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Microbiological Enhanced Oil Recovery : Model of Kinematic Waves and Asymptotic Analysis

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Laboratoire LEMTA : Laboratoire Énergies et Mécanique Théorique et Appliquée
Ecole Doctorale SIMPPÉ: Sciences et Ingénierie des Molécules des Produits des
Procédés et de l'Énergie

Microbiological Enhanced Oil Recovery: Model of Kinematic Waves and Asymptotic Analysis

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Preface

This research was performed at University of Lorraine in Nancy. The first 7 months of this research I worked in the laboratory LEMTA, and then during more than three years in the laboratory of mathematics IECL (Institut Elie Cartan), into which I moved after my scientific supervisor.

I would like to thank all colleagues, students and friends who have accompanied me during this time, especially I would like to thank:

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Summary

One of the strategic objectives of the modern oil industry is the efficient development of high-viscosity oil reserves, which are characterized by low mobility leading to a sharp decline in the oil recovery factor. The development of such reservoirs by traditional methods (natural drives, waterflooding etc.) is frequently not efficient. The alternative is an application of active recovery methods, in other words, enhanced oil recovery methods.

In this thesis we analyse the problems of modelling the displacement of oil by water in presence of bacteria producing some active chemicals that change favourably the properties of oil and water. More strictly, we analyse the bacteria producing biosurfactant that reduces the negative effects of capillary oil trapping in porous media. Such a problem makes part of the general theory of multiphase multicomponent partially miscible flow with chemical reactions, coupled with the dynamics of population.

The general mathematical model of the process is presented, which is reduced next to the model of kinematic waves, due to several admissible simplifications. More exactly, we have obtained the system of five nonlinear partial differential equations of the first order, which can have discontinuous solutions. Such a system can be studied only numerically in the general case. However, we have shown that for a particular case this model can be completely analyzed qualitatively.

For such an analysis, we have introduced the concept of weak bioreactivity. It corresponds to the asymptotic behaviour of the general model as the rate of bacterial kinetics tends to zero. Applying the technique of asymptotic expansions we have obtained the semi-analytical solution to the displacement problem. In particular, this offered us the possibility to detect the discontinuities (chocks) of various types and to analyse exactly their structure.

The general case of arbitrary kinetic rate was studied numerically, by using the code COMSOL MULTIPHYSICS. We analyzed the impact of the microbial growth rate, microbial and nutrient concentrations, the form of kinetic functions and the viscosity ratio on the oil recovery. In the last chapter, we simulated a field case for a Kazakhstani oil field.

The main and unique tool of studying microbiological enhanced oil recovery (MEOR) was the numerical analysis, whilst analytical solutions were missing. The semi-analytical solutions we have obtained fill this gap. They represent exact results that could be used to check the validity of various numerical schemes and codes.

Keywords Microbiological enhanced oil recovery (MEOR), Microbial kinetics, Bacteria, biotic reactions, weak reactivity, kinematic waves, asymptotic solution.

Résumé

L'un des objectifs stratégiques de l'industrie pétrolière moderne est le développement efficace de réserves de pétrole à haute viscosité, caractérisées par une faible mobilité entraînant une forte baisse du facteur de récupération du pétrole. Le développement de tels réservoirs par des méthodes traditionnelles (déplétion naturelle, injection d'eau, etc.) n'est souvent pas efficace. L'alternative est une application de méthodes de récupération actives, en d'autres termes, des méthodes de récupération assistée du pétrole.

Dans cette thèse, nous analysons les problèmes de modélisation du déplacement du pétrole par l'eau en présence de bactéries produisant des agents chimiques actifs qui changent favorablement les propriétés du pétrole et de l'eau. Plus strictement, nous analysons les bactéries produisant un biosurfactant qui réduit les effets négatifs du piégeage d'huile capillaire en milieux poreux. Un tel problème fait partie de la théorie générale de l'écoulement multiphase multicomposant partiellement miscible avec des réactions chimiques, couplé à la dynamique de la population.

Le modèle mathématique général du processus est présenté, qui a été réduit au modèle des ondes cinématiques, grâce à plusieurs simplifications admissibles. Plus exactement, nous avons obtenu le système de cinq équations différentielles en dérivées partielles non linéaires du premier ordre, qui peuvent avoir des solutions discontinues. Un tel système ne peut être étudié que numériquement dans le cas général. Cependant, nous avons montré que pour un cas particulier, ce modèle peut être complètement analysé qualitativement.

Pour une telle analyse, nous avons introduit le concept de faible bioréactivité. Elle correspond au comportement asymptotique du modèle général lorsque le taux de cinétique bactérienne tend vers zéro. En appliquant la technique de développement asymptotique, nous avons obtenu la solution semi-analytique du problème de déplacement. En particulier, cela nous a offert la possibilité de détecter les discontinuités (les chocs) de différents types et d'analyser exactement leur structure.

Le cas général du taux cinétique arbitraire a été étudié numériquement, en utilisant le code COMSOL MULTIPHYSICS. Nous avons analysé l'impact du taux de croissance microbien, des concentrations microbiennes et nutritives, de la forme des fonctions cinétiques et du rapport de viscosité sur la récupération de l'huile. Dans le dernier chapitre, nous avons simulé un cas réel d'un champ pétrolier kazakh.

Le principal et unique outil d'étude de la récupération microbiologique assistée du pétrole (RAMP) était l'analyse numérique, alors qu'il manquait des solutions analytiques. Les solutions semi-analytiques que nous avons obtenues comblent cette lacune.

Ils représentent des résultats exacts qui pourraient être utilisés pour vérifier la validité de divers schémas et codes numériques.

Mots clés Récupération microbiologique assistée du pétrole (RAMP), cinétique microbienne, bactéries, réactions biotiques, faible réactivité, ondes cinématiques, solution asymptotique.

INTRODUCTION GÉNÉRALE

L'une des tâches principales du développement du champ est d'augmenter la récupération du pétrole. Une croissance considérable de la récupération du pétrole est le résultat de l'application de méthodes de récupération améliorées qui sont principalement réalisées dans les formations pétrolières très visqueuses, les couches coupées par l'eau et les réservoirs à faible perméabilité. Actuellement, la majorité de l'énergie mondiale provient du pétrole brut. Une grande partie de cette ressource précieuse et non renouvelable est laissée dans le sol après l'application des méthodes d'extraction de pétrole conventionnelles, où les réservoirs de pétrole doivent être abandonnés car la production n'est plus économiquement réalisable.

Les réserves mondiales prouvées de pétrole ont plus que doublé au cours des 35 dernières années pour atteindre la valeur actuelle de 1,6 billion de barils de pétrole. En utilisant la technologie d'aujourd'hui, entre 20 et 40 % du pétrole d'origine en place dans le réservoir peut être récupéré lors d'une opération de production conventionnelle (par exemple, le forage vertical). La récupération secondaire du pétrole par les techniques d'injection d'eau et d'injection de gaz peut atteindre 15 à 25 % supplémentaires, laissant jusqu'à 55 % de pétrole résiduel non récupérable dans les réservoirs. La région caspienne contient des réserves de 300 à 350 milliards de barils, le Kazakhstan contrôlant environ 110 milliards de barils dans cette région. Parmi ceux-ci, environ 30 % sont facilement récupérables, ce qui représente 34 milliards de barils.

La récupération du pétrole des réservoirs souterrains se fait en plusieurs étapes. De plus, il est urgent de produire plus de pétrole brut pour répondre à la demande énergétique mondiale croissante, ce qui indique la nécessité de promouvoir des procédés de récupération assistée du pétrole (RAP). Les méthodes améliorées de récupération du pétrole consistent en une exposition au processus de formation avec des agents physiques ou chimiques (polymères, tensioactifs, solvants, chaleur, etc.) pour augmenter la mobilité du pétrole ou l'efficacité de nettoyage du processus de déplacement. La théorie de l'écoulement composite biphasé et triphasé à travers des milieux poreux a été développée principalement en fonction de la nécessité de prédire l'efficacité d'une récupération améliorée du pétrole.

En particulier, au Kazakhstan, la plupart des gisements d'hydrocarbures ont déjà été découverts et sont au stade de la production commerciale. De moins en moins de sites de forage présentent désormais un intérêt commercial dans ces réservoirs matures. À cet égard, le pétrole résiduel restant dans ces réservoirs d'hydrocarbures matures provenant de la récupération primaire et secondaire du pétrole permet des processus améliorés de récupération du pétrole, y compris l'utilisation de la technologie de récupération assistée microbienne du pétrole.

Malgré les progrès considérables de la théorie des méthodes améliorées de récupération du pétrole, ces méthodes sont souvent considérées comme économiquement peu attrayantes [1, 2, 3, 4]. Par conséquent, il est nécessaire de développer des méthodes respectueuses de l'environnement et compatibles avec l'infrastructure de l'industrie pétrolière

au Kazakhstan. La récupération assistée microbienne du pétrole est dépourvue de cet inconvénient. Les microbes et leur nourriture sont naturellement présents dans les strates géologiques, il suffit de les activer et de les utiliser. Ainsi, d'un point de vue économique, le processus lui-même est abordable par rapport aux autres processus RAP [3, 2].

Dans RAMP, de l'eau, des bactéries et des nutriments sont injectés dans un réservoir d'huile, et les bactéries produisent un tensioactif, ce qui réduit le rôle négatif des effets capillaires. Par conséquent, une partie du pétrole qui a été piégée lors de l'inondation peut être mobilisée et produite grâce à l'action des microbes.

L'avantage principal de la récupération assistée microbienne du pétrole de la récupération assistée du pétrole chimique, lorsque le tensioactif est directement injecté avec de l'eau, consiste en ce que les microbes se développent en permanence et intensifient leur activité commune, tandis que les méthodes chimiques sont caractérisées par une efficacité à court terme due à la dissolution des produits chimiques injectés par l'eau. Il est à noter qu'un autre avantage de la microbiologique récupération améliorée du pétrole est sa sécurité écologique.

La récupération assistée microbienne du pétrole (RAMP) est une des nouvelles méthodes expérimentales développées. RAMP [5] consiste à utiliser des bactéries et leurs sous-produits utilisés pour manipuler la fonction ou la structure des environnements microbiens existants dans les réservoirs de pétrole pour améliorer la récupération du pétrole piégé dans des milieux poreux tout en augmentant les profits économiques.

Cette méthode a montré son potentiel en tant que méthode de récupération du pétrole tertiaire qui prolonge la durée de vie des réservoirs de pétrole Matures. Par conséquent, les technologies microbiennes sont de plus en plus acceptées dans le monde entier comme des approches rentables et respectueuses de l'environnement pour augmenter la production de pétrole [6].

L'efficacité de la méthode a été prouvée dans plusieurs pays, par exemple, en Russie où pendant cinq ans d'application de la récupération assistée microbienne du pétrole dans le champ Bonduge appartenant à TatNeft il a été obtenu 30 % de toute l'huile produite.

La situation au Kazakhstan est telle que les principaux gisements de pétrole sont en dernière phase de développement et contiennent beaucoup de pétrole non produit piégé par l'eau injectée.

À notre connaissance, il n'existe aucun rapport accessible au public sur l'application de RAMP dans un domaine mature au Kazakhstan. Bien que les prérequis RAMP ne soient pas nouveaux, cette biotechnologie industrielle doit être mise en œuvre localement, une compréhension approfondie de la microbiologie et des conditions biogéochimiques locales est donc requise.

Il est à noter que les technologies basées sur l'utilisation de micro-organismes deviennent de plus en plus populaires non seulement dans l'industrie pétrolière, mais dans d'autres processus industriels et à l'échelle mondiale. Plusieurs processus observés dans les aquifères naturels, stockage souterrain de gaz, ou des réservoirs de pétrole utilisant des micro-organismes, qui consomment certaines espèces chimiques (nutriments) et d'autres produits chimiques (métabolites). Ces métabolites peuvent améliorer le transport et la qualité des fluides. Par exemple, dans les processus de récupération assistée microbienne

du pétrole, l'eau, les bactéries et les éléments nutritifs sont injectés dans un réservoir de pétrole, et les bactéries génèrent un agent tensio-actif qui réduit les effets capillaires. Après l'injection initiale de micro-organismes, nous introduisons des nutriments dans la roche du réservoir pour permettre la croissance de la population bactérienne. Les bactéries produisent eux même des agents chimiques permettant à la partie du pétrole qui a été piégée d'être mobilisée et produite.

Il y a également la technologie de la bioremédiation des eaux souterraines qui utilise des bactéries capables de dégrader divers polluants organiques et inorganiques. Dans la méthanisation souterraine ou le stockage souterrain d'hydrogène, H_2 et CO_2 sont injectés dans un aquifère et transformés en méthane par les bactéries méthanogènes. La technologie de lixiviation de l'uranium utilise également des bactéries qui peuvent produire un acide capable de dissoudre les oxydes d'uranium.

L'objectif de cette thèse est d'étudier le problème du déplacement d'un fluide par un autre dans les milieux poreux. Nous nous focalisons sur la description de la structure interne de fluide et un certain nombre de composants chimiques dissous dans ces phases.

Les principaux objectifs de la thèse sont :

- Développer le modèle mathématique conceptuel des processus observés au cours de la récupération assistée microbienne du pétrole, en couplant le transport réactif multiphasique à la dynamique des populations ;
 - Obtenir des solutions analytiques ou semi-analytiques des problèmes du modèle susceptibles de fournir des résultats qualitatifs ;
 - Révéler les principaux scénarios d'évolution du système régis par la dynamique non linéaire de la population ;
 - Appliquer les modèles développés pour comparer avec les données de cas sur le gisement.

Toute théorie doit avoir des résultats analytiques exacts de base, qui pourraient être utilisés pour vérifier la validité de divers schémas numériques et codes. Le résultat d'une solution analytique est donné en décrivant l'influence sur les profils de saturation déterminant leur contribution à la récupération accrue du pétrole. L'influence sur les profils de saturation devient importante car les caractéristiques du processus de RAMP doivent être déterminées introduisant les équations pour l'équilibre de solubilité et pour les propriétés du matériau. Le test de la modélisation numérique et de la validation de la mise en œuvre sont discutés et analysés pour des modèles de flux de deux phases avec différentes paramétrisations de fluide et un modèle de transport simple en comparant les solutions numériques avec les solutions analytiques fabriquées en discutant de l'effet de la concurrence microbienne sur le système.

La stabilité du modèle mathématique a été étudiée dans le développement à une paire d'équations différentielles. Le modèle mathématique général du processus est présenté, et

a été réduit au modèle des ondes cinématiques, grâce à plusieurs simplifications admissibles. Nous avons obtenu un système de cinq équations aux dérivées partielles non linéaires du premier ordre, qui peuvent avoir des solutions discontinues (les chocs). Nous avons déterminé les types de chocs admissibles et dérivé les conditions de Huguenot-Rankine aux chocs. En négligeant la diffusion et la pression capillaire, et en utilisant certaines propriétés typiques des systèmes biochimiques, nous avons réduit notre système d'équations à un système de quatre équations hyperboliques quasi linéaires de premier ordre. En physique, un tel système est appelé le modèle des ondes cinématiques.

Une brève introduction aux mécanismes de la récupération assistée microbienne du pétrole est décrite et les effets biochimiques possibles sont expliqués. Un modèle d'écoulement et de transport en milieu poreux est mis en œuvre en combinant plusieurs processus tels que la cinétique des réactions microbiennes et la croissance de la population. Ces procédés sont généralement modélisés par un système d'équations fortement couplé, dépendant de l'écoulement est diphasique multi composant, des relations thermodynamiques pour la dissolution des composants dans les différentes phases, des formules cinétiques de réactions chimiques, et une équation de transport non linéaire pour la dynamique des populations bactériennes.

Notre approche, qui nous permet d'obtenir des solutions analytiques, s'appuie sur une spécificité des microorganismes, ce qui nous donne la possibilité d'introduire la notion générale de déséquilibre faible. Et nous sommes arrivés à quelques conclusions:

1. La méthode MEOR ne peut être efficace qu'après une injection à long terme, jusqu'à ce que le contre-choc Sc arrive au puits producteur.
2. L'effet positif du MEOR consiste à retirer du pétrole supplémentaire de la zone inondée, en raison de l'amélioration de la perméabilité relative. L'effet principal est concentré dans la zone proche du puits d'injection.
3. L'effet négatif consiste en la formation du banc de pétrole dans la zone inondée, qui déplace l'eau, et réduit finalement l'amplitude du front avant de déplacement du pétrole par l'eau. A partir de ce moment, l'efficacité du déplacement de l'huile par le front avant a été réduite.
4. Pour améliorer l'efficacité du MEOR, il est nécessaire de réduire le rôle négatif de la banque pétrolière.

De plus, il a été obtenu que les réactions biochimiques ont une influence importante dans les études prédictives en raison de la réduction de la viscosité et de la tension interfaciale huile-eau, qui sont des paramètres bénéfiques pour la récupération du pétrole, etc.

Les quelques études de terrain disponibles dans le domaine public manquent d'informations importantes telles que des détails sur le mélange de nutriments utilisé pour la biostimulation. Par conséquent, dans notre modèle mathématique, la concentration est plus généralisée que d'habitude pour les champs, en raison de la complexité des consortiums microbiens par rapport à la souche unique et des caractéristiques géochimiques uniques du réservoir pétrolier, qui sont souvent mal caractérisées. Cependant, tous les éléments

fondamentaux : carbone, azote, phosphore et oligo-éléments, doivent être apportés à des concentrations adéquates. Le mélange chimique pour la biostimulation dépend de la disponibilité locale d'usines chimiques telles que les raffineries de sucre et les usines d'engrais, qui peuvent fournir des produits dilués adaptés à l'injection dans le réservoir de pétrole. Outre la disponibilité, les caractéristiques qualitatives de ces sous-produits par rapport à leurs teneurs et proportions en éléments nutritifs sont également importantes. En général, des injections répétées sur une période prolongée (par exemple, 6 à 9 mois) sont nécessaires pour obtenir une biostimulation efficace et une récupération assistée du pétrole mesurable.

Nouveauté scientifique et signification:

La récupération assistée microbienne du pétrole (RAMP) a déjà été étudié dans plusieurs articles à la fois expérimentalement et théoriquement. Contrairement à ces travaux, nous avons utilisé des relations plus avancées qui décrivent la cinétique de la population et nous avons montré que l'impact de la forme des fonctions cinétiques est important. Deuxièmement, tous les articles théoriques sont basés sur des simulations numériques. Nous avons obtenu la solution analytique du problème, qui correspond au cas asymptotique de la bio réactivité faible. Nous l'appelons "semi-analytique", car certains paramètres de cette solution doivent être calculés en résolvant des équations algébriques non linéaires les plus simples. Cependant toute la structure de la solution, les types et le nombre de discontinuités (chocs) et les paramètres de l'chocs sont obtenus analytiquement. En troisième lieu, nous avons effectué une analyse détaillée de l'impact de différents paramètres du système sur le comportement du procédé et la récupération finale du pétrole.

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Introduction

One of the main tasks of field development is to increase the oil recovery. A considerable growth in oil recovery is a result of enhanced recovery methods application which are mainly achieved in highly viscous oil formations, water-cut layers and low permeable reservoir. Nowadays the majority of the world's energy comes from crude oil. A large proportion of this valuable and non-renewable resource is left behind in the ground after the application of conventional oil extraction methods, where the oil reservoirs must be abandoned as the production is no longer economically feasible. Recovering oil from subsurface reservoirs is done in multiple stages. Moreover, there is a dreadful need to produce more crude oil to meet the worldwide rising energy demand which illustrates the necessity of progressing Enhanced Oil Recovery (EOR) processes. EOR methods consist of exposure to the formation process with physical or chemical agents (polymers, surfactants, solvents, heat, etc.) in order to increase the mobility of the oil or the efficiency of cleaning the displacement process. The theory of two- and three-phase composite flow through porous media was developed mainly in correspondence with the necessity of predicting the efficiency of enhanced oil recovery. Despite the high advancement in the theory of EOR, these methods are often considered economically unattractive [1, 2, 3, 4]. Microbial enhanced oil recovery (MEOR) is devoid of this drawback. Microbes and food for them are naturally present in geological strata, it is sufficient to activate and use them. So, from an economical point of view, the process itself is affordable compared to other EOR processes [3, 2]

In MEOR, water, bacteria and nutrients are injected into an oil reservoir, and the bacteria produce a surfactant, which reduces the negative role of capillary effects. Consequently, a part of oil that has been trapped during waterflooding, can be mobilized and produced due to the action of microbes.

The principle advantage of MEOR from chemical EOR, when the surfactant is directly injected with water, consists of that the microbes permanently grow and intensify their common activity, while the chemical methods are characterized by short-term efficiency due to the dissolution of the injected chemicals by water. It is to note that another

advantage of MEOR is its ecological safety.

Microbial methods are new in experimental areas of research. MEOR [5] consists of using bacteria and their by-products which utilized for manipulating function or structure of microbial environments existing in oil reservoirs to improve the recovery of oil entrapped in porous media while increasing economic profits. MEOR has shown its potential as a tertiary oil recovery method that extends the life of mature oil reservoirs. Microbial technologies are becoming accepted worldwide as costeffective and environmentally friendly approaches to increase oil production [6].

The efficiency of the method was proved in several countries, for instance, in Russia where during five year of application of MEOR in the field Bonduge belonging to TatNeft it was obtained 30% of all the produced oil.

The situation in Kazakhstan is such that the major oil fields are in the last stage of development and contain a lot of non-produced oil trapped by the injected water.

It is to note that the technologies based on the use of microorganisms becomes increasingly popular not only in petroleum industry, but in other industrial processes and globally in geosciences. Several processes observed in natural aquifers or underground gas storages rely on using microbes. Their action consists of producing some chemicals (the metabolites), which generally improve flow parameters or fluid quality. For example, the in situ bioremediation of groundwater uses bacteria to degrade various organic and inorganic pollutants. In the underground methanation or underground storage of hydrogen, hydrogen and CO₂ are injected into an aquifer and transformed into methane by methanogenic bacteria. The technology of in-situ leaching of uranium also uses bacteria that can produce an acid capable of dissolving uranium oxides.

Objectives of study

The main objectives of the thesis are:

- to develop the conceptual mathematical model of the processes observed in course of MEOR, by coupling the reactive multiphase transport with population dynamics;
- to obtain analytical or semi-analytical solutions of the model problems susceptible to provide qualitative results;
- to reveal the main scenarios of the system evolution governed by nonlinear dynamics of the population.
- To apply the developed models to compare with field case data.

Novelty of the results obtained

MEOR was already studied in several papers both experimentally and theoretically. In contrast to these works, we used more advanced relationships that describes the population kinetics and we have shown that the impact of the form of the kinetic functions is important. Secondly, all the theoretical papers are based on numerical simulations. We have obtained the analytical solution of the problem, that corresponds to the asymptotic case of weak bioreactivity. We call it "semi-analytical", because some parameters of this solution have to be calculated by solving some simplest nonlinear algebraic equations. However all the structure of the solution, the types and the number of discontinuities (the shocks) and the parameters of the shocks are obtained analytically. In third, we performed a detailed analysis of the impact of various parameters of the system on the behaviour of the process and the final oil recovery.

Chapter 1

Literature review on MEOR and bacterial dynamics

This part provides an extended review on MEOR mechanisms, its advantage over conventional EOR methods, its operational problems and engineering challenges.

1.1 Microbial Enhanced Oil Recovery: MEOR

1.1.1 EOR objectives and forms

Oil production from geological strata is performed in three stages:

(i) During the primary stage, oil is produced due to its own potential energy. The initial reservoir pressure is usually high enough to lift the oil up to surface, otherwise it is possible to help oil to lift by pumping it or by injecting a gas into the producing well (gaslift) and reducing the oil density;

(ii) The secondary stage is that of water injection with the objective to maintain oil pressure (or the potential energy) and to displace oil mechanically. This stage begins

when the gaslift or pumping become non rentable economically [7]. Instead of water it is possible to inject gas (immiscible) if the reservoir has a gas cap;

(iii) The tertiary stage (or the stage of EOR) involves the extraction of residual oil after the primary and secondary phases by applying various physicochemical techniques [8]. The objective of these techniques is to modify the properties of fluids or/and rocks, in particular [9]:

- to reduce oil viscosity (injection of miscible gases or thermal EOR);
- to reduce the interfacial tension between oil and water (injection of surfactants);
- to increase water viscosity and the stability of displacement (injection of polymers);
- to maintain pressure at a constant level sufficient to uplift oil in the production wells;
- to displace mechanically oil from injectors to producers.

All these various objectives lead to four main EOR methods: thermal EOR, miscible EOR, chemical EOR and microbiological EOR (MEOR).

1.1.2 Thermal, Chemical and Miscible EOR

Thermal EOR is achieved in three ways: injection of steam, injection of hot water, and the in situ combustion. Steam injection increases oil recovery by 50 percent up to 65 percent of the original oil in place [10]. It is however a difficult method to employ as it requires boiling large quantities of water in the field. The defect of the method is the necessity to have a lot of surrounding water to produce steam, and the air pollution by steam production by burning a part of the produced crude oil [10].

Chemical EOR: chemicals such as surfactants, alkaline solutions, and polymers are added to the displacing water in order to change the physicochemical properties of oil and/or water making the displacement process more effective. Polymer flooding improves the vertical and real sweep efficiency as a consequence of improving the *water/oil* mobility ratio. On the other hand, caustic flooding will result in the production of soap that may lower the interfacial tension enough to increase production if the oil reservoir has naturally occurring organic acids [11]. However, this type of chemical reaction would take place in a poor reservoir so it will also produce oil pollution and the capacity for water absorption would be damaged. Most wells cannot achieve a satisfactory result using this method, making it counter productive, with the negative effects outweighing the benefits. For instance, this method faces significant challenges in light oil reservoirs. One of the reasons is the availability, or lack of, compatible chemicals in high temperature and high salinity environments [11].

Miscible EOR means mainly the injection of CO₂. During the carbon dioxide injection, CO₂ partially or completely dissolves in oil, which can lead to a significant decrease in

the oil viscosity, or even to its partial or complete evaporation. Miscibility between the injected and the initial phases brings the system closer to quasiphase behavior, so that the relative permeability becomes diagonal, the residual saturations tend to zero, the displacement tends to be piston-like. All these tendencies enhance oil recovery [12, 13] .

1.1.3 MEOR technology

MEOR consists of injecting water with bacteria or nutrients for bacteria already inhabiting in rocks, so that the result of bacterial metabolism improves the efficiency of oil displacement [14]. Unlike conventional EOR methods, the MEOR approach targets more than one process simultaneously. Various versions of MEOR are as follows:

- Microbes produce a surfactant that reduces oil/water surface tension and capillary trapping;
- Microbes produce miscible gas (CO_2 , CH_4) that dissolves in oil and reduces its viscosity;
- Microbes produce metabolites that selectively plug pores and force water to bypass these sites and to displace trapped oil;
- Microbes destroy long-chain molecules of oil and reduce its viscosity;
- Microbes produce acids that dissolve rocks improving their permeability.

The interest of the present thesis is focused only on the first mechanisms mentioned above, which is considered as the most promising in petroleum industry.

Biosurfactant synthesized by microbes reduces the interfacial tension between water and oil, thus releasing oil trapped in the rock by capillary forces. Such a system is two-phase (water-oil) [15].

In the case when bacteria produce various immiscible gases, their bubbles expand, as far as the pressure decreases, and displace oil mechanically. Such a gas can be H_2 or CH_4 if the oil is saturated with methane (then an excess amount of methane can no longer dissolve in it).

The production of miscible gas (CO_2) by bacteria, which easily dissolves in oil, reduces its viscosity and increases its fluidity.

In all the cases, bacteria also cause the second effect that is related to pore clogging. Microorganisms develop biofilms on the reservoir rock surfaces, which cause plugging of watered pores, thus redirecting the injected water to up swept areas of higher oil saturations, making waterflooding more effective. The area of preferable water flow and

stagnant oil are called thief zones. Figure 1.1 illustrates the effect of blocking thief zones by microbes and forcing water to flow through new areas, thus driving more oil to producing wells.

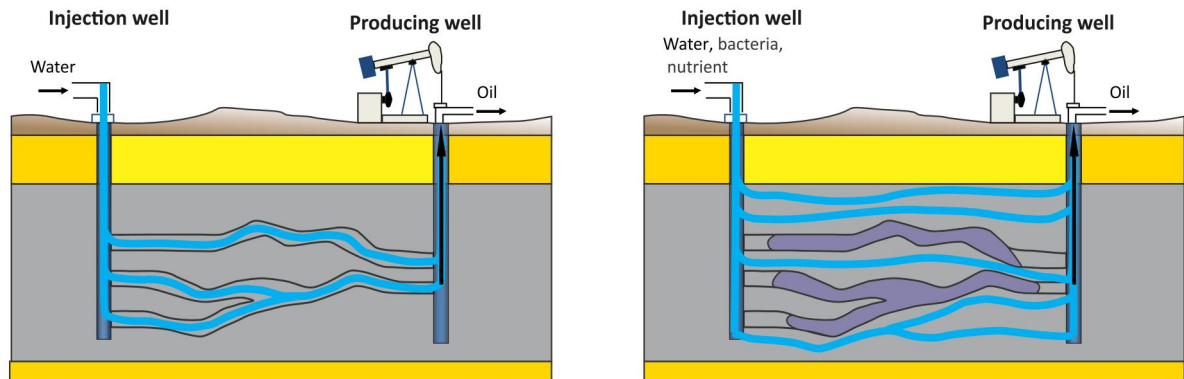


FIG. 1.1 : Visualization of thief zones (left part) and the process of blocking thief zones

1.1.4 Advantages and disadvantages of MEOR

The main advantages of MEOR over conventional enhanced oil recovery (EOR) methods are: much lower energy consumption, lower cost, and low environmental toxicity. Lower costs come from the fact that after the introduction of microorganisms into the reservoir their growth increases exponentially, therefore it is possible to obtain a large number of active substances from a small volume of initial organic material. After the initial injection of microorganisms, it is only necessary to supply nutrients into the reservoir rock to enable the development of the existing bacterial population that will produce the required chemical agents.

More in details, the advantages are:

- Injected microbes and nutrients are cheap; easy to handle in the field and independent of oil prices.
- Economically attractive for mature oil fields before abandonment [5].
- Existing facilities require slight modifications.
- Easy application.
- Less expensive set up.
- Low energy input requirement for microbes to produce MEOR agents.
- More efficient than other EOR methods when applied to carbonate oil reservoirs.

- Microbial activity increases with microbial growth. This is opposite to the case of other EOR additives in time and distance.
- Cellular products are biodegradable and therefore can be considered environmentally friendly.

The disadvantages of MEOR are as follows:

- The oxygen deployed in aerobic MEOR can act as corrosive agent on non-resistant topside equipment and down-hole piping
- Anaerobic MEOR requires large amounts of sugar limiting its applicability in offshore platforms due to logistical problems
- Exogenous microbes require facilities for their cultivation.
- Indigenous microbes need a standardized framework for evaluating microbial activity, e.g. specialized coring and sampling techniques.
- Microbial growth is favoured when: layer permeability is greater than 50 md; reservoir temperature is inferior to 85⁰C, salinity is below 150 g/L and reservoir depth is less than 3500m [16].
- The complexity of the MEOR processes is also a disadvantage. Since there are several reactions occurring that are specific only to MEOR, commercial reservoir simulators, such as Schlumberger's ECLIPSE, do not include MEOR functionality.

Not all reservoir micro-organisms can be applied in MEOR process; some such as the Sulfate reducing bacteria [17] (SRB are some of the oldest microorganisms on Earth) may even be harmful for the exploitation and processing of crude oil. SRB are heterotrophic organisms and absolute anaerobes that use sulfates as well as other oxygenated sulfur compounds (sulfites, thiosulfites, trithionate, tetrathionate, and elemental sulfur) as final electron acceptors in respiration processes [18]. The activity of SRBs can be hampered by the introduction of chemical compounds into the reservoir that would increase the activity of autochthonous microorganism groups to slow down the activity of the SRBs [18]. Very often applications of MEOR can result in the increase of hydrogen sulfide activity and production. This phenomenon is very unfavorable and cannot be controlled entirely due to the complex nature of biochemical and abiotic processes taking place in the reservoir and in the exploitation area.

Another factor that reduces the MEOR efficiency is the adsorption of several metabolites. The polymer and the surfactant no longer flow and do not improve oil recovery during adsorption and become useless.

It is obvious that simultaneously there are occurring several processes, which are influenced by many factors. The challenge is to encapsulate them all into a single mode.

1.1.5 Factors affecting MEOR

The important factors affecting MEOR are the temperature, pressure, pH, salinity, and pore size. At high temperature, the microbes do not function properly, which means no reproduction and no metabolite production. The maximum growth rate was observed at temperatures below 80 °C. However, it is known that some microorganisms can grow at temperatures up to 120 °C ([19]).

Pore size and geometry may affect bacterial motion. Penetration of bacteria through the reservoir is regarded possible with minimum pore diameters of at least $2\mu\text{m}$ [20] and ideally be in the range of 6–10 μm [21]. If this is not the case, microbes may not be transported to the target zone. Even if we assume that the pore spaces are relatively large, losses of injectivity may occur due to well-bore plugging by bacteria.

A reservoir with high pressure allows gases to mix with fluids for which the reservoir may have strong acidic components [22, 23]. At high pressures the DNA double helix becomes denser, and therefore both gene expression and protein synthesis are affected also increasing pressure increases gas solubility, and this may affect the redox potential of gases participating as electron acceptors and donors, such as hydrogen or CO_2 [22].

Nielsen [23] reports that acidity determines the microbial surface charge, which also affects the transport of microbes. A low pH value impairs the ability of microbes to reproduce. Embedded cell proteins (enzymatic activity) play a fundamental role in transporting chemicals across the cell membrane. Their function is highly dependent on their ionization state, which in turn is highly dependent on pH. So far, an understanding of the interaction between pH and the microbial communities of the environment remains unknown, and research is still under development. Moreover, its physiology influences the transport of bacteria. For example, bacteria with a hydrophobic surface tend to stick together and transport in a stream, while hydrophilic bacteria often are suspended and transported alone with [24] fluid.

The thermodynamically favorable oxidative potential is crucial for the survival of microbes. For bacterial growth, electron donors and acceptors must be present, where they are oxidized and reduced in biochemical processes, respectively. In aerobic respiration, oxygen in the form of O_2 is a terminal electron acceptor, where they receive a large amount of energy used in the processes of growth and maintenance. When there is no oxygen, only anaerobic processes occur. Especially for the oil reservoir, the redox potential is very low, and electron acceptors such as ferric ion, nitrate and sulfate are used as the background. The water phase contains sulfate and carbonate in various concentrations, which suggests that the main metabolic processes that occur naturally are sulfate reduction, methanogenesis, acetogenesis and fermentation [22, 20].

It is also necessary to determine whether there are any microorganisms that the injected microbes will have to compete with for nutrients.

Although the fundamental mechanisms for controlling microbial transport, growth, and sedimentation are well understood in laboratory experiments, the results are often not directly related to processes occurring in underground and engineering systems [25]. The physical, chemical, and biological heterogeneity inherent in these systems is a very complex medium for simulating [25].

1.1.6 Previous studies on MEOR

The study of MEOR based on a coupled system of equations of bacterial dynamics and flow in porous media has been actively studied in hydrogeology where, however, only the case of single-phase flow was analysed [26]. In reservoir engineering the hydrodynamic models of MEOR have been analysed for two-phase case of oil and water in [27, 28, 29] and other papers.

Islam, [28] presented a mathematical model for MEOR, where bacterial growth leads to plugging the pores, reducing the oil viscosity and interfacial tension, and producing a gas. Interfacial tension was directly correlated with bacterial concentration avoiding actual surfactant production in the model.

In the work [29], bacteria produce a surfactant as the metabolites. This biosurfactant reduces the oil-water interfacial tension. It was assumed that surfactant can be distributed between both phases being a novel approach in the context of MEOR mathematical modelling. All the analysis was performed only numerically, without developing the general theory.

The case of three-phase flow (oil, water and gas produced by bacteria) was studied in [30] but only on the basis of some semi-empirical tools. The mathematical theory of these processes remains a very complicated problem.

The weak point of all papers devoted to this subject is the model of bacterial kinetics, which is done in very approximate way, by using mechanistic model (Monod) describing just the simplest case of a single stage (exponential in time), a single reaction initiated by microbes, and a single type of metabolism. This is far from the reality of MEOR where the one species initiates several reactions (production of various gases, surfactant, etc.), consumes nutrients through different types of metabolisms, and the kinetics occurs through several stages. The kinetics that takes the double metabolism was suggested in [31] for methanogenic microorganisms in underground storage of hydrogen. In the present thesis we will extend the similar principles on MEOR.

A part of bacteria live in a film of polysaccharides created by them-self on pore walls

(the biofilm). The kinetics of nutrient consumption by a biofilm is qualitatively different from that of microbes freely moving in water. The study of multi-stage and multi-metabolic kinetics for biofilm is a new scientific challenge.

1.2 Capillary trapping of oil and surfactants

1.2.1 Mechanism of capillary trapping

The main losses of oil in underground reservoirs are caused by the phenomenon of capillary trapping, which consists of the following. Two phases (oil and water) are separated by an interface, which possesses a proper energy, called the Helmholtz free energy. The free energy divided by the surface area is an intensive parameter called the surface tension (or the interfacial tension IFT). Due to this energy, the pressures from both sides of a closed interface are distinct. Their difference is called the Laplace capillary pressure. It is proportional to the surface tension, i.e. the capillary pressure disappears if the surface tension tends to zero.

Let us consider an oil drop surrounded by water in a pore channel, Fig. 1.2.

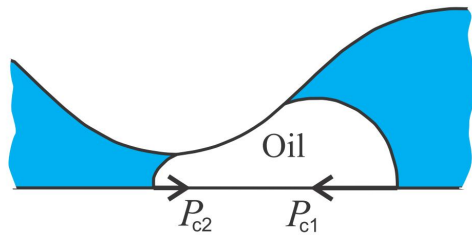


FIG. 1.2 : Mechanism of capillary trapping

For an arbitrary placement of the drop, the capillary pressures at the left and right menisci are distinct. This generates the resultant capillary force of pressure directed to the left. If the water flows to the right, then the resultant capillary force immobilises the oil drop being opposite to the water flow.

This effect is also known as the Jamin effect.

Thus, to reduce the capillary trapping of oil, it is necessary to reduce the capillary pressure, or to reduce the surface tension. This is possible to do by surfactants.

1.2.2 Mechanism of the action of surfactants

A surfactant is a chemical substance, whose molecules prefer to be placed near the interfaces and which reduces significantly the surface tension even at small concentration. The molecule of a typical surfactant consists of two parts: its head has hydrocarbon nature and is hydrophobic, while the opposite part is polar (ionic) and is hydrophilic (Fig. 1.3).

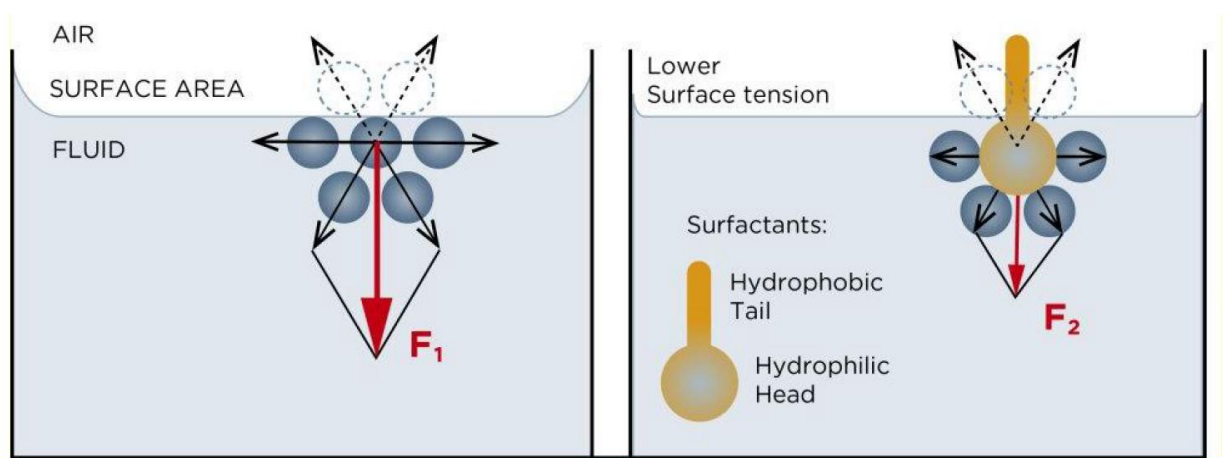


FIG. 1.3 : Effect of surfactant on the surface tension

The hydrophobic head prefers to be dissolved in oil, while the hydrophilic tail prefers to remain in water. As the result, such a molecule occupies the place exactly at the oil-water interface. This reduces the difference between two phases, and, consequently, the surface tension.

In general case, a surfactant can not only reduce the surface tension between oil and water, but also between water and solid, which makes water more adhesive (more wetting). The hydrophobic heads penetrate between the oil and the solid, being followed with water film. Due to this the oil is separated from the solid walls, which creates an emulsion of oil in water. Such an emulsion has specific structure, as oil forms micelles (the drops of oil surrounded by water molecules). This provides a physical mechanism to mobilise oil in a moving aqueous phase [32].

Thus, the surfactant reduces the negative effect of capillary trapping of oil in pores. The liberated oil can now be more easily mobilized and produced. All this can be reached

by dissolving surfactants in water.

1.2.3 Effect of capillary trapping in flow models

Capillary trapping occurs on the pore scale, whilst the flow equations used in reservoir engineering are macroscopic. How this fine effect can appear in coarse models? The principle consists in the following.

The flow of two phases in porous media is governed by the well-known Darcy's law, which says that the flow velocity is proportional to the pressure gradient:

$$\mathbf{V}_i = -\frac{Kk_i(S)}{\mu_i}\nabla P_i, \quad i = w, o \quad (1.1)$$

where \mathbf{V}_i is the Darcy velocity; K is the intrinsic permeability of the medium; μ is the dynamic viscosity; P is the pressure; S is the water saturation (the volume fraction of water in the total fluid). The dimensionless functions $k_w(S)$ and $k_o(S)$ are called the relative permeabilities (RP) and have the form shown in Fig. 1.4.

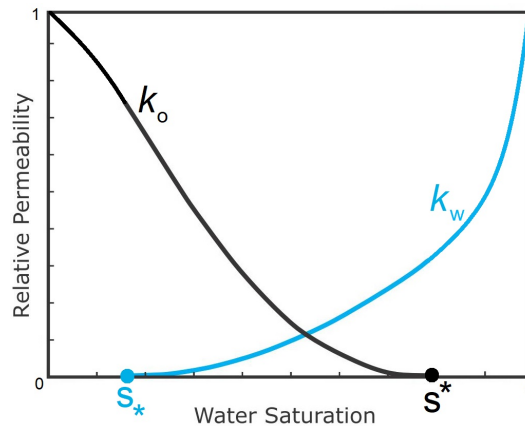


FIG. 1.4 : Capillary trapped phases in the relative permeability curves

They take into account for the presence of the second phase. In particular, if the flow is single-phase (only water) then $k_w = 1$, while $k_o = 0$. If, inversely, only oil flows, then $k_w = 0$ and $k_o = 1$. This explains their form.

As seen, below the value S_* water is immobile ($k_w = 0$), because the saturation of water is so low that it represents a system of drops separated from each other. These drops are immobilized by the effect of capillary trapping. Similarly for oil: above S^* the saturation of oil is so low that oil is immobile.

Thus, in the macroscopic model of fluid flow, the effect of capillary trapping is reflected by the critical values S^* and S^* , which are called the residual saturations or the percolation thresholds.

1.3 Structure of Microbes and their Functions

Oil reservoirs are complex environments containing living (microorganisms) and non-living factors (minerals) which interact with each other in a complicated dynamic network of nutrients and energy fluxes. Since the reservoir is heterogeneous, so do the variety of ecosystems containing diverse microbial communities, which in turn are able to affect reservoir behaviour and oil mobilization [4, 33]. Microbes are living machines whose metabolites, excretion products and new cells may interact with each other or with the environment, positively or negatively, depending on the global desirable purpose, e.g. the enhancement of oil recovery. All these entities, i.e. enzymes, extracellular polymeric substances (EPS) [34] and the cells themselves, may participate as catalyst or reactants. Such complexity is increased by the interplay with the environment, the later playing a crucial role by affecting cellular function, i.e. genetic expression and protein production.

Bacteria inhabiting underground strata frequently play a significant role in transport phenomena and determine important variation of the chemical composition of fluids confined in the rocks. As the bacteria have the characteristic size of $10^{-6}m$ they are considered as colloidal particles, which are transported practically as molecular solutions, but prove some particular features.

1.3.1 Structure of a cell

A microorganism is a single cell without cell nucleus [35]. The main types of microorganisms are Archaea and bacteria, which are similar in many aspects but have some structural differences. In this book we use the terms microorganism, bacteria and Archaea as identical.

The usual size of a microbial cell is $0.5 - 1\mu m$ in diameter and $1 - 2\mu m$ in length. The cell contains a full organism with all functionalities like orientation, mobility, nutrient intake, digestion and reproduction [36].

A bacterial cell consists of three main elements shown in Fig.1.1:

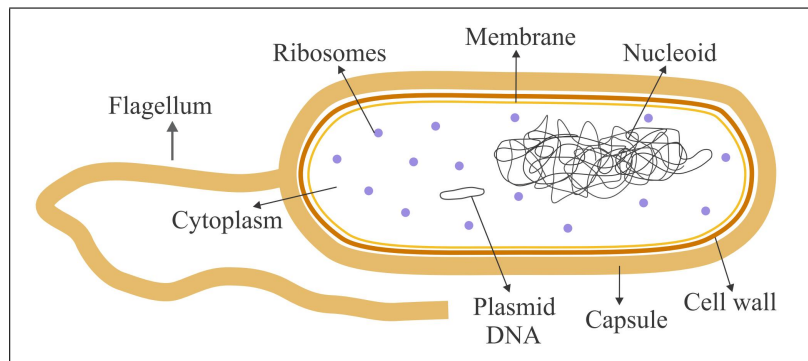


FIG. 1.5 : Structure of a bacterium

- The cytoplasm, which is a viscous immobile liquid.
- The nucleoid (or nuclear region), which is an irregularly shaped region that contains most of the genetic material. This area is occupied essentially by the chromosome, which has a closed ring shape. A bacterium contains a single chromosome (except some particular cases). As mentioned, bacteria do not have nucleus: their nuclear matter is not separated from the cytoplasm by a membrane, but is distributed within the cytoplasm.
- The ribosomes, which are the sites of protein (enzymes) synthesis and the reserve of nutrients.

Bacterial cells are surrounded by a dense membrane (cell wall), which performs protective and supporting functions, and also ensures a constant form to the cells. The cell membrane is permeable: through it nutrients freely pass into the cell, and the metabolic products exit into the environment. Often, over the cell wall, bacteria develop an additional protective layer of mucus called the capsule. Capsule protects the bacteria from drying out. The thickness of the capsule can many times exceed the diameter of the cell itself, but it can also be very small.

On the surface of some bacteria, there are long flagella (one or many) or short, thin villi. The length of flagella can many times exceed the size of the body of bacteria. With the help of flagella and villi bacteria move.

The organism of a cell is capable of performing various functionalities like orientation, mobility, nutrient intake, digestion and reproduction.

Three main forms of existence of micro-organisms are considered in the literature:

- **Biofilm:** it is a slimy film of biomass attached to the solid surface in contact with water, which is the habitat for microorganisms. The extracellular matrix of a biofilm consists of excretates and metabolic waste such as polysaccharides, (Peptidoglycan or

murein, cellulose, etc.) or, more rarely, lipids and proteins, but also of decayed or destroyed cells and consequently contains proteins, lipids, DNA, RNA ... It also contains a large proportion of water. The living microorganisms in a biofilm constitute 5 to 35%. More than 90% of bacteria live in the form of a biofilm.

- **Plankton:** there are the microorganisms freely floating in water.

- **Neuston:** the microorganisms leaving at the interface between water and gas. The standard definition of neuston stands for all the organisms that live at the surface between water and air (this can be not necessary microorganisms). We generalize this concepts to the surfaces between water and gases.

1.3.2 Bacteria movement

To describe various forms of movement, let us introduce the following definitions: n is the total number of bacteria in $1m^3$ of the medium (the bacterial concentration), n_p is the concentration of plankton in water, V is the velocity of water flow, C is the mole fraction of nutriments for bacteria, q is the rate (vector) of transport of bacteria. One distinguishes the following forms of bacterial movement:

- **Advection** or convective transport by water; the rate of transport is:

$$q = Vn$$

Usually bacteria create a biofilm in some stagnant zones, where the flow velocity of water is minimal. But if the flow velocity sharply increases, some fragments of the biofilm can be detached to the surface and transported by water. Once the velocity decreases, the transported fragments can be immobilized again. This kind of transport may be described in terms of the advection with trapping:

$$\frac{\partial n_*}{\partial t} + \nabla \cdot (Vn_*) + \dots = -\lambda Vn_*$$

where n_* is the concentration of biofilm fragments in water, λ_* is the rate of trapping (s^{-1}).

- **Diffusion:** or random movement of bacteria, is generally considered a slow process compared with advective transport [37] :

$$q = -D_b \nabla n$$

where D_b is the diffusion coefficient of bacteria.

The effective diffusion coefficient depends on the porosity, saturation and tortuosity

of the porous medium will be less. Since molecular diffusion is proportional to the concentration gradient, it will initially fast, but when the concentration gradients decrease, its effect is also reduced. It is independent of advective transport, so it can be a process control during periods of inactivity.

• **Chemotaxis** - movement of the motile bacteria towards nutrients. Bacteria have a physiological mechanism to feel an increasing concentration of beneficial substances such as nutrients and away from detrimental substances such as toxins [38, 39]. Mathematically, this movement is directed to the sense of growth of nutrient concentration:

$$q = D_{ch}\nabla C$$

where D_{ch} is the coefficient of chemotaxis [m^2/s].

Movement requires energy, since bacteria use their flagellar motor to rotate in the direction of, for example, a nutrient [40]. Therefore, chemotaxis is closely related to the growth rate of bacteria [41], and note that whatever the physiological mechanism of feeling nutrients, it is only possible via the material contact between the molecules of nutrients with bacterial cells. Thus, the chemotaxis is possible only if the diffusion of nutrients is nonzero. Consequently, two model the bacterial dynamics should be coupled with the diffusion equation for nutrients.

Parameter D_{ch} is non constant, but depends on several values. First of all, In the context of oil reservoirs, the bacteria flux caused by the chemotaxis is proportional to the number of floating bacteria, but the effect from chemotaxis is regarded minimal.

Secondly, it depends on the concentration of the nutrients: in the case of nutrient abundance, the bacterial flux will be zero whatever the gradient of C . In contrast, the movement of bacteria will be intensive, if the average concentration of nutrients is low. Consequently we obtain the following approximations for D_{ch} :

$$D_{ch}(n, C) = \frac{\Lambda_{ch}\nu_0 n}{(K_{ch} + C)_{ch}^{\beta}}, \quad or \quad D_{ch}(n, C) = D_{ch}^m n e^{-\alpha_{ch} C}$$

where ν_0 is the intrinsic velocity of floating, $\Lambda_{ch}[m]$ and $D_{ch}^m[m^2/s]$ are the empirical parameter, K_{ch} is the dimensionless "coefficient of dissociation" [42]; α_{ch} and β_{ch} are the dimensionless empirical parameters.

The first approximation with $\beta_{ch} = 2$ was proposed in [42] as the best approximation for experimental data. This value of parameter β_{ch} also ensures the positive sign of $D_{ch} = 2$ in numerical simulations, where the numerical oscillations can lead to negative values of the concentration. Several examples of calculating the bacterial dynamics with

chemotaxis may be found for instance in [43],[44], [45].

The dependence of $D_{ch} = 2$ on C can not be ignored, otherwise this can lead to qualitative errors.

1.3.3 Metabolism of bacteria

The mass exchange of microbes with the surrounding world is called the metabolism. The theoretical microbiology distinguishes different types of metabolism depending on its products, organic or non-organic origin of the consumed compounds, absorption of light or not, etc. For the purpose of the fluid dynamics, we should know only whether the metabolism leads to the variation of the mass of microbes or not.

The consumption or metabolic process occurs in two different ways:

- The constructive metabolism: microbes consume carbon (and other substances) to reproduce their biomass and to produce new biomass (new cells);
- The respiratory metabolism: microbes consume nutrients to obtain energy. This happens through chemical reactions initiated by microbes between nutrients directly inside their cells. The energy released from such reactions is their single product that is used by microbes. In this case, the nutrients are consumed, but the biomass is not produced.

Respiration

Respiration or the respiratory anabolism, consist of initiating a chemical reaction (introducing external species into the cell body of bacterium) and consuming the energy released from this reaction. The chemical species that participate in the reaction are not consumed by bacteria. Only the energy emitted from such a reaction is consumed, while the reaction products are evacuated outside. This energy is stored in ATP (adenosine triphosphate). Such a mechanism is that of oxidation (of some species already present in the cell). Thus, the respiration consists of destroying some chemical substances and using the emitted energy ('the anabolism'). Note that the respiration does not influence the biomass of microorganisms.

Consequently, the respiration represents the exchange of ions, in which bacteria forcibly detach electrons and remove them from the atoms of a chemical substance (the donor) and transfer these electrons to another substance (the acceptor). Such an exchange is characterized by the release of energy.

In MEOR, bacteria use petroleum hydrocarbons as electron donors, and nitrates, sulfates, and carbonates dissolved in water as electron acceptors.

Respiration can be aerobic or anaerobic. Aerobic bacteria use the reaction of oxidation by the oxygen from air; the respiration consists of using oxygen as the electron donor.

This type is typical for microorganisms inhabiting in soil or in geological strata that have connection to the air of the atmosphere. Anaerobic bacteria use other oxidants. This is typical for bacteria inhabiting at high depth, separated from the access of oxygen.

Although the respiration does not lead directly to the biomass production, the difficulty in respiration can retard the population growth. Indeed, if there is no sufficient reserve of nutrients for respiration, then the population cannot grow even if it has an abundant amount of feed. In this case, the rate of the net population growth may be considered as the basic parameter, which determines all other rates.

This is why, the population dynamics may be very different for aerobic and anaerobic bacteria. For aerobic bacteria, having free access to the oxygen of the atmosphere, the respiration is not constrained and, consequently, does not influence feeding. The population growth rate, in this case, is determined only by the feed resource. In this research, we consider only the aerobic mechanism.

Nutrition and cell duplication

Nutrition or the constructive catabolism, consists of consuming external chemical species and transforming them into the biomass: amino acids, nucleotides, lipids and carbohydrates. The essential chemical element needed to microorganisms is carbon. The nutrition is equivalent to a system of reactions of biomass synthesis while consuming the energy stored in ATP ('the catabolism'). Note that carbon is not unique chemical species needed to microbes. They also need nitrogen, which is the main compound of amino acids, phosphates (the main compound of nucleotides) and other species.

In some cases, the same chemical component can be used by bacteria both for respiration and nutrition. This is the case of CO_2 for methanogenic or acetogenic bacteria.

The nutrition leads to the increase of the biomass, which occurs through several stages. Initially each cell increases its mass, after which, once a critical mass is reached, the cell launches the dichotomy process, whose main part is the DNA replication.

Various microorganisms can consume carbon both in organic and inorganic form (CO_2). The transformation of inorganic carbon to the biomass (organic substances) is called the carbon fixation.

There exists also the third type of metabolism, called the fermentation, which represents the respiration and nutrition simultaneously. As an example, glucose decomposes onto alcohol and CO_2 by bacteria with the release of energy. But this type of metabolism is not considered in this book.

The metabolism provokes cell duplication, which leads, in turn, to population growth. The growth depends upon the ability of the cell to form new protoplasm from nutrients available in the environment. The process consists of several steps. At the first step, one

bacterium increases in cell mass and replicates its DNA, as illustrated in Fig. 1.6. In the middle of the elongated cell, a septum forms and this develops into a cell wall that divides two separate cells [46]. This is a dichotomy process.

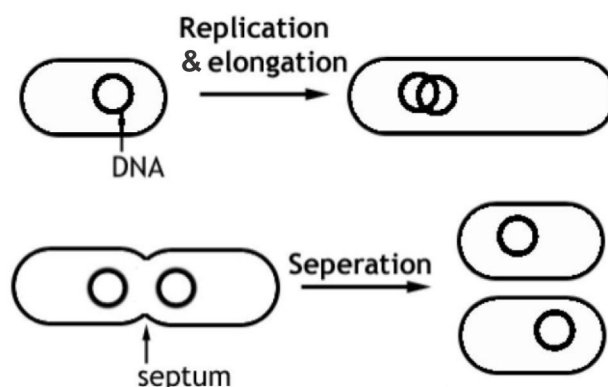


FIG. 1.6 : Duplication of a microbial cell

The process of cell duplication requires a series of chemical reactions within the microbial cell which is referred to as microbial metabolism [36].

1.3.4 Products of metabolism: metabolites

Bacteria is highly resilient and can survive extreme conditions. This is evidenced by the existence of bacteria inside reservoirs that have not had outside exposure since their formation.

Depending on the type of respiration, there exist various families of bacteria which produce various metabolites, which, in turn, have different effects on oil recovery. In general, MEOR aims to get bio-products like surfactant and polymers to name few, which play an important role in altering the reservoir properties and lead to significant improvement in oil recovery [47]. Table 1.1 shows the summarized products of microbes and their effects on the system [48].

In this work we analyse only anaerobic bacteria that produce surfactants.

Biosurfactant produced by *Pseudomonas putida* exhibited interfacial tension between oil and water sufficient to mobilize oil easily [32].

Other metabolites are the biopolymers and gases.

In the case of a polymer, bacteria produce extracellular polymeric substances that plug high permeability zones, resulting in a change of direction of the water flood to oil-rich

Table 1.1: MEOR agents produced by microorganism and their effects [4]

MEOR agents	Microbes	Respiration type	Effects
Surfactant	- Clostridium - Bacillus - Pseudomonas - Corynebacterium - Arthobacter	- Anaerobic - Facultative - Aerobic - Aerobic - Facultative	Interfacial tension reduction and Emulsification and de-emulsification
Polymers	- Xanthomonas - Pseudomonas - Leuconostoc	- Aerobic - Aerobic - Facultative	Mobility control and selective plugging
Gases	- Clostridium - Enterobacter - Desulfovibrio	- Anaerobic - Facultative - Anaerobic	Increased pressure, reduction of oil viscosity; increase permeability
Biomass or biofilms	- Bacillus - Leuconostoc - Xanthomonas	- Facultative - Facultative - Aerobic	Selective plugging and wettability angle alteration
Solvent	- Clostridium - Arthobacter	- Anaerobic - Facultative	Rock dissolution for better permeability, oil viscosity reduction
Acids	- Clostridium - Bacillus - Desulfovibrio - Enterobacter	- Anaerobic - Facultative - Anaerobic - Facultative	Degradation of rock and reduction of oil viscosity

channels, consequently increasing the sweep efficiency of oil recovery with water flooding. The main mechanisms of EOR in polymer water solution injection are:

- water thickening resulting in lower ratio of oil and water motilities, which decreases the ability of water breakthrough into the producing wells;
- clogging high-permeable channels because of polymer adsorption on the rock surface. Sweep efficiency of low-permeable reservoirs increases in this case.

Completeness and rate of polymer dissolution in water determine the efficiency of polymer flooding. The dissolvent may be both fresh and mineralized water with different pH values and general mineralization. However, salts of chlorine ion, calcium chloride and natrium chlorine considerably reduce the viscosity of thickened water. The solution viscosity is also observed to reduce if the total formation water mineralization increases. Polymer destruction has a negative effect on efficient oil displacement. The destruction may be chemical, thermal, mechanical and microbiological.

When a polymer solution is injected into the reservoir, some of the long chain molecules constituting the polymer are adsorbed onto the rock surfaces. In addition, mechanical

entrapment of large molecules at the entrance to small pore throats may also occur. This leads to losses of polymer from the invading solution. Experimentally, the reservoir rock material is believed to retain a specific capacity of polymer. The main effects of polymer loss occur at the leading edges of the polymer slug where a stripped water bank is created and the slug width is gradually reduced in time. Some desorption effects can occur as the trailing edge of the slug passes but these effects are usually small compared with the adsorption losses.

Biopolymer production and the resulting biofilm formation (less 27% cells, 73-98% EPS and void space) are affected by water chemistry, pH, surface charge, microbial physiology, nutrients and fluid flow [49].

In the old practice, the production of gas has a positive effect on oil recovery by increasing the differential pressure driving the oil movement. Anaerobically produced methane from oil degradation has a low effect on MEOR due to its high solubility at high pressures. Carbon dioxide is also a good MEOR agent. The miscible CO₂ is condensed into the liquid phase when light hydrocarbons are vaporised into the gas phase. Immiscible CO₂ helps to saturate oil, resulting in swelling and reduction of viscosity of the liquid phase and consequently improving mobilization by extra driving pressure. Concomitantly, other gases and solvents may dissolve carbonate rock, leading to an increase in rock permeability and porosity.

1.4 Kinetics of population evolution

1.4.1 Population growth

It is important to describe the growth and decay of the microorganisms in an adequate way because this is related to the local rates of the biochemical reactions.

The growth of bacteria or increasing in cell mass in batch culture can be modeled with four different phases: lag phase, log phase or exponential phase, stationary phase, and death phase. The growth behavior of microbial populations can be investigated in the laboratory. The microorganisms are inserted into a bottle containing a nutrient solution [36]. A fixed amount of substrate is added and samples are taken frequently to analyse them with respect to the cell number or biomass [36]. The number of cells or biomass is

subsequently shown in a semi-logarithmic plot versus time. Such experiments are referred to as batch culture experiments. In the Figure 1.7 a conceptual plot of microbial cell concentration versus time.

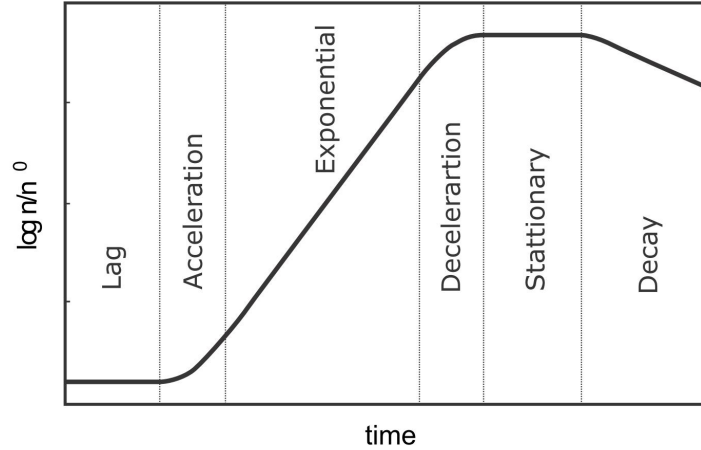


FIG. 1.7 : Growth phases in batch cultures of bacterial cells

The rate of growth reported to a single bacterium is

$$\psi \equiv \frac{1}{n} R_{grow} = \frac{1}{n} \frac{dn}{dt} = \frac{d \ln n}{dt}$$

This is why the function $\ln n(t)$ is used in bacterial kinetics.

Lag phase. The first phase is the lag phase ($\frac{dn}{dt} = 0$), bacteria adapt themselves to growth conditions or synthesis of RNA, enzymes and other molecules occurs. The microorganisms need a certain time to physiologically adjust themselves to the new external conditions [50, 46], as they are inoculated from a medium with a low concentration of nutrients into a medium with a high concentration. During this adaptation, the cells produce the enzymes necessary for the metabolism of available nutrients. The lag period may be long, the length of this lag period depends on many factors such as the type and age of the microorganisms, the size of the inoculum n^0 , and culture conditions.

Note, if a small amount of cells are inoculated into a large volume (small concentration n^0), they will have a long lag phase [51]. For large-scale operation of the cell culture, it is our objective to make this lag phase as short as possible. Therefore, to inoculate a large fermenter, we need to have a series of progressively larger seed tanks to minimize the effect of the lag phase. So the number of cells does not increase, but they may increase in size during this period, in this period the concentration of nutrients decreases, but

the concentration of bacteria does not increase. If bacteria move from a high to a low concentration of nutrients, as a rule, the lag phase is not observed.

Exponential growth. The exponential phase is a period characterized by cell doubling or is a regular dichotomy of the cells. This means that the number of cells increases exponentially with a maximum rate [36]. This rate is specific for each substrate and the external conditions [36]. Under ideal condition this can be every 10 minutes, however, it can also last up to 100 years [46]. This is a regular dichotomy of the cells, for a simple process of dichotomy at a constant rate, the logarithm of the number of bacteria is linear function of time. Mathematically, the growth during this phase can be described by the following differential equation [36]:

$$\frac{dn}{dt} = \mu n \quad (1.2)$$

where n is the number of microbial cells and μ is the maximum growth rate in [1/s].

Remark: it is easy to prove that the dichotomy of bacteria is described by an exponential law. Indeed, let the number of bacteria be n^0 at $t = 0$, and is doubled during the fixed time Δt , so that $n(\Delta t) = 2n_0$, $n(2\Delta t) = 2^2 n_0$, ... , $n(m\Delta t) = 2^m n_0$. Then

$$n(t) = n_0 e^{t/t_{\mathfrak{S}}}, \quad t_{\mathfrak{S}} \equiv \Delta t / \ln 2$$

where parameter $t_{\mathfrak{S}}$ is the characteristic time of population growth.

Stationary phase. The stationary phase is often due to a growth-limiting factor such as the depletion of an essential nutrient, and the formation of an inhibitory product such as an organic acid. During this phase no net growth occurs. However, this does not mean that individual cells do not duplicate. It only means that growth and death are balanced [46]. When microbial cells are using dead microbial cells as carbon source the process is referred to as endogenous metabolism [46]. If the concentration of nutrients is kept constant, then the stationary phase should not exist, and the population will continue to grow exponentially.

Monod [52] introduced two additional phases in the life cycle of a microbial batch culture:

- the acceleration phase, which is the transition between the lag and the exponential growth phase;
- the deceleration phase, which is the transition between the exponential growth phase and the stationary phase. From the Fig.1.8 (the blue curve), we see that the nutrient begins to decrease too rapidly.

1.4.2 Mathematical models of population kinetics

Several approximations to experimental data exist in the literature to describe the behaviour of the microbial growth function, [50]. The most known are:

$$R_{\mathfrak{B}} = \frac{R_{\mathfrak{B}}^*(n)}{t_{\mathfrak{B}}} \zeta(c^n), \quad \zeta(c^n) = \frac{(c^n)^m}{1 + \frac{(c^n)^m}{K_s}} \quad (1.3)$$

where c^n is the concentration of nutrients; $R_{\mathfrak{B}}^*$ depends only on the population size (n) and describes the kinetics of growth in the condition of nutrient abundance; $\zeta(c^n)$ reflects the impact of the amount of nutrients; $t_{\mathfrak{B}}$ is the characteristic time of population growth; m and K_s are empirical parameters.

Function $\zeta(c^n)$ should tend to zero if no nutrients, and to 1 if c^n tends to an abundant value (mathematically: $c^n \rightarrow \infty$). This explains the approximation above for ζ .

Function $R_{\mathfrak{B}}^*/n$, which is the rate of population growth per one bacterium, should be independent of n for a simple dichotomic splitting of cells (this is shown in [53]). Then the function $R_{\mathfrak{B}}^*(n)$ should be linear. However, such a behaviour describes well the exponential phase, but does not capture the lag phase. We select three main models that are of interest:

For $R_{\mathfrak{B}}^*(n) = n$ and $m = 1$, the model is known as Monod's model [52].

For $R_{\mathfrak{B}}^*(n) = n$ and $m = 2$, this is Moser's model [54].

For $R_{\mathfrak{B}}^*(n) = n^2$, and $m = 1$, the model was suggested by Panfilov [53]. It describes better the lag phase.

The more complicated model that describes all the phases (lag, exponential phase, and stabilization) was also suggested in [53]:

$$R_{\mathfrak{B}}^*(n) = \frac{n^2}{1 + \left(\frac{n}{n_*}\right)^\beta} \quad (1.4)$$

We conducted a series of numerical experiments to verify and analyze the convergence of the Monod model, the Moser model and the model suggested recently by Panfilov [53], which is implemented in Matlab (Appendix B). The results of comparison are shown in Fig. 1.8.

As seen, the Monod and the Moser models begin directly with the exponential growth phase and none of these models can predict the lag and the stationary phases. One can improve them by moving the initial moment to the end of the lag phase, but this means that the duration of the lag phase is set a priori, whereas in reality this parameter should

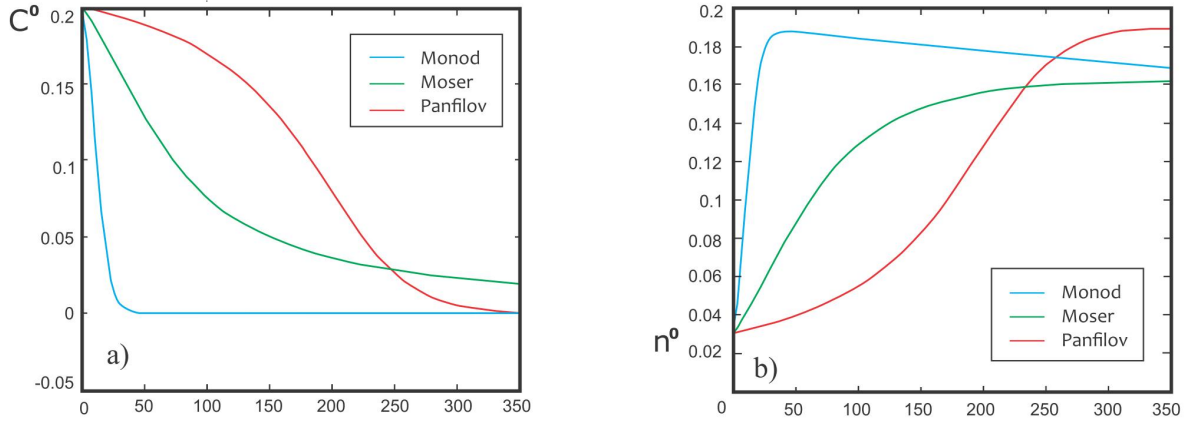


FIG. 1.8 : Comparison of different models of kinetics of population growth with experimental data

be the result of calculation.

Another type of functions which are able to capture the lag phase is the following family ([53]):

$$R_{\mathfrak{B}}^*(n) = n(1 - \exp\{-\lambda n^\alpha\}) \quad (1.5)$$

where t_{grow} , λ , and $\alpha \geq 1$ are empirical parameters.

Its asymptotic behaviour is:

$$R_{\mathfrak{B}}^*(n) \sim \lambda n^{\alpha+1}, \quad \text{when } n \rightarrow 0, \quad (1.6)$$

$$R_{\mathfrak{B}}^*(n) \sim n, \quad \text{when } n \rightarrow \infty, \quad (1.7)$$

In the case when the nutrients are presented by two or more species, the multiplicative rule should be used for ς :

$$\varsigma(c_1^n, c_2^n) = \frac{(c_1^n)^m}{\left(1 + \frac{(c_1^n)^m}{K_{s1}}\right)} \frac{(c_2^n)^m}{\left(1 + \frac{(c_2^n)^m}{K_{s2}}\right)} \quad (1.8)$$

The parameters of bacterial kinetics may be found in the literature. For instance, the data for methanogenic bacteria are presented in [53].

1.4.3 Population decay

The decay of a population (*Death phase*) consists in a decrease in the number of individuals and at death phase bacteria should die, which means an irreversible loss of ability to grow

and multiply. The death necessary leads to the decomposition of the cell or its rotting [51]. This could be caused by:

- lack of nutrients;
- the action of other microorganisms and fungi (the predators) that use bacterium A as a nutrient (the prey);
- environmental temperature above or below the tolerance band for the species;
- the radiation;
- the action of chemicals or other injurious factors to growth bacteria, during which the number of active cells shows a negative exponential decline.

Mathematically the decay phase can be described as:

$$\frac{dn}{dt} = -bn \quad (1.9)$$

where b is the decay rate, [1/s].

However, individual cells can still duplicate. In the work of [51] assume that the death of an individual is an event that does not depend on the amount of nutrients. On the contrary, it may depend on the number of bacteria n . Indeed, the probability of encountering prey for one predator is proportional to n . If the concentration of predators is N , then the probability of all collisions is $\sim Nn$. The predator colony grows in proportion to the prey colony, then $\sim Nn$. Therefore $R_{dec} \sim n^2$, which means that the decay rate reported by one bacterium is proportional to n :

$$R_{dec} = \frac{n^\beta}{t_{dec}} \quad (1.10)$$

where t_{dec} is the characteristic time of decay; $\beta \geq 1$ is an empirical parameter.

Chapter 2

Mathematical model of MEOR

2.1 General mathematical model of MEOR

The general model of MEOR processes is a coupled system of equations of flow of a two-phase multicomponent chemical mixture, thermodynamic relationships for the dissolution of components in phases, kinetic formulae for chemical reactions, and a nonlinear transport equation for bacterial population dynamics.

Based on some specific features of microorganisms, we have the possibility to simplify this model and to perform its qualitative analysis.

2.1.1 Structure of the fluids

We consider the fluid that consists of two phases: liquid water (bacteria cannot live without water), and liquid oil. Both phases consist of five components:

- * H_2O ;
- * head component of oil (**h**);
- * nutrient (**n**);

* active metabolite (**m**);

* bacteria (**b**).

The head component of oil is the heavy hydrocarbon. It can also be dissolved in water, but this part of its mass is negligible.

The component H₂O is present essentially in water phase.

The nutrient can be dissolved in both phases, but its consumption occurs only in water.

The metabolite can also be dissolved in both phases, but its production occurs only in water.

To identify the components, we introduce their mole fractions:

- $c^{\text{H}_2\text{O}}$, c^{n} , and c^{m} : the mole fraction of H₂O, nutrient and metabolite in water respectively;

- c_o^{h} and c_o^{n} are the mole fractions of the head component and nutrient in oil;

- c_o^{m} is the mole fraction of the metabolite in oil;

- n is the volume fraction of bacteria in water ($\text{m}^3 \cdot \text{m}^{-3}$), i.e. the number of cells multiplied by the volume of a single cell and divided by the volume of water;

The repartition of nutrients and metabolites between water and oil is described by Henry's law, if we assume the chemical equilibrium between water and oil:

$$c^{\text{m}} = \tilde{H}^{\text{m}} c_o^{\text{m}} P, \quad c^{\text{n}} = \tilde{H}^{\text{n}} c_o^{\text{n}} P \quad (2.1)$$

where \tilde{H}^{m} and \tilde{H}^{n} are the Henry solubilities for the metabolite and nutrient (Pa^{-1}).

From the Eq.(2.1), the concentration of metabolite and nutrient in oil phase become: $c_o^{\text{m}} = \frac{c^{\text{m}}}{\tilde{H}^{\text{m}} P}$, $c_o^{\text{n}} = \frac{c^{\text{n}}}{\tilde{H}^{\text{n}} P}$, where $H^{\text{n}} \equiv \tilde{H}^{\text{n}} P$ and $H^{\text{m}} \equiv \tilde{H}^{\text{m}} P$ are the dimensionless Henry solubilities.

2.1.2 General properties of micro-biochemical systems (MBS)

1. Bacterial population does not decay. The decay of population is related to the death of individuals, which means an irreversible loss of ability to grow and multiply. The death necessary leads to the decomposition of the cell or its rotting. In contrast to multicell animals, the death of a single cell is considered as practically impossible event. In fact, a cell is dead when it has been eat by other maicroorganisms or fungi. A nutrient deficiency causes the inactivity and hibernation of bacteria, but not their death. This fact was used even in the first models of bacterial population growth [52]. We will consider the decay (the death) of bacteria as negligible effect.

2. The process is isothermal, since all the energy produced by the respiratory reaction is consumed by bacteria to their vital functions. For the displacement of a phase by another one, the pressure may be also considered as almost constant value.
3. The phases are in chemical equilibrium: i.e. the concentrations of all chemical components in water and oil (not bacteria) are controlled by the laws of phase equilibria. According to the Gibbs rule of phases, the thermodynamic degree of freedom of a two-phase equilibrium system with four dissolved chemical components and fixed temperature and pressure is equal to 2. Therefore, only two concentrations are independent. We select them as c^n and c^m . All other concentrations depend on them.
4. The mobility (the relative permeability and viscosity) of the phases depends on the phase saturation and on the phase composition, i.e. on c^n and c^m .
5. Water and oil are dilute solutions: water consists essentially of H₂O, while the oil phase consists essentially of the heavy component.
6. We consider bacteria as a colloidal suspension (or "emulsion") in water, so that the size of a single bacterium is much lower than the size of a pore. The averaged flow of such a system of particles has the same velocity as that of transporting water, so that bacteria behave as a chemical component of water, but not a separated phase.
7. The respiratory component is always present in sufficient amounts, so it is not necessary to take it into account. We also assume that the chemical transformations during the respiration do not produce any substances important for oil recovery. Thus, the respiration process may be entirely ignored.
8. At the first step of study, the gravity, the capillary pressure, and the diffusion of all components and bacteria can be neglected.

2.1.3 Mass balance equations

Under the mentioned assumptions, the balance of mass of chemical components in two phases is described as follows:

$$\phi \partial_t (\rho_w c^{\text{H}_2\text{O}} S) + \partial_x (\rho_w c^{\text{H}_2\text{O}} V F) \approx 0, \quad (2.2a)$$

$$\phi \partial_t (\rho_o c_o^{\text{H}_2\text{O}} (1 - S)) + \partial_x (\rho_o c_o^{\text{H}_2\text{O}} V (1 - F)) \approx 0, \quad (2.2b)$$

$$\phi \partial_t (\rho_w c^n S + \rho_o c_o^n (1 - S)) + \partial_x (\rho_w c^n V F + \rho_o c_o^n V (1 - F)) = -\phi \rho_w S (R_{\mathfrak{N}} + R_{\mathfrak{R}}) \quad (2.2c)$$

$$\phi \partial_t (\rho_w c^{\text{m}} S + \rho_o c_o^{\text{m}} (1 - S)) + \partial_x (\rho_w c^{\text{m}} V F + \rho_o c_o^{\text{m}} V (1 - F)) = \phi \rho_w S R_{\mathfrak{M}}, \quad (2.2d)$$

$$\phi \partial_t (n S) + \partial_x (n V F) = \phi S R_{\mathfrak{B}}, \quad (2.2e)$$

where index w means water phase, and o is oil. Other notations are as follows:

- ρ_i is the molar density ($\text{mol} \cdot \text{m}^{-3}$) of the phase i ;
- S is the water saturation;
- V is the total Darcy velocity: $V = V_w + V_o$;
- F is the fractional flow of water defined as: $V_w = V F$ (then $V_o = V(1 - F)$).

Four kinetic functions have the dimension $[\text{s}^{-1}]$ and stand for the rates of:

- $R_{\mathfrak{R}}$ - nutrient consumption for respiration only ;
- $R_{\mathfrak{N}}$ - nutrient consumption for nutrition only (i.e. for the biomass production);
- $R_{\mathfrak{B}}$ - bacterial population growth;
- $R_{\mathfrak{M}}$ - metabolite production.

Using Darcy's law ($\mathbf{V}_i = -\frac{K k_i}{\mu_i} \nabla P$), we obtain the following expression for F :

$$F(S, c^n, c^{\text{m}}) = \frac{k_w}{k_w + k_o \mu_w / \mu_o}$$

where K is the absolute permeability of the porous medium; k_w and k_o are the relative permeabilities; μ_i is the viscosity of the phase i ; and P is the pressure. According to the property 6 of section 2.1.2, this function depends on the water saturation and the phase composition.

Equations (2.2a), (2.2c), and (2.2e) take into account the fact that components H_2O and the bacteria are present only in water.

Eq. (2.2b) does not take into account water and all the probable transformations of this component, which occur in water.

Eq. (2.2c) describes the nutrient, which can be dissolved in both phases, but its consumption occurs only in water.

Eq. (2.2d) describes the metabolite, which can also be dissolved in both phases, but its production occurs only in water.

Using Henry's law (2.1), we obtain:

$$\phi \partial_t S + \partial_x (VF) = 0, \quad (2.3a)$$

$$-\phi \partial_t S + \partial_x (V(1 - F)) = 0, \quad (2.3b)$$

$$\phi \partial_t \left(\rho_w c^n S + \rho_o \frac{c^n}{\tilde{H}^n} (1 - S) \right) + \partial_x \left(\rho_w c^n VF + \rho_o \frac{c^n}{\tilde{H}^n} V(1 - F) \right) = -\phi \rho_w S (R_{\mathfrak{N}} + R_{\mathfrak{M}}) \quad (2.3c)$$

$$\phi \partial_t \left(\rho_w c^m S + \rho_o \frac{c^m}{\tilde{H}^m} (1 - S) \right) + \partial_x \left(\rho_w c^m VF + \rho_o \frac{c^m}{\tilde{H}^m} V(1 - F) \right) = \phi \rho_w S R_{\mathfrak{M}}, \quad (2.3d)$$

$$\phi \partial_t (nS) + \partial_x (nVF) = \phi S R_{\mathfrak{B}}, \quad (2.3e)$$

2.1.4 Description of the impact of biosurfactant

The surfactant produced by bacteria reduces the surface tension between water and oil, σ , which reduces, in turn, the capillary trapping of oil. The effect of capillary trapping is reflected by the value of the immobile saturation of oil on the relative permeability curve, shown in Fig. 2.1.

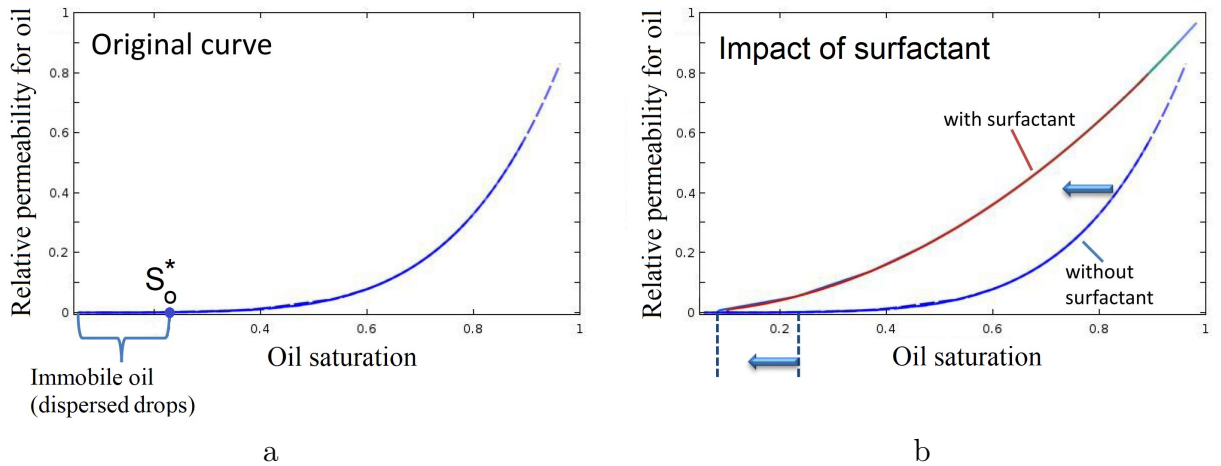


FIG. 2.1 : Relative permeability for oil and the immobile saturation (a). The impact of surfactant on the immobile saturation (b)

The immobile saturation is equal to S_o^* and is also called *the percolation threshold*. So

the main effect of biosurfactant consists in reducing the value S_o^* .

Similarly for water phase, the percolation threshold for water, which is S_{w*} , should be also reduced if the surface tension decreases.

Consequently, the decrease in surface tension caused by the surfactant modifies the relative permeabilities in two ways:

- First of all, the percolation thresholds S_* and $1 - S^*$ (i.e. the maximal immobile saturation) decrease both for oil and water when σ decreases.

- Secondly, the form of RP curves straightens when $\sigma \rightarrow 0$, because the reduction of the surface tension makes the system more miscible.

Thus, a decrease in interfacial tension changes the shape and the position of RP, making them less curved and moving them mostly to the right [55, 13].

As the result, the RP curves become the functions of the water saturation and the surface tension. In turn, the surface tension is the function of the surfactant concentration in water. The following approximation is usually accepted for σ and relative permeabilities:

$$k_w(S, \mathbf{c}^m) = \begin{cases} \left(\frac{S - S_*}{1 - S_*} \right)^{\gamma_*}, & S > S_*; \\ 0, & S \leq S_* \end{cases}; \quad k_o(S, \mathbf{c}^m) = \begin{cases} \left(\frac{S^* - S}{S^*} \right)^{\gamma_*}, & S \leq S^*; \\ 0, & S > S^* \end{cases} \quad (2.4a)$$

$$S_*(\sigma) = 0.5S_{*0} \left(1 + \frac{\sigma}{\sigma^0} \right); \quad S^*(\sigma) = S^{*0} \left(1 + 0.58 \left(1 - \frac{\sigma}{\sigma^0} \right) \right); \quad (2.4b)$$

$$\gamma_*(\sigma) = 1 + (\gamma_{*0} - 1) \frac{\sigma}{\sigma^0}; \quad \sigma(\mathbf{c}^m) = \sigma^0 \exp\{- (\mathbf{c}^m / \mathbf{c}_{cr}^m)^\varkappa\} \quad (2.4c)$$

where σ^0 is the surface tension of the oil-water interface at reservoir conditions without surfactant; parameter $\varkappa \geq 2$. Note that all the formulae (2.4) do not depend on σ^0 , but only on the ratio σ/σ^0 .

Parameters $S_{*0}, S^{*0}, \sigma^0, \varkappa, \gamma_{*0}$, and \mathbf{c}_{cr}^m are empirical.

The variation of the surface tension versus surfactant concentration is shown in Fig. 2.2.

The minimal value of σ is much lower than the maximum, so that one may accept that it is zero.

If \varkappa is large, this function tends to become piece-wise constant:

$$\sigma(\mathbf{c}^m) = \begin{cases} \sigma^0, & \mathbf{c}^m \leq \mathbf{c}_{cr}^m \\ 0, & \mathbf{c}^m > \mathbf{c}_{cr}^m \end{cases}$$

In this case the behaviour of saturation is determined only by two fractional flow

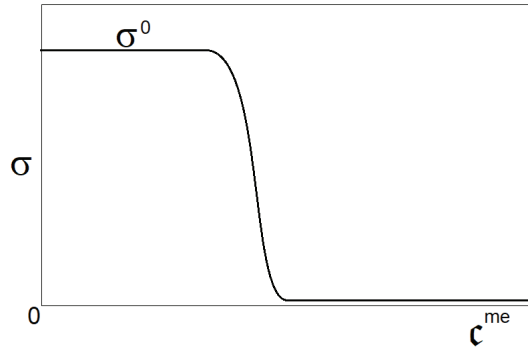


FIG. 2.2 : Surface tension versus the concentration of the biosurfactant

curves that are shown in Fig. 2.3.

The relative permeability and the fractional flow become, thus, the families of curves for various concentrations c^m , as shown in Fig. 2.3.

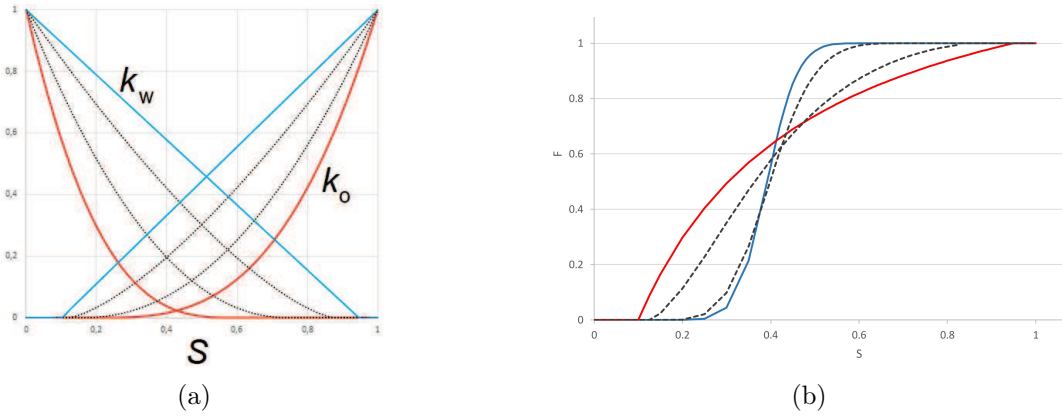


FIG. 2.3 : Relative permeability of oil and water (a) and the fractional flow of water F (b) for various concentrations of metabolite. The red curves mean before microbial treatment ($c^m = 0$); the blue curves mean after the reduction of the surface tension to zero ($c^m > c^m_{cr}$)

The relative permeability curves change even at very low concentration c^m . Thus, the saturation field is affected even by a weak bioreactivity.

2.1.5 Kinetic functions

Four kinetic functions R_B, R_N, R_{N_1} , and R_{N_2} may be defined via the following principles.

For the rate of respiration, we can use the hypothesis of synchronization [51], which says that the higher the nutrition rate, the higher the rate of respiration: $R_{N_1} = \omega_{N_1} R_N$. Parameter ω_{N_1} is an empirical coefficient of synchronization, which is close to 1. If the nutrient is not used for the respiration, then $\omega_{N_1} = 0$.

The rate of population growth is certainly proportional to the rate of nutrient consumption for the biomass production, and the rate of metabolite production is certainly proportional to the rate of population growth. Therefore, various rates can be expressed through parameter $R_{\mathfrak{B}}$, which will be considered as the basic parameter (in practice, one knows how to measure it).

Although the nutrients consumed for feeding ('the feed') are directly transformed into the biomass, the rate of the net population growth is not equal to the rate of feeding. In fact, a part of nutrients is transformed not into the biomass, but into some metabolites. In addition, the biomass growth occurs through several stages and may be sufficiently slower than feeding. Initially each cell increases its mass, after which, once a critical mass is reached, the cell launches the dichotomy process, whose main part is the DNA replication, which may be long. During all this process the bacterium continue to consume nutrients, but the rate of consumption is not at all equivalent to the rate of growth of the number of bacteria. The difference between them is characterized by "the yield".

Let N be the part of nutrients that transforms into the biomass, then $(1 - N)$ part transforms into the metabolite. Then $NR_{\mathfrak{B}}$ and $(1 - N)R_{\mathfrak{B}}$ are the rates of biomass consumption and metabolite production.

These rates are not necessary equal to $R_{\mathfrak{B}}$ and $R_{\mathfrak{m}}$, because the transformation of nutrients into another product is slower process, as mentioned above. Then we obtain:

$$R_{\mathfrak{B}} = Y_{\mathfrak{B}}NR_{\mathfrak{B}} \quad R_{\mathfrak{m}} = Y_{\mathfrak{m}}(1 - N)R_{\mathfrak{B}}$$

where $Y_{\mathfrak{B}}$ and $Y_{\mathfrak{m}}$ is the yield: $0 \leq Y_{\mathfrak{B}} \leq 1$ $0 \leq Y_{\mathfrak{m}} \leq 1$, usually this parameter is accepted to be constant.

Then we can express $R_{\mathfrak{B}}$ and $R_{\mathfrak{m}}$ through $R_{\mathfrak{B}}$:

$$\begin{cases} R_{\mathfrak{B}} = \frac{R_{\mathfrak{B}}}{NY_{\mathfrak{B}}} \\ R_{\mathfrak{m}} = \frac{(1 - N)Y_{\mathfrak{m}}}{NY_{\mathfrak{B}}}R_{\mathfrak{B}} \end{cases}$$

where $Y_{\mathfrak{B}}^* = NY_{\mathfrak{B}}$ and $Y_{\mathfrak{m}}^* = (1 - N)Y_{\mathfrak{m}}$. Their properties are: $0 \leq Y_{\mathfrak{B}}^* \leq N$ and $0 \leq Y_{\mathfrak{m}}^* \leq (1 - N)$.

For the growth rate $R_{\mathfrak{B}}$, we will use the empirical correlations (1.3) in various versions presented in section 1.4.2.

2.2 Reduction to the model of kinematic waves

2.2.1 Ideal mixing and volume concentrations

The first simplification that can be done concerns the fluid compressibility. As the considered systems are dilute solutions (property 6 in section 2.1.2), this gives us the possibility to eliminate the phase densities.

In the coarse approximation, it is sufficient to consider both phases as incompressible. Then their densities are independent of the pressure and phase composition. Dividing by ρ_w , we obtain from (2.3) the equations without densities.

This is however not too right, as the phase density of a liquid is not constant but depends on the phase composition. Then, it is possible to apply another approach. A dilute solution may be considered as an ideal mixture, for which the volume of the mixture equals to the sum of the individual volumes of pure components. For ideal mixtures, one can introduce volume fractions instead of mole fractions, as follows:

$$c_i^k \equiv \frac{\text{volume of component } k \text{ in the pure state}}{\text{volume of mixture}} = \frac{\rho_i c_i^k}{\rho^k}$$

where ρ^k is the density of the pure component. For small variations in pressure, the density ρ^k is constant.

The first two equations (2.3) gives: $\partial_x V = 0; \implies V = V(t)$, and $V = \text{const}$ if we maintain the injection velocity constant.

2.2.2 Kinematic wave model

Under these assumptions, one can transform the system (2.3) of balance equations into the following kinematic wave model:

$$\partial_x U = 0,$$

$$\partial_t S + U \partial_x F = 0,$$

$$\begin{aligned} \phi \partial_t \left(\mathbf{c}^n (S + \gamma(H^n)^{-1}(1 - S)) \right) + \partial_x \left(\mathbf{c}^n V (F + \gamma(H^n)^{-1}(1 - F)) \right) = \\ - \frac{\rho_w}{\rho^n} \phi S Y_{\mathfrak{B}} (1 + \omega_{\mathfrak{B}}) R_{\mathfrak{B}}, \end{aligned}$$

$$\phi \partial_t (nS) + \partial_x (nVF) = \phi S R_{\mathfrak{B}}$$

$$\phi \partial_t \left(\mathbf{c}^m (S + \gamma(H^m)^{-1}(1 - S)) \right) + \partial_x \left(\mathbf{c}^m V (F + \gamma(H^m)^{-1}(1 - F)) \right) = \frac{\rho_w}{\rho^m} \phi S Y_{\mathfrak{m}} R_{\mathfrak{B}}$$

where ρ^n and ρ^m are the densities of the nutrient and metabolite in their pure state; $H^k \equiv \tilde{H}^k P$ (P is an average pressure), and $\gamma = 1$. For the system water-oil, $\mathbf{c}^n = \frac{\rho_w \mathbf{c}^n}{\rho^n}$, $\rho^m = \rho^n = \rho_w$, $\mathbf{c}^k = c^k$, and $\gamma = \rho_o / \rho_w$.

The mass balance equation for nutrients can be simplified, given that:

$$\left(\mathbf{c}^n (S + \gamma(H^n)^{-1}(1 - S)) \right) \cdot H^n = H^n \mathbf{c}^n S + \frac{H^n \gamma \mathbf{c}^n}{H^n} - \frac{H^n \mathbf{c}^n S}{H^n}$$

and

$$\left((1 - H^n) \mathbf{c}^n S + \gamma \mathbf{c}^n \right) / (1 - H^n) = \mathbf{c}^n S + \frac{\gamma \mathbf{c}^n}{(1 - H^n)}$$

Then the system becomes:

$$\partial_x U = 0, \tag{2.5a}$$

$$\partial_t S + U \partial_x F = 0, \tag{2.5b}$$

$$\partial_t (\mathbf{c}^n S + \alpha^n \mathbf{c}^n) + U \partial_x (\mathbf{c}^n F + \alpha^n \mathbf{c}^n) = -\beta^n S R_{\mathfrak{B}} \tag{2.5c}$$

$$\partial_t (nS) + U \partial_x (nF) = S R_{\mathfrak{B}} \tag{2.5d}$$

$$\partial_t (\mathbf{c}^m S + \alpha^m \mathbf{c}^m) + U \partial_x (\mathbf{c}^m F + \alpha^m \mathbf{c}^m) = \beta^m S R_{\mathfrak{B}} \tag{2.5e}$$

where

$$U \equiv V \phi^{-1}, \quad \alpha^n \equiv \frac{\gamma}{H^n - 1}, \quad \alpha^m \equiv \frac{\gamma}{H^m - 1},$$

$$\beta^n \equiv \frac{\phi \rho_w Y_{\mathfrak{B}} H^n (1 + \omega_{\mathfrak{R}})}{\rho^n (H^n - 1)}, \quad \beta^m \equiv \frac{\phi \rho_w Y_{\mathfrak{M}} H^m}{\rho^m (H^m - 1)}$$

The typical problem which is of interest is the problem of injection of nutrients and bacteria in an initially immobile system, which is formulated in terms of the following one-dimensional Riemann problem:

$$\text{at } t = 0 : \quad S = S^0, \quad n = n^0, \quad \mathbf{c} = \mathbf{c}^{n0}, \quad \mathbf{c}^m = \mathbf{c}^{m0} \quad (2.6a)$$

$$\text{at } x = 0 : \quad S = S^*, \quad n = n^*, \quad \mathbf{c}^n = \mathbf{c}^{n*}, \quad \mathbf{U} = \mathbf{U}^* \quad (2.6b)$$

where $x = 0$ is the point of injection, and S^*, n^*, \mathbf{c}^* are the injected values. The value \mathbf{U}^* is the injected total velocity.

If $\mathbf{U}^* = \text{const}$, then in 1D case the velocity \mathbf{U} is simply a constant value, which results from (2.5a).

Another simplification consists in multiplying Eq.(2.5d) by β^n and adding it to Eq.(2.5c), which yields:

$$\partial_t ((\mathbf{c}^n + n\beta^n)S + \alpha^n \mathbf{c}^n) + \mathbf{U} \cdot \nabla ((\mathbf{c}^n + n\beta^n)F + \alpha^n \mathbf{c}^n) = 0$$

2.2.3 Conditions at the shocks

The formulated problem can have discontinuous solutions. The global structure of a solution consists of a combination between rarefaction waves (continuous nontrivial curves), shocks (discontinuities), and plateaux (values constant in space).

We note the following properties of the considered system:

- a discontinuity of a phase concentration provokes the discontinuity of all other concentrations of chemicals, as they are related by the conditions of phase equilibria;
- a discontinuity of the phase composition provokes the discontinuity of the fractional flow.

These properties make possible the existence of only two types of shocks. To describe them, we will use the following notations: let "+" and "-" mean "ahead of" and "behind" the shock, while U_f be the shock velocity. The two types of shocks and the Hugoniot-Rankine conditions (the mass balance of each component at a shock) are as follows:

- Mechanical shock: S is discontinuous, while n , \mathbf{c}^n , and \mathbf{c}^m are continuous:

$$\frac{U_f}{U} = \frac{F^+ - F^-}{S^+ - S^-} \quad (2.7)$$

- Biochemical shock: all the functions, S , \mathbf{c}^n , \mathbf{c}^m and n are discontinuous:

$$\frac{U_f}{U} = \frac{F^+ - F^-}{S^+ - S^-}, \quad \frac{U_f}{U} = \frac{F^-}{S^-} = \frac{F^+}{S^+}; \quad \frac{U_f}{U} = \frac{F^- + \alpha^n}{S^- + \alpha^n}, \quad \frac{U_f}{U} = \frac{F^- + \alpha^m}{S^- + \alpha^m} \quad (2.8)$$

which is self-consistent only if $\alpha^n = \alpha^m = 0$. Otherwise, biochemical shocks do not exist.

2.2.4 Derivation of the conditions at the shocks

The first condition for the biochemical shock results directly from (2.5b). The last condition is obtained in the following way. Eq. (2.5c) yields:

$$\begin{aligned} U_f(S^+ - S^-) &= U(F^+ - F^-) \\ U_f(\mathbf{c}^{m+}S^+ + \alpha^m\mathbf{c}^{m+} - \mathbf{c}^{m-}S^- - \alpha^m\mathbf{c}^{m-}) &= U(\mathbf{c}^{m+}F^+ + \alpha^m\mathbf{c}^{m+} - \mathbf{c}^{m-}F^- - \alpha^m\mathbf{c}^{m-}) \end{aligned}$$

Multiplying the first equation by \mathbf{c}^{m+} and subtracting it from the second one, we obtain:

$$\begin{aligned} U_f(\mathbf{c}^{m+}S^+ - \mathbf{c}^{m-}S^- - \mathbf{c}^{m+}S^+ + \mathbf{c}^{m+}S^- + \alpha^m(\mathbf{c}^{m+} - \mathbf{c}^{m-})) &= \\ U(\mathbf{c}^{m+}F^+ - \mathbf{c}^{m-}F^- - \mathbf{c}^{m+}F^+ + \mathbf{c}^{m+}F^- + \alpha^m(\mathbf{c}^{m+} - \mathbf{c}^{m-})) &= \\ U_f((\mathbf{c}^{m+} - \mathbf{c}^{m-})S^- + \alpha^m(\mathbf{c}^{m+} - \mathbf{c}^{m-})) &= U((\mathbf{c}^{m+} - \mathbf{c}^{m-})F^- + \alpha^m(\mathbf{c}^{m+} - \mathbf{c}^{m-})) \end{aligned}$$

which yields the last condition in (2.8).

The second and the third conditions in (2.8) results from (2.5d) and (2.5c) in the similar way.

The mechanical shock can arise if the fractional flow is independent of the phase composition. Then we have a single curve $F(S)$ whatever the phase composition. Then $\alpha \neq 0$, which makes impossible the existence of a biochemical shock, since the four conditions (2.8) become incompatible. Then only the saturation may be discontinuous, while n , \mathbf{c}^n , and \mathbf{c}^m are continuous functions. The shock of saturation is simply a forward front of oil displacement by water.

The biochemical shock may arises when the metabolite is soluble only in water. In this case $\alpha = 0$, which is the single possibility that ensures the compatibility of four conditions (2.8) for a biochemical shock.

Note that a biochemical shock separates two different two-phase systems, but the composition of water (and oil) is different ahead of this shock and behind it.

In addition, any shock should satisfy the entropy conditions, which say that its velocity

should be not higher than the transport velocity of the rarefaction wave just ahead of it, and not lower than the transport velocity of the rarefaction wave just behind it.

The geometrical image of shocks is given in the diagram $F(S)$, where two curves correspond to two different phase compositions. In Fig. 2.4, the images of the shocks are red straight lines.

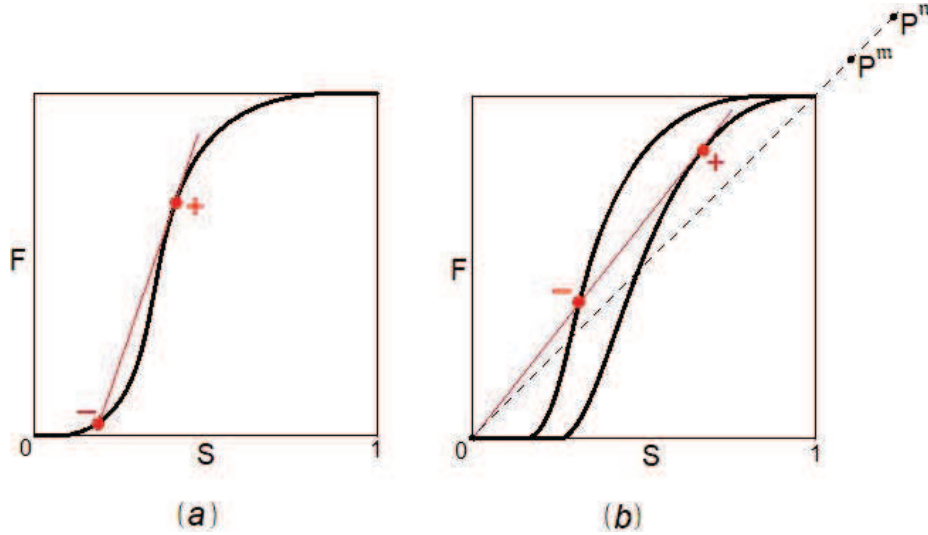


FIG. 2.4 : Geometrical image of mechanical (a) and biochemical (b) shocks

A mechanical shock is given on the image (a) by the red line that is tangent to one of these curves and connects two points "+" and "-", which are on the same curve (since the phase composition is continuous through this shock). The tangent of this line is the dimensionless shock velocity U_f/U , according to (2.7).

A biochemical shock (b) is represented by a straight line tangent to one of the curves, connecting the two points "+" and "-" lying on different curves (since the phase composition + and - are different), passes through the origin (0,0) and through two poles P^n and P^m , which have coordinates $(-\alpha^n, -\alpha^n)$ and $(-\alpha^m, -\alpha^m)$ respectively, according to (2.8). These conditions are compatible between them only if both poles lie in the origin of coordinates, i.e. $\alpha^n = \alpha^m = 0$. So the biochemical pole exists only if $\alpha^n = \alpha^m = 0$.

Chapter 3

Asymptotic solution at weak bioreactivity

3.1 Method of Solution

3.1.1 Concept of weak bioreactivity

We consider only the class of processes, in which the characteristic rate of bacterial population growth is much lower than the rate of fluid flow in the reservoir. This can be caused by two circumstances, according to (??):

- a low concentration of nutrients;
- a large characteristic time of population growth $t_{\mathfrak{B}}$.

According to [56], the time of population growth is between 1 and 10 days. To have the characteristic time of fluid flow much lower than the population growth time, the flow parameters should be as follows: the permeability is between 1 and 10 Darcy (10^{-12} – 10^{-11} m²), the distance between the wells ~ 100 m, the pressure drop ~ 30 bar ($30 \cdot 10^5$ Pa), the viscosity is 1 cP (0.001 Pa·s), the porosity is 0.1. Then the flow time is between 1 and 10 h. These estimations give the idea of applicability of the developed method.

These conditions lead to a significant simplification in the mathematical model of the

process.

Indeed, system (2.5) may be presented in the following form:

$$\partial_t S + U \partial_x F = 0 \quad (3.1a)$$

$$\partial_t \psi + U \frac{F}{S} \partial_x \psi = 0 \quad (3.1b)$$

$$\partial_t \mathbf{c} + U \frac{F}{S} \partial_x \mathbf{c} = -\beta r(n) \mathbf{c} \quad (3.1c)$$

$$\partial_t \mathbf{c}^m + U \frac{F}{S} \partial_x \mathbf{c}^m = \beta_m r(n) \mathbf{c} \quad (3.1d)$$

where

$$\mathbf{c} \equiv \mathbf{c}^n, \quad U = \text{const}, \quad \psi \equiv \mathbf{c} + n\beta, \quad F = F(S, \mathbf{c}^m), \quad R_b = r(n)\mathbf{c},$$

$$\beta = \frac{\rho_w Y}{\rho^n}, \quad \beta_m = \frac{\rho_w Y_m}{\rho^m}$$

The weak bioreactivity means that the term R_b is small: $R_b \sim \varepsilon$, where ε is a small parameter. This means, in turn, for the concentration of nutrients \mathbf{c} that:

$$\mathbf{c} \sim \varepsilon \quad (3.2)$$

We consider the Riemann problem (2.6).

3.1.2 Asymptotic expansion

The method of solution consists in developing functions S , \mathbf{c} , \mathbf{c}^m , and n in the form of the asymptotic expansion:

$$S = S_0 + \varepsilon S_1 + \varepsilon^2 \dots$$

and similar to other functions.

In the zero term, we should obtain the solution that corresponds to the transport without bacterial growth, i.e. bacteria are transported but no nutrients are injected. Consequently, any activity of bacteria is zero, so that $R_{\mathfrak{B}} = 0$, $\mathbf{c}^n \equiv 0$, and $\mathbf{c}^m \equiv 0$. For

zero approximation, we obtain two equations from (3.1):

$$\partial_t S_0 + U \partial_x F_0 = 0, \quad F_0 \equiv F(S_0), \quad (3.3)$$

$$\partial_t \mathbf{c}_0 + U \frac{F}{S} \partial_x \mathbf{c}_0 = 0 \quad (3.4)$$

$$(3.5)$$

The problem for saturation S_0 is independent and represents the classical Buckley-Leverett problem of immiscible displacement. The concentrations of bacteria behaves as a continuous travelling wave, as shown in Fig. 3.1.

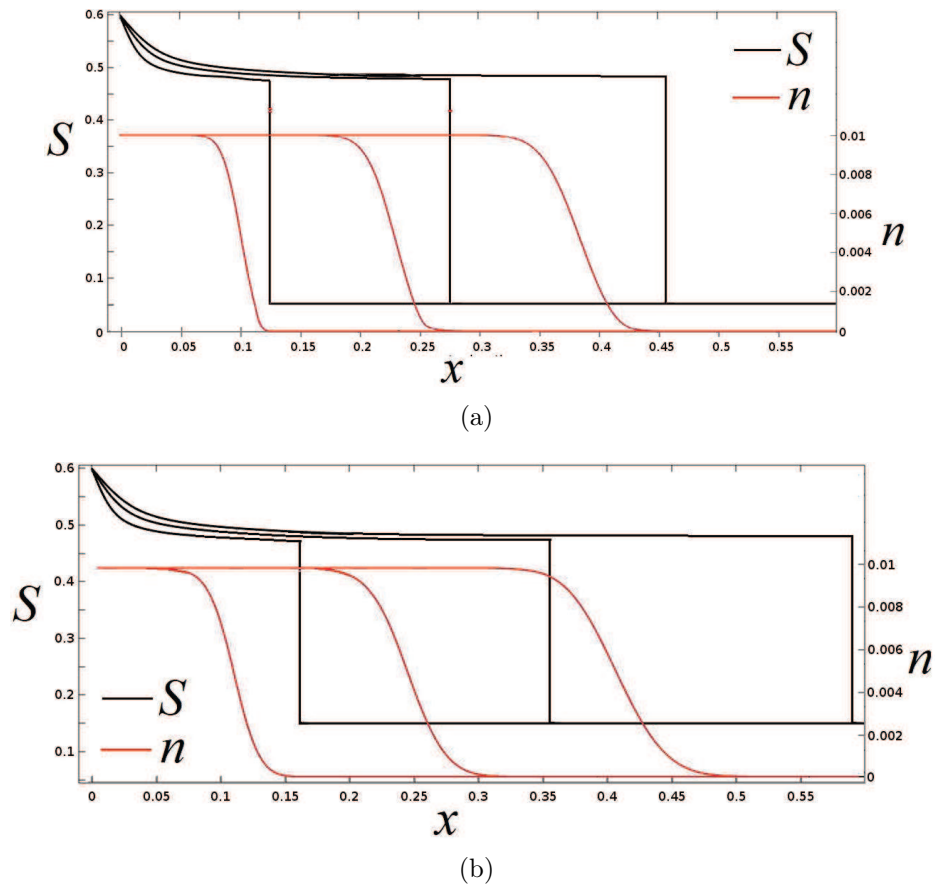


FIG. 3.1 : Saturation of water (S) and concentration of bacteria (n) without nutrients: initial water saturation 0.05 (a) and 0.15 (b)

Despite the fact that the bacteria are simply transported by water, the forward front of their propagation may be delayed with respect to the front of water, as seen in Fig. 3.1(b). This is due to the fact that the reservoir contained water at the initial moment.

This water is displaced by the injected water, which creates a bank of pure water (without bacteria) ahead of the injected water (with bacteria).

When the amount of the initial water is negligible, the front of water and bacteria is identical (Fig. 3.1(a)).

3.1.3 Expected structure of the solution

The expected solution should contain at least two zones behind the forward front of oil displacement:

- the forward zone of mechanical displacement of oil by water; this zone should also contain bacteria, transported with water, but does not contain necessarily nutrients and metabolites;

- the back zone of biochemical action, in which bacteria consume nutrients and, thus, produce the metabolite; the oil is more mobile in this zone. The forward front of this zone should be a biochemical shock.

As the concentration of nutrients is low, then the production of the metabolite for bacteria will be also low. However it cannot be neglected, because the RP curves change even at very low concentration c^m . This causes a change in the water saturation profile during oil displacement by water. Thus, the saturation field will be affected even by a weak bioreactivity.

In other words, the expected structure of the solution, in terms of water saturation S , is as shown in Fig. 3.2.

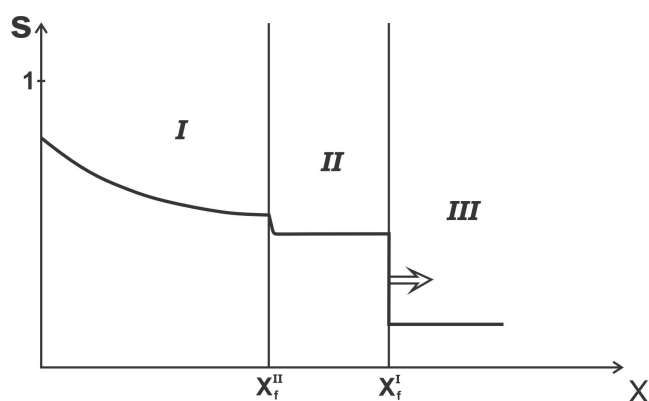


FIG. 3.2 : Expected structure of the solution: I - zone of the action of surfactant; II - zone of mechanical displacement without surfactant; III - initial oil

Thus, the bacterial activity can occur only in zone I.

Such a structure is expected to arise from the moment t^* , when the metabolite concentration in zone I reaches the critical value. Before t^* , the solution corresponds to the classical Buckley-Leverett problem.

As it is shown in the next sections, the true scenario is more complicated, since the transformation of the lowly mobile non-displaced oil into a highly mobile liquid changes qualitatively the behaviour of the back zone and leads to arising of an oil bank in the flooded area.

3.2 Semi-analytical solution to the MEOR problem

3.2.1 Behaviour of concentrations

Let us consider equation (3.1b) with Riemann conditions. As F does not depend on \mathbf{c} and n in zero approximation, then the equation is linear with respect to ψ . Then its solution is then piece-wise constant. The discontinuity corresponds to the front of water injection $x_f(t)$:

$$\psi = \begin{cases} \psi^{\text{inj}}, & x \leq x_f(t) \\ \psi^0, & x > x_f(t) \end{cases}$$

As the nutrient concentration \mathbf{c} is small, then the concentration of bacteria n may be presented in the form of the asymptotic expansion:

$$n = n_0 + \varepsilon n_1 + \varepsilon^2 \dots$$

where n_0 satisfies the linear equation w.r.t. n_0 :

$$\partial_t n_0 + U(F/S)\partial_x n_0 = 0$$

Its solution is a piece-wise constant function:

$$n_0 = \begin{cases} n^{\text{inj}}, & x \leq x_f^{II}(t) \\ n^0, & x > x_f^{II}(t) \end{cases}$$

Then we obtain for the concentration of nutrients (\mathbf{c}) from (3.1c) a linear equation w.r.t. \mathbf{c} :

$$\partial_t \mathbf{c} + U(F/S)\partial_x \mathbf{c} = -\beta r(n_0)\mathbf{c}$$

Its solution is an exponential function in space:

$$\mathbf{c} = \begin{cases} \mathbf{c}^{\text{inj}} \exp\left(-\frac{r(n^{\text{inj}})}{U} \int_0^x \frac{S}{F} dx'\right), & x \leq x_f^{II}(t) \\ \mathbf{c}^0, & x > x_f^{II}(t) \end{cases} \quad (3.6)$$

Then we obtain for n , since $\psi = \mathbf{c} + \beta n$, where $n = \frac{\psi}{\beta} - \frac{\mathbf{c}}{\beta}$:

$$n = \begin{cases} n^{\text{inj}} + \frac{\mathbf{c}^{\text{inj}}}{\beta} \left(1 - \exp\left(-\frac{r(n^{\text{inj}})}{U} \int_0^x \frac{S}{F} dx'\right)\right), & x \leq x_f^{II}(t) \\ n^0, & x > x_f^{II}(t) \end{cases} \quad (3.7)$$

Assuming that $F/S \approx \text{const}$, we obtain approximate explicit formulae, for $x \leq x_f^{II}(t)$:

$$n \approx n^{\text{inj}} + \frac{\mathbf{c}^{\text{inj}}}{\beta} \left(1 - e^{-r^{\text{inj}}x/W}\right), \quad \mathbf{c} \approx \mathbf{c}^{\text{inj}} e^{-r^{\text{inj}}x/W}; \quad W \equiv \frac{UF}{S}, \quad r^{\text{inj}} \equiv r(n^{\text{inj}}) \quad (3.8)$$

Equation (3.1d) for then metabolite is nonlinear with respect to \mathbf{c}^{m} , since the function F depends on \mathbf{c}^{m} . However, we can obtain the approximate solution, assuming that $F/S \approx \text{const}$. It becomes linear, and taking into account (3.8), we obtain its analytical solution by the method of characteristics:

$$\mathbf{c}^{\text{m}} \approx \beta_m \mathbf{c}^{\text{inj}} e^{-r^{\text{inj}}x/W} \left(e^{r^{\text{inj}}t} - 1\right) \quad (3.9)$$

Note that the ratio F/S is different at different stages of the process and in different zones of the domain.

3.2.2 Stage 1: before the impact of surfactant

The expected scenario of the evolution of $S(x, t)$ described above is just a coarse approximation. The real scenario, in all the details, is much more complicated.

The first stage the surfactant concentration is not sufficient to produce any impact on the flow. This is the immiscible Buckley-Leverett displacement [57], without any action of the surfactant. At the same time, the number of bacteria grows, and the metabolite

concentration grows too, while the concentration of nutrients should decrease. This stage is observed until the metabolite concentration reaches the critical value c_{cr}^m at least at a single point of the reservoir. The saturation field is shown in Fig. 3.3(a).

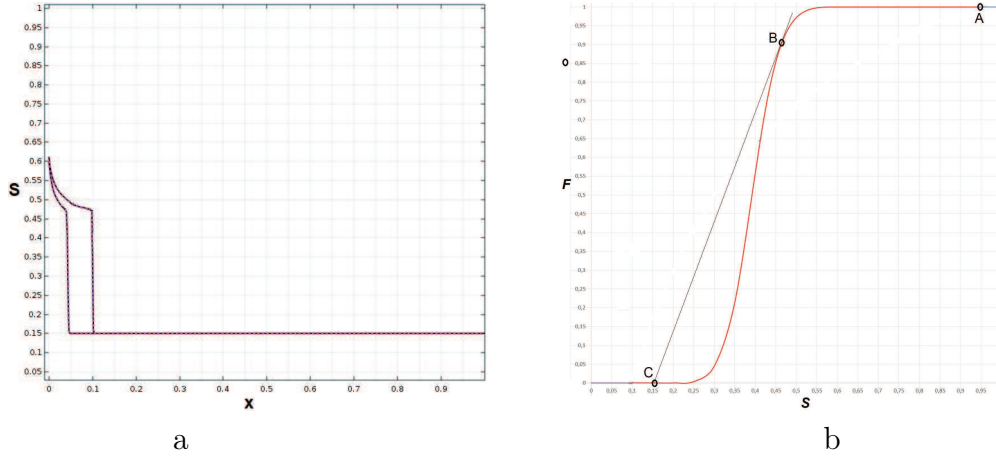


FIG. 3.3 : Saturation before the surfactant action (a) and the solution pathway in the diagram $F(S)$ (b)

The process is controlled by a single fractional flow curve (b) that corresponds to the initial concentration of the surfactant in the reservoir. The injection point is A, the initial point is C. The rarefaction wave AB is followed by a mechanical shock BC (the line CB is tangent to the curve).

The chemical shock does not exist at this stage, but the ratio F/S is discontinuous, therefore the solutions (3.8) and (3.9) is also discontinuous. As seen in Fig. 3.3, F/S varies between 1 and 1.3 behind the mechanical shock and is zero ahead of it. Consequently, we obtain:

Fig. 3.4 shows the behaviour of the exact functions (the solid curves) and the approximation (3.8) (the dotted curves).

3.2.3 Stage 2: formation of an oil bank

At the moment t_1 , the metabolite concentration exceeds the critical value. This corresponds to a new curve $F(S)$ (blue curve in Fig. 3.7-b). A chemical shock should appear, which ensures the transition between two curves.

It corresponds to the straight line OED. So, the solution has the form of a rarefaction wave AD and the shock DE.

Ahead of this shock, the behaviour of the solution is not impacted by surfactant, so the solution should return to the non-perturbed curve (dotted curve in magenta colour).

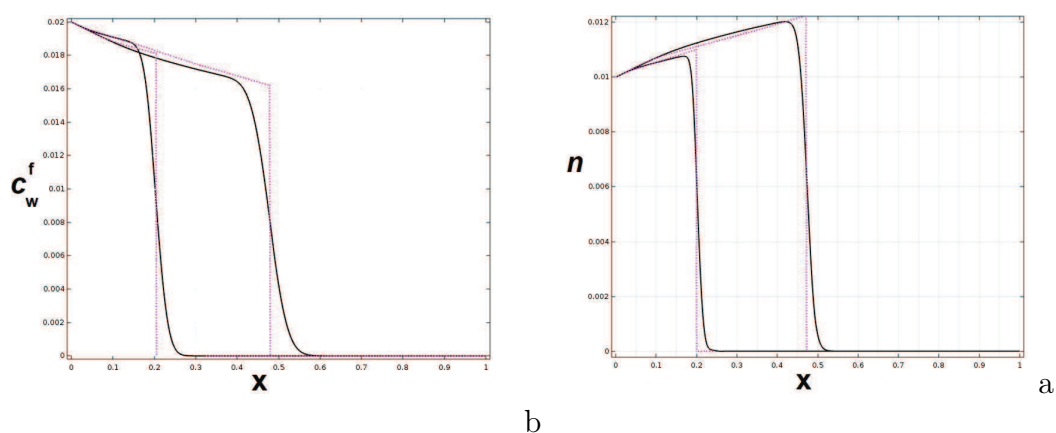


FIG. 3.4 : Variation of the concentration of nutrients (a) and bacteria (b) in space, for two moments of time

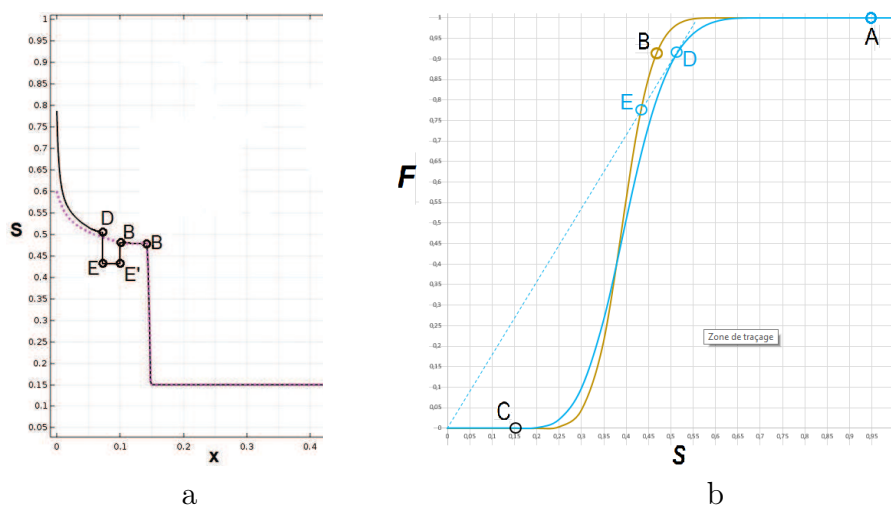


FIG. 3.5 : Appearance of the chemical shock (a) and the solution pathway in the diagram $F(S)$ (b)

The pathway along the initial curve EB in the diagram $F(S)$ is prohibited, as the transport velocity along this path progressively decreases, which leads to a non single-valued solution. Then the only possibility is to return via a new shock E'B. This corresponds to the following construction in the diagram $F(S)$ shown in Fig. 3.8.

The new shock E'B cannot pass through any point G above B, as the subsequent pathway GB is prohibited - the derivative decreases). It is a mechanical shock, as it is placed on the same curve F-S, i.e. at a fixed concentration of the surfactant. The pathway is:

- the rarefaction wave AD,
- the chemical shock DE,

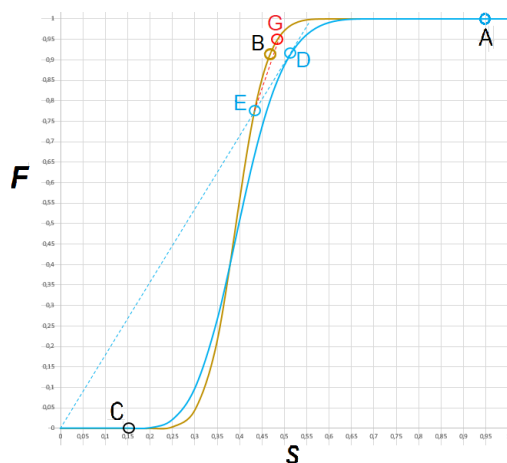


FIG. 3.6 : Fractional flow during the formation of an oil bank

- the plateau EE' (at point E the shock changes its velocity, which corresponds to a plateau);
- the mechanical shock E'B;
- the plateau BB,
- the mechanical shock BC.

Note that the new shock E'B is faster than the forward shock BC. As this shock is mechanical, its physical meaning is the front of displacement of water by oil. Indeed, the element D-E-E'-B in the figure is the oil bank, which is created behind the forward front.

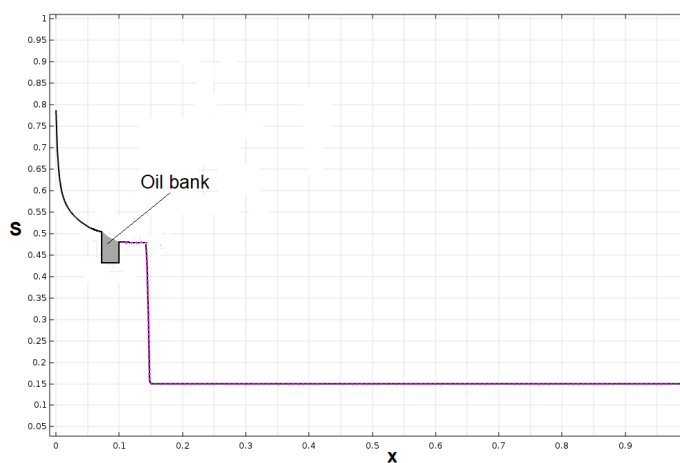


FIG. 3.7 : Water saturation with an oil bank

This bank of oil grows and progressively displaces water in the flooded zone. From where does this oil come? From the areas near the injected well (one sees that water

saturation increases herein). On the MEOR curve, there is a second water front beyond the primary water front. Thus, water breakthrough occurs twice and has a lower effect on oil production. Therefore, a second water front allows to create an oil bank. Part of this oil bank represents the capillary-trapped, which was released and mobilized by the IFT surfactant reduction effect. Thus, the surfactant forms near the injected well and displaces oil from the zone already flooded by water. This leads to the formation of an oil bank which displaces water in the flooded zone.

The moment of the appearance of the biochemical shock, its velocity and saturations behind and ahead of it, as well as the velocity and saturations of the third shock can be strictly determined.

3.2.4 Stage 2: moment of the appearance of the biochemical shock (t_1)

Let t_1 be the moment of the appearance of the chemical shock. This corresponds to the moment when the concentration of metabolite reaches the critical value near the injection well:

$$c_w^m(x, t_1) = c_{w,cr}^m \tag{3.10}$$

for any x .

3.2.5 Moment of appearance of the third shock (t_2)

The biochemical shock corresponds to the straight line coming from the origin of the coordinate system $F - S$ and is tangent to the initial curve $F(S)$. The point of tangency is denoted as D_1 in Fig. 3.8.

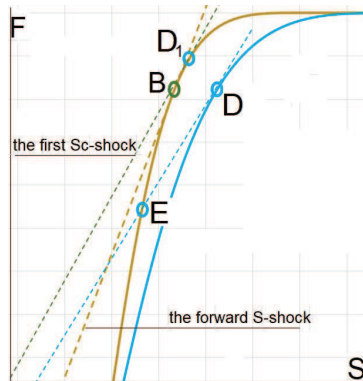


FIG. 3.8 : Fractional mass flow function at the moment of appearance of the third shock (t_2)

After this, the curve $F(S)$ displaces towards the blue curve and the chemical shock also displaces, so that its minimum saturation (E) lies between B and D_1 (or M_1), while its maximum saturation D (M) lies above D_1 (or M_1). As far as the saturation of point D progressively decreases, saturation of point E (N) displaces below the saturation of point B, which corresponds to arising of the third shock E'B (N'B) (Fig. 3.7), t_2 . Two shocks are connected through a plateau EE' (NN'). We obtain the situation when the returning to the initial curve $F(S)$ occurs through a plateau NN':

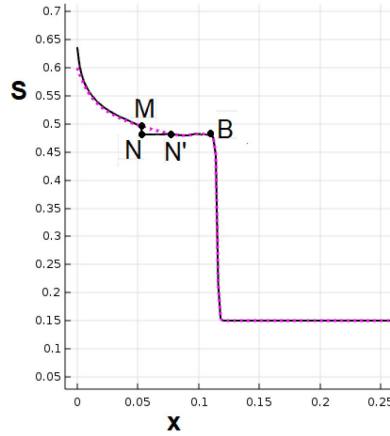


FIG. 3.9 : Water saturation at the moment of appearance of the third shock (t_2)

As far as the saturation M progressively decreases, point N' displaces below the point B and, thus, cannot longer connect the initial curve $F(S)$. Starting from this moment, the third shock appears at point N'. Let t_2 be the moment of the appearance of the third shock. Then we obtain for $S(x_{N'}, t_2)$ the following equation:

$$S(x_{N'}, t_2) = S_B \quad (3.11)$$

where S_B is determined from the Hugoniot-Rankine conditions for the forward S-shock:

$$\frac{\partial F}{\partial S}(S_B, c_w^{m,0}) = \frac{F(S_B, c_w^{m,0}) - F(S^0, c_w^{m,0})}{S_B - S^0} \quad (3.12)$$

$$\frac{U_f^{\text{III}}}{U} = \frac{F(S_{E'}, c_w^{m,0}) - F(S_B, c_w^{m,0})}{S_{E'} - S_B}, \quad (3.13)$$

$$\frac{dx_{E'}}{dt} = U_f^{\text{III}}$$

where (3.12) and (3.13) are the Hugoniot-Rankine conditions for the forward shock and the third shock, respectively. Both shocks are mechanical.

Eq. (3.11) determines the moment t_2 , if the coordinate of point N' is determined. This may be done from the fact that point N' is the connection of the plateau and the rarefaction wave lying on the initial curve $F(S)$, so their transport velocities are identical for any time moment. The transport velocity of the rarefaction wave belonging to the initial curve $U \frac{\partial F}{\partial S}(S, c_w^{m,0})$, where S is the saturation at the contact point. As this saturation varies in time, then the coordinate

$$x_{N'}(t_2) = x_N(t_1)U \int_{t_1}^{t_2} \frac{\partial F}{\partial S}(S_N, c_w^{m,0})dt \quad (3.14)$$

where we taken into account that $S_{N'} = S_N$ for a plateau NN' . Saturation S_N is determined as the solution of the Hugoniot-Rankine condition for the Sc-shock:

$$\frac{\partial F}{\partial S}(S_N, c_w^m(t)) = \frac{F(S_N, c_w^m(t))}{S_N} \quad (3.15)$$

where c_w^m is the metabolite concentration behind the Sc-shock (it is almost constant in space).

The coordinate $x_N(t_1)$ is determined as the distance of transport of saturation S_N from the injection point during period t_1 :

$$x_N(t_1) = U \int_0^{t_1} \frac{\partial F}{\partial S}(S_N, c_w^m(t))dt \quad (3.16)$$

System of five equations (3.11) – (3.16) contains five unknown values: $S_N, t_2, x_N(t_1), x_N(t_2)$ and S_B . The system is closed.

3.2.6 Velocity of the third shock

As mentioned, the third shock appears when the saturation at point N (and N') reaches the value as saturation at point B . The corresponding fragment of the diagram F - S is shown below.

The upper curve is the initial one, the blue curve corresponds to the moment of the appearance of the third shock. The dashed blue line is the tangent line corresponding to the Sc-shock. The solution has the form of a rarefaction wave AM , the Sc-shock MN , the plateau NB , and the S-shock BC :

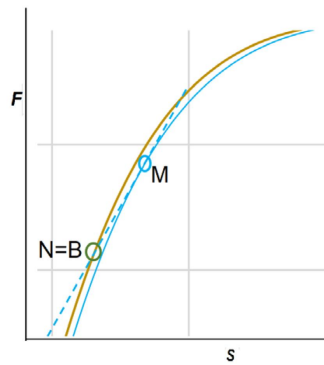


FIG. 3.10 : Evolution of the chemical shock in the diagram $F(S)$

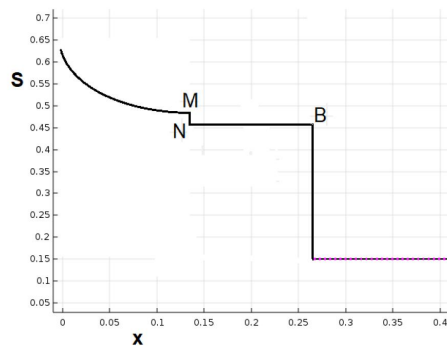


FIG. 3.11 : Evolution of the chemical shock in water saturation profile

After this moment, point N displaces below point B.

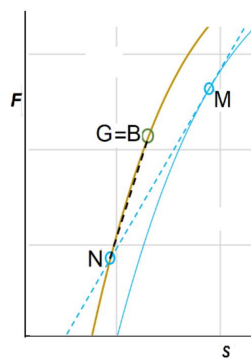


FIG. 3.12 : Illustration of point N displaces below point B in the diagram $F(S)$

The third shock N'G that appears has to relate the plateau NN' and the upper plateau with point B.

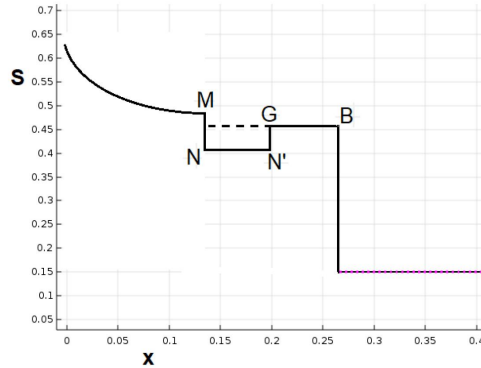


FIG. 3.13 : Water saturation profile of point N displaces below point B

Thus, it should go from point N to point B in the diagram F-S. This is shown above by the dashed black line.

The velocity of this shock is then defined by the tangent of the line N'G. As seen, tite velocity progressively growth while point N goes down. The initial position of the shock N'G is the same as the shock MN at the moment t_2 . It is calculated as:

$$x_N(t_2) = x_M(t_1)U \int_{t_1}^{t_2} \frac{F(S_N, c_w^m(t))}{S_N} dt \quad (3.17)$$

where the distance travelled by point M, which belongs to the rarefaction wave, is:

$$x_M(t_1) = U \int_0^{t_1} \frac{\partial F}{\partial S}(S_M, c_w^m(t)) dt \quad (3.18)$$

and the saturation at point M is obtained from the conditions at the Sc-shock:

$$\frac{\partial F}{\partial S}(S_M, c_w^m(t)) = \frac{F(S_M, c_w^m(t))}{S_M} \quad (3.19)$$

3.2.7 Stage 3: collision of two mechanical shocks

The subsequent evolution is shown below for two moments of time (curves 1 and 2). Point E goes to point N, while point D goes to point M. Saturation behind this chemical shock and ahead of it varies (decreases) in time, as shown in Fig. 3.14.

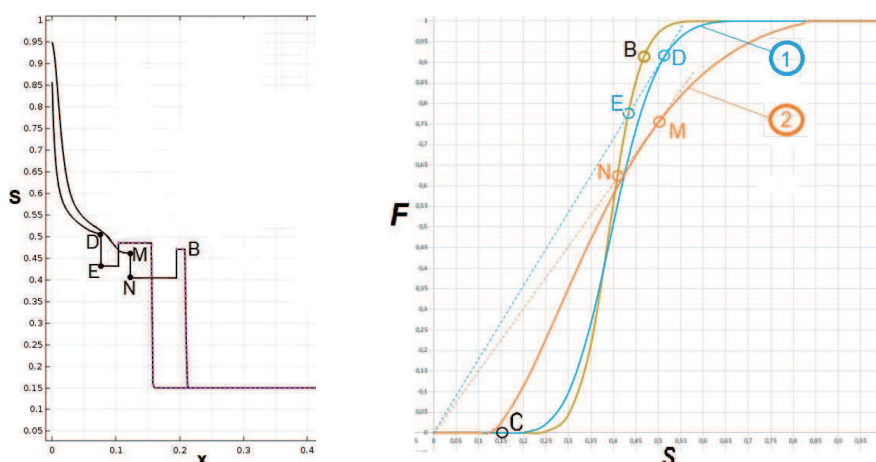


FIG. 3.14 : Evolution and collision of the oil bank

Oil production will also be more efficient with the effective combination of the two fronts. This is because the saturation at the primary water front DE is lower than that of the waterflooding front MN. Therefore, after primary water breakthrough, the water saturation behind the front will be just slightly higher than the irreducible water saturation S_{wi} . Thus water production after primary breakthrough will be limited to this saturation and does not increase until the time of secondary breakthrough; hence less water is produced alongside the oil bank.

As the intermediate shock is faster than the forward shock BC, they necessarily collide at a moment t_3 . This situation is shown in Fig. 3.15.

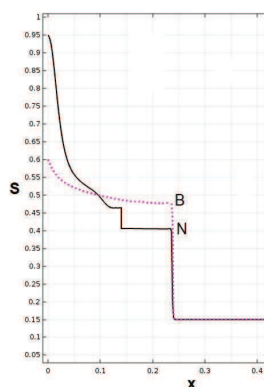


FIG. 3.15 : Collision of two mechanical shocks

Physically, this means the end of water displacement by the oil bank in the flooded zone. The saturation at the forward front drops, which means a less efficient subsequent oil displacement by the forward front. This is a small defect of the bacterial EOR.

The moment t_3 is determined from the condition that the distance travelled by the

forward shock without bacteria is the same as the distance propagated by the third shock + the distance at the moment of its appearance.

Stage 4: after the shock collision – stabilization of the Sc-shock

The subsequent increase in surfactant saturation in the back area leads to the evolution of the curve $F - S$ from the blue to the green curve in Fig. 3.16.

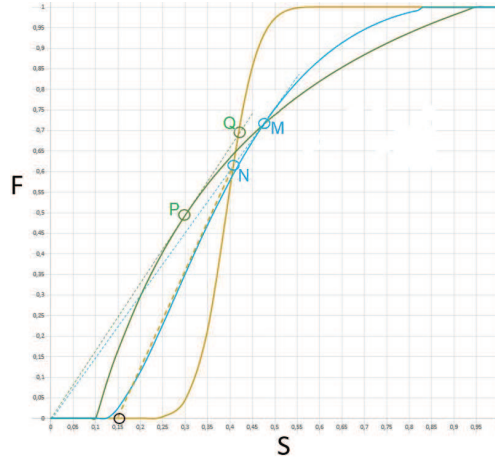


FIG. 3.16 : Evolution of the chemical shock at the stage 4

For the chemical shock, the scenario changes dramatically. Indeed, as seen in the figure, the shock MN should evolve to PQ, which is an “inverse” shock, i.e. the saturation behind it (point P) is lower than the saturation ahead of it (point Q). If such scenario is realized, then point Q would be a plateau between two shocks: PQ and QC. The saturation of this plateau is S_Q is higher than the saturation S_N at the forward front at the moment of shock collision. This means that water saturation behind the forward shock suddenly increases at all the points between two fronts, which does not correspond to mass conservation, and, thus, is impossible physically.

This is why, the scenario of the evolution changes. As usually for a forward mechanical shock, the saturation behind it (S_N) remains constant. Therefore, the saturation ahead of the chemical shock, which is also equal to S_N , also remains constant. Then its velocity, determined by the tangent of straight line ON, also is constant. In contrast, the saturation behind such a shock (point S_M) is not constant in general case, and can rather increase than decrease. The connection of the preceding rarefaction waves AM with this shock is not smooth (the shock is faster), so that a plateau arises between the rarefaction wave and the chemical shock. The solution has the form shown in Fig. 3.17.

The red dotted curves correspond to the displacement without bacteria.

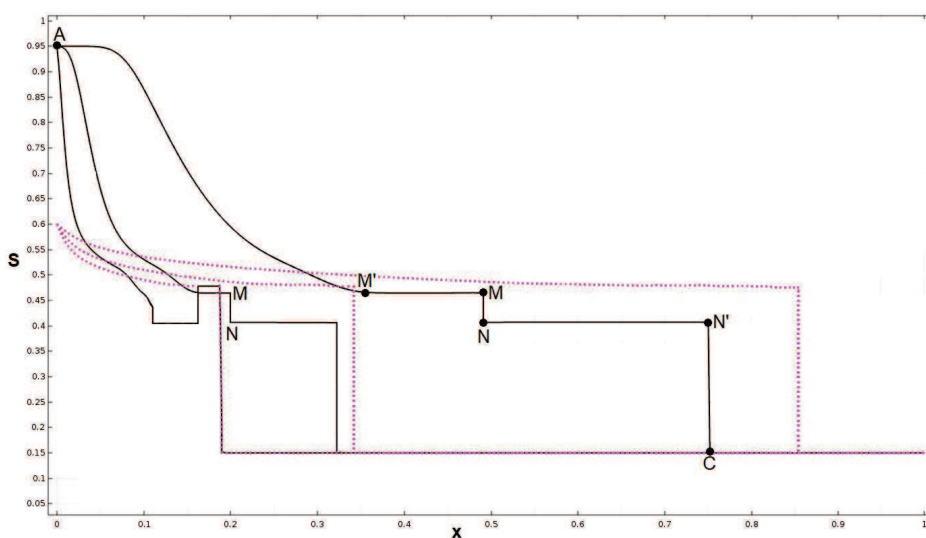


FIG. 3.17 : Evolution of the water saturation: all stages

The pathway is: the rarefaction wave AM' , the plateau $M'M$ (point M in the diagram $F - S$), the chemical shock MN , the plateau NN' (point N), and the mechanical shock $N'C$.

We see that behind the chemical shock we have a wide zone containing much lower amount of oil than in the case of purely immiscible displacement without bacteria.

3.2.8 Behaviour at large times

The main effect is reached at large times, and is presented by the last curve in Fig. 3.17; which is characterized by the movement at constant velocity of the forward mechanical shock and the back chemical shock. The saturations behind both shocks, S_N and S_M determine the process efficiency. They may be estimated analytically.

Let us consider the case when the value S_N is formed before the moment of collision between two mechanical shocks. This saturation can only decrease or be constant, otherwise, the transport velocity of the rarefaction wave behind it will decrease, which will lead to a non single-valued solution. Then S_N is defined in the following way.

Let us consider the infinite set of straight lines coming from point O and tangent to the curves $F_i(S)$, where i is the number of the straight line and the curve in chronological order. The corresponding concentration of metabolite is c_i^m . The points of tangency are W_i . These straight lines cross the curve $F(S, C^{m,0})$ at points N_i . Then S_N is the minimal saturation among all points N_i . Let the number of this minimal point be k .

This straight line number k crosses the curves $F_i(S)$, for $i > k$ at points M_i , which

gives the saturation S_M (variable in time).

Conclusions

1. The MEOR method can be efficient only after a long-term injection, until the back Sc-shock arrives to the producing well.
2. The positive effect of MEOR consists of removing additional oil from the flooded zone, due to the improved relative permeability. The main effect is focused in the area close to the injection well.
3. The negative effect consists of the formation of the oil bank in the flooded zone, which displaces water, and finally reduces the amplitude of the forward front of oil displacement by water. From this moment the efficiency of the displacement of oil by the forward front has been reduced.
4. To improve the efficiency of MEOR, it is necessary to reduce the negative role of the oil bank.

Chapter 4

Numerical analysis of MEOR scenarios

4.1 Numerical qualitative analysis of the process

The asymptotic solutions obtained in the previous chapter concern only the case of slow bioreactions. For the general case of an arbitrary rate of bioreactivity, the solution can be obtained only numerically, which is the objective of this chapter.

We used the simulator COMSOL Multiphysics. The spacial discretization is based on the finite element method. The numerical schemes are implicit in time.

To avoid numerical instability while solving the hyperbolic system (2.5), we used the regularization through the small fictitious diffusion.

4.1.1 Description of the numerical code COMSOL Multiphysics

The realization of the flow model was performed using the COMSOL Multiphysics software [58].

COMSOL Multiphysics [59] is a commercial tool which contains the following basic elements:

- the mathematical solver for systems of partial differential equations; it is based on the finite element method, with irregular discretization of domain;

- a solver that constructs an optimal numerical grid for various forms of elements (triangles, rectangles, pyramids,...);
- a post-processor that ensures the visualisation of the results of simulation, the constructions of various plots, and post-calculation of various parameters.

COMSOL shows a higher flexibility compared to commercial reservoir simulators what makes it interesting for scientific use. It is possible to couple any kind of physical or chemical process with the flow equations. Additionally, the mathematical interfaces allow to enter user-defined equation systems. Thereby, the discretization and solving is still carried out by COMSOL and no extended programming is required. However, the finite element method which is used for the spatial discretization shows some disadvantages for the modeling of flow and transport processes. Oscillations tend to occur around discontinuities in saturations or concentrations. A stabilization of the solution is possible but potentially COMSOL is only adequate for simple conceptual simulations in one- or two-dimensional domains and not for three-dimensional field scale simulations [60] .

4.1.2 Input parameters

The input parameters close to the reality were found in [51] and [23]:

$$\begin{aligned} \mu_w &= 0.001(Pa \cdot s), & \mu_o &= 0.003(Pa \cdot s), & \rho_w &= 1000(kgm^{-3}), & \rho_o &= 900(kgm^{-3}), \\ S^0 &= 0.15, & c_w^{bo} &= c_w^{fo} = 0, & c_w^{b,inj} &= 0.01, & c_w^{f,inj} &= 0.02, & S^{inj} &= 1, \\ \frac{t_{grow}}{t_*} &= 0.01, & \frac{t_{dec}}{t_*} &= 0.05, & K_s &= 0.5, & \alpha_\sigma &= 0, \\ Y_{grow} &= 0.6, & Y_{met} &= 0.3, & \phi &= 0.2, & \gamma_m &= 0, & \frac{D^b}{(UL)} &= 0.0001 \end{aligned}$$

For the surface tension, we used the following relationship:

$$\sigma(c_w^m) = \sigma^0 \exp(-(\zeta c_w^m)^\xi), \quad \zeta = 5000$$

The relative permeabilities are presented in Fig. 4.1.

4.1.3 Comparison with net waterflooding

The surface above the curve behind the front (shock) corresponds to the volume of undisplaced oil. Thus, the displacement efficiency depends on the saturation behind the shock: the higher the amplitude of the forward front, the higher the displacement efficiency.

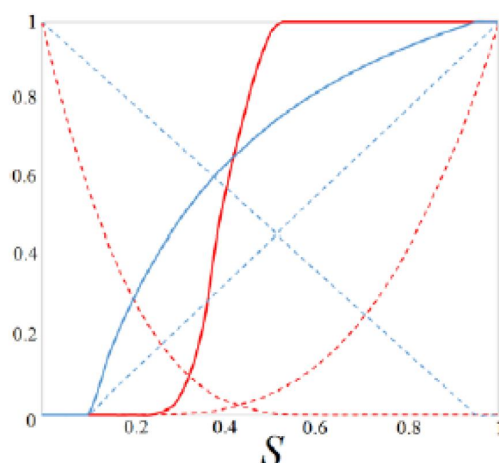


FIG. 4.1 : Relative permeability of oil and water (the dashed curves) and the fractional flow of water F (the solid curves) before microbial treatment (light and red curves) and after reduction of the surface tension to zero (dark and blue curves)

The main parameters that can affect the efficiency of flooding are: the viscosity ratio, the injection rate, the form of RP that affects the shape of the fractional flow, and the dependence of the surface tension on the concentration of metabolites.

The behaviour of the water saturation, compared to the case without bacteria, is shown in Fig. 4.2 by the dashed curves.

One sees that the effect of microbial activity leads to the more complete, practically perfect, displacement, but the zone of the complete displacement is sufficiently delayed with respect to the forward front. This is why the effect may be reached much later than the water breakthrough to the producing well. The behaviour of other functions is presented in Fig. 4.2.

4.1.4 Impact of viscosity ratio

The viscosity ratio changes the shape of the fractional flow, as shown in Fig.4.3. The peculiarity of the two limiting cases is that the Wedge tangent does not exist.

The main component of the water phase is H_2O . Therefore, the viscosity of the water phase is considered constant. At high concentrations of bacteria, viscosity may be affected more, but it is assumed that this is not so. Similarly, the main component of the oil phase is oil, and the viscosity of the oil is considered constant.

The case $\mu_w/\mu_o \ll 1$ corresponds to heavy oil. The saturation profile for this case is shown in Fig.4.4a. The MEOR curve resembles a typical curve for viscous waterflooding.

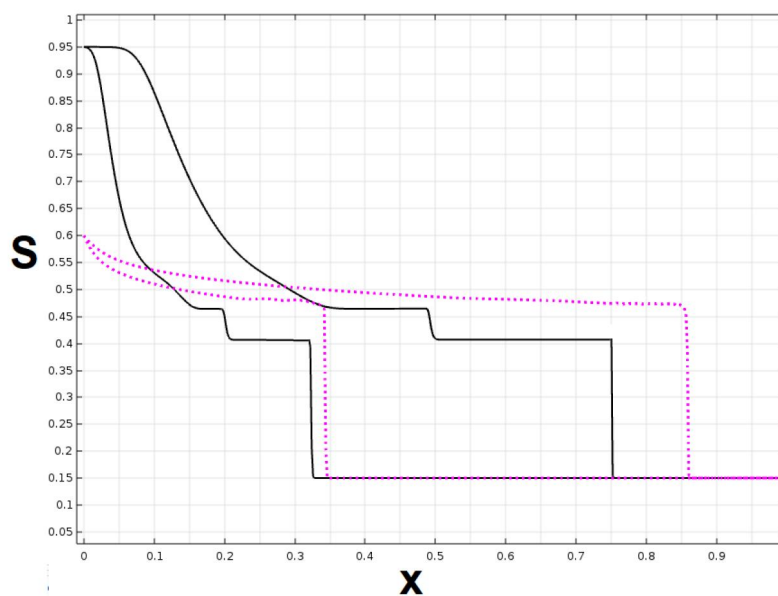


FIG. 4.2 : Water saturation without bacteria (the dotted curve) and with bacteria (the solid curve) for two moments of time: 0.4 and 1

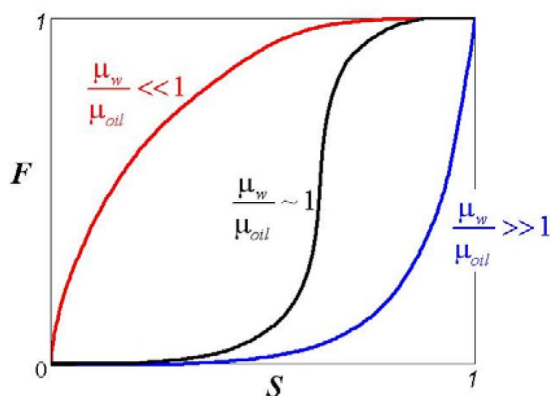


FIG. 4.3 : Structure of the fractional flow for various viscosity ratios

The displacement front corresponds to the corner point when the saturation derivative becomes discontinuous. The rate of this point corresponds to the velocity of the rarefaction wave, and the impact velocity is determined from the Hugoniot condition. The efficiency of this process is lower than in the case $\mu_w/\mu_o \gg 1$, but the rate of this process is high.

The case $\mu_w/\mu_o \gg 1$ corresponds to a light oil or to the injection of viscous solutions of water (polymers). The saturation profile for this case is shown in Fig. 4.4c. The dis-

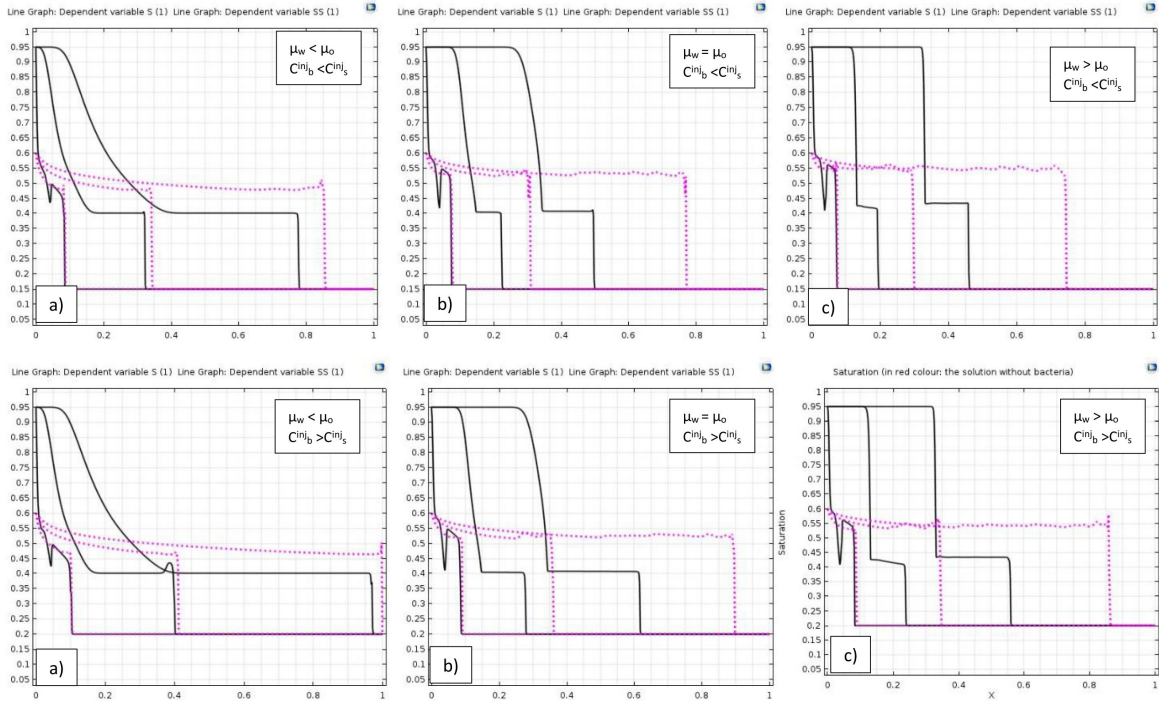


FIG. 4.4 : Behaviour of water saturation for various viscosity ratios

placement is piston-like, so that all the oil is displaced. A larger water viscosity influences the fractional flow of oil positively, giving rise to the increase in oil recovery.

4.1.5 Numerical results for low injection concentration

This case corresponds to the weak bioreactivity. This asymptotic case was analyzed in the previous chapter. The results obtained in this section have the objective to validate the asymptotic analysis performed previously.

We considered two situations:

- (I) $c^{n,inj} > n^{inj}$: one injects more nutrients than bacteria;
- (II) $c^{n,inj} < n^{inj}$: one injects more bacteria than nutrients.

The first case is presented in Fig. 4.8.

The case II is shown in Fig. 4.6.

One sees that both scenarios leads to approximately identical results. The results obtained correspond well to the analytical results obtained in the previous chapter for the weak bioreactivity.

It would be natural to expect that for lower amount of bacteria (case I), the amount of

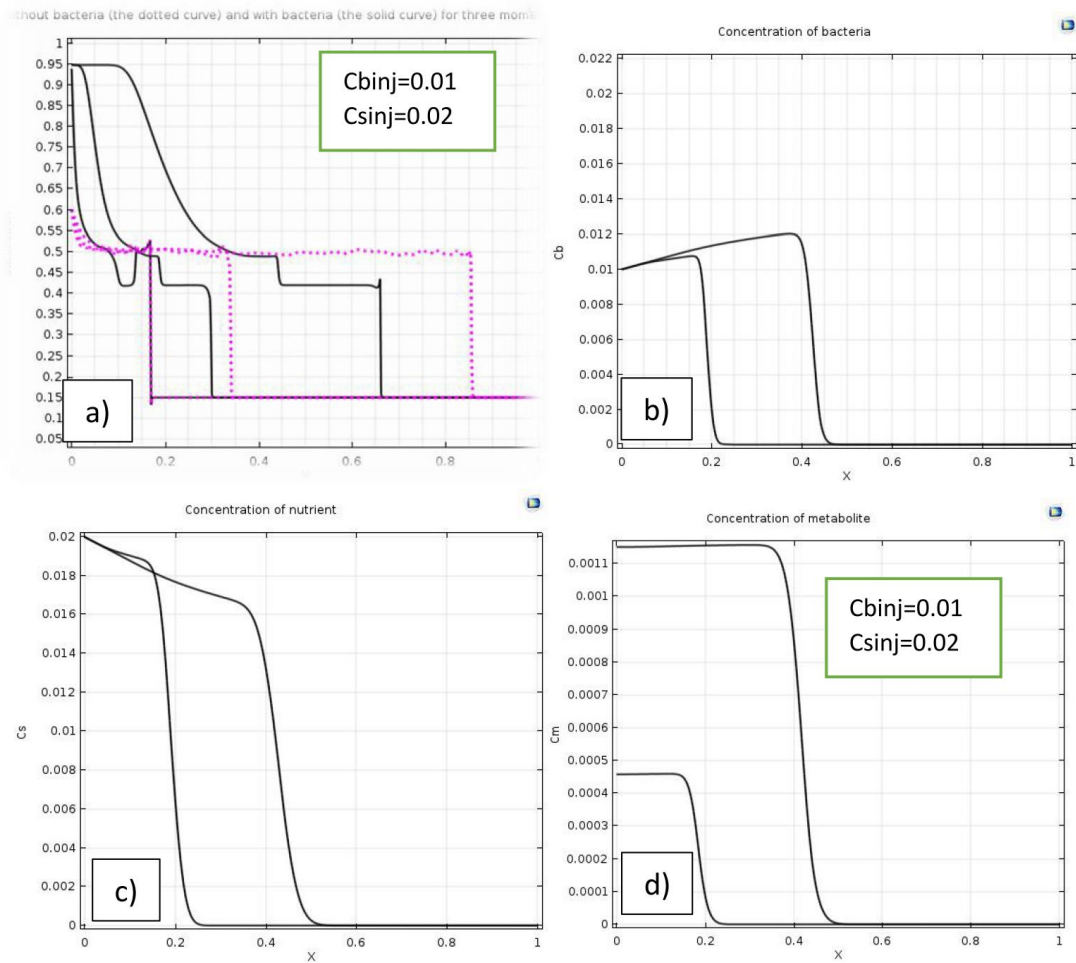


FIG. 4.5 : Water saturation (a) and concentrations of nutrients (b), bacteria (c), and metabolite (d) in the case I: $n^{inj} = 0.01$, $c^{n,inj} = 0.02$

the produced metabolite would be also lower. This is not the case, however. The amount of metabolite produced in these two cases is practically identical. This is why the impact of bacteria on oil displacement is practically identical.

4.1.6 Numerical results for high injection concentration

This case is beyond the asymptotic case analyzed in the previous chapter and can be analyzed only numerically. As in section refsec:tru, we analyzed two cases:

- (I) $c^{n,inj} > n^{inj}$: one injects more nutrients than bacteria;
- (II) $c^{n,inj} < n^{inj}$: one injects more bacteria than nutrients.

The results are presented in Figs. 4.7 and 4.8.

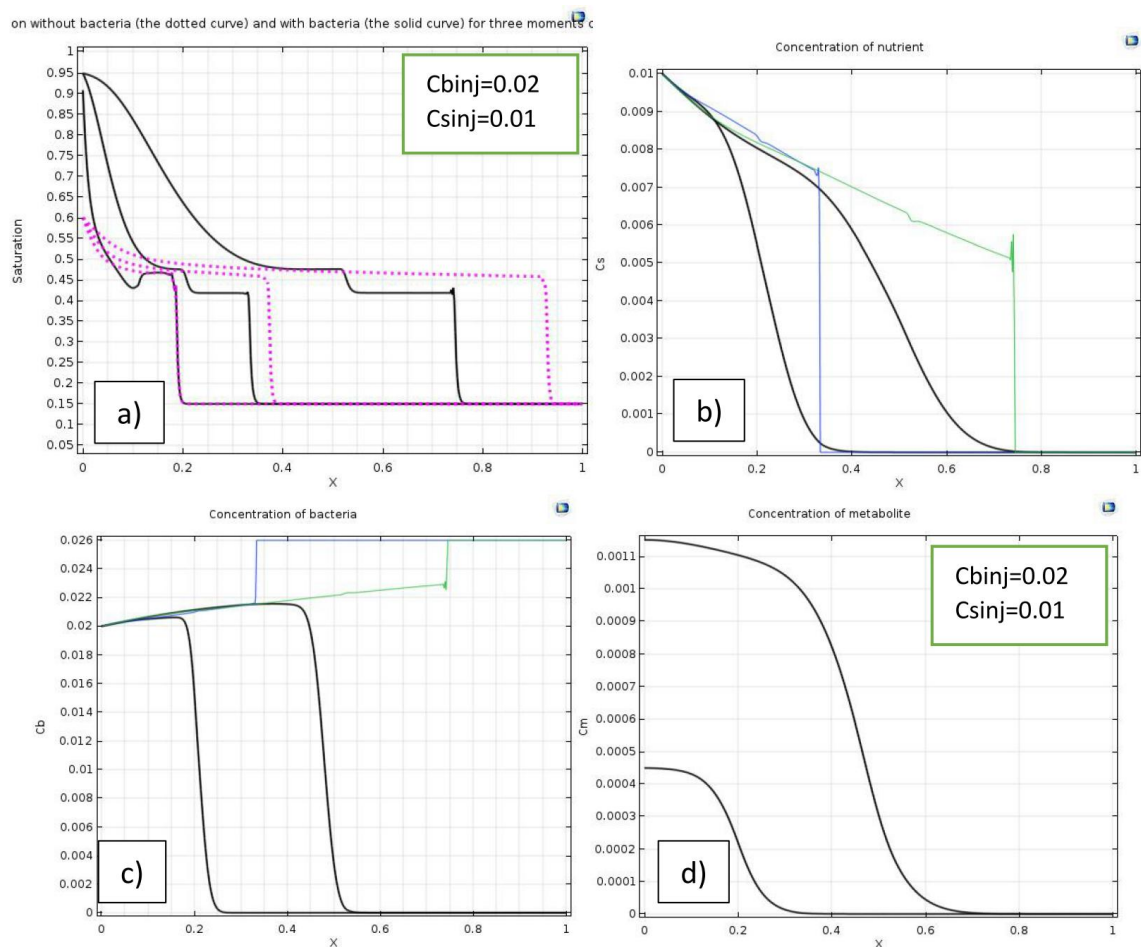


FIG. 4.6 : Water saturation (a) and concentrations of nutrients (b), bacteria (c), and metabolite (d) in the case I: $n^{inj} = 0.02$, $c^{n,inj} = 0.01$

As expected, the impact of bacteria is much more important in this case. In particular, the amount of injected nutrients sharply decreases four times faster than at low concentrations. A lot of metabolites are produced during a very short time, and, therefore, the oil bank does not appear.

The observed numerical oscillations can be suppressed by higher diffusion.

As seen, the increase in the injected concentration of bacteria leads to increasingly smoother behaviour of all concentrations. This means that the conditions for the biochemical shock become incompatible and it cannot exist at high concentrations n .

Due to the high number of injected bacteria, the bacteria rapidly consume the nutrients and their evolution stops. Consequently, the impact on the saturation behaviour is not so high as expected.

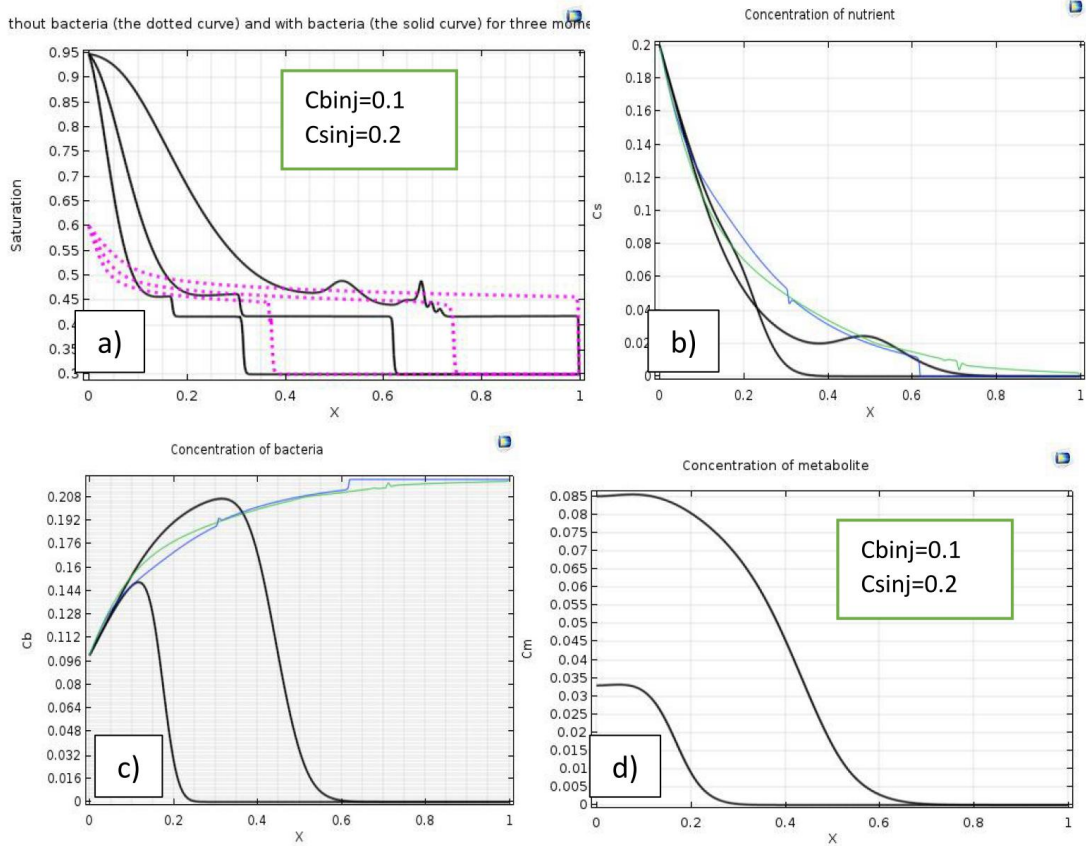


FIG. 4.7 : Water saturation (a) and concentrations of nutrients (b), bacteria (c), and metabolite (d) for high injection concentrations. Case I: $n^{inj} = 0.1$, $c^{n,inj} = 0.2$

4.1.7 Impact of the form of the population kinetics

We analyzed the impact of the mathematical approximation used to describe the bacterial kinetics among formulae of Monod (Eq. 1.3), Moser (Eq. 1.8) and Panfilov (Eq. 1.4). The respective results of comparison are presented in Fig. 4.9.

In the case of Monod (b) we do not observe the oil bank, which is the main difference with a and c.

4.1.8 Impact of the characteristic kinetic rate

The characteristic time t_{grow} of population growth in the equation (1.3), or the characteristic rate, which is the inverse value: $\mathfrak{R} \equiv t_{grow}^{-1}$, is one of the essential parameters impacting the EOR process. Fig. 4.10 shows the results of simulation for different values of \mathfrak{R} .

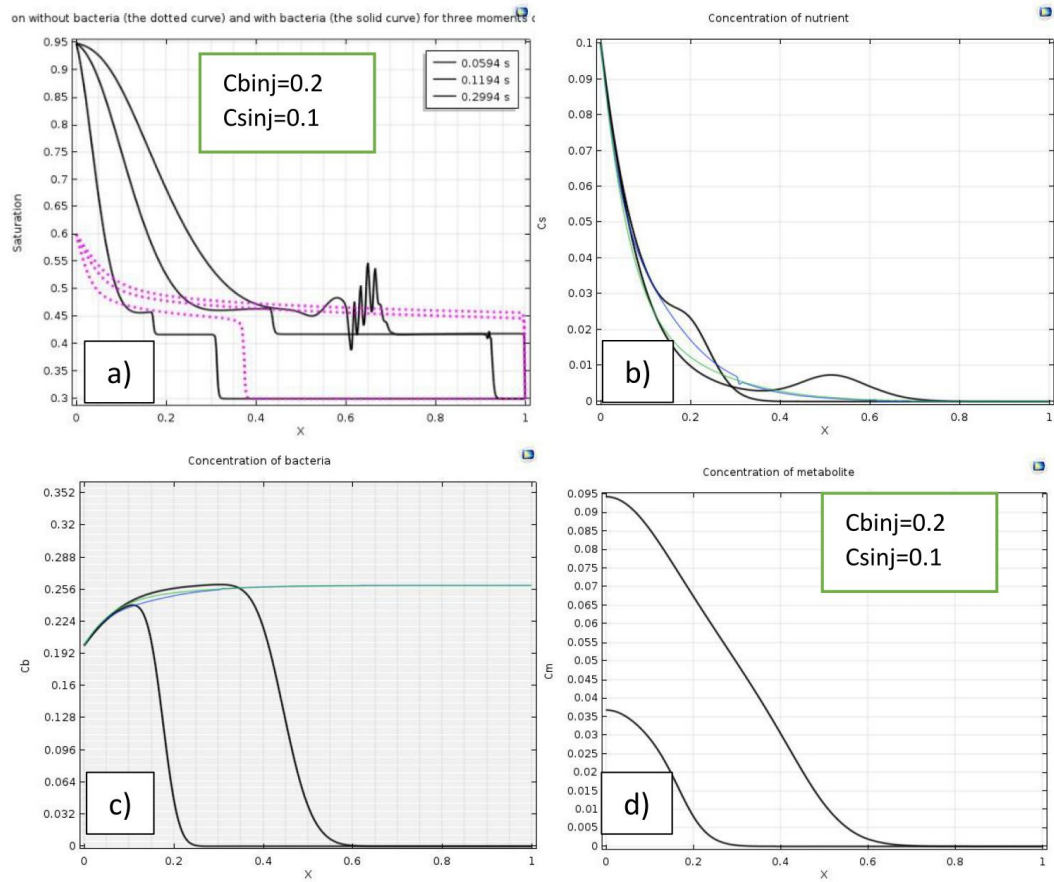


FIG. 4.8 : Water saturation (a) and concentrations of nutrients (b), bacteria (c), and metabolite (d) for high injection concentrations. Case II: $n^{inj} = 0.2$, $c^{n, inj} = 0.1$

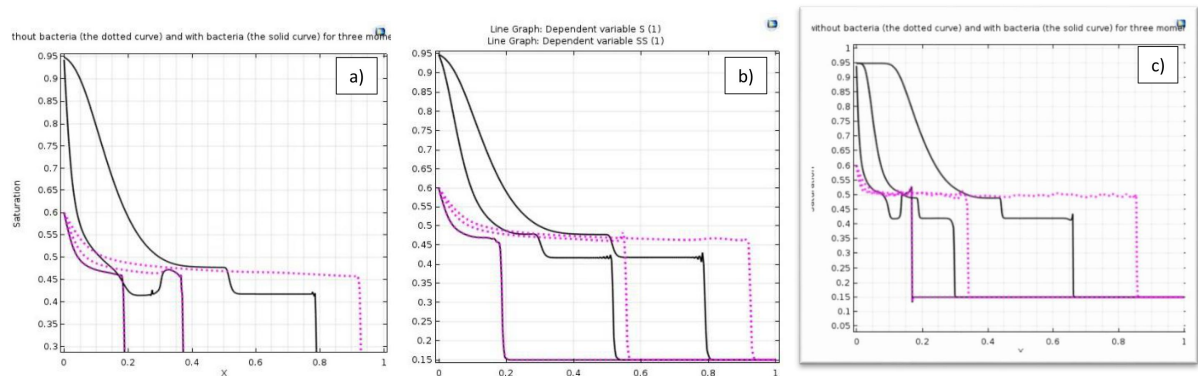


FIG. 4.9 : water saturation at different formulae for the population growth rate

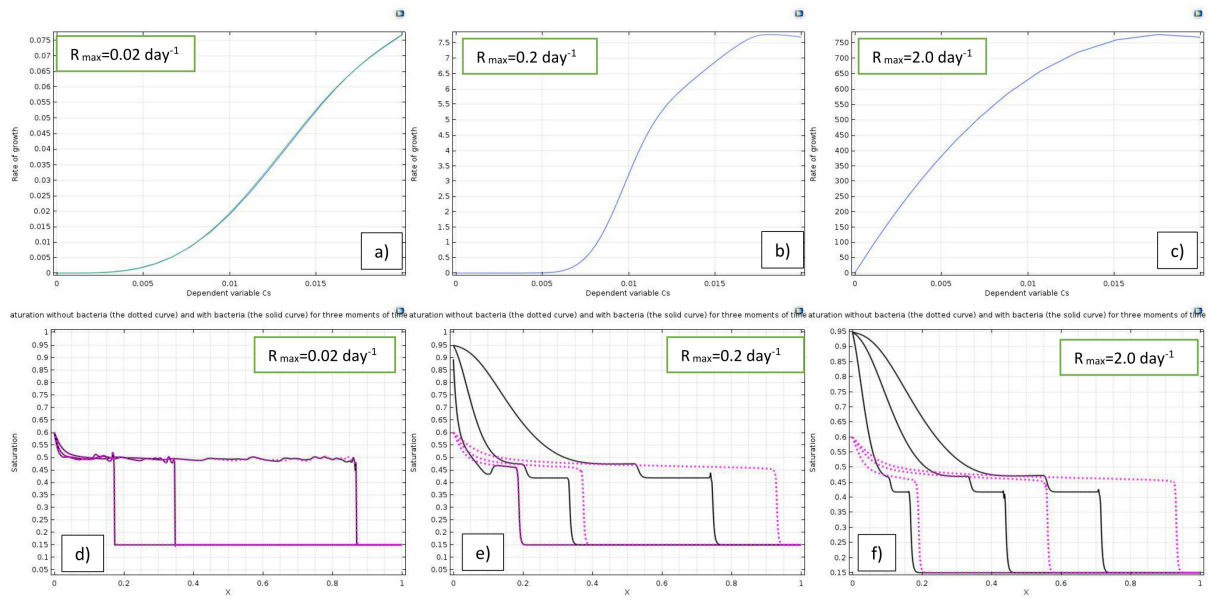


FIG. 4.10 : Water saturation at different characteristic rates of population growth: 0.02 days⁻¹, 0.2 days⁻¹, and 2 days⁻¹

If the kinetics is very slow (the left-hand image), the impact of bacteria is neglecting. On the other hand, if the kinetics is too fast (the right-hand image), no significant differences are observed between the moderate case (the central image). So, starting from some critical value of \mathfrak{R} the efficiency of MEOR will not change. This means that an optimal injection rate exist that ensures the maximal efficiency. This optimal injection rate should be no higher than \mathfrak{R} , in order to give bacteria the time to perform their useful action.

4.2 Simulation of a case study

4.2.1 Description of the Kazakhstan oil fields

The Major hydrocarbon reserves in Kazakhstan (over 90%) are concentrated in 15 largest oil fields: Tengiz, Kashagan, Karachaganak, Uzen, Zhetybai, Janajol, Kalamkas, Kenkiyak,

Karazhanbas Kumkol North Buzachi, Alibekmola, Prorva, Kenbai, Royal Fig.4.11. According to the Ministry of Oil and Gas, the recoverable oil reserves are estimated at 17 bln. Tons.

Many of the southern part of the Caspian deposits discovered in 30-40s of the last century, are in the late stage of development, their proficiency reaches 75,6-98,9%. Basically these fields are developed by such companies as "Embamunaigas" and PF "Uzenmunaigaz", whose activity is of a great socio-economic importance to the regions.

The majority of deposits in Western Kazakhstan (30) are characterized by a high content of paraffin (up to 26%), resins and asphaltenes (20%), as well as corrosive gases (H_2S , CO_2). Operation of such deposits initially hampered extremely complex geological and physical conditions: a large number (20) of non-homogeneous multilayer (up to 10-12) horizons, the proximity of the initial reservoir pressure to the saturation pressure, and proximity of the reservoir temperature to the temperature of paraffin crystallization (Uzen oil field).

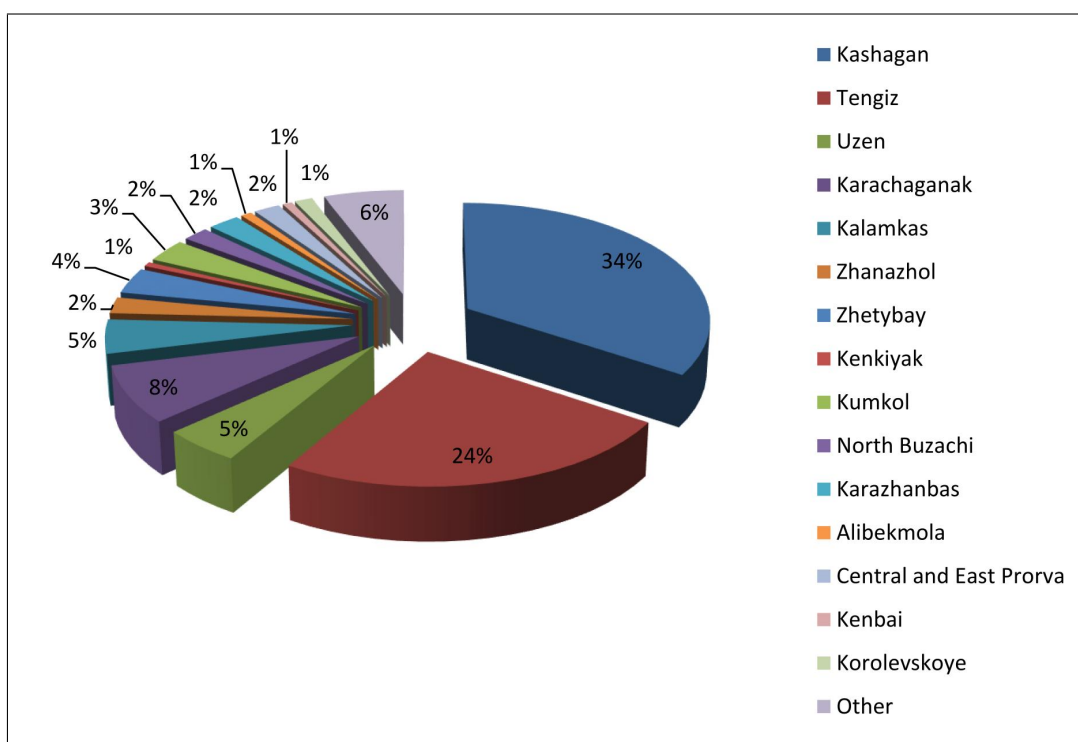


FIG. 4.11 : Distribution of recoverable oil reserves in Kazakhstan, %

4.2.2 On the implementation of MEOR in Kazakhstan's oil fields

To implement MEOR, it is necessary to create a microbiological solution (culture fluid), beneficial microorganisms, oxygen sources, nutrients for creating orthophotomaps in the natural zone, as well as the subsequent development and production of useful microorganisms in producing wells. In reservoir conditions, microorganisms are able to synthesize organic solvents, such as alcohols, polymers, acids.

The technology can be used in areas both in fresh and in mineralized water, using available domestic reagents or waste products that do not require sophisticated equipment for implementation. Through the use of naturally occurring microorganisms and completely reusable in nature reagents, this technology is economical and safe [61, 62].

The microbial flora of several oil strata in the fields of Kazakhstan is characterized by the presence of abundant vegetable organics, which means a favorable environment for growth of microorganisms. Such situation is very favourable to the application of MEOR. MEOR has already been studied in the field Kalamkas, located on the Peninsula Buzachi and give excellent results: just in cours of 5 days, the oil recovery in the treated zone was increased from 20% to 25% [63] .

Microbiological analysis by using data of Uzen field. From the whole variety of microorganisms, the possibilities of exposure and the type of biological effect can be identified. The criterion may be the number of hydrocarbon-oxidizing, sulfate-reducing and heterotrophic bacteria. The second initiators of hydrogen sulfide corrosion, the third are indicators of the conditions for the development of microbiological processes. The aim of the research is to determine the above microorganisms in the formation water samples. 1 ml of test water, followed by a higher impact on the electric nutrient medium.

In the process of microbiological activity, due to the decomposition of petroleum hydrocarbons and others, oxidation products are formed, including low-caloric acids , alcohols, bio-surfactants, biopolymers, gases (CO_2 , N_2 , CH_4), which are good oil displacing agents. An increase in the gas phase of formation fluids of carbon dioxide and methane, products that completely mineralize hydrocarbons and other compounds, can also contribute to an increase in in-situ pressure. To this end, plastic microphones are activated, which can be used for microorganisms, including in natural substrates, among the most promising for whey and active and treatment facilities [61, 64].

4.2.3 Data for Uzen oil field

We performed simulations for a segment of the Uzen oil field with the objective to compare the results obtained with the in-situ data. This will give us the possibility to obtain several

Table 4.1: Initial geological and physical characteristics of Uzen oil field

Parameter	Properties
Collector type	Terrigenous
Reservoir depth (top of productive zone)	402 m
Initial reservoir pressure, MPa	10.08-15.77
Initial reservoir temperature, °C	57-58
Oil viscosity at reservoir conditions, mPas	4.51
Density of oil in reservoir conditions, kg/m^3	781.6
Density of water in reservoir conditions, kg/m^3	1018.44
Porosity, %	26.5
pH	5.4-7.71
The concentration of asphaltenes in the oil, %	17.57

parameters of the process, which are difficult to obtain in laboratory experiments. The data for Uzen oil field are presented in Table 4.1.

4.2.4 Calculation of the field case for Uzen oil field

Figure 4.12 illustrates and compares a typical saturation plot for standard waterflooding (the red curves) and one generated by the action of biosurfactants.

As seen, the forward front of mechanical displacement gives approximately the same breakthrough oil recovery as simple waterflooding. The biomechanical front, which arrives later, brings a small contribution in the oil recovery. The main positive impact of MEOR is observed at high times, larger than the time of the arrival of the biomechanical front, when the smooth wave (non shock) of biochemical action arrives. For this moment, the oil recovery tends to be perfect (as the water saturation in the reservoir tends to 1). The propagation velocity of this wave is four times lower than the velocity of the forward mechanical front. Therefore, it is necessary to inject four water volumes to reach the MEOR effect.

This conclusion can be considered as a practical recommendation to the implementation of MEOR.

This analysis is purely physical and technical, and does not concern any economical considerations.

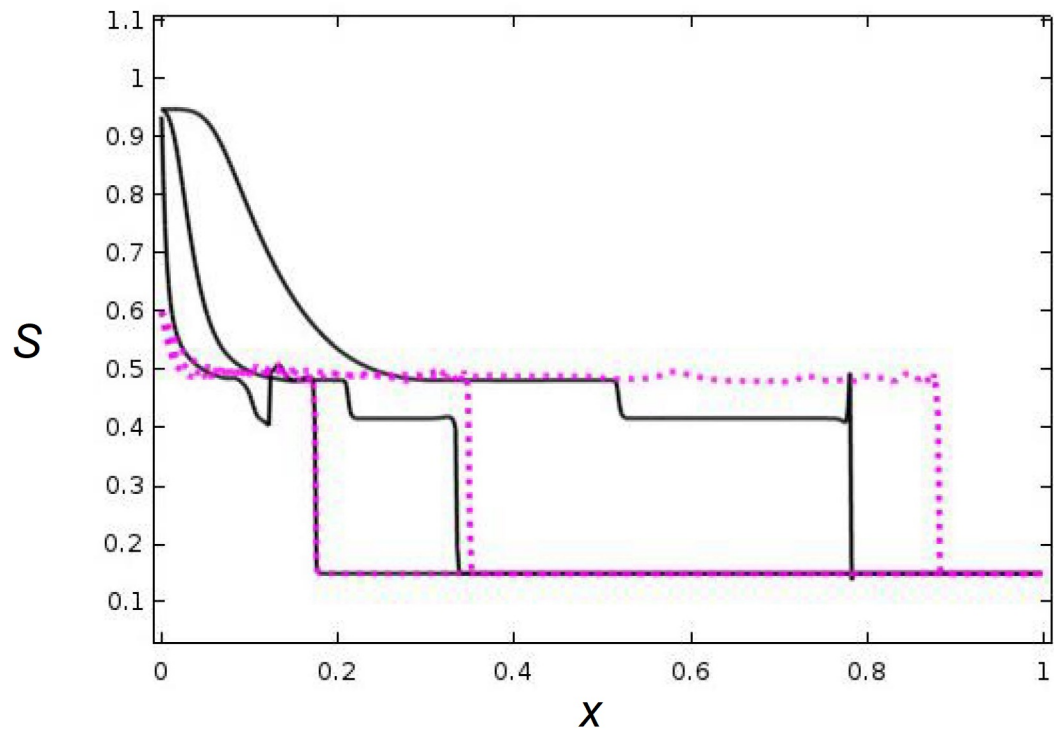


FIG. 4.12 : Water saturation during implementation of MEOR in Uzen oil field: MEOR - black curves; simple waterflooding - red curves

Conclusions

In the present thesis we analyzed the complex physical process that consists of two-phase flow (water and oil) in porous media of fluids containing several (four) chemical components and bacteria. Some components are injected from the exterior, while other components are the results of chemical reactions initiated by bacteria. The components can cross the interfaces between water and oil and change their concentrations depending on the global phase composition and thermodynamic conditions. The main component produced by bacteria is a surfactant that changes the surface tension between water and oil and has a significant impact on the flow. IN particular, it reduces the effect of capillary trapping of oil, which allows to liberate immobile oil and produce it. The transport of chemical species is coupled with the dynamics of the bacterial population that consume nutrients, grows and intensifies the production of surfactant.

Such system is described by five partial differential equations. Neglecting the diffusion and capillary pressure, and using some typical properties of biochemical systems, we have reduced it to a system of four quasi-linear hyperbolic equations of first order. In physics, such a system is called the model of kinematic waves. Such a system has discontinuous solutions (the shocks). We determined the admissible types of shocks and derived the Hugoniot-Rankine conditions at the shocks.

To analyse such a system we suggested the asymptotic approach based on the concept of the weak bioreactivity. This means that the characteristic rate of population growth is assumed to be a small parameter of the system. The solution can be found in the form of the asymptotic series over this small parameter. The zero term of such a series corresponds to the two-phase flow accompanied with the transport of bacteria, but without chemical reactions and without population growth. In such a system, the production of surfactant is also low, however, its impact on the surface tension becomes important even at very low concentrations of surfactant. This approach enables us to obtain the solution of the displacement problem (a Riemann problem) in the analytical form and to perform the complete qualitative analysis of this solution.

The case of high growth rate was analyzed numerically. We used the code Comsol

Multiphysics existing in the laboratory IECL. Due to numerical simulations, we also confirmed the analytical results obtained by the asymptotic method.

The developed solutions has been applied to calculate an example of MEOR in a field case: the oil field Uzen of Kazakhstan. This given us the possibility to estimate some parameters of this technology that ensure the efficient oil recovery.

Appendices

Appendix A

Analytical solution to the Buckley-Leverett equation

The case of simple waterflooding without bacteria is described by a single equation known as the Buckley-Leverett equation:

$$\begin{aligned}\frac{\partial S}{\partial t} + U \frac{\partial F(S)}{\partial x} &= 0 \\ U &= -\frac{\lambda_w + \lambda_o}{\phi} K \frac{\partial P}{\partial x} \\ F(S) &= \frac{\lambda_w(S)}{\lambda_w(S) + \lambda_o(S)}, \quad \lambda_i = \frac{k_i}{\mu_i}\end{aligned}$$

The fractional flow F is S-shaped, as shown in Fig. [A.1](#)

So $F(S)$ is neither convex nor concave, which leads to the appearance of a shock. This shock is determined by the Hugoniot-Rankine and entropy condition, so that

$$\frac{\partial F}{\partial S}(S^*) = \frac{F(S^*) - F(S^0)}{S^* - S^0}$$

Graphically this means that the shock corresponds to the red tangent line to the curve $F(S)$ coming from the initial point S^0 .

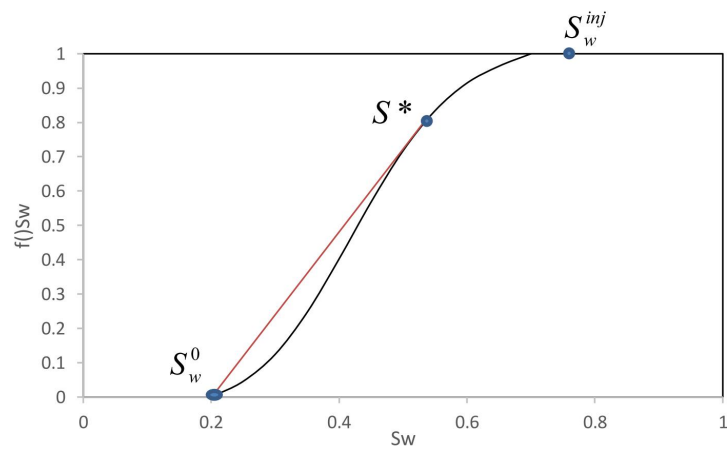


FIG. A.1 : Fractional flow for immiscible displacement

Appendix B

Numerical code Matlab: population growth for various kinetic functions

```
Calculation of bacterial population growth :
%=====
% differential equations:
%      dC/dt = -f(C,n)
%      dn/dt = ka*f(C,n) - coef*n^beta*(1-C)
% where C is the concentration of nutrients (co2)
%      n is the number of bacteria
% for three kinetic functions f(C,n):
%      - Monod: f=n*C/(1+C)
%      - Moser: f=n*C^2/(1+C^2)
%      - Panfilov: f=n^2*C/(1+(n/nend)^2)
%
% Solution of three ODU.
% Resulting plots:
%      - C(t)
%      - n(t)

close all;
clear all;

global ka nend coef beta
```

```
%
nend=10;    % constant for Panfilov's kinetics
ka=0.8;    % coefficient of conversion of nutriments to biomass
n0=0.03;   %initial colony
co20=0.2;  %initial CO2
coef=0.002; % coef=0 to eliminate bacterial decay
tend=350;  % final time
beta=2;
%
[T1,y_Monod]=ode45(@rMonod,[0 tend],[co20 n0]);
[T2,y_Moser]=ode45(@rMoser,[0 tend],[co20 n0]);
[T3,y_Panf] =ode45(@rPanf, [0 tend],[co20 n0]);
%
plot(T1,y_Monod(:,1), T2,y_Moser(:,1), T3,y_Panf(:,1))
%
figure;
plot(T1,y_Monod(:,2), T2,y_Moser(:,2), T3,y_Panf(:,2))

function dy=rExp(t,y)
global ka nend coef beta
dy=zeros(2,1);
co2=y(1);
n=y(2);
psi=0.000002*exp(-t^4/5000000000);
fi=0*coef*n^beta*(1-co2);
dy(1)=-psi;
dy(2)=2*ka*t*psi - fi;
end

%      1) The kinetic functions of Monod
%=====
function dy=rMonod(t,y)
global ka nend coef beta
dy=zeros(2,1);
co2=y(1);
```

```
n=y(2);
psi=n*(co2/(1+co2));
fi=coef*n^beta*(1-co2);
dy(1)=-psi;
dy(2)=ka*psi - fi;
end
```

```
%      2) The kinetic functions of Moser
```

```
%=====
```

```
function dy=rMoser(t,y)
global ka nend coef beta
dy=zeros(2,1);
co2=y(1);
n=y(2);
psi=n*(co2^2/(1+co2^2));
fi=coef*n^beta*(1-co2);
dy(1)=-psi;
dy(2)=ka*psi - fi;
end
```

```
%      3) The kinetic functions of Panfilov
```

```
%=====
```

```
function dy=rPanf(t,y)
global ka nend coef beta
dy=zeros(2,1);
co2=y(1);
n=y(2);
%psi=n^2*co2/(1+(n/nend)^2);
psi=n^2*co2/(1+(n/100)^2);
fi=coef*n^beta*(1-co2);
dy(1)=-psi;
%dy(2)=ka*psi - fi;
dy(2)=ka*psi;
```

Symbols

C_i^j - Overall component concentration in the phase [kg/m^3PV]

F_i - Overall component flux

f_j - Fractional flow function for phase j

i - Index for component

j - Index for phase

K - Absolute permeability [mDa]

K_i - Partitioning coefficient for surfactant

K_s - Half saturation constant in Monod expression [kg/m^3]

kr_j - Phase relative permeability

k_{rowi} - Endpoint relative permeability for oil at s_{wi}

k_{rwor} - Endpoint relative permeability for water at $(1 - s_{or})$

L - Length of the reservoir [m]

P - Pressure [kPa]

Q - Volumetric flow rate [m^3/day]

s_f - Saturation of the water front [m^3/m^3]

s_j - Saturation of phase j [m^3/m^3]

s_{or} - Residual oil saturation [m^3/m^3]

s_{wi} - Initial water saturation [m^3/m^3]

t - Time [day]

u_d - Dimensionless velocity

u_j - Phase velocity [m/s]

u_t - Total flow velocity [m/s]

U_{inj} - Injection velocity [m/day]

U - Linear velocity [m^3/day]

V_j - Volume of a phase in a block [m^3]

x - Horizontal axis in sample reservoir [m]

Y_{sb} - Yield of bacteria on substrate [kg/kg]

Y_{sm} - Yield of surfactant/metabolite on substrate [kg/kg]

Greeks

μ_i - Viscosity for component i [Pa·s]

ρ_i - Density for component i [kg/m^3]

σ_{ow} - Interfacial tension between oil and water [mN/m]

τ - Dimensionless time [PVI]

ϕ - Porosity

λ_j - Phase mobility

Λ - Total mobility

Subscripts/superscripts

b - Bacteria

k - Index for position/block

inj - Index indicating injection

m - Metabolite/surfactant

o - Oil

s - Substrate

w - Water

0 - Index indicating the initial state

* - modified/calculated values

Abbreviations

CMC - Critical micelle concentration

EOR - Enhanced Oil Recovery

EPS - Extracellular polymeric substances

ESPB - Exogenous surfactant-producing bacteria

IFT - Interfacial Tension

Matlab - Matrix Laboratory

MEOR - Microbial enhanced oil recovery

NRB - Nitrate reducing bacteria

OOIP - Original oil in place

ODE - Ordinary Differential Equations

REV - Representative elementary volume

PDE - Partial Differential Equations

PV - Pore volume

SRB - Sulphate reducing bacteria

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