1. The material of this chapter is developed from the following work:
[1]. Phuong Anh Nguyen, Hoai An Le Thi. *Efficient DCA based Algorithms for Simultaneous Wireless Information and Power Transfer in MISO Secrecy Channel*. Submitted.

3.1 Introduction

As an effective solution to cope with the energy scarcity, simultaneous wireless information and power transfer (SWIPT) have attracted much research attention recently. The essence of SWIPT technology is exploiting the features of radio signals for carrying energy and information in the meantime [58]. To guarantee the performance of SWIPT systems, the transmitter needs to raise the information-bearing signal power which would lead to risk of information leakage [5]. Besides, energy receivers are normally located near the transmitter to meet energy requirements [56]. These may cause the secrecy rate to fall below zero since energy receiver can be potential eavesdroppers [107, 106]. Hence, security in SWIPT systems is a critical issue and severer than conventional wireless networks. Physical layer security was proven to ensure secrecy for wireless information transmission [90]. The secure SWIPT system was original studied by Ng et al. [63] and has become a trend recently.

Some secure information transmission schemes in SWIPT systems were studied in [56, 63, 101, 97, 64] where channel state information (CSI) knowledge was assumed to be perfect. With the assumption of imperfect CSI at eavesdroppers, authors in [67, 8, 51, 88, 16] investigated robust transmit designs. In addition, some robust secure transmission schemes were developed in [4, 89, 7, 17, 96, 80, 6] to handle the imperfect CSI models of all channels. There are two main strategies in order to improve secrecy performance as well as guarantee energy harvesting demands: secure beamforming approach in [4, 89, 7, 17, 6] and joint beamforming and artificial noise scheme in [96, 80, 6].

[4] considered transmit beamforming design for multiple input-single output (MISO) SWIPT systems. The optimization problems in [4] consists of minimizing the system power, maximizing the sum secrecy rate, and maximizing the minimal secrecy rate. These problems are nonconvex and were solved by successive convex approximation (SCA)based algorithm. It was demonstrated that joint beamforming and artificial noise schemes are more effective to significantly enhance security than conventional beamforming approaches [80]. Nonetheless, the security optimization problems of joint beamforming and artificial noise schemes are more complicated and difficult to solve than that of beamforming. In [6], Chu et al. considered both beamforming without artificial noise and that with artificial noise under scenarios of perfect and imperfect CSI of all channels. In such four scenarios, four optimization problems are formulated which maximize the secrecy rate, subject to constraints of the transmit power consumption and the harvested energy at each energy receiver. The considered problems are nonconvex, thus, hard to solve. Chu et al. first solved the problems based on two-level algorithms (bisection search (BS) algorithm and one-dimensional line-search-based two-level optimization (1D) algorithm): the problems of external level are coped with linear search while the internal problems are solved by semidefinite relaxation technique. In order to replace these complicated two-level schemes, Chu et al. then developed SCA algorithms. However, SCA algorithms lead to solve convex subproblems with exponential constraints, which should be avoided by simpler constraints. It is well-known that SCA is a special version of DCA.

As mentioned before, each DC function has an infinite number of DC decompositions which have crucial impacts on the qualities of DCA (speed of convergence, robustness, efficiency, globality of computed solutions). Hence, finding a suitable DC decomposition is vital but also difficult. The flexible property of DCA motivates us to investigate new DC formulations for the considered problems in [6] and develop the insightful schemes based on DCA. Our main contributions lie in developing algorithms based on DC programming and DCA to circumvent the nonconvexity of four secrecy rate maximization problems in [6]. We first study the special structure of the considered problems in a suitable way to convert them into general DC programs by utilizing the semidefinite relaxation technique and S-procedure [95]. Then, efficient algorithms based on general DCA are carefully studied to solve the relative DC programs. We investigate the relaxation technique to deal with the infeasibility of subproblems in DCA schemes, which is caused by linearization of concave part of DC decompositions of DC constraints. Numerical results verify the efficiency and superiority of our proposed methods in comparing them with the existing methods (BS, 1D, and SCA).

The rest of this chapter is organized as follows. The considered secrecy rate maximization problems are described in Section 3.2. The numerical solution methods are studied in Section 3.3. Section 3.4 provides numerical experiments and finally, Section 3.5 concludes the chapter.

3.2 System model

In this section, we briefly describe four secrecy rate maximization problems according to four scenarios, which were introduced in [6].

Consider a MISO SWIPT wiretap channel, which comprises one multi-antenna transmitter, one single-antenna information receiver and L single-antenna energy receivers in presence of M single-antenna eavesdroppers. The transmitter is equipped with K antennas. The channel coefficients from the transmitter to the information receiver, the l th energy receiver, and i th eavesdropper are denoted by $\mathbf{h}_s \in \mathbb{C}^{K \times 1}$, $\mathbf{h}_l \in \mathbb{C}^{K \times 1}$, $l \in \mathcal{L} \triangleq 1, \dots, L$ and $\mathbf{g}_i \in \mathbb{C}^{K \times 1}$, $i \in \mathcal{M} \triangleq 1, \dots, M$, respectively.

The received signals at the information receiver and the i th eavesdropper can be, respectively, given as

$$\begin{aligned} y_s &= \mathbf{h}_s^H \mathbf{x} + n_s, \\ y_{e,i} &= \mathbf{g}_i^H \mathbf{x} + n_i, \quad i \in \mathcal{M}, \end{aligned}$$

where $\mathbf{x} \in \mathbb{C}^{K \times 1}$ represents the transmit signal vector while $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ and $n_i \sim \mathcal{CN}(0, \sigma_i^2)$ are the additive noises at the information receiver and the i th eavesdropper, respectively.

The considered problems aim to maximize the worst-case secrecy rate subject to constraints on the total transmit power and the harvested energy at each energy receiver. Both transmit beamforming without artificial noise and that with artificial

noise for secrecy SWIPT system are considered.

A. Secrecy rate optimization without artificial noise

The signal transmitted takes the following form:

$$\mathbf{x} = \mathbf{w}s,$$

where $\mathbf{w} \in \mathbb{C}^{K \times 1}$ is the transmit beamforming vector, s is the information intended for the legitimate user.

The achieved secrecy rate is given as

$$R_s = \left\{ \log \left(1 + \frac{|\mathbf{h}_s^H \mathbf{w}|^2}{\sigma_s^2} \right) - \max_i \log \left(1 + \frac{|\mathbf{g}_i^H \mathbf{w}|^2}{\sigma_i^2} \right) \right\}^+, \quad i \in \mathcal{M}.$$

The transmit power of all antennas at transmitter is calculated by

$$P_s = \text{Tr}(\mathbf{w}\mathbf{w}^H).$$

The harvested energy at the l th energy receiver is written as

$$E_l = |\mathbf{h}_l^H \mathbf{w}|^2, \quad l \in \mathcal{L}.$$

Two secrecy rate maximization problems according to perfect and imperfect CSI scenarios were considered.

- Problem I: The first secrecy rate maximization problem under the assumption of perfect CSI was given in [6] as below

$$\begin{aligned} & \max_{\mathbf{w}} \left\{ \log \left(1 + \frac{|\mathbf{h}_s^H \mathbf{w}|^2}{\sigma_s^2} \right) - \max_i \log \left(1 + \frac{|\mathbf{g}_i^H \mathbf{w}|^2}{\sigma_i^2} \right) \right\}, \\ & \text{s. t. } \text{Tr}(\mathbf{w}\mathbf{w}^H) \leq P, \\ & \quad |\mathbf{h}_l^H \mathbf{w}|^2 \geq E, \quad l \in \mathcal{L}. \end{aligned} \quad (3.1)$$

- Problem II: In case of imperfect CSI of all channels, the channel errors are bounded in spheres

The actual coefficient channel vectors take the following forms:

$$\begin{aligned} \mathbf{h}_s &= \bar{\mathbf{h}}_s + \mathbf{e}_s, \quad \|\mathbf{e}_s\| \leq \epsilon_s, \\ \mathbf{g}_i &= \bar{\mathbf{g}}_i + \mathbf{e}_{g_i}, \quad \|\mathbf{e}_{g_i}\| \leq \epsilon_{g_i} \quad i \in \mathcal{M}, \\ \mathbf{h}_l &= \bar{\mathbf{h}}_l + \mathbf{e}_{h_l}, \quad \|\mathbf{e}_{h_l}\| \leq \epsilon_{h_l} \quad l \in \mathcal{L}, \end{aligned} \quad (3.2)$$

where $\bar{\mathbf{h}}_s$, $\bar{\mathbf{g}}_i$, $\bar{\mathbf{h}}_l$ are the corresponding estimated channels while \mathbf{e}_s , \mathbf{e}_{g_i} , \mathbf{e}_{h_l} denote the channel errors.

The authors in [6] formulated the secrecy rate maximization problem in this case as follows.

$$\begin{aligned} & \max_{\mathbf{w}} \left\{ \min_{\mathbf{e}_s} \log \left(1 + \frac{|(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{w}|^2}{\sigma_s^2} \right) - \max_i \max_{\mathbf{e}_{g_i}} \log \left(1 + \frac{|(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{w}|^2}{\sigma_i^2} \right) \right\}, \\ & \text{s. t. } \text{Tr}(\mathbf{w}\mathbf{w}^H) \leq P, \\ & \min_{\mathbf{e}_{h_l}} \{ |(\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})^H \mathbf{w}|^2 \} \geq E, \quad l \in \mathcal{L}. \end{aligned} \quad (3.3)$$

B. Secrecy rate optimization with artificial noise

In order to enhance the reliable and power efficient transmission, the system employs joint beamforming and AN strategy. As a result, the signal transmitted takes the following form:

$$\mathbf{x} = \mathbf{w}s + \mathbf{v},$$

where $\mathbf{v} \sim \mathcal{CN}(0, \mathbf{V})$ is the artificial noise vector.

The achieved secrecy rate can be formulated as

$$R_s = \left\{ \log \left(1 + \frac{\mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s}{\mathbf{h}_s^H \mathbf{V} \mathbf{h}_s + \sigma_s^2} \right) - \max_i \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i}{\mathbf{g}_i^H \mathbf{V} \mathbf{g}_i + \sigma_i^2} \right) \right\}^+, \quad i \in \mathcal{M},$$

where $\mathbf{Q} = \mathbf{w}\mathbf{w}^H$, $\text{rank}(\mathbf{Q}) = 1$, $\mathbf{Q} \succeq \mathbf{0}$, and \mathbf{Q} is a hermitian matrix.

The transmit power of all antennas and the k th antenna at transmitter is given by

$$\begin{aligned} P_s &= \text{Tr}(\mathbf{Q} + \mathbf{V}) \\ P_n &= [\mathbf{Q}]_{(k,k)} + [\mathbf{V}]_{(k,k)}, \quad k \in \mathcal{K} \triangleq 1, \dots, K. \end{aligned}$$

The harvested energy at the l th energy receiver is written as

$$E_l = \mathbf{h}_l^H (\mathbf{Q} + \mathbf{V}) \mathbf{h}_l, \quad l \in \mathcal{L}.$$

- Problem III: In case of perfect CSI of all channels, the secrecy rate maximization problem was given in [6] as follows.

$$\begin{aligned} & \max_{\mathbf{Q}, \mathbf{V}} \left\{ \log \left(1 + \frac{\mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s}{\mathbf{h}_s^H \mathbf{V} \mathbf{h}_s + \sigma_s^2} \right) - \max_i \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i}{\mathbf{g}_i^H \mathbf{V} \mathbf{g}_i + \sigma_i^2} \right) \right\}, \\ & \text{s. t. } \mathbf{Q} \succeq \mathbf{0}, \quad \mathbf{V} \succeq \mathbf{0}, \\ & \text{Tr}(\mathbf{Q} + \mathbf{V}) \leq P, \\ & \text{Tr}[\mathbf{e}_k \mathbf{e}_k^H (\mathbf{Q} + \mathbf{V})] \leq P_k, \quad k \in \mathcal{K}, \\ & \mathbf{h}_l^H (\mathbf{Q} + \mathbf{V}) \mathbf{h}_l \geq E, \quad l \in \mathcal{L}, \end{aligned} \quad (3.4)$$

where \mathbf{e}_k is a unit vector with the k th entry being one.

- Problem IV: Consider the scenario with spherical CSI uncertainties of all channels. The channel models are formulated as (3.2).

Taking into account the channel uncertainty models, the authors in [6] formulated the secrecy rate maximization problem as follows.

$$\begin{aligned}
& \max_{\mathbf{Q}, \mathbf{V}} \left\{ \min_{\mathbf{e}_s} \log \left(1 + \frac{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{Q} (\bar{\mathbf{h}}_s + \mathbf{e}_s)}{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{V} (\bar{\mathbf{h}}_s + \mathbf{e}_s) + \sigma_s^2} \right) \right. \\
& \quad \left. - \max_i \max_{\mathbf{e}_{g_i}} \log \left(1 + \frac{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{Q} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})}{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{V} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i}) + \sigma_i^2} \right) \right\}, \\
& \text{s. t. } \text{Tr}(\mathbf{Q} + \mathbf{V}) \leq P, \\
& \quad \text{Tr}[\mathbf{e}_k \mathbf{e}_k^H (\mathbf{Q} + \mathbf{V})] \leq P_k, \quad k \in \mathcal{K}, \\
& \quad \min_{\mathbf{e}_{h_l}} \{ (\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})^H (\mathbf{Q} + \mathbf{V}) (\bar{\mathbf{h}}_l + \mathbf{e}_{h_l}) \} \geq E, \quad l \in \mathcal{L}, \\
& \quad \mathbf{Q} \succeq \mathbf{0}, \quad \mathbf{V} \succeq \mathbf{0}.
\end{aligned} \tag{3.5}$$

These aforementioned optimization problems are intractable to solve due to their highly nonconvexity, channel models and fractional form. Our main contributions lie in investigating DC programming and DCA in efficient ways to solve these problems.

3.3 DCA for solving the SRM problems in the MISO SWIPT system

3.3.1 DC programming and DCA for solving the problem (3.1)

The problem (3.1) is intractable in its original form owing to the maximization and minimization of log objective function. We first reformulate the problem as a general DC program and then develop a scheme based on general DCA to solve it.

- 1) Reformulate the problem (3.1)

To start with, by defining $\mathbf{Q} = \mathbf{w}\mathbf{w}^H$, $\log v = \log \left(1 + \frac{|\mathbf{h}_s^H \mathbf{w}|^2}{\sigma_s^2} \right)$,

$\log u = \max_i \log \left(1 + \frac{|\mathbf{g}_i^H \mathbf{w}|^2}{\sigma_i^2} \right)$ and exploiting the semidefinite relaxation tech-

nique, problem (3.1) can be equivalently transformed into

$$\min_{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v} \log u - \log v, \quad (3.6)$$

$$\text{s. t. } \mathbf{Q} \succeq \mathbf{0}, u \geq 1, v \geq 1, \quad (3.7)$$

$$\text{Tr}(\mathbf{Q}) \leq P, \quad (3.8)$$

$$\mathbf{h}_l^H \mathbf{Q} \mathbf{h}_l \geq E, l \in \mathcal{L}, \quad (3.9)$$

$$\log v \leq \log \left(1 + \frac{\mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s}{\sigma_s^2} \right), \quad (3.10)$$

$$\log u \geq \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i}{\sigma_i^2} \right), i \in \mathcal{M}, \quad (3.11)$$

$$\mathbf{Q} = \mathbf{w} \mathbf{w}^H. \quad (3.12)$$

As can be seen, the inequalities constraints (3.10) and (3.11) hold with equalities at the optimal solution. Otherwise, we can increase v and decrease u until the equalities in these constraints hold such that the objective value can be decreased. Resorting to the increase of the binary logarithm function, the constraints (3.10) and (3.11) are rewritten as

$$\begin{aligned} \sigma_s^2 + \mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s &\geq \sigma_s^2 v, \\ \sigma_i^2 + \mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i &\leq \sigma_i^2 u \end{aligned}$$

We rely on the following lemma to deal with the constraint (3.12)

Lemma 3.1. (*[74]*) *Given \mathbf{X}_0 , \mathbf{X}_1 , and \mathbf{X}_2 of appropriate dimensions, the equality $\mathbf{X}_2 = \mathbf{X}_0 \mathbf{X}_1$ is equivalent to the following constraints in additional matrix variables Y_1 and Y_2 :*

$$\left\{ \begin{array}{l} \begin{bmatrix} \mathbf{Y}_1 & \mathbf{X}_2 & \mathbf{X}_0 \\ \mathbf{X}_2^H & \mathbf{Y}_2 & \mathbf{X}_1^H \\ \mathbf{X}_0^H & \mathbf{X}_1 & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \\ \text{Tr}(\mathbf{Y}_1) - \text{Tr}(\mathbf{X}_0 \mathbf{X}_0^H) \leq 0. \end{array} \right.$$

Then, we get the reformulation of the problem (3.1) as follows.

$$\min_{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v} -\log v + \log u, \quad (3.13)$$

$$\text{s. t. } (3.7) - (3.9), \quad (3.14)$$

$$\sigma_s^2 + \mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s \geq \sigma_s^2 v, \quad (3.15)$$

$$\sigma_i^2 + \mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i \leq \sigma_i^2 u, i \in \mathcal{M}, \quad (3.16)$$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{Q} & \mathbf{w} \\ \mathbf{Q}^H & \mathbf{A}_2 & \mathbf{w} \\ \mathbf{w}^H & \mathbf{w}^H & 1 \end{bmatrix} \succeq \mathbf{0}, \quad (3.17)$$

$$\text{Tr}(\mathbf{A}_1) - \mathbf{w}^H \mathbf{w} \leq 0. \quad (3.18)$$

For convenience, we denote $\mathbf{X} = \{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v\}$.

Note that the reformulated problem is nonconvex owing to objective function and constraint (3.18). It is easy to see that the objective function and the left side of constraint (3.18) have the form as DC functions. Consequently, we propose DC decompositions for them as follows.

The objective function is expressed as

$$F_0(\mathbf{X}) = G_0(\mathbf{X}) - H_0(\mathbf{X}), \text{ where} \\ G_0(\mathbf{X}) = -\log v, \quad H_0(\mathbf{X}) = -\log u,$$

while the left side of constraint (3.18) is rewritten as

$$F_1(\mathbf{X}) = G_1(\mathbf{X}) - H_1(\mathbf{X}), \text{ where} \\ G_1(\mathbf{X}) = \text{Tr}(\mathbf{A}_1), \quad H_1(\mathbf{X}) = \mathbf{w}^H \mathbf{w}.$$

The linear approximations of $H_0(\mathbf{X})$ and $H_1(\mathbf{X})$ at the point (\mathbf{X}^k) are given by

$$\bar{H}_0(\mathbf{X}) = -\log u^k - \frac{u - u^k}{u^k \ln 2}, \\ \bar{H}_1(\mathbf{X}) = (\mathbf{w}^k)^H \mathbf{w}^k + 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H \mathbf{w}^k\}.$$

Since $G_1(\mathbf{X}) - H_1(\mathbf{X}) \leq G_1(\mathbf{X}) - \bar{H}_1(\mathbf{X})$, it can lead to infeasibility of the convex subproblems. Thus, the relaxation technique is proposed to deal with this challenge.

2) General DCA for solving the problem (3.13)

Algorithm 3.1. DCA₁ for solving the problem (3.1)

Initialization: Let (u^0, v^0, \mathbf{w}^0) be an initial point, $Rs^0 = 0$, $k \leftarrow 0$.

repeat

1. Compute $(u^{k+1}, v^{k+1}, \mathbf{w}^{k+1})$ by solving the following subproblem

$$\min_{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v} -\log v + \log u^k + \frac{u - u^k}{u^k \ln 2} + ts_1 \\ \text{s. t. (3.14) - (3.17),} \\ \text{Tr}(\mathbf{A}_1) - (\mathbf{w}^k)^H \mathbf{w}^k - 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H \mathbf{w}^k\} \leq s_1.$$

2. Update $Rs^{k+1} = -\log v^{k+1} + \log u^k + \frac{u^{k+1} - u^k}{u^k \ln 2}$.

3. $k \leftarrow k + 1$,

until $|R_s^k - R_s^{k-1}| \leq \tau_0$ and $|s_1| \leq \tau_1$.

3.3.2 DC programming and DCA for solving the problem (3.3)

In this section, imperfect CSI scenario has been considered. This scenario leads to the intractable optimization problem due to the infinitely constraints with the CSI

variations bounded in spheres. We first transform the problem into a tractable form. Then, we propose an scheme based on DC programming and DCA to deal with the transformed problem.

1) Reformulate the problem (3.3)

Let us introduce some slack variables: $\log v = \min_{\mathbf{e}_s} \log \left(1 + \frac{|(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{w}|^2}{\sigma_s^2} \right)$,

$\log u = \max_i \max_{\mathbf{e}_{g_i}} \log \left(1 + \frac{|(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{w}|^2}{\sigma_i^2} \right)$, $\mathbf{Q} = \mathbf{w}\mathbf{w}^H$.

Following the semidefinite relaxation approach and applying Lemma 3.1, we transform the problem (3.3) as follows.

$$\min_{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v} -\log v + \log u, \quad (3.19)$$

$$\text{s. t. } \mathbf{Q} \succeq \mathbf{0}, u \geq 1, v \geq 1, \quad (3.20)$$

$$\text{Tr}(\mathbf{Q}) \leq P, \quad (3.21)$$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{Q} & \mathbf{w} \\ \mathbf{Q}^H & \mathbf{A}_2 & \mathbf{w} \\ \mathbf{w}^H & \mathbf{w}^H & 1 \end{bmatrix} \succeq \mathbf{0}, \quad (3.22)$$

$$\text{Tr}(\mathbf{A}_1 - \mathbf{w}\mathbf{w}^H) \leq 0, \quad (3.23)$$

$$\min_{\mathbf{e}_{h_l}} \{(\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})^H \mathbf{Q} (\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})\} \geq E, l \in \mathcal{L}, \quad (3.24)$$

$$\log v \leq \min_{\mathbf{e}_s} \log \left(1 + \frac{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{Q} (\bar{\mathbf{h}}_s + \mathbf{e}_s)}{\sigma_s^2} \right), \quad (3.25)$$

$$\log u \geq \max_{\mathbf{e}_{g_i}} \log \left(1 + \frac{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{Q} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})}{\sigma_i^2} \right), i \in \mathcal{M}. \quad (3.26)$$

It is easy to verify that the inequality constraints (3.25), (3.26) hold with equalities at the optimal solution.

Next, we handle the CSI variations at constraints (3.24), (3.25), and (3.26) by applying S-procedure (Lemma (2.1)).

Constraints (3.24) are transformed into following linear matrix inequality with S-procedure

$$\begin{bmatrix} \alpha_l \mathbf{I}_K + \mathbf{Q} & \mathbf{Q} \bar{\mathbf{h}}_l \\ \bar{\mathbf{h}}_l^H \mathbf{Q} & \bar{\mathbf{h}}_l^H \mathbf{Q} \bar{\mathbf{h}}_l - E - \alpha_l \epsilon_{h_l}^2 \end{bmatrix} \succeq \mathbf{0}, l \in \mathcal{L}. \quad (3.27)$$

Since the binary logarithm function is increasing function, we get

$$\min_x \log(f(x)) = \log \min_x f(x), \max_x \log(f(x)) = \log \max_x f(x).$$

Thus, the constraints (3.25), (3.26) can be rewritten as

$$\log v \leq \log \min_{\mathbf{e}_s} \left(1 + \frac{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{Q} (\bar{\mathbf{h}}_s + \mathbf{e}_s)}{\sigma_s^2} \right),$$

$$\log u \geq \log \max_{\mathbf{e}_{g_i}} \left(1 + \frac{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{Q} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})}{\sigma_i^2} \right), i \in \mathcal{M}.$$

By equivalent transformations, we achieve

$$(v-1)\sigma_s^2 \leq \min_{\mathbf{e}_s} \{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{Q}(\bar{\mathbf{h}}_s + \mathbf{e}_s)\}, \quad (3.28)$$

$$(u-1)\sigma_i^2 \geq \max_{\mathbf{e}_{g_i}} \{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{Q}(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})\}, \quad i \in \mathcal{M}. \quad (3.29)$$

Constraints (3.28), (3.29) are expressed as following linear matrix inequalities with S-procedure

$$\begin{bmatrix} \alpha_s \mathbf{I}_K + \mathbf{Q} & \mathbf{Q} \bar{\mathbf{h}}_s \\ \bar{\mathbf{h}}_s^H \mathbf{Q} & \bar{\mathbf{h}}_s^H \mathbf{Q} \bar{\mathbf{h}}_s - (v-1)\sigma_s^2 - \alpha_s \epsilon_s^2 \end{bmatrix} \succeq \mathbf{0}, \quad (3.30)$$

$$\begin{bmatrix} \beta_i \mathbf{I}_K - \mathbf{Q} & -\mathbf{Q} \bar{\mathbf{g}}_i \\ -\bar{\mathbf{g}}_i^H \mathbf{Q} & -\bar{\mathbf{g}}_i^H \mathbf{Q} \bar{\mathbf{g}}_i + (u-1)\sigma_i^2 - \beta_i \epsilon_{g_i}^2 \end{bmatrix} \succeq \mathbf{0}, \quad i \in \mathcal{M}. \quad (3.31)$$

In summary, we obtain transformation of the problem as following (3.3)

$$\min_{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v} -\log v + \log u, \quad (3.32)$$

$$\text{s. t. (3.20) - (3.22), (3.27), (3.30), (3.31),} \quad (3.33)$$

$$\text{Tr}(\mathbf{A}_1 - \mathbf{w}\mathbf{w}^H) \leq 0. \quad (3.34)$$

This transformed problem is still challenging due to nonconvex objective function and nonconvex constraint (3.27). We propose similar DC decompositions for the objective function and the left side of constraint (3.27) to that in subsection 3.3.1.

2) General DCA for solving the problem (3.32)

Algorithm 3.2. DCA₂ for solving the problem (3.32)

Initialization: Let (u^0, v^0, \mathbf{w}^0) be an initial point, $R_s^0 = 0$, $k \leftarrow 0$.

repeat

1. Compute $(u^{k+1}, v^{k+1}, \mathbf{w}^{k+1})$ by solving the following subproblem

$$\min_{\mathbf{Q}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{w}, u, v} -\log v + \log u^k + \frac{u - u^k}{u^k \ln 2} + t s_1$$

$$\text{s. t. (3.20) - (3.22), (3.27), (3.30), (3.31),}$$

$$\text{Tr}(\mathbf{A}_1) - (\mathbf{w}^k)^H \mathbf{w}^k - 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H \mathbf{w}^k\} \leq s_1.$$

2. Update $R_s^{k+1} = -\log v^{k+1} + \log u^k + \frac{u^{k+1} - u^k}{u^k \ln 2}$.

3. $k \leftarrow k + 1$,

until $|R_s^k - R_s^{k-1}| \leq \tau_0$ and $|s_1| \leq \tau_1$.

3.3.3 DC programming and DCA for solving the problem (3.4)

The problem (3.4) is very difficult to solve directly. We first devote to convert this problem as a general DC program and then propose an algorithm based on general DCA to solve it.

1) Reformulate the problem (3.4)

$$\text{Let us introduce some slack variables: } \log v = \log \left(1 + \frac{\mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s}{\mathbf{h}_s^H \mathbf{V} \mathbf{h}_s + \sigma_s^2} \right),$$

$$\log u = \max_i \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i}{\mathbf{g}_i^H \mathbf{V} \mathbf{g}_i + \sigma_i^2} \right).$$

By exploiting semidefinite relaxation approach, the problem (3.4) can be reformulated as

$$\begin{aligned} & \min_{\mathbf{Q}, \mathbf{V}, u, v} -\log v + \log u, \\ \text{s. t. } & \mathbf{Q} \succeq \mathbf{0}, \mathbf{V} \succeq \mathbf{0}, u \geq 1, v \geq 1, \end{aligned} \quad (3.35)$$

$$\text{Tr}(\mathbf{Q} + \mathbf{V}) \leq P, \quad (3.36)$$

$$\text{Tr}[\mathbf{e}_k \mathbf{e}_k^H (\mathbf{Q} + \mathbf{V})] \leq P_k, k \in \mathcal{K}, \quad (3.37)$$

$$\mathbf{h}_l^H (\mathbf{Q} + \mathbf{V}) \mathbf{h}_l \geq E, l \in \mathcal{L}, \quad (3.38)$$

$$\log v \leq \log \left(1 + \frac{\mathbf{h}_s^H \mathbf{Q} \mathbf{h}_s}{\mathbf{h}_s^H \mathbf{V} \mathbf{h}_s + \sigma_s^2} \right), \quad (3.39)$$

$$\log u \geq \log \left(1 + \frac{\mathbf{g}_i^H \mathbf{Q} \mathbf{g}_i}{\mathbf{g}_i^H \mathbf{V} \mathbf{g}_i + \sigma_i^2} \right), i \in \mathcal{M}. \quad (3.40)$$

At the optimal solution, constraints (3.39), (3.40) must hold with equalities, otherwise, we can decrease the value of objective function by decreasing u and increasing v until the equality happens.

We rewrite the aforementioned problem based on the increase of the binary logarithm function as follows.

$$\begin{aligned} & \min_{\mathbf{Q}, \mathbf{V}, u, v, x} -\log v + \log u, \end{aligned} \quad (3.41)$$

$$\text{s. t. (3.35) - (3.38),}$$

$$\mathbf{h}_s^H (\mathbf{Q} - (v - 1)\mathbf{V}) \mathbf{h}_s \geq (v - 1)\sigma_s^2, \quad (3.42)$$

$$\mathbf{g}_i^H (\mathbf{Q} - (u - 1)\mathbf{V}) \mathbf{g}_i \leq (u - 1)\sigma_i^2, i \in \mathcal{M}. \quad (3.43)$$

The problem (3.41) is difficult to solve due to DC constraints (3.42) and (3.43). We first transform these constraints by resorting to Lemma 3.1. To proceed, let us define $\mathbf{V}_1 = (v - 1)\mathbf{V}$, $\mathbf{V}_2 = (u - 1)\mathbf{V}$. The problem (3.41) is converted as follows

$$\min_{\substack{\mathbf{Q}, \mathbf{V}, \mathbf{V}_1, \mathbf{V}_2, \mathbf{A}_1, \\ \mathbf{A}_2, \mathbf{B}_1, \mathbf{B}_2, u, v}} -\log v + \log u, \quad (3.44)$$

$$\text{s. t. (3.35) - (3.38),} \quad (3.45)$$

$$\mathbf{h}_s^H (\mathbf{Q} - \mathbf{V}_1) \mathbf{h}_s \geq (v - 1) \sigma_s^2, \quad (3.46)$$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{V}_1 & (v - 1) \mathbf{I}_K \\ \mathbf{V}_1^H & \mathbf{A}_2 & \mathbf{V}^H \\ (v - 1) \mathbf{I}_K & \mathbf{V} & \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \quad (3.47)$$

$$\mathbf{g}_i^H (\mathbf{Q} - \mathbf{V}_2) \mathbf{g}_i \leq (u - 1) \sigma_i^2, \quad i \in \mathcal{M}, \quad (3.48)$$

$$\begin{bmatrix} \mathbf{B}_1 & \mathbf{V}_2 & (u - 1) \mathbf{I}_K \\ \mathbf{V}_2^H & \mathbf{B}_2 & \mathbf{V}^H \\ (u - 1) \mathbf{I}_K & \mathbf{V} & \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \quad (3.49)$$

$$\text{Tr}(\mathbf{A}_1) - \text{Tr}((v - 1)^2 \mathbf{I}_K) \leq 0, \quad (3.50)$$

$$\text{Tr}(\mathbf{B}_1) - \text{Tr}((u - 1)^2 \mathbf{I}_K) \leq 0. \quad (3.51)$$

Here, the problem (3.44) is still nonconvex due to objective functions, constraints (3.50) and (3.51).

For convenience, we denote $\mathbf{X} = \{\mathbf{Q}, \mathbf{V}, \mathbf{V}_1, \mathbf{V}_2, \mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1, \mathbf{B}_2, u, v\}$.

Note that the objective function and the left sides of these constraints are DC functions. Hence, DC decompositions for them are expressed as follows. The objective function is expressed as

$$F_2(\mathbf{X}) = G_2(\mathbf{X}) - H_2(\mathbf{X}), \quad \text{where} \\ G_2(\mathbf{X}) = -\log v, \quad H_2(\mathbf{X}) = -\log u,$$

while DC decomposition for the left side of constraint (3.50) is proposed as

$$F_3(\mathbf{X}) = G_3(\mathbf{X}) - H_3(\mathbf{X}), \quad \text{where} \\ G_3(\mathbf{X}) = \text{Tr}(\mathbf{A}_1), \quad H_3(\mathbf{X}) = K(v - 1)^2,$$

and DC decomposition for the left side of constraint (3.51) can be

$$F_4(\mathbf{X}) = G_4(\mathbf{X}) - H_4(\mathbf{X}), \quad \text{where} \\ G_4(\mathbf{X}) = \text{Tr}(\mathbf{B}_1), \quad H_4(\mathbf{X}) = K(u - 1)^2.$$

The linear approximations of $H_2(\mathbf{X})$, $H_3(\mathbf{X})$, and $H_4(\mathbf{X})$ at the point (\mathbf{X}^k) are given by

$$\begin{aligned} \overline{H}_2(\mathbf{X}) &= -\log u^k - \frac{u - u^k}{u^k \ln 2}, \\ \overline{H}_3(\mathbf{X}) &= K(v^{(k)} - 1)^2 + 2K(v^{(k)} - 1)(v - v^{(k)}), \\ \overline{H}_4(\mathbf{X}) &= K(u^{(k)} - 1)^2 + 2K(u^{(k)} - 1)(u - u^{(k)}). \end{aligned}$$

Since $G_3(\mathbf{X}) - H_3(\mathbf{X}) \leq G_3(\mathbf{X}) - \overline{H}_3(\mathbf{X})$ and $G_4(\mathbf{X}) - H_4(\mathbf{X}) \leq G_4(\mathbf{X}) - \overline{H}_4(\mathbf{X})$, it can lead to infeasibility of the convex subproblems. Thus, the relaxation technique is proposed to deal with this challenge. Based on above analysis, we develop a scheme based on general DCA as follows.

2) General DCA for solving the problem (3.44)

Algorithm 3.3. DCA₃ for solving the problem (3.44)

Initialization: Let (u^0, v^0) be an initial point, $Rs^0 = 0$, $k \leftarrow 0$.
repeat

1. Compute (u^{k+1}, v^{k+1}) by solving the following subproblem

$$\begin{aligned} & \min_{\substack{\mathbf{Q}, \mathbf{V}, \mathbf{V}_1, \mathbf{V}_2, \mathbf{A}_1, \\ \mathbf{A}_2, \mathbf{B}_1, \mathbf{B}_2, u, v}} -\log v + \log u^k + \frac{u - u^k}{u^k \ln 2} + ts_2 \\ \text{s. t. } & (3.35) - (3.38), (3.46) - (3.49), \\ & \text{Tr}(\mathbf{B}\mathbf{1}) - K(u^{(k)} - 1)^2 - 2K(u^{(k)} - 1)(u - u^{(k)}) \leq s_2, \\ & \text{Tr}(\mathbf{A}\mathbf{1}) - K(v^{(k)} - 1)^2 - 2K(v^{(k)} - 1)(v - v^{(k)}) \leq s_2. \end{aligned}$$

2. Update $Rs^{k+1} = -\log v^{k+1} + \log u^k + \frac{u^{k+1} - u^k}{u^k \ln 2}$.

3. $k \leftarrow k + 1$,

until $|R_s^k - R_s^{k-1}| \leq \tau_0$ and $|s_2| \leq \tau_2$.

3.3.4 DC programming and DCA for solving the problem (3.5)

Since the problem (3.5) is very complicated due to its fractional form and CSI uncertainty models, we transform this problem into a DC program. Then, an algorithm based on general DCA is presented to solve this DC program.

1) Reformulate the problem (3.5) as a DC program

We begin with defining some additional variables as follows.

$$\begin{aligned} \log v &= \min_{e_s} \log \left(1 + \frac{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{Q} (\bar{\mathbf{h}}_s + \mathbf{e}_s)}{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{V} (\bar{\mathbf{h}}_s + \mathbf{e}_s) + \sigma_s^2} \right), \\ \log u &= \max_i \max_{e_{g_i}} \log \left(1 + \frac{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{Q} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})}{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{V} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i}) + \sigma_i^2} \right), \quad i \in \mathcal{M}. \end{aligned}$$

By using semidefinite relaxation approach, the problem (3.5) is transformed into

$$\min_{\mathbf{Q}, \mathbf{V}, u, v} -\log v + \log u, \quad (3.52)$$

$$\text{s. t. } \text{Tr}(\mathbf{Q} + \mathbf{V}) \leq P, \quad (3.53)$$

$$\text{Tr}[\mathbf{e}_k \mathbf{e}_k^H (\mathbf{Q} + \mathbf{V})] \leq P_k, \quad k \in \mathcal{K}, \quad (3.54)$$

$$\mathbf{Q} \succeq \mathbf{0}, \quad \mathbf{V} \succeq \mathbf{0}, \quad u \geq 1, \quad v \geq 1, \quad (3.55)$$

$$\min_{\mathbf{e}_{h_l}} \{(\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})^H (\mathbf{Q} + \mathbf{V}) (\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})\} \geq E, \quad l \in \mathcal{L}, \quad (3.56)$$

$$\log v \leq \min_{\mathbf{e}_s} \log \left(1 + \frac{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{Q} (\bar{\mathbf{h}}_s + \mathbf{e}_s)}{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H \mathbf{V} (\bar{\mathbf{h}}_s + \mathbf{e}_s) + \sigma_s^2} \right), \quad (3.57)$$

$$\log u \geq \max_{\mathbf{e}_{g_i}} \log \left(1 + \frac{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{Q} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})}{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H \mathbf{V} (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i}) + \sigma_i^2} \right), \quad i \in \mathcal{M}. \quad (3.58)$$

It is easy to verify that the relaxed inequality constraint (3.57) and (3.58) must hold with equalities at the optimum.

Since the binary logarithm function is an increase function, the transformed problem can be rewritten as

$$\min_{\mathbf{Q}, \mathbf{V}, u, v} -\log v + \log u,$$

$$\text{s. t. } (3.53) - (3.55),$$

$$\min_{\mathbf{e}_{h_l}} \{(\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})^H (\mathbf{Q} + \mathbf{V}) (\bar{\mathbf{h}}_l + \mathbf{e}_{h_l})\} \geq E, \quad l \in \mathcal{L},$$

$$\min_{\mathbf{e}_s} \{(\bar{\mathbf{h}}_s + \mathbf{e}_s)^H (\mathbf{Q} - (v-1)\mathbf{V}) (\bar{\mathbf{h}}_s + \mathbf{e}_s)\} \geq (v-1)\sigma_s^2,$$

$$\max_{\mathbf{e}_{g_i}} \{(\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})^H (\mathbf{Q} - (u-1)\mathbf{V}) (\bar{\mathbf{g}}_i + \mathbf{e}_{g_i})\} \leq (u-1)\sigma_i^2, \quad i \in \mathcal{M}.$$

Let us define $\mathbf{V}_1 = (v-1)\mathbf{V}$, $\mathbf{V}_2 = (u-1)\mathbf{V}$. By applying Lemma 3.1 and Lemma 2.1, the aforementioned problem is transformed into

$$\min_{\substack{\mathbf{Q}, \mathbf{V}_1, \mathbf{V}_2, \mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1, \\ \mathbf{B}_2, u, v, \beta_i, \alpha_s, \alpha_l}} -\log v + \log u, \quad (3.59)$$

$$\text{s. t. } (3.53) - (3.55), \quad (3.60)$$

$$\begin{bmatrix} \alpha_l \mathbf{I}_K + \mathbf{Q} + \mathbf{V} & (\mathbf{Q} + \mathbf{V}) \bar{\mathbf{h}}_l \\ \bar{\mathbf{h}}_l^H (\mathbf{Q} + \mathbf{V}) & \bar{\mathbf{h}}_l^H (\mathbf{Q} + \mathbf{V}) \bar{\mathbf{h}}_l - E - \alpha_l \epsilon_{h_l}^2 \end{bmatrix} \succeq \mathbf{0}, \quad l \in \mathcal{L}, \quad (3.61)$$

$$\begin{bmatrix} \alpha_s \mathbf{I}_K + (\mathbf{Q} - \mathbf{V}_1) & (\mathbf{Q} - \mathbf{V}_1) \bar{\mathbf{h}}_s \\ \bar{\mathbf{h}}_s^H (\mathbf{Q} - \mathbf{V}_1) & \bar{\mathbf{h}}_s^H (\mathbf{Q} - \mathbf{V}_1) \bar{\mathbf{h}}_s - (v-1)\sigma_s^2 - \alpha_s \epsilon_s^2 \end{bmatrix} \succeq \mathbf{0}, \quad (3.62)$$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{V}_1 & (v-1)\mathbf{I}_K \\ \mathbf{V}_1^H & \mathbf{A}_2 & \mathbf{V}^H \\ (v-1)\mathbf{I}_K & \mathbf{V} & \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \quad (3.63)$$

$$\begin{bmatrix} \beta_i \mathbf{I}_K - (\mathbf{Q} - \mathbf{V}_2) & -(\mathbf{Q} - \mathbf{V}_2) \bar{\mathbf{g}}_i \\ -\bar{\mathbf{g}}_i^H (\mathbf{Q} - \mathbf{V}_2) & -\bar{\mathbf{g}}_i^H (\mathbf{Q} - \mathbf{V}_2) \bar{\mathbf{g}}_i + (u-1)\sigma_i^2 - \beta_i \epsilon_{g_i}^2 \end{bmatrix} \succeq \mathbf{0}, \quad i \in \mathcal{M}, \quad (3.64)$$

$$\begin{bmatrix} \mathbf{B}_1 & \mathbf{V}_2 & (u-1)\mathbf{I}_K \\ \mathbf{V}_2^H & \mathbf{B}_2 & \mathbf{V}^H \\ (u-1)\mathbf{I}_K & \mathbf{V} & \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \quad (3.65)$$

$$\text{Tr}(\mathbf{A}\mathbf{1}) - \text{Tr}((v-1)^2 \mathbf{I}_K) \leq 0, \quad (3.66)$$

$$\text{Tr}(\mathbf{B}\mathbf{1}) - \text{Tr}((u-1)^2 \mathbf{I}_K) \leq 0. \quad (3.67)$$

The problem is still challenging due to objective function and constraints (3.66)-(3.67). DC decompositions for the objective functions and the left sides of these constraints can be taken as those in Subsection 3.3.3.

2) General DCA for solving the problem (3.59)

Algorithm 3.4. DCA₄ for solving the problem (3.59)

Initialization: Let (u^0, v^0) be an initial point, $Rs^0 = 0$, $k \leftarrow 0$.

repeat

1. Compute (u^{k+1}, v^{k+1}) by solving the following subproblem

$$\begin{aligned} & \min_{\substack{\mathbf{Q}, \mathbf{V}, \mathbf{V}_1, \mathbf{V}_2, \mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1, \\ \mathbf{B}_2, u, v, \beta_i, \alpha_s, \alpha_l}} -\log v + \log u^k + \frac{u - u^k}{u^k \ln 2} + ts_2 \\ \text{s. t. } & (3.53) - (3.55), (3.61) - (3.65), \\ & \text{Tr}(\mathbf{B}\mathbf{1}) - K(u^{(k)} - 1)^2 - 2K(u^{(k)} - 1)(u - u^{(k)}) \leq s_2, \\ & \text{Tr}(\mathbf{A}\mathbf{1}) - K(v^{(k)} - 1)^2 - 2K(v^{(k)} - 1)(v - v^{(k)}) \leq s_2. \end{aligned}$$

2. Update $Rs^{k+1} = -\log v^{k+1} + \log u^k + \frac{u^{k+1} - u^k}{u^k \ln 2}$.

3. $k \leftarrow k + 1$,

until $|R_s^k - R_s^{k-1}| \leq \tau_0$ and $|s_2| \leq \tau_2$.

3.4 Numerical experiments

In this section, we study the performance of the proposed algorithms by comparing them with the existing algorithms (BS, 1D and SCA).

Experiment settings

All algorithms were implemented in MATLAB R2017b, and executed on a PC Intel(R) Core(TM)i5-2450M CPU 2.50 GHz of 4GB RAM. The CVX toolbox [10] (based on sedumi solver) is used to solve the subproblems.

We follow the experiment setting in [6]. All the channel vectors are modeled by large-scale fading and small scale fading. The simplified large-scale fading model can be expressed as $D = A_0 \left(\frac{d}{d_r}\right)^{-\alpha}$ where α denotes the path-loss exponent, A_0 is a constant set to be 1, the reference distance is $d_r = 20$ m while the distance d between the transmitter and the information receiver, the eavesdroppers, and the energy receivers are 100 m, 50 m and 25 m, respectively. For the small-scale fading, the Rician factor is assumed to be 5. We set simulation parameter as follows: $\alpha = 3$, $K = 4$, $L = 2$, $M = 3$, $P = 20$ dBm, $P_k = \frac{P}{K}$, $E = 1$ mW unless otherwise specified. For simplicity, we assume that the noise covariances $\sigma_s^2 = \sigma_i^2 = -40$ dBm and the error bounds $\epsilon = \epsilon_s = \epsilon_{g_i} = \epsilon_{h_l} = 0.01$, $i \in \mathcal{M}$, $l \in \mathcal{L}$.

We examine the impact of system parameters on secrecy rate (the transmit power in Table 3.1-3.3, the distance between transmitter and information receiver in Table 3.4, the distance between transmitter and information receiver and error bounds in Table 3.5, and the distance between the transmitter and energy receivers in Table 3.6). Then, we compare the performance of our schemes with the existing approaches. All algorithms are tested on 10 dependent channel realizations. For each value of these parameters, the average values of the secrecy rate (SSR) and CPU time in seconds (CPU(s)) are reported. We set $\tau_0 = 10^{-3}$, $\tau_1 = 7e - 06$ and $\tau_2 = 1e - 10$.

Comments on numerical results

- *Secrecy rate versus the transmit power budget P* : From Table 3.1-3.3, we have the following observations: firstly, the secrecy rate increases as the transmit power increases. Secondly, our proposed algorithms yield the best rate performance among all the existing methods on both quality and rapidity.

For the case of employing beamforming without artificial noise, DCA₁ and DCA₂ are much more efficient than BS. Regarding the quality of solutions: the gain of DCA₁ versus BS and DCA₂ versus BS varies from 59.6% to 125.3% and from 64.6% to 95.3%, respectively. Concerning CPU time: DCA₁ and DCA₂ are faster than BS - the ratio of gain varies from 6.0 to 18.4 times and from 3.3 to 7.6 times, respectively.

For the case of joint beamforming and artificial noise, we observe that DCA₃ outperforms 1D and SCA on both secrecy rate and CPU time. In terms of secrecy rate: the gain of DCA₃ versus 1D, SCA varies from 14.5% to 37.4%, 14.8% to 37.5%, respectively. It can be seen that DCA₃ is faster than 1D and SCA.

- *Secrecy rate versus distance between the transmitter and the information receiver d_s* for joint beamforming and artificial noise strategy: Table 3.4–3.5 present the secrecy rate under different d_s in case of perfect and imperfect CSI. Clearly, the increase of d_s causes the secrecy rates decreasing. In addition, the secrecy rates of all methods decrease along with the increase of the error bounds. These tables show the performance superiority of DCA₃ and DCA₄ as compared with 1D and SCA on both secrecy rate and CPU time. In terms of secrecy rate: the gain of DCA₄ versus 1D, SCA varies from 5.5% to 50.6%, 14.5% to 70.5%, respectively.

Both DCA₃ and DCA₄ run faster than 1D and SCA. In terms of CPU time, the ratio of gain of DCA₄ versus 1D, SCA varies from 3.4 to 3.8 times, 5.7 to 8.3 times, respectively.

- *Impacts of distance between the transmitter and the energy receivers d_k* : Table 3.6 shows that the harvested energy declines with the increasing of d_k . In comparison, DCA₃ has an advantage in gaining the harvested energy over 1D and SCA. Harvested energy achieved from DCA₃ always exceeds that from 1D, SCA – the gain varies from 290.8% to 323.1%, 311.0% to 332.9%, respectively. DCA₃ still gives beneficial in terms of CPU time as compared to 1D and SCA.

In summary, the proposed algorithms achieve the best performance as compared with the existing methods.

Table 3.1 – Secrecy rate with different transmit power in case of perfect CSI and without artificial noise

$P(\text{dBm})$	DCA ₁		BS	
	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)
16	4.7634	3.6731	2.9839	25.9343
18	5.2706	4.6272	3.0046	27.6652
20	5.7859	3.1579	3.0065	30.6847
22	6.3152	2.9425	3.0193	54.0179
24	6.8634	3.3926	3.0466	35.0581

Table 3.2 – Secrecy rate with different transmit power in case of imperfect CSI and without artificial noise

$P(\text{dBm})$	DCA ₂		BS	
	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)
16	4.4763	11.1201	2.7197	84.6727
18	4.8922	11.5454	2.8842	74.8979
20	5.3331	13.0667	3.0301	61.9731
22	5.7661	12.7569	3.1127	54.0179
24	6.1994	15.0703	3.1741	50.1447

Table 3.3 – Artificial noise assisted secrecy rate with different transmit power values in case of perfect CSI

$P(\text{dBm})$	DCA ₃		1D		SCA	
	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)
16	6.1304	2.7112	4.4617	10.1708	4.4577	10.6652
18	6.4932	2.8607	4.9854	9.7739	4.9996	10.0329
20	6.7829	2.6606	5.5432	10.8771	5.5487	10.4409
22	7.0240	2.7296	6.1321	10.0770	6.1206	7.6046

Table 3.4 – Secrecy rate with distance between the transmitter and the information receiver in case of perfect CSI and artificial noise-aided

d_s (m)	DCA ₃		1D		SCA	
	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)
50	6.7829	2.6606	5.5432	10.8771	5.5487	10.4409
60	6.4932	2.7620	4.7739	10.5195	4.7481	11.1566
70	6.1304	2.8152	4.1341	10.4050	4.1223	9.8345
80	5.9080	2.6864	3.5916	10.6998	3.5870	10.5119

Table 3.5 – Secrecy rate with distance between the transmitter and the information receiver in case of imperfect CSI

$\epsilon = 0.02$						
d_s (m)	DCA ₄		1D		SCA	
	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)
60	3.8928	2.8807	3.6909	11.4424	3.3985	16.8179
70	3.7300	2.6067	2.9948	11.6568	2.7083	18.4346
80	3.6165	2.7010	2.4021	11.3656	2.1206	20.8313
$\epsilon = 0.03$						
d_s (m)	DCA ₄		1D		SCA	
	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)	SSR(bps/Hz)	CPU(s)
60	3.6739	3.4440	3.1251	11.7170	2.8658	19.5473
70	3.5865	2.5468	2.3904	9.7158	2.1217	20.7583
80	3.4472	2.6020	1.7767	9.9503	1.5010	21.6872

Table 3.6 – Harvested energy with different distances between the transmitter and the energy receivers in case of perfect CSI and artificial noise-aided

d_k (m)		DCA ₃	1D	SCA
5	E(mW)	4131.4060	1057.0970	964.8695
	SSR(bps/Hz)	5.4710	5.3615	5.3447
	CPU(s)	2.8626	12.3821	10.0414
10	E(mW)	512.8800	119.5980	124.7852
	SSR(bps/Hz)	5.4672	5.3615	5.3443
	CPU(s)	2.5927	12.1280	8.3111
15	E(mW)	150.4268	35.55502	34.7510
	SSR(bps/Hz)	5.4672	5.3615	5.3577
	CPU(s)	2.5031	12.0051	11.1713
20	E(mW)	63.4590	14.9874	14.9691
	SSR(bps/Hz)	5.4673	5.3615	5.3469
	CPU(s)	2.4859	12.0989	11.3256
25	E(mW)	32.4105	7.6838	7.4284
	SSR(bps/Hz)	5.4672	5.3615	5.3596
	CPU(s)	2.4781	12.0789	11.0122

3.5 Conclusion

We have investigated DC programming and DCA to solve the SRM problems for secure MISO SWIPT system where scenarios of perfect and imperfect CSI knowledge have been considered. Both transmit beamforming without artificial noise and that with artificial noise have been studied. The considered problems are intractable due to their fraction form and CSI uncertainty models. By utilizing semidefinite technique and S-procedure, we have devoted to reformulate them as general DC programs. The efficient schemes based on general DCA have been developed to solve these DC programs. The numerical results have demonstrated the efficiency and the superiority of our proposed approaches versus the existing algorithms on both quality and rapidity.

Chapter 4

A DCA-Based Approach for Probability Constrained Robust Secure Power-Splitting SWIPT MISO System

Abstract: This chapter studies a probability constrained secrecy rate optimization problem based on the assumption of statistical channel state information to eavesdroppers and imperfect channel coefficients to legitimate users in a multiple-input single-output multiuser system. Simultaneous wireless information and power transfer with the power splitting scheme is studied. The objective is to maximize the minimal secrecy rate while guaranteeing constraints of total transmit power, energy harvesting and secrecy rate probability through a robust beamforming design. The presence of the probabilistic constraint and channel uncertainty models make the problem nonconvex, thus, difficult to solve. The difficulty can be overcome based on DC programming and DCA, an innovative approach in nonconvex programming. Exploiting the special structure of the problem, we reformulate the considered problem as a general DC program. A highly efficient algorithm, namely, alternating DCA- ρ is developed to solve the resulting DC program. The convergence property of alternating DCA- ρ is proven. The computational results confirm the efficiency and superiority of the proposed approach over the existing algorithm.

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1. The material of this chapter is developed from the following work:
[1]. Phuong Anh Nguyen, Hoai An Le Thi. *A DCA-Based Approach for Outage Constrained Robust Secure Power-Splitting SWIPT MISO System*. In: Le Thi H. A., Le H. M., Pham Dinh T. (eds) *Optimization of Complex Systems: Theory, Models, Algorithms and Applications*. WCGO 2019. *Advances in Intelligent Systems and Computing*, vol 991, pp. 289–298, Springer, Cham.

4.1 Introduction

Simultaneous wireless information and power transfer (SWIPT) technique is efficient to mitigate energy scarcity [20]. The secrecy rate is degraded in SWIPT system since radio-frequency signals carry not only information but also energy [47]. In addition, energy receivers that may be potential eavesdroppers are normally located closer to the transmitter than information receiver [56]. In this case, the secrecy rate could down to zero and the transmission from the transmitter to the receiver becomes insecure [107, 106]. Physical layer security has been emerging as a promising mean to secure communication. The challenge to physical layer security is that transmitter needs to know channel state information (CSI) of all links. In practice, it is hard to gain perfect CSI, especially of eavesdroppers. The works [80, 6, 88, 8] considered robust secure transmission based on deterministic norm-bounded uncertainty model of all links for SWIPT systems. These works correspond to the active eavesdropping scenario. In case of passive eavesdropping, the channel error norm-bound cannot be gained exactly. The concept of probability is an effective approach in such case.

The problems with probability constraints in secure SWIPT systems such as transmit power minimization [87, 22, 19], secrecy rate maximization [59, 49, 4] are nonconvex. Thus, it is hard to solve them. To overcome the nonconvexity, the semi-definite relaxation technique was applied in [87, 22, 19] where a convex semi-definite programming approximation could be directly generated. In addition, Chen et al. transformed the probability constrained secrecy rate maximization problem in [4] into tractable formulation. Then, the authors proposed successive convex approximation (SCA) algorithm to solve the transformed problem. However, this transformed problem contain DC exponential constraints that should be avoided. SCA algorithm is a special version of DCA.

It is well known that DCA is a powerful approach in nonconvex programming framework (see e.g. [41]). There are as many DCA as there are DC decompositions. Finding a suitable DC decomposition is the key issue which plays a crucial role in determining the efficiency of resulting DCA (the speed of convergence, robustness, globality of optimal solutions). Motivated by this flexibility property of DCA, we propose a new DC formulation for the considered problem in [4] and develop an efficient scheme based on DCA for solving it.

Our contributions are multiple. Firstly, we explore and exploit the special structure of the considered problem in a suitable way to transform it into a tractable form. S-procedure is applied to deal with the uncertainty channel models. Secondly, one of the major contribution of the chapter is to propose an extension of alternating DCA, namely, alternating DCA- ρ scheme. Since it is hard to optimize all variables at the same time, a natural idea is to decouple the variables into two variable subsets and then apply the alternating optimization procedure. At each iteratively step, the variable subsets are alternately fixed and we solve the considered nonconvex problem respect to the remaining variable subset. Two general DC programs are proposed for two nonconvex subproblems at each iteratively step. In general, DCA scheme requires

computing a parameter ρ to guarantee the DC components (g and h) convex. In practice, it is difficult to efficiently compute the parameter ρ . Hence, it is usually estimated by a quite large value that could deteriorate the performance of DCA. Therefore, we develop general DCA- ρ scheme to solve the general DC programs at each iteratively step, in which ρ is updated at each iteration and the convexity of h is not necessary. We prove that the convergence of alternating DCA- ρ is guaranteed. Numerical results validate the efficiency and superiority of our method as compared to the existing method (SCA).

The rest of this chapter is organized as follows. The system model is described in Section 4.2. Section 4.3 shows an efficient approach based on alternating DCA- ρ to solve the considered optimization problem. Section 4.4 provides numerical experiments and finally, Section 4.5 concludes the chapter.

4.2 System model

In this section, we briefly describe the optimization problem formulated in [4].

Consider a MISO SWIPT network with K transmission links. Each link consists of one transmitter, one legitimate receiver in presence of one eavesdropper. All transmitters are equipped with N antennas while all legitimate receivers and eavesdroppers have single antenna. The channel coefficients from j th transmitter to k th legitimate user and k th eavesdropper are denoted by $\mathbf{h}_{jk} \in \mathbb{C}^{N \times 1}$ and $\mathbf{g}_{jk} \in \mathbb{C}^{N \times 1}$, $j, k \in \mathcal{K} \triangleq 1, \dots, K$, respectively. The received signal at the k th intended receiver and the k th eavesdropper can be expressed as

$$\begin{aligned} y_{Ik} &= \mathbf{h}_{kk}^H \mathbf{w}_k x_k + \sum_{j \neq k} \mathbf{h}_{jk}^H \mathbf{w}_j x_j + n_{1k}, \\ y_{Ek} &= \mathbf{g}_{kk}^H \mathbf{w}_k x_k + \sum_{j \neq k} \mathbf{g}_{jk}^H \mathbf{w}_j x_j + n_{3k}, \end{aligned}$$

It is assumed that intended receivers employ the power splitting scheme to simultaneously information decoding and energy harvesting. The received signal y_{Ik} at k th legitimate receiver is split by a power splitter $\rho_k \in (0, 1)$. The information decoding at k th intended receiver is given as

$$y_{Ik}^{ID} = \sqrt{\rho_k} (\mathbf{h}_{kk}^H \mathbf{w}_k x_k + \sum_{j \neq k} \mathbf{h}_{jk}^H \mathbf{w}_j x_j + n_{1k}) + n_{2k},$$

where $x \sim \mathcal{CN}(0, 1)$ represents the transmit signal at k th transmitter while $n_{ik} \sim \mathcal{CN}(0, \sigma_{ik}^2)$, $i = 1, 2, 3$, is the additive noises. Let $\mathbf{w}_k \in \mathbb{C}^{N \times 1}$ be the transmit beamforming vector.

Suppose that CSI of k th legitimate receiver \mathbf{h}_{jk} can be obtained. The imperfect is modeled as

$$\mathbf{h}_{jk} = \bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk}, \quad \Delta \mathbf{h}_{jk}^H \mathbf{Q}_{jk} \Delta \mathbf{h}_{jk} \leq 1, \quad \mathbf{Q}_{jk} = \delta_{jk}^{-2} \mathbf{I}_N, \quad j, k \in \mathcal{K}.$$

where $\bar{\mathbf{h}}_{jk}$ is the estimate of the true CSI \mathbf{h}_{jk} while $\Delta\mathbf{h}_{jk}$ denotes the error. δ_{jk} called coefficient error. In addition, the transmitter only knows some statistical information about CSI of eavesdroppers:

$$\mathbf{g}_{jk} \sim \mathcal{CN}(0, \mathbf{G}_{jk}), \quad \mathbf{G}_{jk} \succ \mathbf{0}.$$

Accordingly, the secrecy rate $R_{sec,k}$ of the k th legitimate receiver can be formulated as

$$\begin{aligned} R_{sec,k} &= R_{I,k} - R_{E,k}, \text{ where} \\ R_{I,k} &= \log \left(1 + \frac{\rho_k \mathbf{h}_{kk}^H \mathbf{W}_k \mathbf{h}_{kk}}{\rho_k \sum_{j \neq k} \mathbf{h}_{jk}^H \mathbf{W}_j \mathbf{h}_{jk} + \rho_k \sigma_{1k}^2 + \sigma_{2k}^2} \right), \\ R_{E,k} &= \log \left(1 + \frac{\rho_k \mathbf{g}_{kk}^H \mathbf{W}_k \mathbf{g}_{kk}}{\sum_{j \neq k} \mathbf{g}_{jk}^H \mathbf{W}_j \mathbf{g}_{jk} + \sigma_{3k}^2} \right), \end{aligned}$$

where $\mathbf{W}_j = \mathbf{w}_j \mathbf{w}_j^H$, $\text{rank}(\mathbf{W}_j) = 1$, $\mathbf{W}_j \succeq \mathbf{0}$, $\mathbf{W}_j \in \mathbb{H}^{N \times N}$, $j \in \mathcal{K}$.

The harvested energy of the k th legitimate receiver and the total power consumption at k th transmitter can be given by

$$\begin{aligned} E_{EH,k} &= (1 - \rho_k) \sum_{j=1}^K \mathbf{h}_{jk}^H \mathbf{W}_j \mathbf{h}_{jk}, \\ P_k &= \sum_{j=1}^K \text{Tr}(\mathbf{W}_j). \end{aligned}$$

The considered problem is to maximize the minimal secrecy rate under constraints of harvested energy at each receiver, total power, and secrecy rate probability of each legitimate receiver. Mathematically, the problem was formulated in [4] as follows.

$$\begin{aligned} &\max \min_k R_k, \\ \text{s. t. } &(1 - \rho_k) \sum_{j=1}^K \mathbf{h}_{jk}^H \mathbf{W}_j \mathbf{h}_{jk} \geq E_k, \quad 0 < \rho_k < 1, \quad j, k \in \mathcal{K}, \\ &\sum_{j=1}^K \text{Tr}(\mathbf{W}_j) \leq P_{max}, \quad \mathbf{W}_j \succeq \mathbf{0}, \quad \mathbf{W}_j \in \mathbb{H}^{N \times N}, \quad j \in \mathcal{K}, \\ &\text{Pr}\{R_{sec,k} \geq R_k\} \geq 1 - p_k, \end{aligned}$$

The above problem can't be solved directly due to the secrecy rate probability constraint. The authors in [4] applied Bernstein equality to deal with this probabilistic

constraint. Then, the problem was reformulated as

$$\max_{\rho_k, \mathbf{W}_k, R, R_k, R_{I,k}} R, \quad (4.1a)$$

$$\text{s. t. } \sum_{j=1}^K \text{Tr}(\mathbf{W}_j) \leq P_{max}, \quad \mathbf{W}_j \succeq \mathbf{0}, \quad \mathbf{W}_j \in \mathbb{H}^{N \times N}, \quad j \in \mathcal{K}, \quad (4.1b)$$

$$R_k \geq R, \quad R \geq 0, \quad 0 < \rho_k < 1, \quad k \in \mathcal{K}, \quad (4.1c)$$

$$(1 - \rho_k) \sum_{j=1}^K \mathbf{h}_{jk}^H \mathbf{W}_j \mathbf{h}_{jk} \geq E_k, \quad j, \quad k \in \mathcal{K}, \quad (4.1d)$$

$$\log \left(1 + \frac{\rho_k \mathbf{h}_{kk}^H \mathbf{W}_k \mathbf{h}_{kk}}{\rho_k \sum_{j \neq k} \mathbf{h}_{jk}^H \mathbf{W}_j \mathbf{h}_{jk} + \rho_k \sigma_{1k}^2 + \sigma_{2k}^2} \right) \geq R_{I,k}, \quad k \in \mathcal{K}, \quad (4.1e)$$

$$-\ln p_k - \frac{(2^{R_{I,k} - R_k} - 1) \sigma_{3k}^2}{\text{Tr}(\mathbf{G}_{kk} \mathbf{W}_k)} - \sum_{j \neq k} \ln \left[1 + \frac{(2^{R_{I,k} - R_k} - 1) \text{Tr}(\mathbf{G}_{jk} \mathbf{W}_j)}{\text{Tr}(\mathbf{G}_{kk} \mathbf{W}_k)} \right] \leq 0, \quad k \in \mathcal{K}. \quad (4.1f)$$

The problem (4.1) is still challenging due to the nonconvex constraints (4.1d)-(4.1f). Our main contribution is studying an efficient approach based on DC programming and DCA for solving this problem aiming to enhance the minimal secrecy rate as much as possible.

4.3 Solution method based on DC programming and DCA

4.3.1 Alternating general DCA $_{-\rho}$ (ADCA $_{-\rho}$) algorithm

Based on standard alternating DCA in [29], we developed alternating general DCA $_{-\rho}$ algorithm for general DC program as follows.

Consider a general DC program of the form:

$$\begin{aligned} & \min_{x,y} f_0(x,y), \\ & \text{s. t. } f_i(x,y) \leq 0, \quad i = 1, \dots, m, \\ & (x,y) \in C, \end{aligned} \quad (4.2)$$

where $C \subseteq \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ is a partial convex set; $f_i : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$, $\forall i = 0, 1, \dots, m$, are partially DC functions, i.e., $f_i(x,y) = g_i(x,y) - h_i(x,y)$ with f_i , g_i , h_i being differential, g_i being partial convex function and h_i being satisfied $h_i(x^{k+1}, y^k) \geq h_i(x^k, y^k) + \langle \nabla_x h_i(x^k, y^k), x^{k+1} - x^k \rangle$ and $h_i(x^{k+1}, y^{k+1}) \geq h_i(x^{k+1}, y^k) + \langle \nabla_y h_i(x^{k+1}, y^k), y^{k+1} - y^k \rangle$ at each iteration of the convex subproblem.

ADCA $_{-\rho}$ algorithm

1. Initialization: Let (x^0, y^0) be an initial point, $k \leftarrow 0$.
2. For $k = 1, 2, \dots$, generating the sequences (x^{k+1}, y^{k+1}) by
 - (a) x^{k+1} is a solution of the convex program

$$\begin{aligned} \min_x & g_0(x, y^k) - \langle \nabla_x h_0(x^k, y^k), x \rangle, \\ \text{s. t.} & g_i(x, y^k) - h_i(x^k, y^k) - \langle \nabla_x h_i(x^k, y^k), x - x^k \rangle \leq 0, \quad \forall i = 1, \dots, m. \end{aligned} \quad (4.3)$$
 - (b) y^{k+1} is a solution of the convex program

$$\begin{aligned} \min_y & g_0(x^{k+1}, y) - \langle \nabla_y h_0(x^{k+1}, y^k), y \rangle, \\ \text{s. t.} & g_i(x^{k+1}, y) - h_i(x^{k+1}, y^k) - \langle \nabla_y h_i(x^{k+1}, y^k), y - y^k \rangle \leq 0, \quad \forall i = 1, \dots, m. \end{aligned} \quad (4.4)$$
3. If stopping criterion is met, then stop and we have (x^{k+1}, y^{k+1}) is the computed solution, otherwise, set $k = k + 1$ and go to Step 2.

The following theorem shows the convergence of ADCA- ρ algorithm.

- Theorem 4.1.** 1) Suppose that $\{w^k\} = \{x^k, y^k\}$ is a solution sequence generated by ADCA- ρ algorithm. Then we have $f_0(x^k, y^k) - f_0(x^{k+1}, y^{k+1}) \geq \frac{\rho_x}{2} \|x^{k+1} - x^k\|^2 + \frac{\rho_y}{2} \|y^{k+1} - y^k\|^2$, $\forall k = 1, 2, \dots$ where $\rho_x = \rho_x(g_0) + \lambda_{x_i}^{k+1} \min\{\rho_x(g_i)\}$ and $\rho_y = \rho_y(g_0) + \lambda_{y_i}^{k+1} \min\{\rho_y(g_i)\}$ while $\rho(g_i)$ $i = 0, \dots, m$ are the convex parameters and λ_i is the nonnegative scalars.
- 2) Let $\{x^k, y^k\}$ be a sequence of solutions generated by ADCA- ρ algorithm. If the algorithm terminates after a finite number of k iterations, (x^k, y^k) is a KKT point of the problem (4.2). If the algorithm generates an infinite sequence $\{x^k, y^k\}$, such that $\lim_{k \rightarrow \infty} \|x^k - x^{k+1}\| = 0$, $\lim_{k \rightarrow \infty} \|y^k - y^{k+1}\| = 0$ and every limit point of $\{x^k, y^k\}$ is a KKT point of the problem (4.2).

Proof. □

- 1) • Since the Karush-Kuhn-Tucker (KKT) optimality condition holds for solution (x^{k+1}) of subproblem (4.3), there exist $\lambda_{x_i}^{k+1} \geq 0$, $i = 1, \dots, m$ such that

$$\begin{cases} \sum_{i=1}^m \lambda_{x_i}^{k+1} (g_i(x^{k+1}, y^k) - h_i(x^k, y^k) - \nabla_x h_i^T(x^k, y^k)(x^{k+1} - x^k)) = 0, \\ 0 \in \nabla_x g_0(x^{k+1}, y^k) - \nabla_x h_0(x^k, y^k) + \sum_{i=1}^m \lambda_{x_i}^{k+1} (\nabla_x g_i(x^{k+1}, y^k) - \nabla_x h_i(x^k, y^k)) + N(C, x^{k+1}). \end{cases} \quad (4.5)$$

$$i = 1, \dots, m \text{ and } N(C, x^{k+1}) = \{z \in \mathbb{R}^{n_1} : \langle z, y_1 - x^{k+1} \rangle \leq 0, \forall y_1 \in \mathbb{R}^{n_1}\}.$$

- By the assumption of convexity of g_0 and the special condition for h_0 , we obtain

$$g_0(x^k, y^k) \geq g_0(x^{k+1}, y^k) + \nabla_x g_0^T(x^{k+1}, y^k)(x^k - x^{k+1}) + \frac{\rho_x(g_0)}{2} \|x^{k+1} - x^k\|^2, \quad (4.6)$$

$$h_0(x^{k+1}, y^k) \geq h_0(x^k, y^k) + \nabla_x h_0^T(x^k, y^k)(x^{k+1} - x^k), \quad (4.7)$$

where $\rho_x(g_0)$ is the convexity parameter of g_0 respecting to variable x . Combining (4.6) and (4.7), we have

$$\begin{aligned} & (\nabla_x g_0^T(x^{k+1}, y^k) - \nabla_x h_0^T(x^k, y^k))(x^k - x^{k+1}) \\ & \leq f_0(x^k, y^k) - f_0(x^{k+1}, y^k) - \frac{\rho_x(g_0)}{2} \|x^k - x^{k+1}\|^2. \end{aligned} \quad (4.8)$$

- By the assumption of convexity of g_i , $i = 1, \dots, m$ we have

$$\begin{aligned} & g_i(x^k, y^k) \\ & \geq g_i(x^{k+1}, y^k) + \nabla_x g_i^T(x^{k+1}, y^k)(x^k - x^{k+1}) + \frac{\rho_x(g_i)}{2} \|x^{k+1} - x^k\|^2, \end{aligned}$$

which further lead to

$$\begin{aligned} & (\nabla_x g_i^T(x^{k+1}, y^k) - \nabla_x h_i^T(x^k, y^k))(x^k - x^{k+1}) \\ & \leq -(g_i(x^{k+1}, y^k) - h_i(x^k, y^k) - \nabla_x h_i^T(x^k, y^k)(x^{k+1} - x^k)) \\ & \quad - \frac{\rho_x(g_i)}{2} \|x^{k+1} - x^k\|^2, \end{aligned} \quad (4.9)$$

where $\rho_x(g_i)$ is the convexity parameter of g_i respecting to variable x .

Multiply two sides of (4.9) by λ_{xi}^{k+1} , take sum of m inequalities ($i = 1, \dots, m$), and apply the first inclusion of (4.5), we gain

$$\begin{aligned} & - \sum_{i=1}^m \lambda_{xi}^{k+1} (\nabla_x g_i^T(x^{k+1}, y^k) - \nabla_x h_i^T(x^k, y^k))(x^k - x^{k+1}) \\ & \geq \lambda_{x\varphi}^{k+1} \frac{\rho_x(g_\varphi)}{2} \|x^k - x^{k+1}\|^2, \end{aligned} \quad (4.10)$$

with $\rho_x(g_\varphi) = \min\{\rho_x(g_i) : i = 1, \dots, m\}$

- From the second inclusion of KKT condition and the definition of $N(C, x^{k+1})$, we achieve

$$\begin{aligned} & (\nabla_x g_0^T(x^{k+1}, y^k) - \nabla_x h_0^T(x^k, y^k))(x^k - x^{k+1}) \\ & \geq - \sum_{i=1}^m \lambda_{xi}^{k+1} (\nabla_x g_i^T(x^{k+1}, y^k) - \nabla_x h_i^T(x^k, y^k))(x^k - x^{k+1}), \end{aligned} \quad (4.11)$$

- Substituting (4.8) into (4.11) leads to

$$\begin{aligned} f_0(x^k, y^k) - f_0(x^{k+1}, y^k) & \geq - \sum_{i=1}^m \lambda_{xi}^{k+1} (\nabla_x g_i^T(x^{k+1}, y^k) - \nabla_x h_i^T(x^k, y^k))(x^k - x^{k+1}) \\ & \quad + \frac{\rho_x(g_0)}{2} \|x^k - x^{k+1}\|^2. \end{aligned} \quad (4.12)$$

- Combining (4.10) and (4.12) yields

$$f_0(x^k, y^k) - f_0(x^{k+1}, y^k) \geq \left(\frac{\rho_x(g_0)}{2} + \lambda_{x\varphi}^{k+1} \frac{\rho_x(g_\varphi)}{2} \right) \|x^k - x^{k+1}\|^2. \quad (4.13)$$

Similarly, one has

$$f_0(x^{k+1}, y^k) - f_0(x^{k+1}, y^{k+1}) \geq \left(\frac{\rho_y(g_0)}{2} + \lambda_{y\varphi}^{k+1} \frac{\rho_y(g_\varphi)}{2} \right) \|y^k - y^{k+1}\|^2. \quad (4.14)$$

By adding (4.13) and (4.14), one obtains

$$f_0(x^k, y^k) - f_0(x^{k+1}, y^{k+1}) \geq \frac{\rho_x}{2} \|x^{k+1} - x^k\|^2 + \frac{\rho_y}{2} \|y^{k+1} - y^k\|^2, \quad (4.15)$$

where $\frac{\rho_x}{2} = \frac{\rho_x(g_0)}{2} + \lambda_{x\varphi}^{k+1} \frac{\rho_x(g_\varphi)}{2}$ and $\frac{\rho_y}{2} = \frac{\rho_y(g_0)}{2} + \lambda_{y\varphi}^{k+1} \frac{\rho_y(g_\varphi)}{2}$.

(4.15) shows that the sequence $f_0(x^k, y^k)$ is decreasing through the following assumption.

Assumption 4.1. g_0 is partial strongly convex.

To justify this assumption, we consider the DC function

$$\begin{aligned} f_0(x) &= (g_0(x) + \frac{\tau_x}{2} \|x\|^2) - (h_0(x) + \frac{\tau_x}{2} \|x\|^2), \\ f_0(y) &= (g_0(y) + \frac{\tau_y}{2} \|y\|^2) - (h_0(y) + \frac{\tau_y}{2} \|y\|^2), \end{aligned}$$

for any $\tau_x > 0$ and $\tau_y(g_0) > 0$. Hence, we can always find the DC decomposition of f_0 where g_0 is partial strongly convex.

- 2) • If the algorithm terminates after k iterations, $x^{k+1} = x^k$ and x^{k+1} is a feasible point of subproblem (4.3). Since g_i and h_i are differential, $\nabla_x f_i(x^{k+1}, y^k) = \nabla_x g_i(x^{k+1}, y^k) - \nabla_x h_i(x^{k+1}, y^k)$, $i = 0, \dots, m$. Substituting these relations into (4.5), we obtain

$$\begin{cases} f_i(x^k, y^k) \leq 0, \lambda_{x_i}^{k+1} \geq 0, i = 1, \dots, m, \lambda_{x_i}^{k+1} f_i(x^k, y^k) = 0, \\ 0 \in \nabla_x f_0(x^k, y^k) + \sum_{i=1}^m \lambda_{x_i}^{k+1} \nabla_x f_i(x^k, y^k) + N(C, x^k). \end{cases}$$

Therefore, x^k is a partial KKT point of the problem (4.2).

Similarly, y^k is a partial KKT point of the problem (4.2).

- Now, we move to the second scenario.

From (4.15), one has

$$f_0(x^1, y^1) - f_0(x^{n+1}, y^{n+1}) \geq \frac{\rho_x}{2} \sum_{k=1}^n \|x^{k+1} - x^k\|^2.$$

Let α be the optimal value of the considered problem. Assume that α is finite. Consequently, $f_0(x^1, y^1) - \alpha \geq \frac{\rho_x}{2} \sum_{k=1}^n \|x^{k+1} - x^k\|^2$. Therefore, $\lim_{k \rightarrow \infty} \|x^k - x^{k+1}\| = 0$.

Similarly, one has $\lim_{k \rightarrow \infty} \|y^k - y^{k+1}\| = 0$.

Assumption that there exist a subsequence $\{x^{k_j}, y^{k_j}\}$:

$\lim_{j \rightarrow \infty} x^{k_j} = x^\infty$, $\lim_{j \rightarrow \infty} y^{k_j} = y^\infty$. We prove that (x^∞, y^∞) is a partial KKT point of problem (4.2).

To prove that x^∞ is a partial KKT point, we need to rely on the claim: the sequence $\{\|\lambda_x^{k+1}\|_1\}$ is bounded with $\|\lambda_x^{k+1}\|_1 = \sum_{i=1}^m \lambda_{x_i}^{k+1}$. We assume the contrary: $\lim_{j \rightarrow \infty} \|\lambda_x^{k_j+1}\|_1 = +\infty$ for some subsequence.

Let $p(x, y) = \max f_i(x, y)$, $I(x, y) = \{i \in \{1, \dots, m\} : f_i(x, y) = p(x, y)\}$.

If $p(x^\infty) < 0$ then when k_j is sufficiently large, we achieve

$$g_i(x^{k_j+1}, y^k) - h_i(x^{k_j}, y^k) - \nabla_x h_i^T(x^{k_j}, y^k)(x^{k_j+1} - x^{k_j}) < 0.$$

From the first inclusion of (4.5), we obtain $\lambda_{x_i}^{k_j+1} = 0$, $i = 1, \dots, m$, which contradicts $\|\lambda_x^{k_j+1}\|_1 \rightarrow +\infty$.

Hence, $p(x^\infty) \geq 0$. We assume

$$\lim_{j \rightarrow \infty} \frac{\lambda_{x_i}^{k_j+1}}{\|\lambda_x^{k_j+1}\|_1} = \eta_i, \quad i = 1, \dots, m.$$

Such that $\eta_i = 0$ if $i \notin I(x^\infty, y^\infty)$ and $\sum_{i \in I(x^\infty, y^\infty)} \eta_i = 1$

Assume that $g_0(\cdot, y)$ and $h_0(\cdot, y)$ are Lipschitz uniformly in y . Divide the second inclusion of (4.5) by $\|\lambda_x^{k_j+1}\|_1$ and take $j \rightarrow +\infty$, we achieve

$$0 \in \sum_{i \in I(x^\infty, y^\infty)} \eta_i \nabla_x f_i(x^\infty, y^\infty) + N(C, x^\infty). \quad (4.16)$$

Moreover, according to the extended Mangasarian-Fromowitz constraint qualification [28]: with $p(x^\infty) \geq 0$ at $x^\infty \in C$, for $\forall i \in I(x^\infty)$, there exists a vector $d \in \text{cone}(C - x^\infty)$ such that

$$\langle \nabla_x f_i(x^\infty, y^\infty), d \rangle < 0. \quad (4.17)$$

(4.16) contradicts (4.17) when we multiple two sides of (4.16) by d . Hence, $\|\lambda_x^{k_j+1}\|_1$ is bounded, we can assume that $\lim_{j \rightarrow \infty} \lambda_{x_i}^{k_j+1} = \lambda_i^\infty$, $i = 1, \dots, m$.

Since $\lim_{k \rightarrow \infty} \|x^k - x^{k+1}\| = 0$ and $\lim_{j \rightarrow \infty} x^{k_j} = x^\infty$, $\lim_{j \rightarrow \infty} x^{k_j+1} = x^\infty$.

Replace k in (4.5) by k_j and let $j \rightarrow \infty$, we obtain

$$\begin{cases} f_i(x^\infty, y^\infty) \leq 0, \lambda_{x_i}^\infty \geq 0, i = 1, \dots, m, x^\infty \in C, \lambda_i^\infty f_i(x^\infty, y^\infty) = 0, \\ 0 \in \nabla f_0(x^\infty, y^\infty) + \sum_{i=1}^m \lambda_i^\infty \nabla_x f_i(x^\infty, y^\infty) + N(C, x^\infty). \end{cases}$$

It shows that x^∞ is a partial KKT point of the problem (4.2).

Similarly, y^∞ is a partial KKT point of the problem (4.2).

4.3.2 An approach based on ADCA- ρ for solving the problem (4.1)

The problem is difficult to solve directly due to the bounded CSI error models of legitimate receivers and its fraction form. We first devote to transform the problem

(4.1) into a tractable form and then present an efficient scheme based on ADCA- ρ algorithm to solve it.

1) Reformulate the problem (4.1)

a) First, we transform the EH constraint (4.1d) which can be rewritten as

$$\sum_{j=1}^K (\bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk})^H \mathbf{W}_j (\bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk}) \geq \frac{E_k}{1 - \rho_k}, \quad 0 < \rho_k < 1, \quad j, k \in \mathcal{K}.$$

Denote $\mathbf{D}_{jk} = \bar{\mathbf{h}}_{jk} \Delta \mathbf{h}_{jk}^H + \Delta \mathbf{h}_{jk} \bar{\mathbf{h}}_{jk}^H + \Delta \mathbf{h}_{jk} \Delta \mathbf{h}_{jk}^H$. By applying the triangle inequality and the Cauchy-Schwarz inequality, we have

$$\begin{aligned} \|\mathbf{D}_{jk}\| &\leq \|\bar{\mathbf{h}}_{jk}\| \|\Delta \mathbf{h}_{jk}^H\| + \|\Delta \mathbf{h}_{jk}\| \|\bar{\mathbf{h}}_{jk}^H\| + \|\Delta \mathbf{h}_{jk}\| \|\Delta \mathbf{h}_{jk}^H\|, \\ \text{then, } \|\mathbf{D}_{jk}\| &\leq \delta_{jk}^2 + 2\|\bar{\mathbf{h}}_{jk}\| \delta_{jk} = \epsilon_{jk}. \end{aligned}$$

Lemma 4.1. *If $\|\mathbf{D}_{jk}\| \leq \epsilon_{jk}$, we have*

$$\begin{aligned} (\bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk})^H \mathbf{W}_j (\bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk}) &= \text{Tr} [(\bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk})(\bar{\mathbf{h}}_{jk} + \Delta \mathbf{h}_{jk})^H \mathbf{W}_j], \\ &= \text{Tr} [(\bar{\mathbf{h}}_{jk} \bar{\mathbf{h}}_{jk}^H + \mathbf{D}_{jk}) \mathbf{W}_j] \\ &\geq \text{Tr}(\bar{\mathbf{H}}_{jk} \mathbf{W}_j - \epsilon_{jk} \mathbf{W}_j), \quad \text{where } \bar{\mathbf{H}}_{jk} = \bar{\mathbf{h}}_{jk} \bar{\mathbf{h}}_{jk}^H. \end{aligned}$$

See the proof in [100]

Thus, the EH constraint (4.1d) is recast as

$$\sum_{j=1}^K \text{Tr}(\bar{\mathbf{H}}_{jk} \mathbf{W}_j - \epsilon_{jk} \mathbf{W}_j) \geq \frac{E_k}{(1 - \rho_k)}.$$

b) Next, we transform (4.1e) by using auxiliary variables as follows.

$$\log \left(1 + \frac{\rho_k \mathbf{h}_{kk}^H \mathbf{W}_k \mathbf{h}_{kk}}{\rho_k \sum_{j \neq k} \mathbf{h}_{jk}^H \mathbf{W}_j \mathbf{h}_{jk} + \rho_k \sigma_{1k}^2 + \sigma_{2k}^2} \right) \geq R_{I,k}$$

can be converted into

$$\beta_k \geq \frac{1}{\rho_k},$$

$$\mathbf{h}_{kk}^H \mathbf{W}_k \mathbf{h}_{kk} \geq f_{kk}, \quad (4.18)$$

$$\mathbf{h}_{jk}^H \mathbf{W}_k \mathbf{h}_{jk} \leq f_{jk}, \quad j \neq k, \quad (4.19)$$

$$\sum_{j=1}^K f_{jk} + \sigma_{1k}^2 + \beta_k \sigma_{2k}^2 \geq a_k, \quad \sum_{j \neq k} f_{jk} + \sigma_{1k}^2 + \beta_k \sigma_{2k}^2 \leq b_k,$$

$$0 < u \leq a_k, \quad v \geq b_k > 0, \quad x \geq R_{I,k}, \quad 2^x v - u \leq 0.$$

We apply Lemma 2.1 (S-procedure) to deal with the channel error model of \mathbf{h}_{jk} . Then, (4.18) and (4.19) are rewritten as

$$\begin{aligned} &\begin{bmatrix} \lambda_{kk} \mathbf{Q} + \mathbf{W}_k & \mathbf{W}_k \bar{\mathbf{h}}_{kk} \\ \bar{\mathbf{h}}_{kk}^H \mathbf{W}_k & \bar{\mathbf{h}}_{kk}^H \mathbf{W}_k \bar{\mathbf{h}}_{kk} - f_{kk} - \lambda_{kk} \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_{kk} \geq 0, \\ &\begin{bmatrix} \lambda_{jk} \mathbf{Q} - \mathbf{W}_j & -\mathbf{W}_j \bar{\mathbf{h}}_{jk}, \\ -\bar{\mathbf{h}}_{jk}^H \mathbf{W}_j & -\bar{\mathbf{h}}_{jk}^H \mathbf{W}_j \bar{\mathbf{h}}_{jk} + f_{jk} - \lambda_{jk} \end{bmatrix} \succeq \mathbf{0}, \quad j \neq k, \quad \lambda_{jk} \geq 0. \end{aligned}$$

c) By using slack variables ξ_k, s_k , the constraint (4.1f)

$$-\ln p_k - \frac{(2^{R_{I,k}-R_k} - 1)\sigma_{3k}^2}{\text{Tr}(\mathbf{G}_{kk}\mathbf{W}_k)} - \sum_{j \neq k} \ln \left[1 + \frac{(2^{R_{I,k}-R_k} - 1) \text{Tr}(\mathbf{G}_{jk}\mathbf{W}_j)}{\text{Tr}(\mathbf{G}_{kk}\mathbf{W}_k)} \right] \leq 0$$

is transformed into

$$\left\{ \begin{array}{l} \xi_k \leq \frac{2^{R_{I,k}-R_k} - 1}{s_k}, \quad k \in \mathcal{K}, \\ \text{Tr}(\mathbf{G}_{kk}\mathbf{W}_k) \leq s_k, \quad 0 < s_k, \quad 0 < \xi_k, \quad k \in \mathcal{K}, \\ -\ln p_k - \xi_k \sigma_{3k}^2 - \sum_{j \neq k} \ln(1 + \xi_k \text{Tr}(\mathbf{G}_{jk}\mathbf{W}_j)) \leq 0, \quad k \in \mathcal{K}. \end{array} \right.$$

At the optimum, $\xi_k = \frac{2^{R_{I,k}-R_k} - 1}{\text{Tr}(\mathbf{G}_{kk}\mathbf{W}_k)}$, if not, we can increase R_k . In addition, $\text{Tr}(\mathbf{G}_{kk}\mathbf{W}_k) = s_k$ at the optimum, otherwise, we can decrease s_k leading to increase R_k due to $\xi_k = \frac{2^{R_{I,k}-R_k} - 1}{s_k}$. Hence, if the relaxed constraints do not hold with equalities at the optimum, the objective function can be further increased.

Furthermore, we can rewrite the above reformulation as

$$\left\{ \begin{array}{l} R_k - R_{I,k} + \log_2(1 + \xi_k s_k) \leq 0, \quad k \in \mathcal{K}, \\ \text{Tr}(\mathbf{G}_{kk}\mathbf{W}_k) \leq s_k, \quad 0 < s_k, \quad 0 < \xi_k, \quad k \in \mathcal{K}, \\ -\ln p_k - \xi_k \sigma_{3k}^2 - \sum_{j \neq k} \ln(1 + \xi_k \text{Tr}(\mathbf{G}_{jk}\mathbf{W}_j)) \leq 0, \quad k \in \mathcal{K}. \end{array} \right.$$

Based on the above derivations, we arrive at the following reformulation of the

problem (4.1):

$$\begin{aligned} & \max_{\{\rho_k\}, \{\mathbf{W}_k\}, \{\beta_k\}, \{\lambda_{jk}\},} R, \\ & \{\mathbf{f}_{jk}\}, \{R_{I,k}\}, \{s_k\}, \{a_k\}, \\ & \{b_k\}, \{R_k\}, R, u, v, x, \{\xi_k\} \end{aligned} \quad (4.20)$$

$$\text{s. t. (4.1b) - (4.1c),} \quad (4.21)$$

$$\sum_{j=1}^K \text{Tr}(\bar{\mathbf{H}}_{jk} \mathbf{W}_j - \epsilon_{jk} \mathbf{W}_j) \geq \frac{E_k}{(1 - \rho_k)}, \quad k \in \mathcal{K}, \quad (4.22)$$

$$\beta_k \geq \frac{1}{\rho_k}, \quad 0 < u \leq a_k, \quad v \geq b_k > 0, \quad x \geq R_{I,k}, \quad s_k > 0, \quad k \in \mathcal{K}, \quad (4.23)$$

$$\sum_{j=1}^K f_{jk} + \sigma_{1k}^2 + \beta_k \sigma_{2k}^2 \geq a_k, \quad \sum_{j \neq k} f_{jk} + \sigma_{1k}^2 + \beta_k \sigma_{2k}^2 \leq b_k, \quad k \in \mathcal{K}, \quad (4.24)$$

$$\begin{bmatrix} \lambda_{kk} \mathbf{Q} + \mathbf{W}_k & \mathbf{W}_k \bar{\mathbf{h}}_{kk} \\ \bar{\mathbf{h}}_{kk}^H \mathbf{W}_k & \bar{\mathbf{h}}_{kk}^H \mathbf{W}_k \bar{\mathbf{h}}_{kk} - f_{kk} - \lambda_{kk} \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_{kk} \geq 0, \quad k \in \mathcal{K}, \quad (4.25)$$

$$\begin{bmatrix} \lambda_{jk} \mathbf{Q} - \mathbf{W}_j & -\mathbf{W}_j \bar{\mathbf{h}}_{jk} \\ -\bar{\mathbf{h}}_{jk}^H \mathbf{W}_j & -\bar{\mathbf{h}}_{jk}^H \mathbf{W}_j \bar{\mathbf{h}}_{jk} + f_{jk} - \lambda_{jk} \end{bmatrix} \succeq \mathbf{0}, \quad \lambda_{jk} \geq 0, \quad j \neq k, \quad k \in \mathcal{K}, \quad (4.26)$$

$$\text{Tr}(\mathbf{G}_{kk} \mathbf{W}_k) \leq s_k, \quad k \in \mathcal{K}, \quad (4.27)$$

$$-\ln p_k - \xi_k \sigma_{3k}^2 - \sum_{j \neq k} \ln(1 + \xi_k \text{Tr}(\mathbf{G}_{jk} \mathbf{W}_j)) \leq 0, \quad k \in \mathcal{K}, \quad (4.28)$$

$$R_k - R_{I,k} + \log(1 + \xi_k s_k) \leq 0, \quad k \in \mathcal{K}, \quad (4.29)$$

$$2^x v - u \leq 0. \quad (4.30)$$

We divide the variable set into two variable subsets: $\{\xi_k\}$ and $\mathbf{X} = (\{\rho_k\}, \{\mathbf{W}_k\}, \{\beta_k\}, \{\lambda_{jk}\}, \{\mathbf{f}_{jk}\}, \{R_{I,k}\}, \{s_k\}, \{a_k\}, \{b_k\}, \{R_k\}, R, u, v, x)$.

The reformulated problem is still difficult due to the nonconvex constraints (4.28)-(4.30). This problem enjoys an interesting property: when $\{\xi_k\}$ (or \mathbf{X}) is fixed, the constraint (4.28) becomes convex and the constraint (4.29) is simpler to deal with. Consequently, we develop an efficient scheme based on ADCA algorithm for solving this problem.

2) ADCA- ρ scheme for solving the transformed problem (4.20)

At each iteration we do two steps:

- a) First, we fix $\{\xi_k\}$ and compute \mathbf{X} by solving

$$\begin{aligned} & \max_{\mathbf{X}} R, \\ & \text{s. t. (4.21) - (4.28),} \end{aligned} \quad (4.31)$$

$$R_k - R_{I,k} + \log(1 + \xi_k s_k) \leq 0, \quad k \in \mathcal{K}, \quad (4.31)$$

$$2^x v - u \leq 0. \quad (4.32)$$

We express DC constraints for the nonconvex constraints (4.31) and (4.32) as follows.

The constraint (4.31) can be rewritten as

$$\begin{aligned} F_0(\mathbf{X}) &= G_0(\mathbf{X}) - H_0(\mathbf{X}) \leq 0, \text{ where,} \\ G_0(\mathbf{X}) &= R_k - R_{I,k}, \quad H_0(\mathbf{X}) = -\log(1 + \xi_k s_k), \end{aligned}$$

while the constraint (4.32) is transformed into

$$\begin{aligned} F_1(\mathbf{X}) &= G_1(\mathbf{X}) - H_1(\mathbf{X}) \leq 0, \text{ where,} \\ G_1(\mathbf{X}) &= \frac{1}{2}\rho(x^2 + u^2 + v^2), \quad H_1(\mathbf{X}) = \frac{1}{2}\rho(x^2 + u^2 + v^2) + u - 2^x v. \end{aligned}$$

It is not easy to estimate ρ such that H_1 is convex, thus, we investigate general DCA- ρ to solve this subproblem. We rewrite the subproblem in this first step as a following DC program

$$\begin{aligned} &\max_{\mathbf{X}} R, \\ &\text{s. t. (4.21) - (4.28),} \\ &\quad G_0(\mathbf{X}) - H_0(\mathbf{X}) \leq 0, \\ &\quad G_1(\mathbf{X}) - H_1(\mathbf{X}) \leq 0. \end{aligned}$$

The linear approximations of $H_0(\mathbf{X})$ and $H_1(\mathbf{X})$ at the point (\mathbf{X}^t) are given by

$$\begin{aligned} \overline{H}_0(\mathbf{X}) &= H_0(\mathbf{X}^t) + \langle \nabla H_0(\mathbf{X}^t), \mathbf{X} - \mathbf{X}^t \rangle, \\ &= -\log(1 + \xi_k^t s_k^t) - \frac{\xi_k^t (s_k - s_k^t)}{(1 + \xi_k^t s_k^t) \ln 2}, \\ \overline{H}_1(\mathbf{X}) &= H_1(\mathbf{X}^t) + \langle \nabla H_1(\mathbf{X}^t), \mathbf{X} - \mathbf{X}^t \rangle, \\ &= \frac{1}{2}\rho((x^2)^t + (u^2)^t + (v^2)^t) + u^t - 2^{x^t} v^t + (\rho x^t - 2^{x^t} v^t \ln 2)(x - x^t) \\ &\quad + (\rho u^t + 1)(u - u^t) + (\rho v^t - 2^{x^t})(v - v^t). \end{aligned}$$

b) Fix \mathbf{X} and compute $\{\xi_k\}$ by solving

$$\begin{aligned} &\max_{\{\xi_k\}} R, \\ &\text{s. t. } -\ln p_k - \xi_k \sigma_{3k}^2 - \sum_{j \neq k} \ln(1 + \xi_k \text{Tr}(\mathbf{G}_{jk} \mathbf{W}_j)) \leq 0, \quad k \in \mathcal{K}, \\ &\quad R_k - R_{I,k} + \log(1 + \xi_k s_k) \leq 0, \quad k \in \mathcal{K}. \end{aligned} \tag{4.33}$$

A DC constraint for the nonconvex constraint (4.33) is given as

$$\begin{aligned} F_2(\xi_k) &= G_2(\xi_k) - H_2(\xi_k) \leq 0, \text{ where,} \\ G_2(\xi_k) &= R_k - R_{I,k}, \quad H_2(\xi_k) = -\log(1 + \xi_k s_k). \end{aligned}$$

The linear approximation of $H_2(\xi_k)$ at the point (ξ_k^t) is calculated as

$$\overline{H}_2(\xi_k) = -\log(1 + \xi_k^t s_k^*) + \frac{s_k^* (\xi_k - \xi_k^t)}{(1 + \xi_k^t s_k^*) \ln 2}.$$

Our proposed algorithm based on ADCA $_{-\rho}$ is described in Algorithm 4.1.

Algorithm 4.1. ADCA $_{-\rho}$ algorithm for solving the problem (4.20)

Initialization: Let $(\mathbf{X}^0, \{\xi_k^0\})$ be an initial point, $t \leftarrow 0$.

repeat

1. Fix $\xi_k = \xi_k^t$, compute X^{t+1} . Let $l \leftarrow t$.

repeat

1.1. Compute \mathbf{X}^{l+1} by solving the following subproblem

$$\begin{aligned} & \max_{\mathbf{X}} R \\ & \text{s. t. (4.21) - (4.28),} \\ & G_0(\mathbf{X}) - \bar{H}_0(\mathbf{X}^l) \leq 0, \\ & G_1(\mathbf{X}) - \bar{H}_1(\mathbf{X}^l) \leq 0. \end{aligned}$$

1.3. while $H_1(\mathbf{X}^{l+1}) < H_1(\mathbf{X}^l) + \langle \nabla H_1(\mathbf{X}^l), \mathbf{X}^{l+1} - \mathbf{X}^l \rangle$
 $\rho \leftarrow \eta\rho$ and update \mathbf{X}^{l+1} by step 1.1.

1.4. $l \leftarrow l + 1$,

until stopping condition.

Set $X^{t+1} = X^{l+1}$.

2. Fix $X = X^{t+1}$, compute ξ_k^{t+1} . Let $l \leftarrow t$.

repeat

2.1. Compute ξ_k^{l+1} by solving the problem

$$\begin{aligned} & \max_{\{\xi_k\}} R^{t+1} \\ & \text{s. t. } -\ln p_k - \xi_k \sigma_{3k}^2 - \sum_{j \neq k} \ln(1 + \xi_k \text{Tr}(\mathbf{G}_{jk} \mathbf{W}_j^{t+1})) \leq 0, \quad k \in \mathcal{K}, \\ & R^{t+1} - R_{l,k}^{t+1} + \log(1 + \xi_k^l s_k^{t+1}) + \frac{s_k^{t+1}(\xi_k - \xi_k^l)}{(1 + \xi_k^l s_k^{t+1}) \ln 2} \leq 0, \quad k \in \mathcal{K}. \end{aligned}$$

2.2. $l \leftarrow l + 1$,

until stopping condition.

Set $\xi_k^{t+1} = \xi_k^{l+1}$, $k \in \mathcal{K}$.

3. $t \leftarrow t + 1$,

until stopping condition.

4.4 Numerical experiments

The numerical experiments aim to evaluate the performance of ADCA $_{-\rho}$ scheme by comparing it with SCA algorithm.

Experiments settings:

All algorithms were implemented in Matlab R2016b, and executed on a PC Intel(R) Core(TM)2 Quad (2.83 GHz) of 4GB RAM. The CVX toolbox [10] is used to solve the subproblems of algorithm 4.1. The elements of all the legitimate channel vectors are generated by complex Gaussian distribution with zero mean and unit variance. For channel error models, $\mathbf{G}_{jk} = \gamma_{jk} \mathbf{I}$, where γ_{jk} denotes the average channel correlation gain and ϵ denotes channel coefficient error. We assume that $E = E_k$, $p = p_k$, $\sigma^2 = \sigma_{1k}^2 = \sigma_{2k}^2 = \sigma_{3k}^2$, $\gamma = \gamma_{jk}$, $\delta = \delta_{jk}$ for all k, j . Unless otherwise specified, we set by default $K = 5$, $n = 4$, $P = 5$ W, $E = 0.001$ W, $p = 0.1$, $\sigma = 0.01$, $\gamma = 1$, $\epsilon = 0.05$. The stopping criterion for all algorithms is $|R^t - R^{t-1}| < 10^{-3}$.

We are interested in the effect of the coefficient error δ (Figure 4.1), channel correlation gain γ (Figure 4.2) and secrecy rate probability p (Figure 4.3). All the algorithms are tested on 10 independent channel realizations. For each value of these parameters, average values of minimal secrecy rate and computing time in seconds (Time(sec.)) are reported.

Comments on numerical results:

It is seen from Figure 4.1 that the larger the channel coefficient error is, the smaller the minimal secrecy rate is. In Figure 4.2, as we expected, with the increase of the channel correlation gain, the minimal secrecy rate of all approaches tend to decrease. Figure 4.3 shows the sensitivity of the minimal secrecy rate to the secrecy outage probability. Moreover, as justification for the performance of our proposed algorithms, we provide some observations as follows.

In Figure 4.1, we compare the minimal secrecy rate of ADCA- ρ scheme and SCA algorithm under different coefficient errors. It can be seen that ADCA- ρ is much more efficient than SCA on both minimal secrecy rate and computing time. In terms of minimal secrecy rate, the gain varies from 94.0% to 131.5%. Concerning computing time, the ratio of gain varies from 3.9 to 4.4 times.

Figure 4.2 presents the minimal secrecy rate under different channel correlation gains. ADCA- ρ always provides the better solutions. Regarding minimal secrecy rate, the gain varies from 117.1% to 135.3%. Concerning computing time, the ratio of gain is around 4.3 times.

Figure 4.3 shows the results of increasing secrecy rate probability. This figure show again the performance superiority of ADCA- ρ as compared to SCA on both minimal secrecy rate and computing time. In terms of minimal secrecy rate, the gain varies from 131.5% to 229.3%. As for computing time, the ratio of gain varies from 4.2 to 5.6 times.

As aforementioned analysis, numerical results confirmed the efficiency and superiority of ADCA- ρ algorithm as compared to SCA algorithm.

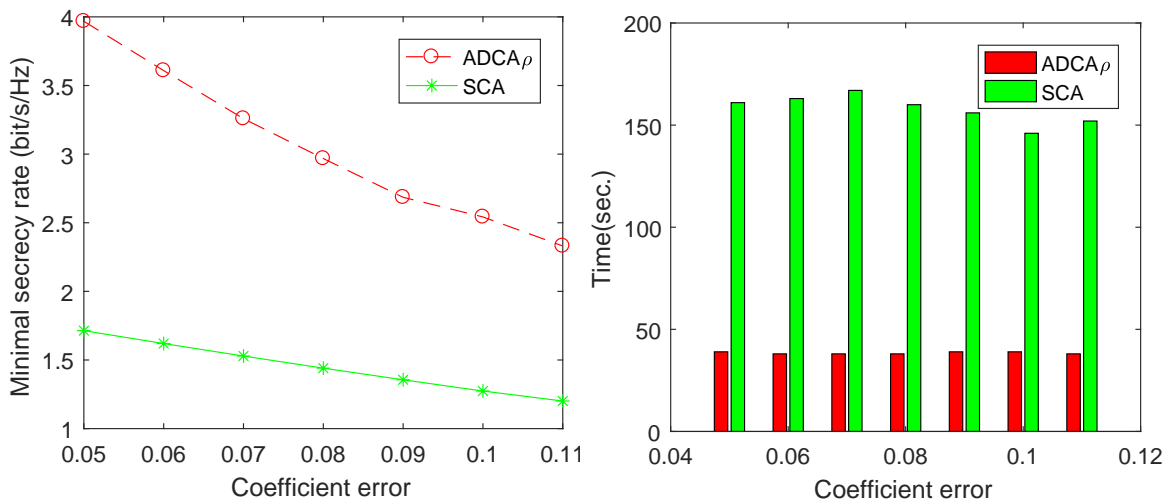


Figure 4.1 – Coefficient channel versus minimal secrecy rate and computing time (in seconds)

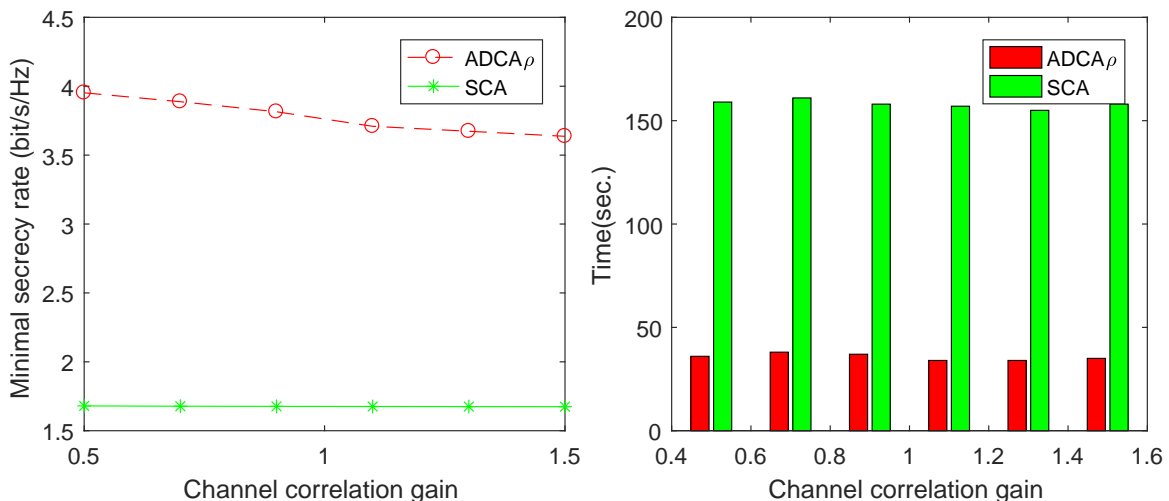


Figure 4.2 – Channel correlation gain versus minimal secrecy rate and computing time (in seconds)

4.5 Conclusion

We have proposed an efficient approach, namely, ADCA ρ to solve the minimal secrecy rate maximization problem under constraints of total transmit power, energy harvesting, and secrecy rate probability. The problem is very complicated because of its fractional form, channel error models, nonconvexity. By studying the special structure of this problem, we have transformed the problem into a tractable form. Since it is hard to optimize all variables, a natural idea is to decouple into two variable subsets and then use the alternating optimization procedure. At each step of the alternating scheme, we have alternately computed these two variable subsets based on DC programming and DCA. The proposed algorithm, ADCA ρ , have been proven to guarantee the convergence property. Numerical experiments have confirmed the

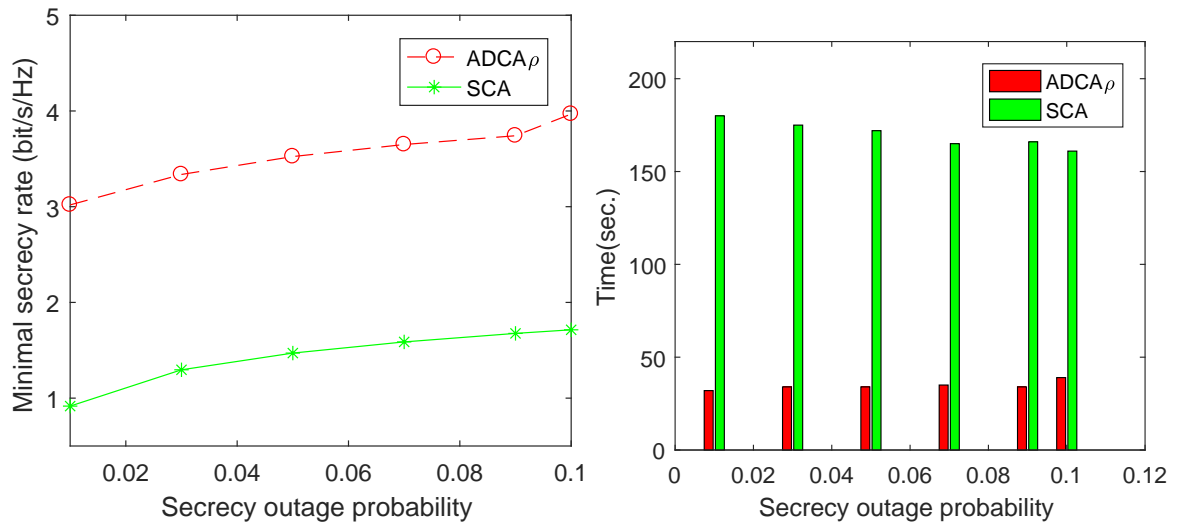


Figure 4.3 – Secrecy outage probability versus minimal secrecy rate and computing time (in seconds)

efficiency of our proposed algorithm as compared to the existing method.

Part II

Transmit power minimization

Chapter 5

DC programming and DCA for Probabilistic Robust SWIPT Beamforming in Secure MISO Channel

Abstract: Wireless power transfer over radio frequency has emerged as a promising technology in which wireless devices can harvest energy from the power transmitter. This chapter investigates the secure transmission issue for simultaneous wireless information and power transfer (SWIPT) in a multiple-input single-output (MISO) wireless communication system. The system consists of one multi-antenna transmitter, one single-antenna information receiver, multiple single-antenna energy receivers, and multiple single-antenna eavesdroppers. Under the assumption of statistical channel state information to eavesdroppers and energy receivers, we focus on the transmit power minimization while guaranteeing the predefined probability of the secrecy rate and the energy harvesting. The problem is nonconvex and difficult to solve because of the probabilistic constraints. We first apply the Bernstein inequalities to transform these probabilistic constraints into deterministic ones. Then, the nonconvex transformed problem is efficiently solved by DC programming and DCA. We reformulate the transformed problem as general DC programs for which the corresponding general DCA based algorithms are developed for solving them. The experimental results illustrate the efficiency of the proposed algorithms.

1. The material of this chapter is developed from the following work:
[1]. Hoai An Le Thi, Phuong Anh Nguyen. *DC programming and DCA for Probabilistic Robust SWIPT Beamforming in Secure MISO Channel*. Submitted.

5.1 Introduction

Simultaneous wireless information and power transfer (SWIPT) technique has emerged as an effective solution to break down the limitation of battery capacity for wireless devices. The concept of SWIPT was first studied in [84], where information and power are transported simultaneously via radio signals. In general, SWIPT systems consist of transmitter, information receiver, and energy receivers in presence of eavesdroppers. Energy receivers, that are usually located closer to the transmitter than information receiver [93], may be potential eavesdroppers. Consequently, energy receivers can successfully eavesdrop the desired information due to the better channels. Thus, security in SWIPT systems is severer than conventional wireless networks. Physical layer security can guarantee secure information transmission in SWIPT systems by exploiting the characteristics of wireless channels. One of the most important concepts in physical layer security is secrecy capacity which measures the maximum information transmission rate without information leakage to an eavesdropper. Beamforming allows the transmitter to steer its signal in a particular direction toward legitimate receiver. In this way, the secrecy capacity based on beamforming can be improved significantly. To employ beamforming, channel state information must be taken into consideration.

In most cases, the channel state information between the transmitter and the information receiver is quite accurate based on channel training and feedback at both transmitter and information receiver side. However, it is difficult to achieve perfect channel state information between transmitter and other users (especially eavesdroppers) due to the limited cooperation among them. Besides, the eavesdroppers try to hide their existence by avoiding cooperation in the stage of channel estimation. There are two classes of channel uncertainty models: deterministic and stochastic models. Some secure SWIPT systems were constructed in [62, 7, 80, 99] under the assumption of deterministic uncertainty models. These works assume that the channel errors are norm-bounded. In the case of sophisticated eavesdroppers, the channel coefficient error bound cannot be gained perfectly. In order to overcome this issue, the channel errors are investigated as stochastic models. Accordingly, the probability constrained methods were investigated in [19, 18, 22, 105] based on stochastic channel error models. Specifically, the secrecy rate maximization problems were studied in [19, 18] while the secrecy rate constrained power minimization problems in multiple-input single-output (MISO) SWIPT systems were considered in [22, 105]. These works only focused on the assumption that eavesdroppers are part of system. In fact, there are eavesdroppers who don't belong to any node in system. Rank-one relaxation method was exploited in [19, 18, 22] to reformulate the nonconvex optimization problems as convex ones. As discussed in [105], the main disadvantage of this relaxation is that rank-one solution does not always happen at the optimum. To prevent rank-one relaxation, successive convex approximation (SCA, a special version of DCA) method was developed in [105] to solve the considered optimization problems.

Aforementioned analysis motivates us to investigate a practical scenario, in which eavesdroppers are not users of system. Specifically, we consider the MISO SWIPT system where the transmitter has some imprecise information about the channels of

energy receivers and channel of eavesdroppers are not known. Statistical channel errors of energy receivers and statistical information about channels of eavesdroppers are available. We design beamforming strategy for this system based on the power minimization problem. The proposed system is closely related to the work in [105], with the important difference at the channel model of eavesdroppers.

Our contributions are multiple.

Firstly, we formulate the power minimization problem subject to probabilistic constraints of the secrecy rate and the harvested energy. The difference of this mathematical formulation from [105] is the secrecy rate probabilistic constraint. The formulated problem is intractable due to the nonconvex secrecy rate function and the probabilistic constraints. We resort to two Bernstein inequalities [13, 92] to convert the probabilistic constraints into deterministic ones. The transformed problem is still difficult due to its nonconvexity.

Secondly, we investigate DC programming and DCA to solve the above transformed problem. We reformulate the considered problem as a DC problem with DC constraints, which is solved by DCA₁. However, this first DC problem contains DC exponential constraints which should be replaced by simpler ones.

Thirdly, we propose another DC problem for the transformed problem, which aims to avoid DC exponential constraints. Difference from the first DC problem, this second one replaces DC constraints of exponential form by quadratic constraints. The general DCA scheme (called DCA₂) is proposed to solve the second DC problem. Furthermore, since DCA performance depends on DC decomposition, we study another DC decomposition for the second DC problem. Particularly, we simplify the approximation part of DC function in the second DC problem (the second DC component - h function), for which results in the third DC program. DCA₃ is developed for solving the third DC program. The relaxation technique is investigated to cope with the infeasibility of subproblems in DCA₂ and DCA₃, which is caused by linearization of concave part of DC decompositions of DC constraints. Numerical experiments show that DCA₂ and DCA₃ are much more efficient than DCA₁ on both quality and rapidity while DCA₃ is better than DCA₂ on quality.

The rest of this chapter is organized as follows. The considered power minimization problem is presented in Section 5.2 while the solution methods based on DC programming and DCA are presented in Section 5.3. Numerical experiments are reported in Section 5.4, and finally, Section 5.5 concludes the chapter.

5.2 System model

We consider a MISO wiretap channel, which comprises one multi-antenna transmitter, one single-antenna information receiver, L single-antenna energy receivers in presence of K single-antenna eavesdroppers. The transmitter is equipped with N antennas.

The channel coefficients from the transmitter to information receiver, l th energy receiver, and i th eavesdropper are denoted by $\mathbf{h}_s \in \mathbb{C}^{N \times 1}$, $\mathbf{h}_l \in \mathbb{C}^{N \times 1}$, $l \in \mathcal{L} \triangleq 1, \dots, L$, and $\mathbf{g}_i \in \mathbb{C}^{N \times 1}$, $i \in \mathcal{K} \triangleq 1, \dots, K$, respectively. Let $\mathbf{w} \in \mathbb{C}^{N \times 1}$ be the transmit beamforming vector. The received signal at the information receiver and the i th eavesdropper can be given as

$$\begin{aligned} y_s &= \mathbf{h}_s^H \mathbf{w} s + n_s, \\ y_{e,i} &= \mathbf{g}_i^H \mathbf{w} s + n_{e,i}, \quad i \in \mathcal{K}, \end{aligned}$$

where $n_s \sim \mathcal{CN}(0, \sigma_s^2)$ and $n_{e,i} \sim \mathcal{CN}(0, \sigma_{e,i}^2)$, $i \in \mathcal{K}$ are the additive Gaussian noise of signal for information receiver and i th eavesdropper, respectively.

We assume that the transmitter has perfect channel coefficient of information receiver and imperfect channel coefficient of energy receivers. The imperfect is modeled as

$$\mathbf{h}_l = \bar{\mathbf{h}}_l + \Delta \mathbf{h}_l, \quad \Delta \mathbf{h}_l \sim \mathcal{CN}(0, \mathbf{E}_l), \quad l \in \mathcal{L},$$

where $\bar{\mathbf{h}}_l$ is the estimate of the true channel state information \mathbf{h}_l while $\Delta \mathbf{h}_l$ denotes the error. $\Delta \mathbf{h}_l$ can be rewritten as $\Delta \mathbf{h}_l = \mathbf{E}_l^{\frac{1}{2}} \tilde{\mathbf{h}}_l$, $\tilde{\mathbf{h}}_l \sim \mathcal{CN}(0, \mathbf{I})$. In addition, the transmitter only knows some statistical information about channel coefficients of eavesdroppers. The channels of eavesdroppers are modeled as

$$\mathbf{g}_i \sim \mathcal{CN}(0, \mathbf{Z}_i), \quad i \in \mathcal{K}.$$

Accordingly, the secrecy rate R_s can be formulated as

$$R_s = \log \left(1 + \frac{|\mathbf{h}_s^H \mathbf{w}|^2}{\sigma_s^2} \right) - \max_i \log \left(1 + \frac{|\mathbf{g}_i^H \mathbf{w}|^2}{\sigma_{e,i}^2} \right).$$

The harvested energy at the l th energy receiver and total power consumption at transmitter can be written as

$$\begin{aligned} E_{er,l} &= |\mathbf{h}_l^H \mathbf{w}|^2, \quad l \in \mathcal{L}, \\ P &= \|\mathbf{w}\|^2. \end{aligned}$$

We aim to minimize the transmit power consumption under the secrecy rate probabilistic constraint and the harvested energy probabilistic constraint. Employing the above channel uncertainty model, we introduce the following probability constrained power minimization (PCPM) problem

$$\min_{\mathbf{w}} \|\mathbf{w}\|^2, \quad (5.1)$$

$$\text{s. t. } \Pr\{|\mathbf{h}_l^H \mathbf{w}|^2 \geq \bar{E}\} \geq 1 - \rho_l, \quad l \in \mathcal{L}, \quad (5.2)$$

$$\Pr\left\{ \log \left(1 + \frac{|\mathbf{h}_s^H \mathbf{w}|^2}{\sigma_s^2} \right) - \log \left(1 + \frac{|\mathbf{g}_i^H \mathbf{w}|^2}{\sigma_{e,i}^2} \right) \geq R \right\} \geq 1 - p_{e,i}, \quad i \in \mathcal{K}. \quad (5.3)$$

The PCPM problem can't be solved directly because of probabilistic constraints. We transform the PCPM problem into deterministic form by applying Bernstein inequalities. The transformed problem is still difficult because of its nonconvexity. Since DC programming and DCA are powerful tools to cope with the nonconvex problems, we investigate them to solve the considered problem.

5.3 Solution methods based on DC programming and DCA

The PCPM problem is first transformed from its probabilistic form into the deterministic one. Then, we investigate DC programming and DCA to solve the resulting problem.

5.3.1 Reformulation of the PCPM problem

We reformulate the probabilistic constraints (5.2) and (5.3) as deterministic ones by applying the Bernstein inequalities (BTIs). The BTIs are presented in the following lemmas.

Lemma 5.1. (Bernstein inequality 1 [92]) *Pr*{ $x^H \mathbf{A}x + \Re.[x^H \mathbf{u}] + c \geq 1 - p$ } can be rewritten as

$$\begin{cases} \text{Tr}(\mathbf{A}) - \sqrt{2 \ln(1/p)}a + \ln(p)a_1 + c \geq 0, \\ \sqrt{\|\mathbf{A}\|^2 + 2\|\mathbf{u}\|^2} \leq a, \\ a_1 \mathbf{I} + \mathbf{A} \succeq \mathbf{0}, \quad a_1 \geq 0. \end{cases}$$

The constraint (5.2) is rewritten as

$$\text{Pr}\{\tilde{\mathbf{h}}_l^H \{\mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \mathbf{E}_l^{\frac{1}{2}}\} \tilde{\mathbf{h}}_l + 2\Re.\tilde{\mathbf{h}}_l^H \{\mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \bar{\mathbf{h}}_l\} + \bar{\mathbf{h}}_l^H \mathbf{w} \mathbf{w}^H \bar{\mathbf{h}}_l \geq \bar{E}\} \geq 1 - \rho_l.$$

Then, we apply Lemma 5.1

$$\begin{cases} \text{Tr}\{\mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \mathbf{E}_l^{\frac{1}{2}}\} - \sqrt{2 \ln(1/\rho_l)}n_l + \ln(\rho_l)m_l + \bar{\mathbf{h}}_l^H \mathbf{w} \mathbf{w}^H \bar{\mathbf{h}}_l - \bar{E} \geq 0, \\ \sqrt{\|\mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \mathbf{E}_l^{\frac{1}{2}}\|^2 + 2\|\mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \bar{\mathbf{h}}_l\|^2} = \sqrt{\mathbf{w}^H \mathbf{E}_l \mathbf{w} \mathbf{w}^H (\mathbf{E}_l + 2\bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}} \leq n_l, \\ m_l \mathbf{I} + \mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \mathbf{E}_l^{\frac{1}{2}} \succeq \mathbf{0}, \quad m_l \geq 0. \end{cases}$$

It is easy to see that $m_l \mathbf{I} + \mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \mathbf{E}_l^{\frac{1}{2}} \succeq \mathbf{0}$, $m_l \geq 0$ always satisfies.

Lemma 5.2. (Bernstein inequality 2 [13])

$$\text{Pr}\left\{ \frac{p_k |\mathbf{h}_{k,k}^H \mathbf{u}_k|^2}{\sum_{j \neq k} p_j |\mathbf{h}_{k,j}^H \mathbf{u}_j|^2 + \sigma_k} \leq \beta_k \right\} = 1 - \exp^{-\frac{\beta_k \sigma_k}{p_k C_{k,k}} \prod_{j \neq k} \left(1 + \frac{\beta_k p_j C_{k,j}}{p_k C_{k,k}}\right)^{-1}},$$

where $\mathbf{h}_{k,j} \sim \mathcal{CN}(0, \mathbf{G}_{k,j})$, $c_{k,j} = \mathbf{u}_j^H \mathbf{G}_{k,j} \mathbf{u}_j$.

By applying Lemma 5.2, the left side of the constraint (5.3) is transformed as

$$\begin{aligned}
& \Pr\left\{\log\left(1 + \frac{|\mathbf{h}_s^H \mathbf{w}|^2}{\sigma_s^2}\right) - \log\left(1 + \frac{|\mathbf{g}_i^H \mathbf{w}|^2}{\sigma_{e,i}^2}\right) \geq R\right\} \\
&= \Pr\left\{\frac{\sigma_{e,i}^2(\sigma_s^2 + |\mathbf{h}_s^H \mathbf{w}|^2)}{\sigma_s^2(\sigma_{e,i}^2 + |\mathbf{g}_i^H \mathbf{w}|^2)} \geq 2^R\right\} \\
&= \Pr\left\{\frac{\sigma_{e,i}^2(\sigma_s^2 + |\mathbf{h}_s^H \mathbf{w}|^2)}{\sigma_s^2 2^R} \geq \sigma_{e,i}^2 + |\mathbf{g}_i^H \mathbf{w}|^2\right\} \\
&= \Pr\left\{|\mathbf{g}_i^H \mathbf{w}|^2 \leq \frac{\sigma_{e,i}^2(|\mathbf{h}_s^H \mathbf{w}|^2 + \sigma_s^2 - \sigma_s^2 2^R)}{\sigma_s^2 2^R}\right. \\
&\quad \left. - \frac{\sigma_{e,i}^2(|\mathbf{h}_s^H \mathbf{w}|^2 + \sigma_s^2 - \sigma_s^2 2^R)}{\sigma_s^2 2^R \mathbf{w}^H \mathbf{Z}_i \mathbf{w}}\right\} \\
&= 1 - \exp\left\{-\frac{\sigma_{e,i}^2(|\mathbf{h}_s^H \mathbf{w}|^2 + \sigma_s^2 - \sigma_s^2 2^R)}{\sigma_s^2 2^R \mathbf{w}^H \mathbf{Z}_i \mathbf{w}}\right\}.
\end{aligned}$$

Thus, the PCPM problem is reformulated as

$$\begin{aligned}
& \min_{\{n_l\}, \{m_l\}, \mathbf{w}} \|\mathbf{w}\|^2, \tag{5.4} \\
& \text{s. t. } m_l \geq 0, \quad l \in \mathcal{L}, \\
& \text{Tr}\{\mathbf{E}_l^{\frac{1}{2}} \mathbf{w} \mathbf{w}^H \mathbf{E}_l^{\frac{1}{2}}\} - \sqrt{2 \ln(1/\rho_l)} n_l + \ln(\rho_l) m_l + \bar{\mathbf{h}}_l^H \mathbf{w} \mathbf{w}^H \bar{\mathbf{h}}_l - \bar{E} \geq 0, \\
& \sqrt{\mathbf{w}^H \mathbf{E}_l \mathbf{w} \mathbf{w}^H (\mathbf{E}_l + 2\bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}} \leq n_l, \quad l \in \mathcal{L}, \\
& \sigma_{e,i}^2 \left(1 - \frac{1}{2^R}\right) - \mathbf{w}^H \left(\mathbf{Z}_i \ln p_{e,i} + \frac{\sigma_{e,i}^2}{\sigma_s^2 2^R} \mathbf{h}_s \mathbf{h}_s^H\right) \mathbf{w} \leq 0, \quad i \in \mathcal{K}.
\end{aligned}$$

We should choose the channel coefficients such that $\lambda\left(\mathbf{Z}_i \ln p_{e,i} + \frac{\sigma_{e,i}^2}{\sigma_s^2 2^R} \mathbf{h}_s \mathbf{h}_s^H\right) > 0$, $i \in \mathcal{K}$ to guarantee the feasibility of the considered problem [59]. We will present DCA schemes to solve the nonconvex problem (5.4) in the following subsections.

5.3.2 The first DCA scheme for solving the problem (5.4)

The problem (5.4) is rewritten as

$$\min_{\{n_l\}, \{m_l\}, \{x_l\}, \{y_l\}, \mathbf{w}} \|\mathbf{w}\|^2, \tag{5.5}$$

$$\text{s. t. } m_l \geq 0, \quad l \in \mathcal{L}, \tag{5.6}$$

$$2^{x_l + y_l} \leq n_l, \quad l \in \mathcal{L}, \tag{5.7}$$

$$\mathbf{w}^H \mathbf{E}_l \mathbf{w} - 2^{2x_l} \leq 0, \quad l \in \mathcal{L}, \tag{5.8}$$

$$\mathbf{w}^H (\mathbf{E}_l + 2\bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w} - 2^{2y_l} \leq 0, \quad l \in \mathcal{L}, \tag{5.9}$$

$$\sqrt{2 \ln(1/\rho_l)} n_l - \ln(\rho_l) m_l + \bar{E} - \mathbf{w}^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w} \leq 0, \quad l \in \mathcal{L}, \tag{5.10}$$

$$\sigma_{e,i}^2 \left(1 - \frac{1}{2^R}\right) - \mathbf{w}^H \left(\mathbf{Z}_i \ln p_{e,i} + \frac{\sigma_{e,i}^2}{\sigma_s^2 2^R} \mathbf{h}_s \mathbf{h}_s^H\right) \mathbf{w} \leq 0, \quad i \in \mathcal{K}. \tag{5.11}$$

Let us define $\mathbf{D} = \mathbf{Z}_i \ln p_{e,i} + \frac{\sigma_{e,i}^2}{\sigma_s^2 2^R} \mathbf{h}_s \mathbf{h}_s^H$, $\mathbf{X} = (\{n_l\}, \{m_l\}, \{x_l\}, \{y_l\}, \mathbf{w})$. The problem is nonconvex due to the constraints (5.8)-(5.11). It can be seen that the left side of constraints (5.8)-(5.11) are DC functions. Thus, we propose DC constraints for these constraints as follows.

The constraint (5.8) is rewritten as

$$F_{1l}(\mathbf{X}) = G_{1l}(\mathbf{X}) - H_{1l}(\mathbf{X}) \leq 0, \text{ where,}$$

$$G_{1l}(\mathbf{X}) = \mathbf{w}^H \mathbf{E}_l \mathbf{w}, \quad H_{1l}(\mathbf{X}) = 2^{2x_l},$$

while the constraint (5.9) is transformed into

$$F_{2l}(\mathbf{X}) = G_{2l}(\mathbf{X}) - H_{2l}(\mathbf{X}) \leq 0, \text{ where,}$$

$$G_{2l}(\mathbf{X}) = \mathbf{w}^H (\mathbf{E}_l + 2\bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}, \quad H_{2l}(\mathbf{X}) = 2^{2y_l}.$$

The constraint (5.10) is reformulated as

$$F_{3l}(\mathbf{X}) = G_{3l}(\mathbf{X}) - H_{3l}(\mathbf{X}) \leq 0, \text{ where,}$$

$$G_{3l}(\mathbf{X}) = \sqrt{2 \ln(1/\rho_l)} n_l - \ln(\rho_l) m_l + \bar{E}, \quad H_{3l}(\mathbf{X}) = \mathbf{w}^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w},$$

and the constraint (5.11) is recast as

$$F_{4i}(\mathbf{X}) = G_{4i}(\mathbf{X}) - H_{4i}(\mathbf{X}) \leq 0, \text{ where,}$$

$$G_{4i}(\mathbf{X}) = \sigma_{e,i}^2 \left(1 - \frac{1}{2^R}\right), \quad H_{4i}(\mathbf{X}) = \mathbf{w}^H \mathbf{D} \mathbf{w}.$$

We rewrite the problem as a following DC program

$$\begin{aligned} & \min_{\mathbf{X}} \|\mathbf{w}\|^2, & (5.12) \\ & \text{s. t. (5.6) - (5.7),} \\ & G_{1l}(\mathbf{X}) - H_{1l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}, \\ & G_{2l}(\mathbf{X}) - H_{2l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}, \\ & G_{3l}(\mathbf{X}) - H_{3l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}, \\ & G_{4i}(\mathbf{X}) - H_{4i}(\mathbf{X}) \leq 0, \quad i \in \mathcal{K}. \end{aligned}$$

The linear approximations of $H_{1l}(\mathbf{X})$, $H_{2l}(\mathbf{X})$, $H_{3l}(\mathbf{X})$ and $H_{4i}(\mathbf{X})$ at the point (\mathbf{X}^k) are given by

$$\begin{aligned} \bar{H}_{1l}(\mathbf{X}) &= 2^{2x_l^k} (2x_l - 2x_l^k + 1), \\ \bar{H}_{2l}(\mathbf{X}) &= 2^{2y_l^k} (2y_l - 2y_l^k + 1), \\ \bar{H}_{3l}(\mathbf{X}) &= (\mathbf{w}^k)^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}^k + 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}^k\}, \\ \bar{H}_{4i}(\mathbf{X}) &= (\mathbf{w}^k)^H \mathbf{D} \mathbf{w}^k + 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H \mathbf{D} \mathbf{w}^k\}. \end{aligned}$$

The general DCA scheme for the problem (5.12) is given as follows.

Algorithm 2. DCA₁ for solving the problem (5.12)

Initialization: Let (\mathbf{X}^0) be an initial point, $k \leftarrow 0$.

repeat

1. Compute \mathbf{X}^{k+1} by solving the following subproblem

$$\begin{aligned} & \min_{\mathbf{X}} \|\mathbf{w}\|^2, \\ \text{s. t. } & (5.6) - (5.7), \\ & G_{1l}(\mathbf{X}) - \bar{H}_{1l}(\mathbf{X}^k) \leq 0, \quad l \in \mathcal{L}, \\ & G_{2l}(\mathbf{X}) - \bar{H}_{2l}(\mathbf{X}^k) \leq 0, \quad l \in \mathcal{L}, \\ & G_{3l}(\mathbf{X}) - \bar{H}_{3l}(\mathbf{X}^k) \leq 0, \quad l \in \mathcal{L}, \\ & G_{4i}(\mathbf{X}) - \bar{H}_{4i}(\mathbf{X}^k) \leq 0, \quad i \in \mathcal{K}. \end{aligned}$$

2. $k \leftarrow k + 1$.

until stopping criterion.

This first DC formulation contains DC exponential constraints, which are more complicated than other constraints (cubic, quadratic,...). For this reason, we reformulate the problem (5.4) as a simpler form to avoid DC exponential constraints in the next subsections.

5.3.3 The second DCA scheme for solving the problem (5.4)

By replacing variable 2^{2x_l} with x_l and variable 2^{2y_l} with y_l , the problem (5.4) is transformed into

$$\min_{\mathbf{X}} \|\mathbf{w}\|^2, \tag{5.13}$$

$$\text{s. t. } x_l \geq 0, y_l \geq 0, m_l \geq 0, n_l \geq 0, l \in \mathcal{L}, \tag{5.14}$$

$$\mathbf{w}^H \mathbf{E}_l \mathbf{w} \leq x_l, l \in \mathcal{L}, \tag{5.15}$$

$$\mathbf{w}^H (\mathbf{E}_l + 2\bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w} \leq y_l, l \in \mathcal{L}, \tag{5.16}$$

$$\sqrt{2 \ln(1/\rho_l)} n_l - \ln(\rho_l) m_l + \bar{E} - \mathbf{w}^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w} \leq 0, l \in \mathcal{L}, \tag{5.17}$$

$$\sigma_{e,i}^2 \left(1 - \frac{1}{2R}\right) - \mathbf{w}^H \mathbf{D} \mathbf{w} \leq 0, i \in \mathcal{K}, \tag{5.18}$$

$$x_l y_l - n_l^2 \leq 0, l \in \mathcal{L}. \tag{5.19}$$

The problem is nonconvex due to the constraints (5.17)-(5.19), that can be expressed as the following DC constraints.

Since the left sides of (5.17) and (5.18) are concave, the constraint (5.17) is recast as

$$F_{3l}(\mathbf{X}) = G_{3l}(\mathbf{X}) - H_{3l}(\mathbf{X}) \leq 0, \text{ where,}$$

$$G_{3l}(\mathbf{X}) = \sqrt{2 \ln(1/\rho_l)} n_l - \ln(\rho_l) m_l + \bar{E}, \quad H_{3l}(\mathbf{X}) = \mathbf{w}^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w},$$

while the constraint (5.18) is rewritten as

$$F_{4i}(\mathbf{X}) = G_{4i}(\mathbf{X}) - H_{4i}(\mathbf{X}) \leq 0, \text{ where,}$$

$$G_{4i}(\mathbf{X}) = \sigma_{e,i}^2 \left(1 - \frac{1}{2R}\right), \quad H_{4i}(\mathbf{X}) = \mathbf{w}^H \mathbf{D} \mathbf{w},$$

and the constraint (5.19) is transformed into

$$F_{5l}(\mathbf{X}) = G_{5l}(\mathbf{X}) - H_{5l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}, \text{ where}$$

$$G_{5l}(\mathbf{X}) = \frac{1}{2}(x_l^2 + y_l^2), \quad H_{5l}(\mathbf{X}) = \frac{1}{2}(x_l^2 + y_l^2) - x_l y_l + n_l^2.$$

Based on the above DC constraints, we rewrite the problem (5.13) as

$$\begin{aligned} & \min_{\mathbf{X}} \|\mathbf{w}\|^2, & (5.20) \\ & \text{s. t. (5.14) - (5.16),} \\ & G_{3l}(\mathbf{X}) - H_{3l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}, \\ & G_{4i}(\mathbf{X}) - H_{4i}(\mathbf{X}) \leq 0, \quad i \in \mathcal{K}, \\ & G_{5l}(\mathbf{X}) - H_{5l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}. \end{aligned}$$

The linear approximations of $H_{3l}(\mathbf{X})$, $H_{4i}(\mathbf{X})$ and $H_{5l}(\mathbf{X})$ at the point \mathbf{X}^k are given by

$$\begin{aligned} \bar{H}_{3l}(\mathbf{X}) &= (\mathbf{w}^k)^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}^k + 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H (\mathbf{E}_l + \bar{\mathbf{h}}_l \bar{\mathbf{h}}_l^H) \mathbf{w}^k\}, \\ \bar{H}_{4i}(\mathbf{X}) &= (\mathbf{w}^k)^H \mathbf{D} \mathbf{w}^k + 2\Re\{(\mathbf{w} - \mathbf{w}^k)^H \mathbf{D} \mathbf{w}^k\}, \\ \bar{H}_{5l}(\mathbf{X}) &= H_{5l}(\mathbf{X}^k) + 2n_l^k (n_l - n_l^k) + (y_l^k - x_l^k)(y_l - y_l^k) + (x_l^k - y_l^k)(x_l - x_l^k). \end{aligned}$$

This linearization introduces an inner convex approximation of the feasible set of the problem (5.20). Since $G(\mathbf{X}) - H(\mathbf{X}) \leq G(\mathbf{X}) - \bar{H}(\mathbf{X})$, it can lead to infeasibility of the convex subproblems. Thus, the relaxation technique is proposed as follows.

Algorithm 3. DCA₂ for solving the problem (5.20)

Initialization: Let (\mathbf{X}^0) be an initial point, $k \leftarrow 0$.
repeat

1. Compute \mathbf{X}^{k+1} by solving the following subproblem

$$\begin{aligned}
& \min_{\mathbf{X}, s} \|\mathbf{w}\|^2 + t_0 s_0 + t_1 s_1, \\
& \text{s. t. (5.14) - (5.16),} \\
& G_{3l}(\mathbf{X}) - \bar{H}_{3l}(\mathbf{X}^k) \leq s_1, \quad l \in \mathcal{L}, \\
& G_{4i}(\mathbf{X}) - \bar{H}_{4i}(\mathbf{X}^k) \leq s_1, \quad i \in \mathcal{K}, \\
& G_{5l}(\mathbf{X}) - \bar{H}_{5l}(\mathbf{X}^k) \leq s_0, \quad l \in \mathcal{L}.
\end{aligned}$$

2. $k \leftarrow k + 1$.

until stopping criterion.

DCA₂ solves a sequence of convex subproblems which a minimization of a quadratic function with quadratic constraints. In the next subsection, we propose the the third DC program, that differs from the second DC program at the expression of the constraint (5.19). We simplify the approximation part of $F_{5l}(\mathbf{X})$ function ($H_{5l}(\mathbf{X})$), that can improve the quality of DCA scheme.

5.3.4 The third DCA scheme for solving the problem (5.13)

For the third DC program, we replace $F_{5l}(\mathbf{X}) = G_{5l}(\mathbf{X}) - H_{5l}(\mathbf{X})$ with the following DC constraint.

$$\begin{aligned}
F_{6l}(\mathbf{X}) &= G_{6l}(\mathbf{X}) - H_{6l}(\mathbf{X}) \leq 0, \quad \text{where,} \\
G_{6l}(\mathbf{X}) &= \frac{1}{2}(x_l + y_l)^2, \quad H_{6l}(\mathbf{X}) = \frac{1}{2}(x_l^2 + y_l^2) + n_l^2.
\end{aligned}$$

It can be seen that $H_{6l}(\mathbf{X})$ is simpler than $H_{5l}(\mathbf{X})$ one element (x_l, y_l) . The problem (5.13) is rewritten as a DC program

$$\begin{aligned}
& \min_{\mathbf{X}} \|\mathbf{w}\|^2, \tag{5.21} \\
& \text{s. t. (5.14) - (5.16),} \\
& G_{3l}(\mathbf{X}) - H_{3l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}, \\
& G_{4i}(\mathbf{X}) - H_{4i}(\mathbf{X}) \leq 0, \quad i \in \mathcal{K}, \\
& G_{6l}(\mathbf{X}) - H_{6l}(\mathbf{X}) \leq 0, \quad l \in \mathcal{L}.
\end{aligned}$$

The linear approximations of $H_{6l}(\mathbf{X})$ at the point \mathbf{X}^k is given by

$$\bar{H}_{6l}(\mathbf{X}) = H_{6l}(\mathbf{X}^k) + 2n_l^k(n_l - n_l^k) + y_l^k(y_l - y_l^k) + x_l^k(x_l - x_l^k).$$

DCA scheme is proposed to solve the DC program (5.21) as follows.

Algorithm 4. DCA₃ for solving the problem (5.21)

Initialization: Let (\mathbf{X}^0) be an initial point, $k \leftarrow 0$.

repeat

1. Compute \mathbf{X}^{k+1} by solving the following subproblem

$$\begin{aligned} & \min_{\mathbf{X}, s} \|\mathbf{w}\|^2 + t_0 s_0 + t_1 s_1, \\ \text{s. t. } & (5.14) - (5.16), \\ & G_{3l}(\mathbf{X}) - \bar{H}_{3l}(\mathbf{X}^k) \leq s_1, \quad l \in \mathcal{L}, \\ & G_{4i}(\mathbf{X}) - \bar{H}_{4i}(\mathbf{X}^k) \leq s_1, \quad i \in \mathcal{K}, \\ & G_{6l}(\mathbf{X}) - \bar{H}_{6l}(\mathbf{X}^k) \leq s_0, \quad l \in \mathcal{L}. \end{aligned}$$

2. $k \leftarrow k + 1$.

until stopping criterion.

5.4 Numerical experiments

In this section, we study the performance of the proposed DCA based algorithms corresponding to three proposed general DC programs.

Experiment settings

All algorithms were implemented in MATLAB R2016b, and executed on a PC Intel(R) Core(TM)2 Quad CPU 2.83 GHz of 4GB RAM. The CVX toolbox [10] is used to solve the subproblems.

We follow the experiment setting in [105]. All the channel vectors are modeled by large-scale fading and small-scale fading. The large-scale fading model is $D = A_0 \left(\frac{d}{d_r}\right)^{-\alpha}$ where α denotes the path-loss exponent and d_r is the reference distance. The distance d between the transmitter and the information receiver, the energy receivers are 50 and 25 m, respectively. Rician fading is used to model the channel coefficients. The Rician factor is 5. We set parameter as follows: $\alpha = 2$, $A_0 = 1$, $d_r = 20$ m, $N = 4$, $L = 2$, $K = 2$, $R = 2$, $\mathbf{E}_l = 0.1^2 \mathbf{I}$ $l \in \mathcal{L}$, $\bar{E} = 1.5$ dBm, $\rho_l = 0.1$ $l \in \mathcal{L}$, $p_{e,i} = 0.1$, $\mathbf{Z}_i = 0.1i \mathbf{I}$ $i \in \mathcal{K}$, unless otherwise specified. For simplicity, we assume that the noise covariances $\sigma_s^2 = \sigma_{e,i}^2 = -10$ dBm.

We are interested in the effect of the harvested power \bar{E} (Figure 5.1), the target secrecy rate R (Figure 5.2) and the number of transmit antenna N (Figure 5.3). For each value of these parameters, all the algorithms are tested on 10 independent channel realizations. The stopping criterion is $|\|\mathbf{w}^k\|^2 - \|\mathbf{w}^{k-1}\|^2| \leq 10^{-3}$, $s_0 < 10^{-6}$, and $s_1 < 10^{-6}$.

Comments on numerical results

It is seen from Figure 5.1 that higher harvested power can be achieved with the increase of transmit power. In Figure 5.2, as we expected, a larger target secrecy rate results in a higher transmit power. Figure 5.3 shows that the larger the number of transmit antenna is, the smaller the transmit power is.

Note that DCA_1 solves the reformulated problem based on [105] while DCA_2 and DCA_3 solve the reformulated problem (5.13)-(5.19). We first compare DCA_2 and DCA_3 to DCA_1 . Then, we experimentally evaluate the effect of DC decomposition by the comparison between DCA_2 and DCA_3 .

- DCA_1 versus DCA_2 , DCA_3

In terms of the transmit power, DCA_2 and DCA_3 are much more efficient than DCA_1 . Specifically, DCA_3 is the best - the gain varies from 32.5% to 33.8% between DCA_3 and DCA_1 (Figure 5.1). The second is DCA_2 which outperforms DCA_1 - the gain varies from 30.8% to 33.4% (Figure 5.1).

In terms of the computing time, it can be seen that DCA_2 and DCA_3 are much faster than DCA_1 . Specifically, the ratio of gain is from 5.3 to 6.1 times between DCA_1 and DCA_3 (Figure 5.2). DCA_2 is faster than DCA_1 from 5.4 to 6.9 times (Figure 5.2). This reason is because the subproblems of DCA_2 and DCA_3 consist of quadratic constraints while the subproblems of DCA_1 contains exponential constraints.

In summary, DCA_2 and DCA_3 are much more efficient than DCA_1 on both the quality and the rapidity.

- DCA_2 versus DCA_3

Regarding the transmit power, DCA_3 is slightly more efficient than DCA_2 in almost cases. However, there is trade-off between the quality and rapidity. In terms of the computing time, DCA_3 is slightly slower than DCA_2 . Thus, we can say that the alternative DC decomposition can lead to the different performance of the corresponding DCA.

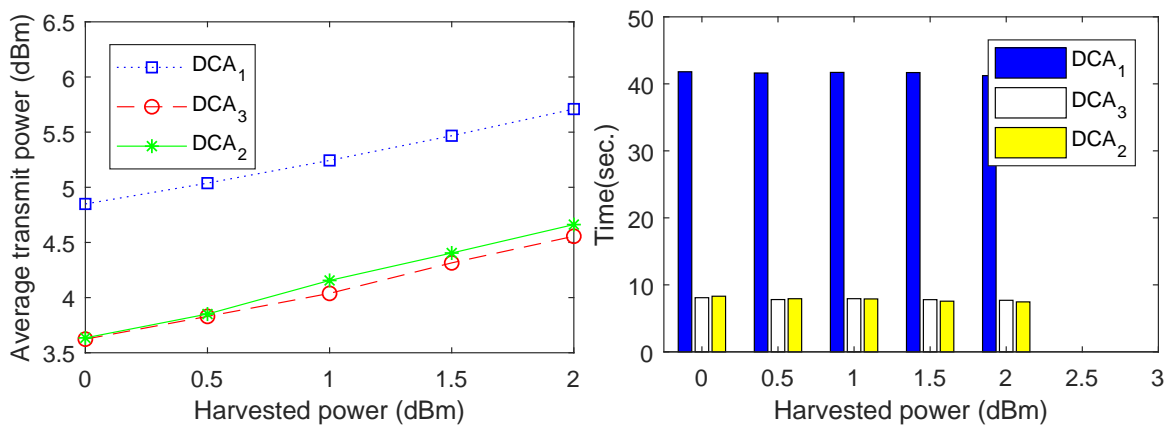


Figure 5.1 – Harvesting power versus transmit power and computing time (in seconds)

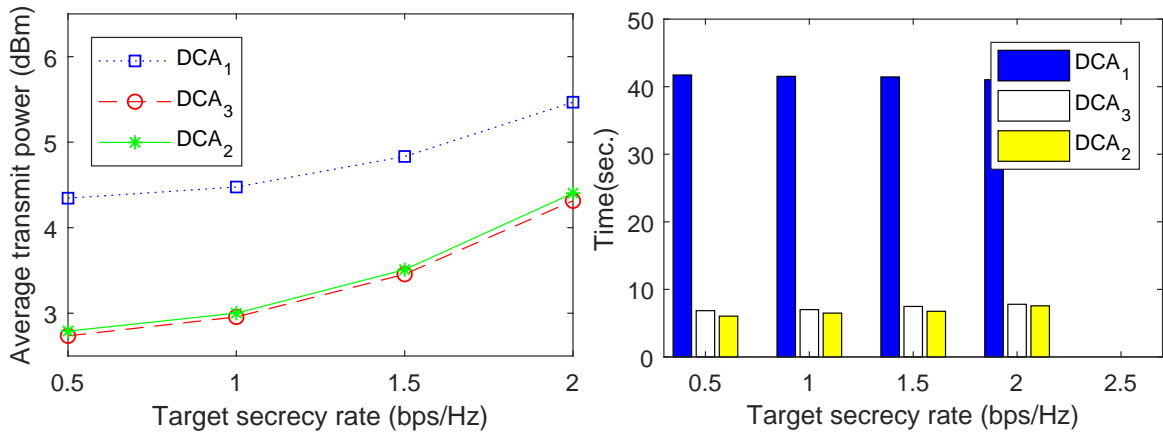


Figure 5.2 – Target secrecy rate gain versus transmit power and computing time (in seconds)

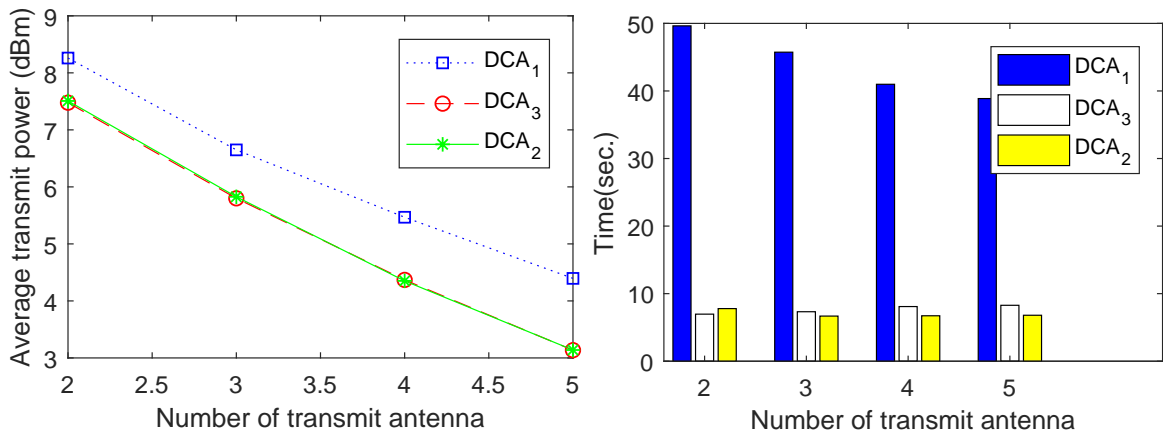


Figure 5.3 – Number of antenna versus transmit power and computing time (in seconds)

5.5 Conclusion

In this paper, we have proposed the MISO SWIPT system employing beamforming strategy, where statistical channel errors of energy receivers and statistical channel knowledge of eavesdroppers are available. The power minimization problem has been investigated subject to probability constraints of the target secrecy rate and harvested power. By applying Bernstein inequalities, we have transformed the probability constraints into deterministic forms. The transformed problem is nonconvex, thus, DC programming and DCA have been studied to solve it. We have first reformulated the deterministic transformed problem as a DC problem, which has been solved by DCA₁. However, this first DC problem contains DC exponential constraints. Hence, the second DC problem has been proposed to replace the DC exponential constraints by quadratic ones. DCA₂ scheme has been developed for solving the second DC problem. Exploiting the nice effect of DC decomposition, we have studied another DC decomposition for the second DC problem, in which results to DCA₃. Numerical results have illustrated the superiority of DCA₂ and DCA₃ compared with DCA₁ on both quality

and rapidity. Moreover, DCA_2 has provided slightly better result than DCA_3 in terms of transmit power.

Conclusion

We have studied two important classes of problems on physical layer security in wireless networks: secrecy rate maximization and transmit power minimization. Basing on a unified DC programming and DCA approach we have formulated all the considered problems as general DC programs and investigated various DCA schemes for them. As mentioned in the introduction, these classes of optimization problems are challenging due to several difficulties: the double non-convexity of the objective and constraints, the presence of DC constraints, in particular DC fractional constraints, and of probabilistic constraints, the complexity and the scalability of the convex subproblems in the algorithms, and the infeasibility of subproblems, etc.

We have investigated several techniques to deal with these challenges by exploiting the special structure of each considered problem and basing on the flexibility and the power of DCA as well as the effect of DC decomposition. Throughout the thesis, we have carefully studied crucial questions while developing the models and solution methods: what is the appropriate formulation that makes the problem tractable and less difficult, what is the "good" DC decomposition (the one corresponds to a simple and efficient DCA scheme), how to effectively solve the convex subproblems, how to overcome the infeasibility of the convex subproblems.

Firstly, the reformulation techniques have been used to get appropriate and tractable formulations. We have resorted to semidefinite relaxation technique with the use of auxiliary variables in a suitable way to eliminate fractional constraints (this type of constraints represents in all considered problems). Besides, another difficulty of the considered problems is that transmitter only knows imperfect channel station information (CSI) with bounded errors or statistical distribution of channels. To deal with constraints contained imperfect CSI bounded in spheres, we have applied S-Procedure to convert them as linear matrix inequalities (chapter 2, chapter 3, and chapter 4). In case of CSI modeled as statistical distribution, the probabilistic constraints have been approximated as their corresponding deterministic ones by using Bernstein inequalities (chapter 5). Here, the transformed problems are still difficult due to their nonconvexity, in particular the DC exponential constraints.

Secondly, the reformulation techniques have been used again to tackle DC exponential constraints in the transformed problems. In the first new model the number of DC exponential constraints is reduced to a half of the one in existing DC formulations (chapter 2). The second new model is more interesting than the first in the sense

that all DC exponential constraints are replaced by quadratic constraints, and the number of constraints is reduced. Those lead to enhance remarkably performance of corresponding DCA schemes. The flexibility of DC decomposition gives birth to the third new DC program.

Thirdly, we have developed the highly efficient schemes based on DCA to solve the proposed DC programs. The effect of DC decomposition has been exploited that results to efficient DCA schemes with convex quadratic subproblems. The relaxation technique has been used to deal with the infeasibility of the subproblems. In case of optimizing simultaneously all variables is too difficult, we have proposed an alternating scheme based on general DCA. At each iteration of the alternating scheme, general DCA $_{\rho}$ has been investigated to solve subproblems, where ρ has been updated iteratively without guaranteeing the second DC component (h function) convex. The convergence properties of the proposed alternating general DCA $_{\rho}$ scheme has been proved. Numerical experiments have demonstrated the efficiency of our DCA based algorithms. Once again, our works have confirmed the flexibility and the effect of DC decompositions, the scalability and, overall, the power of DCA for challenging nonconvex nonsmooth problems.

In summary, this dissertation has explored some issues relating to the secrecy rate maximization and the power transmit minimization problems. These optimization problems are challenging due to complicated DC constraints. By exploiting the special structure of the considered problems and basing on the reformulation, the DC decomposition and the relaxation techniques we have reformulated them as appropriate DC programs. Efficient algorithms based on general DCA have been proposed for solving the proposed DC programs. Numerical results have confirmed the efficiency and superiority of our proposed schemes on both quality and rapidity as compared to the existing methods.

In our ongoing works, DC programming and DCA will continue to be developed for solving optimization problem classes for other structural systems on physical layer security in wireless networks. Specifically, we will study the fact that transmitter does not have any knowledge of eavesdropper's channel state information. In literature, the eavesdropper's channel uncertainty is modeled as bounds and distribution which may not available to sophisticated eavesdroppers.

For a further goal, we will study the trade-offs between security and reliability. In wireless networks, security guarantees the right receivers, the accuracy of the message, and the message access to intended receivers only while reliability means that the certain message is successfully transmitted to the intended receivers. However, there are trade-offs between security and reliability in the presence of eavesdroppers in wireless communications [108]. In literature, the designs guaranteeing security and reliability are investigated separately. It would be of interest to study the security performance maximization problems under constraints of a certain reliability level and transmit power. As powerful tools in optimization, how to develop algorithms based DC programming and DCA should be addressed for solving the future problems.

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