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Decarbonization strategies: a game theory perspective

Bouchra Mroué

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Decarbonization strategies: a game theory perspective

THÈSE

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par

Bouchra Mroué

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Mis en page avec la classe thesul.

To Baba and tante Zainab

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Abstract

This thesis delves into the study of climate change by the tools of game theory, investigating strategies for global emissions reduction. The journey begins with an informal introduction, providing insights into the modeling of climate change problems using game theory and optimization—the twin pillars of the thesis. Chapter 2 provides a formal treatment, presenting preliminaries which are instrumental for the subsequent chapters. Graph theory definitions and key concepts in game theory, pivotal to the thesis, are revisited in this foundational chapter.

Chapter 3 outlines the motivation for the research subject addressed in this thesis and summarizes the proposed contributions. It critically examines the limitations of the current emission reduction strategies, primarily driven by short-term economic considerations, blocking global efforts. Next, we present a thorough survey of the climate change models and the theoretical frameworks employed for their analysis. Climate models, inspired by IPCC projections, are examined to unravel the circumstances influencing nations' behavior in emissions reduction.

In Chapter 4, we formulate a static game model in the strategic form, allowing us to study the decision-making process and exhibit environmentally conscious behavior. To reach this goal, we conduct a complete analysis of the Nash equilibria of the game. We provide conditions under which the equilibrium is unique and numerically analyze the impact of different modeling choices. The Chapter ends with a brief discussion summarizing the takeaway messages of this part.

The study continues in Chapter 5, where the climate change problem is cast as an imitation game. In this setup, the players are minimizing a utility function including socio-economic benefits, global climate change damage, and an imitation term capturing the influence of the other players. The Nash equilibrium analysis unveils a variety of players' behaviors under climate change awareness and mutual influence. Key parameters, such as the

weight of imitation, are numerically analyzed, emphasizing the importance of coordination and cooperation in mitigating climate change.

This thesis explores the complex interplay between climate considerations and game theory, providing a nuanced understanding of global emissions reduction strategies. The provided insights contribute to the broad subject of finding sustainable solutions for addressing the urgent issue of global warming.

Résumé

Cette thèse étudie l'impact socio-économique du changement climatique à l'aide de la théorie des jeux et a pour but de déterminer des stratégies de réduction d'émission au niveau des gouvernements. La thèse commence par une introduction informelle, offrant des perspectives sur la modélisation des problèmes liés au changement climatique à l'aide de la théorie des jeux et de l'optimisation, les deux piliers méthodologiques de la thèse. Le Chapitre 2 propose un traitement formel, présentant des préliminaires essentiels pour les chapitres suivants. Les définitions de la théorie des graphes et les concepts clés de la théorie des jeux, cruciaux pour la thèse, sont présentés dans ce chapitre fondamental.

Le Chapitre 3 présente la motivation du sujet de recherche abordé dans cette thèse et résume les contributions proposées. Il examine de manière critique les limites des stratégies actuelles de réduction des émissions, principalement motivées par des considérations économiques à court terme, empêchant les efforts mondiaux. Ensuite, nous présentons une synthèse approfondie sur les modèles de changement climatique et les cadres théoriques utilisés pour leur analyse. Les modèles climatiques, inspirés des projections du GIEC, font l'objet d'une analyse afin de mettre en lumière les circonstances qui influencent le comportement des nations en matière de réduction des émissions.

Dans le Chapitre 4, nous formulons un jeu statique sous forme stratégique, nous permettant d'étudier le processus de prise de décision et de mettre en évidence un comportement écologiquement responsable. Pour atteindre cet objectif, nous effectuons une analyse complète des équilibres de Nash du jeu. Nous fournissons des conditions sous lesquelles l'équilibre est unique et analysons numériquement l'impact de différents choix de modélisation. Le chapitre se termine par une brève discussion résumant les messages clés de cette partie.

L'étude se poursuit dans le Chapitre 5, où le problème du changement climatique est présenté sous la forme d'un jeu d'imitation. Dans cette configuration, les joueurs minimisent une fonction d'utilité comprenant des avantages socio-économiques, des dommages liés au chan-

gement climatique globale et un terme d'imitation capturant l'influence des autres joueurs. L'analyse de l'équilibre de Nash révèle une variété de comportements des joueurs face à la prise de conscience du changement climatique et à l'influence mutuelle. Des paramètres clés, tels que le poids de l'imitation, sont analysés numériquement, soulignant l'importance de la coordination et de la coopération dans l'atténuation du changement climatique.

Cette thèse étudie l'interaction complexe entre les considérations climatiques et la théorie des jeux, offrant une compréhension nuancée des stratégies mondiales de réduction des émissions. Les perspectives fournies contribuent au domaine de la recherche de solutions durables pour faire face au réchauffement climatique.

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Acronyms

BAU	Business-As-Usual
AOC	All other countries
BR	Best response
NE	Nash equilibrium
FPT	Fixed-Point Theorem
EPG	Exact potential game
WPG	Weighted potential game
OPG	Ordinal potential game
GPG	Generalized potential game
BRPG	Best-response potential game
COP	Conference of parties
IPCC	Intergovernmental Panel on Climate Change
GCM	Global Climate Models
GHG	Greenhouse Gas
IAMs	Integrated Assessment Models
SCMs	Simple Climate Models

Acronyms

ESMs	Earth system models
ECS	Equilibrium Climate Sensitivity
GDP	Gross domestic product
CM	Climate model
CC	Carbon cycle
TD	Temperature dynamics

Introduction

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This chapter provides an informal introduction to the modeling of climate change problems in game theory and optimization, which are the two pillars of the thesis. A more formal treatment is provided in Chapter 2. Then, the motivation for the accomplished work and a summary of the proposed contributions are given, together with the associated scientific outcomes.

1.1 Background and Motivation

During the 2019 COVID-19 pandemic, a notable reduction in emissions, highlighting a growing awareness among people about their role in averting global warming. This period cast its influence over the entire Anthropocene, leaders worldwide were compelled to envision a transformed future marked by environmental sustainability, heightened social connectivity, and increased equity. Arising from official government statements, these commitments naturally emerged as a global response to heightened awareness of inequality and the diverse

challenges confronted during the crisis. This pivotal moment presented a unique chance for collective reflection, urging humanity to reassess the world and establish the groundwork for a new path in anticipation of the imminent crisis: climate change.

However, the current state reveals a stark divergence from the promised engagements, as the world appears to revert to familiar patterns, succumbing to the allure of the Business-As-Usual (BAU) trajectory. The carbon emissions of most of the countries have rebounded to pre-pandemic levels, underscoring the disconcerting reality that the transformative visions articulated in the wake of COVID-19 are being overshadowed by a return to established norms. This regression is particularly worrying in the context of a summer that announced that natural disasters are awaiting humanity if decisive action on the climate crisis is not undertaken promptly.

Recent catastrophic events bear testament to the urgency of addressing climate change, with devastating floods submerging a third of Pakistan, water shortages plaguing U.S. cities in the aftermath of floods, and record-breaking heatwaves scorching regions across the Middle East, Africa, Asia, and Europe. The historian Maximiliano Herrera, monitoring extreme temperatures globally, remarked: "There is nothing in world climate history even minimally comparable to what is happening in China," highlighting the unprecedented two months of extreme heat endured by the nation [10].

This paradoxical situation raises a fundamental query: "If governments possess knowledge of the imminent climate crisis and its potential impact, why persist in Business-As-Usual actions?" To address this enigma, we turn to the tools of game theory to gain some insights. This thesis serves as a prelude to a forthcoming thesis delving into the design of decarbonization strategies.

1.2 Problem Statement

Climate change stands as one of the most pressing challenges of our time, posing significant threats to ecosystems, economies, and societies worldwide. The intricate dynamics of the climate system, driven by complex interactions between atmospheric and oceanic components, make mitigating climate change a complex and multifaceted problem. While the nations understand the need to reduce greenhouse gas emissions, a critical challenge emerges in the form of coordinated decision-making. The benefits and costs of climate change mitigation efforts are not evenly distributed among countries, leading to a complex trade-off that involves short-term economic loss, ecological impact, and political considerations.

Motivated by the urgency of addressing climate change and the need for a comprehensive understanding of the decision-making processes involved, this thesis adopts a game-theoretic approach. Game theory provides a valuable framework for studying the interdependence of

decisions made by different countries in the context of climate change. Through a strategic lens, this research aims to unravel the complexities of global efforts to reduce greenhouse gas emissions and enhance our understanding of the decision-making strategies employed by nations facing this global challenge.

The central problem addressed in this thesis revolves around the strategic decision-making of countries in the context of climate change mitigation. The mitigation strategies adopted by each country are influenced by a delicate balance between economic considerations, ecological impact, and international political dynamics. To capture this complexity, we employ a game-theoretic model that integrates both the geophysical aspects of climate change and the strategic considerations of individual countries.

We give particular attention to the problem of understanding how the imitation behavior among countries, represented as a network, influences their decision-making strategies. The imitation term in the utility functions captures the alignment or deviation of emission behaviors with neighboring countries. This introduces a novel element of strategic interaction, where countries not only consider the direct impact of their decisions but also respond to the actions of their peers.

1.3 Objectives

Game theory, a mathematical framework widely employed in the analysis of decision-making processes, has found increasing relevance within the study of spreading phenomena, particularly in the context of climate change. This application extends beyond traditional scenarios, incorporating various stakeholders involved in climate change decision-making, including governments, environmental organizations, industries, and the public. Game theory emerges as a valuable tool for identifying strategic behaviors that lead to equilibrium and evaluating their impact on the overall climate outcomes.

Examining Nash equilibrium in the context of climate change games provides a scenario where no stakeholder has an incentive to unilaterally deviate from their chosen strategy. This equilibrium concept is crucial for understanding the stability of strategic decisions made by various entities involved in addressing climate change. Identifying Nash equilibrium allows decision-makers and researchers to comprehend the long-term consequences of chosen strategies and assess their impact on the overall climate landscape.

The games examined in this thesis follow a methodology commonly employed in game theory. The analysis begins with investigating whether a climate change game possesses an equilibrium. Subsequently, attention is given to determining the uniqueness of the equilibrium and its determination. These questions are essential for understanding the stability and predictability of strategic decisions made by stakeholders in the face of climate change

challenges.

Once equilibrium-related questions are addressed, numerical analysis becomes a crucial step in evaluating the effectiveness of the identified solutions. This involves employing computational methods to simulate and analyze the consequences of different strategies adopted by stakeholders. The numerical analysis provides insights into the practical implications of equilibrium strategies, enabling decision-makers to make informed choices in climate change mitigation and adaptation efforts.

The methodology employed in this thesis serves as a powerful tool for comprehending the strategic dynamics of climate change games. It enables the identification of optimal strategies for various stakeholders, contributing to the fields of environmental science, policy-making, and climate economics. This approach is extensively used in disciplines including economics, political science, and business, allowing the analysis of a wide range of climate change scenarios. From simple two-player interactions to complex multi-player games involving nations and international cooperation, game theory offers a versatile framework for understanding and addressing the challenges posed by climate change.

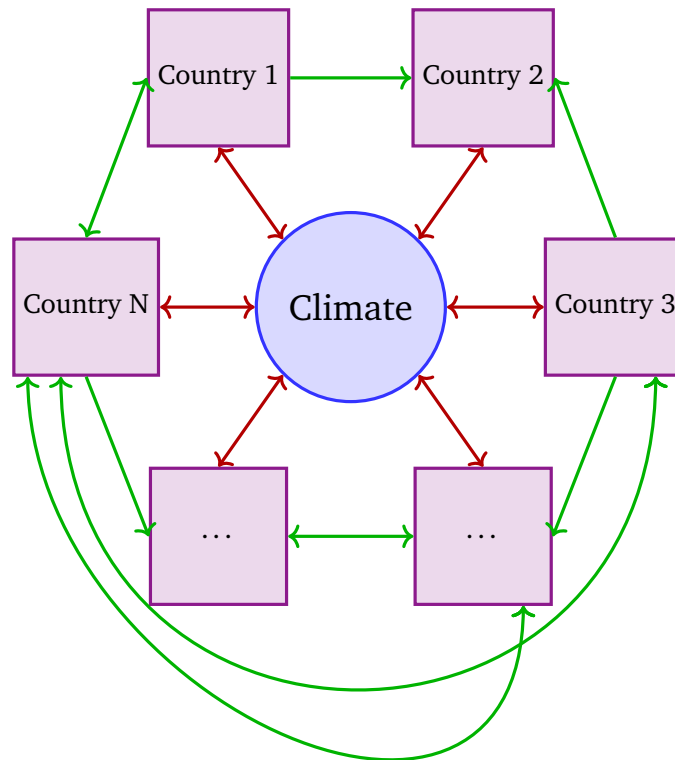


FIGURE 1.1 – Climate network and influence on the players.

In this thesis, we study the influence of the climate on the players, and how it affects

their utility and their decision regarding their emissions. In addition, we use climate models that model the influence of the players' emissions on the carbon concentrations, and then on the temperature indeed on the climate. We consider a complex network where there exist connections between countries and between countries and climate. Indeed, countries can interact and influence each other, as seen by the influences illustrated by the green arrows in Figure 1.1. Additionally, each country affects the climate with its greenhouse gas emissions, as represented by the red arrows going from the countries. Also, the climate has an effect on countries, which is economically represented by losses or costs incurred due to rising atmospheric temperatures, illustrated by the red arrows going from the climate towards the countries. To formulate the game between countries, which models the climate problem, we use dynamic systems of CO_2 concentrations and temperature.

1.4 Thesis Structure

This thesis is structured to provide a comprehensive investigation of the climate change mitigation problem based on game theoretic results. The subsequent chapters unfold as follows:

1. Chapter 2: Preliminaries

In this chapter, we present some preliminaries that are helpful to understand the rest of the thesis. In particular, we recall some useful game theory definitions and theorems.

2. Chapter 3: Problem Formulation

We present the theoretical framework, including the game-theoretic model and the climate dynamics considered. The integration of geophysical aspects and strategic interactions is discussed in detail.

3. Chapter 4: Static Game

Building on the theoretical framework, this chapter delves into a comprehensive analysis of the static game within the context of climate change. Our focus lies on establishing rigorous conditions for the existence and uniqueness of a pure Nash equilibrium, shedding light on the stability and strategic dynamics of interconnected countries. By thoroughly examining the strategic decisions influenced by individual benefits, weighted global damage, and imitation behaviors, we provide valuable insights into the intricacies of emission strategies. This exploration not only contributes to the foundational aspects of theoretical game theory but also extends its practical implications for climate change decision-makers. Moreover, the chapter includes a detailed numerical assessment, offering a quantitative dimension to our theoretical findings and enhancing the understanding of the numerical implications of the proposed model.

4. Chapter 5: Imitation Game

This study focuses on the static imitation game within the context of climate change, where we rigorously establish conditions for the existence and uniqueness of a pure Nash equilibrium. Investigating the strategic decisions of interconnected countries, we analyze the influence of individual benefits, weighted global damage, and imitation on emission strategies. Numerical simulations provide visual insights into the dynamics of the proposed model, validating theoretical findings and offering tangible representations of strategic behaviors. This exploration contributes to both theoretical game theory foundations and practical insights for climate change decision-makers.

5. Chapter 6: Conclusion and Future Work

The final chapter summarizes the findings, draws conclusions, and suggests avenues for future research. We reflect on the implications of our study and its potential contributions to the field.

1.5 Contributions

The key contributions of this thesis are as follows:

1. Introduction of novel static game models that can incorporate imitation behavior among countries in their decision-making strategies.
2. Development of a theoretical framework that integrates geophysical aspects of climate change with strategic interactions, providing a unique perspective on the global challenge.
3. Establishment of the existence of at least one pure Nash equilibrium in the proposed imitation game, with proof of uniqueness under certain conditions.
4. Derivation of the expression for the unique pure Nash equilibrium, particularly in the case of quadratic utility functions.
5. Numerical assessment of the impact of imitation functions on connected players, offering insights into the practical implications of the proposed model.

The subsequent chapters delve into each of these contributions, providing a detailed exploration of the theoretical foundations, analytical results, and practical implications.

1.6 Publications

In this section, we provide a list summarizing the publications that arose from this thesis, detailed by categories.

1.6.1 Journal papers

1. A. Couthures, B. Mroué, S. Lasaulce, I.C. Morărescu: *A Planning Game for Global Decarbonization Strategies*, In preparation
2. B. Mroué, A. Couthures, S. Lasaulce, I.C. Morărescu: *On the impact of cross-country imitation on climate change: A game-theoretical analysis*, Submission phase

1.6.2 Peer Conference papers

1. B. Mroué, A. Couthures, S. Lasaulce, I.C. Morărescu: *The power of influence: a game theory approach for climate change*, ENOC, Delft, Netherlands, 2024. Accepted
2. B. Mroué, A. Couthures, S. Lasaulce, I.C. Morărescu: *A quadratic static game model for assessing the impact of climate change*, Complex Networks Conference, Menton, France, 2023. [11]
3. B. Mroué, A. Couthures, S. Lasaulce, I.C. Morărescu: *Une approche par la théorie des jeux du problème du choix des niveaux nationaux d'émissions de CO₂*, GRETSI, Grenoble, France, Août 2023. [12]

1.6.3 Workshops

1. A. Couthures, B. Mroué, S. Lasaulce, I.C. Morărescu: *A Planning Game for Global Decarbonization Strategies*, PGMO days, Paris, France, 2023.
2. A. Couthures, B. Mroué, S. Lasaulce, I.C. Morărescu: *A static game model for analyzing country carbon emissions*, PGMO days, Paris, France, 2022.

Preliminaries

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In this chapter, we present preliminaries, which are useful for the rest of the thesis. In Section 2.1, we recall some definitions and concepts considered and studied in this thesis.

2.1 Game Theory

Game theory, a branch of mathematics, has proven to be a robust tool for modeling strategic interactions among agents across diverse fields. The seminal work by John von Neumann and Oskar Morgenstern in 1944 [13] laid the foundation, and subsequent contributions by John Nash in the 1950s, especially his concept of Nash equilibrium [7], significantly advanced the field.

In essence, game theory provides a framework for analyzing systems involving interacting agents, each striving to maximize their individual utility. This utility is influenced not only by their own actions but also by the actions of other agents within the system. This complexity distinguishes game theory from classical optimization frameworks, where an agent's utility is independent of others.

Despite methodological differences, game theory maintains a close relationship with classical optimization theory. For example, potential games [2] emphasize the maximum achievable utility for agents through a function intrinsically linked to the game, known as the potential function.

Applications of game theory span various fields, including economics, political science, biology, computer science, and climate [14, 15]. The overarching goal of game theory is to construct models that capture the intricacies of player interactions, establish solution concepts, predict potential outcomes based on specific information and behavioral assumptions, analyze feasible outcomes and design effective strategies to achieve specific goals. Now, let us proceed to delve into the fundamental concepts of game theory.

2.1.1 Some Useful Definitions and Theorems

The foundation of the thesis relies on a key concept: the one-shot game. Understanding the definitions is essential for comprehending the subsequent analysis and findings.

In game theory, a one-shot game involves simultaneous decision-making without knowledge of others' choices. Players independently select strategies, and outcomes are based on the combined strategies. No revisions are allowed after decisions are made.

2.1.1.1 Normal form

Definition 2.1. A game represented in normal form can be described by an ordered vector denoted by $\Gamma := (\mathcal{N}, (\mathcal{A}_n)_{n \in \mathcal{N}}, (u_n)_{n \in \mathcal{N}})$. In this representation:

- $\mathcal{N} := \{1, \dots, N\}$ refers to the finite set of players involved in the game with $N > 0$ players.
- $\mathcal{A} := \mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_N$ denotes the Cartesian product of the strategy sets, where \mathcal{A}_n is the strategy set of player $n \in \mathcal{N}$.
- $(u_n)_{n \in \mathcal{N}} := (u_1, \dots, u_N)$ the utility profile and u_n represents the utility function (in the case of maximization convention) or cost function (in the case of minimization convention) for each player $n \in \mathcal{N}$, where $u_n : \mathcal{A} \rightarrow \mathbb{R}$.

Within this framework, each player $n \in \mathcal{N}$ selects an action a_n from their respective strategy set, \mathcal{A}_n , and receives a utility, $u_n(a_1, \dots, a_N)$, contingent upon the collective action choices made by all players involved in the game. We denote the control action profile as

$a := (a_1, \dots, a_N) \in \mathcal{A}$, and use the notation a_{-n} to refer to the reduced action profile $a_{-n} := (a_1, \dots, a_{n-1}; a_{n+1}, \dots, a_N) \in \mathcal{A}_{-n}$ where $\mathcal{A}_{-n} := \mathcal{A}_1 \times \dots \times \mathcal{A}_{n-1} \times \mathcal{A}_{n+1} \times \dots \times \mathcal{A}_N$.

The concept of best response shifts the focus to the individual level, where players strive to maximize their utilities (or minimize their own cost functions) based on the strategies chosen by others.

2.1.1.2 Best response

Definition 2.2. The best response (BR) of player $n \in \mathcal{N}$ to the reduced strategy profile $a_{-n} \in \mathcal{A}_{-n}$ is

$$BR_n(a_{-n}) := \arg \max_{a_n \in \mathcal{A}_n} u_n(a_n, a_{-n}), \text{ (or } \arg \min \text{ for minimization)}. \quad (2.1)$$

The best response map of a player forms the foundation for understanding Nash equilibrium in game theory. A best response is an optimal strategy for a player, given the strategies of the other players. It represents a choice that maximizes (or minimizes) a player's utility or payoff, taking into account the actions of others. Nash equilibrium, on the other hand, extends the notion of best response to a solution concept where each player's strategy is a best response to the strategies chosen by others. In a Nash equilibrium, no player has an incentive to unilaterally deviate from their chosen strategy, as doing so would result in a lower payoff.

2.1.1.3 Nash equilibrium

Definition 2.3. The action profile a^{NE} is a Nash equilibrium (NE) strategy of the game Γ if: $\forall n \in \mathcal{N}$ and $a_n \in \mathcal{A}_n$,

$$u_n(a^{NE}) \geq u_n(a_n, a_{-n}^{NE}), \text{ (} \leq \text{ for minimization)}. \quad (2.2)$$

A Nash equilibrium can be characterized by the concept of best response.

Definition 2.4. A strategy profile a^{NE} is a Nash equilibrium if

$$a_n^{NE} \in BR_n(a_{-n}^{NE}), \quad \forall n \in \mathcal{N}, \quad (2.3)$$

where

$$BR_n : \mathcal{A}_{-n} \rightarrow \mathcal{A}_n$$

$$a_{-n} \rightarrow BR_n(a_{-n}) := \{a_n \in \mathcal{A}_n : u_n(a_n, a_{-n}) \geq u_n(\tilde{a}_n, a_{-n}), \forall \tilde{a}_n \in \mathcal{A}_n\}.$$

The characterization of Nash equilibria as solutions to a fixed-point problem was formalized by Nash in [7]. This perspective reveals why standard existence theorems for Nash

equilibria rely on topological and geometrical assumptions. The Debreu-Fan-Glicksberg theorem, as described in [16], provides a valuable existence theorem for Nash equilibria. This theorem builds upon the contributions of [17]. The authors in [16] further elaborate on the specific requirements and conditions that need to be satisfied for the application of the Debreu-Fan-Glicksberg theorem. To refresh our understanding of the terminology, let's recall the definition of quasi-concave and quasi-convex functions before discussing the Debreu-Fan-Glicksberg theorem, and different theorems that insures the existence and uniqueness of the Nash equilibrium.

Definition 2.5. *Let a function $f : \mathcal{X} \rightarrow \mathbb{R}$, where \mathcal{X} is a convex set. The function f is said to be:*

- *Quasi-concave function if, for any points $x, y \in \mathcal{X}$ and $\lambda \in [0, 1]$, the following condition holds:*

$$f(\lambda x + (1 - \lambda)y) \geq \min\{f(x), f(y)\}. \quad (2.4)$$

- *Quasi-convex function if, for any points $x, y \in \mathcal{X}$ and $\lambda \in [0, 1]$, the following condition holds:*

$$f(\lambda x + (1 - \lambda)y) \leq \max\{f(x), f(y)\}. \quad (2.5)$$

Building upon this notion, we provide ways to prove the existence of at least one pure Nash equilibrium in the game. For more details, see [18].

2.1.2 Existence of Equilibrium

In mathematics, one way to demonstrate the existence of a Nash equilibrium is by proving a solution to a fixed-point problem, as originally proposed by Nash in [7]. Game theorists invest significant effort in formulating fixed-point theorems, as symbolized by the contributions of Border [19] and Smart [20]. Therefore, it is unsurprising that the theorems establishing equilibrium existence are deeply rooted in the topological properties of players' strategy sets, as well as the topological and geometrical attributes of their utility functions. The impact of fixed-point theorems (FPT), particularly those introduced by Lefschetz [21], Hopf [22], Brouwer [23], and Kakutani [24], on game theory is indeed significant.

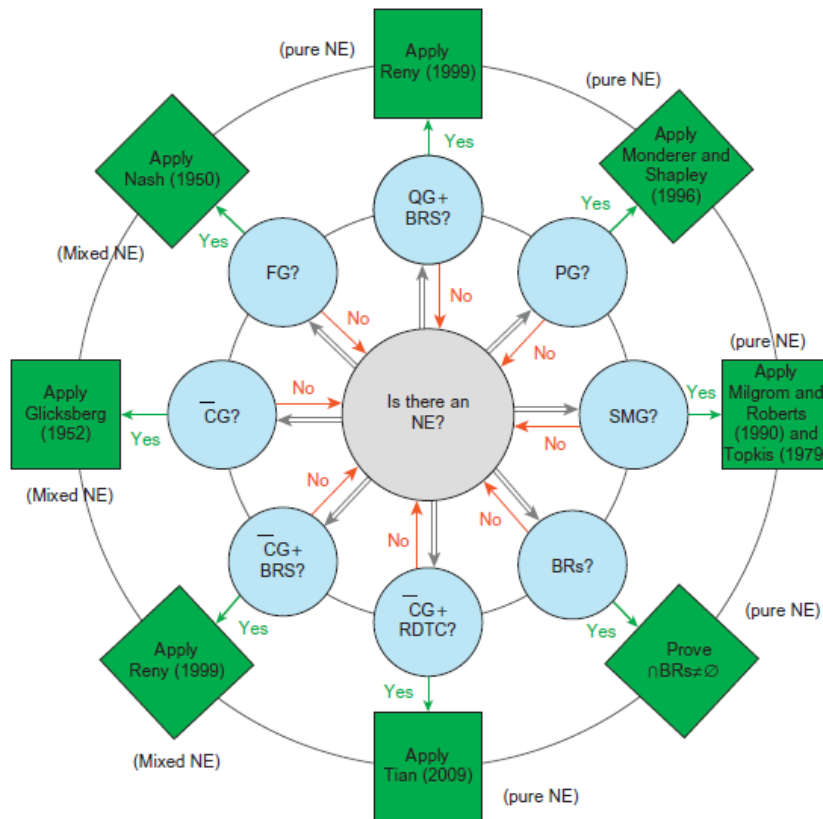


FIGURE 2.1 – A (non-exhaustive) methodology for proving the existence of a Nash equilibrium in strategic form games. The meaning of the used acronyms is as follows: QG (quasi-concave games), BRS (better-reply secure; Reny, [1]), PG (potential games; Monderer and Shapley, [2]), SMG (S-modular games; Milgrom and Roberts, [3]; Topkis, [4]), BRs? (Are the best-responses known?), CG (compact games; Glicksberg, [5]), RDTTC (recursive diagonally transferable continuity; Tian, [6]), FG (finite games; Nash, [7]).

To confirm the presence of equilibrium in a specific problem, the typical approach involves seeking a Fixed-Point Theorem (FPT), which serves as a comprehensive method. Nevertheless, numerous existing theorems prove adequate for various scenarios, particularly in environmental games. For example, established theorems concerning equilibrium existence find practical application as demonstrated by Finus in [25], with benefit function or damage function characterized by the desired convexity properties. This section elaborates on several such existence theorems and illustrates their utilization in the literature, adhering to the structure outlined in Figure 2.1, in this figure, we represent the cases where there exists at least one NE by the green arrow with "Yes".

2.1.2.1 Better-Reply Secure and Quasi-Concave Games

Prior to introducing Reny's theorem [1], it is essential to establish the definitions of better-reply security and quasi-concavity. The symbols $a = (a_1, \dots, a_N)$ and $u = (u_1, \dots, u_N)$ are employed to represent strategy and utility profiles, respectively. We highlight that a pair (a, u) is in the closure of the graph of the utility profile function if u is the limit of the vector of utilities corresponding to some sequence of strategies converging to a .

Definition 2.6. Better-Reply Secure Games

A strategic game $\Gamma = (\mathcal{N}, \{\mathcal{A}_n\}_{n \in \mathcal{N}}, \{u_n\}_{n \in \mathcal{N}})$ is better-reply secure (BRS) if, for any strategy profile a that is not a Nash equilibrium, and the corresponding utility profile u , the following holds: If (a, u) is in the closure of the graph of the game's utility profile function, then for some player n , there exists a'_n such that for all a'_{-n} close to a_{-n} , $u_i(a_n, a_{-n}) > u_n(a'_n, a'_{-n})$.

So, a game is better-reply secure if for every nonequilibrium strategy a and every utility vector u resulting from strategies approaching a , some player $n \in \mathcal{N}$, has a strategy yielding a utility strictly above u_n even if the others deviate slightly from a .

Games with continuous utility functions are better-reply secure since any better reply will secure a utility strictly above a player's inferior nonequilibrium utility and those generated by nearby strategies. Many discontinuous economic games are better-reply secure. A classical illustration is found in Bertrand competition [26].

Now, we proceed to define quasi-concavity.

Definition 2.7. Quasi-Concave Functions

A function ψ is quasi-concave on a convex set \mathcal{S} if, for all $\alpha \in \mathbb{R}$, the upper contour set $U_\alpha = \{x \in \mathcal{S}, f(x) \geq \alpha\}$ is convex. Games in which utilities are quasi-concave are denoted by QG.

The Reny existence theorem for pure Nash equilibria [1] can now be stated.

Theorem 2.1. Reny (1999)

Let Γ be a strategic form game. If the following conditions hold: $\forall n \in \mathcal{N}$, \mathcal{A}_n is a compact and convex set; u_n is quasi-concave in a_n ; and Γ is better-reply secure, then the game Γ has at least one pure Nash equilibrium.

While this theorem has been applied in the game theory literature, particularly in auction scenarios, its application in games studying a climate problem appears to be limited. The subsequent section explores a less general version, assuming continuous utility functions.

2.1.2.2 Continuous Quasi-Concave Games

A highly valuable existence theorem, known as the Debreu-Fan-Glicksberg theorem, is presented in Fudenberg and Tirole [16], stemming from the contributions of Debreu [17],

Fan [27], and Glicksberg [5]. This theorem is applicable when users' utilities are both continuous and quasi-concave, provided that certain additional conditions are met.

Theorem 2.2. Debreu (1952) [16]

Let Γ be a strategic form game. If the following conditions are met: $\forall n \in \mathcal{N}$, \mathcal{A}_n is a compact and convex set; and u_n is a continuous function in the action profile a and quasi-concave in a_n , then the game Γ has at least one pure Nash equilibrium.

A particular instance of this theorem emerges when assuming the utility functions to be concave. Rosen's [8] Theorem 1, applicable to concave N -person games can be viewed as a corollary of Theorem (2.2).

Of course, not all games need to exhibit quasi-concave utilities. Even if a game is non-quasi-concave, it may still possess properties ensuring the existence of a pure Nash equilibrium. This is the case for potential games [2] or S -modular games [3, 4], which encompass sub-modular games [28] and super-modular games [3, 4].

The following discussion introduces the definition, an existence theorem, and an example of these game types. The notation $a \geq b$ indicates that each entry of the vector a is greater than or equal to the corresponding entry in the vector b .

2.1.3 Potential Games

Potential games, initially proposed by Monderer and Shapley [2], include four main types: weighted potential games, exact potential games, ordinal potential games, and generalized potential games. Subsequently, additional variations of potential games have been introduced. For example, Voorneveld introduced best-response potential games in [29]. Distinguishing between these types in relation to the existence of a Nash equilibrium is not significant. The inclusion of various types is driven by two factors: the simplicity of proving potentiality for a specific type based on the characteristics of the game under consideration and the distinct properties exhibited by each type, particularly concerning convergence.

Definition 2.8. Exact Potential Games (EPG)

The game Γ is an exact potential game if there exists a function ϕ such that: $\forall n \in \mathcal{N}, \forall a = (a_n, a_{-n}) \in \mathcal{A}, \forall a'_n \in \mathcal{A}_n$,

$$u_n(a) - u_n(a'_n, a_{-n}) = \phi(a) - \phi(a'_n, a_{-n}). \quad (2.6)$$

As in physics, there can be an infinite number of potential functions; what matters is whether one can be found. Alternatively, one can test one of the following conditions:

- the weighted potential game (WPG) condition: there exists a function ϕ such that, $\forall n \in \mathcal{N}, \forall a = (a_n, a_{-n}) \in \mathcal{A}, \forall a'_n \in \mathcal{A}_n$,

$$u_n(a) - u_n(a'_n, a_{-n}) = w_n [\phi(a) - \phi(a'_n, a_{-n})] \quad (2.7)$$

for some vector of positive numbers $w = (w_1, w_2, \dots, w_N)^\top$;

- ordinal potential game (OPG) condition: there exists a function ϕ such that, $\forall n \in \mathcal{N}, \forall a = (a_n, a_{-n}) \in \mathcal{A}, \forall a'_n \in \mathcal{A}_n$,

$$u_n(a) - u_n(a'_n, a_{-n}) > 0 \iff \phi(a) - \phi(a'_n, a_{-n}) > 0; \quad (2.8)$$

- generalized potential game (GPG) condition: in the ordinal potential game condition, replace the equivalence with a right implication;
- best-response potential game (BRPG) condition: there exists a function ϕ such that,

$$\arg \max_{a_n \in \mathcal{A}_n} u_n(a_n, a_{-n}) = \arg \max_{a_n \in \mathcal{A}_n} \phi(a_n, a_{-n}). \quad (2.9)$$

Denoting Γ^{XPG} as the set of strategic potential games with $X \in E, W, O, G, BR$, we establish the relations $\Gamma^{\text{EPG}} \subseteq \Gamma^{\text{OPG}} \subseteq \Gamma^{\text{GPG}}$ and $\Gamma^{\text{EPG}} \subseteq \Gamma^{\text{OPG}} \subseteq \Gamma^{\text{BRPG}}$ [2, 29]. It is crucial to emphasize that the potential function remains independent of the user index. In exact potential games, ϕ quantifies the impact of a unilateral deviation on all users' utilities, while in ordinal potential games, it indicates the sign of the difference in utilities. This parallels the definition of a potential in physics. For instance, in a field E with a potential function ϕ , we have $E = -\text{grad}(\phi)$. This implies that a N -dimensional problem (with $N = \dim E$) can be simplified to a one-dimensional problem using ϕ . In certain games with a continuum of players, such as those discussed in [30], the exact potential game precisely takes the form $u = -\text{grad}(\phi)$. Finally, when the type of potential game is unspecified, exact potential games are generally considered.

The following existence theorem is applicable for EPG/WPG/OPG/GPG/BRPG, [2, 29].

Theorem 2.3. Monderer–Shapley–Voorneveld

If Γ is a potential game with a finite number of players and:

- either non-empty compact strategy sets and continuous utilities
- or finite non-empty strategy sets,

then it has at least one pure NE.

When a potential function can be found, the theorem 2.3 is very important, and simplifies the study of the game. In instances where finding such a function poses challenges, alternative results can be employed to confirm the potentiality of the game. As highlighted in Monderer [2], a simple scenario where the verification of potentiality is straightforward occurs when the strategy sets are intervals of \mathbb{R} . In such cases, the following theorem applies:

Theorem 2.4. Monderer (1996) *Let Γ be a game in which the strategy sets are intervals of real numbers. Assume the utilities are continuously differentiable twice. Then Γ is an exact potential game if and only if:*

$$\forall (n, m) \in N^2, \frac{\partial^2 (u_n - u_m)}{\partial a_n \partial a_m} = 0. \quad (2.10)$$

2.1.4 Uniqueness of Equilibrium

Once the existence of an equilibrium is established, a natural inquiry arises regarding its uniqueness. The desire for uniqueness depends on the context; in scenarios where predicting system performance or dynamic process outcomes (e.g., games involving learning automata) is crucial, uniqueness is often preferred. Conversely, in situations like repeated games with a finite number of stages, it's known (refer to Benoît and Krishna [31,32]) that Nash equilibria of the repeated game cannot be subgame perfect if the constituent (static) game has only one Nash equilibrium. Indeed, a strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game. Informally, this means that at any point in the game, the players' behavior from that point onward should represent a Nash equilibrium of the continuation game (*i.e.* of the subgame), no matter what happened before. Unfortunately, there are limited general studies on equilibrium uniqueness. In this section, for clarity, we distinguish between situations where the best responses of players can be explicitly expressed and those where they cannot. The methodology for proving uniqueness is summarized in the chart in Figure 2.2.

2.1.4.1 Best Responses Do Not Need to be Explicit

Unfortunately, Debreu-Fan-Glicksberg theorem does not have a general uniqueness theorem for quasi-concave N-player games. However, for proving the uniqueness of a pure Nash equilibrium when players' utilities are concave, there exists a powerful tool: the uniqueness theorem derived by Rosen [8]. This theorem states that if a certain condition, known as diagonally strict concavity (DSC), is met, uniqueness is guaranteed. The theorem is as follows:

Theorem 2.5. Rosen (1965)

Assume that: $\forall n \in \mathcal{N} : \mathcal{A}_i$ is a non-empty, compact, and convex set; $u_n(a)$ is a continuous function in $a \in \mathcal{A}$ and concave in a_n . Let $r = (r_1, \dots, r_N)^\top$ be an arbitrary vector of fixed

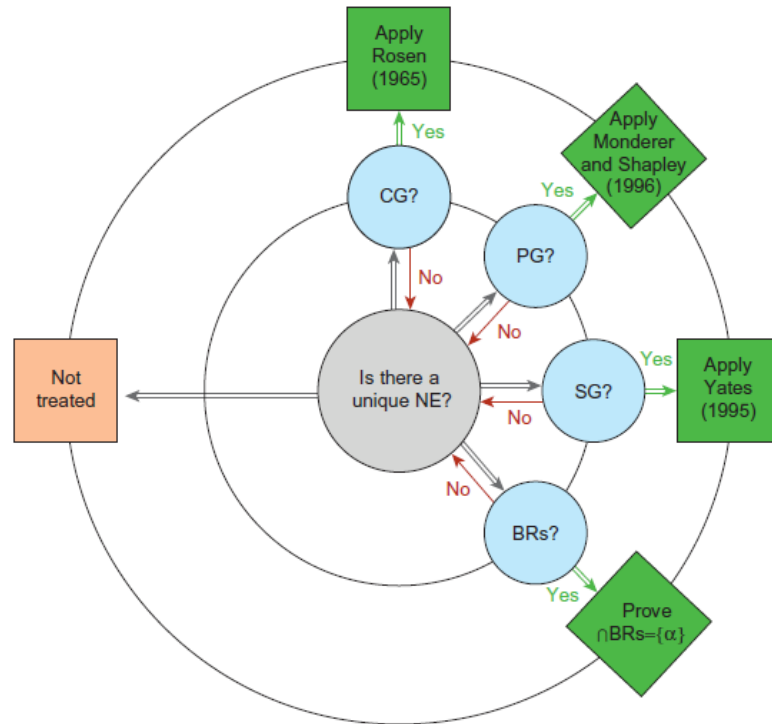


FIGURE 2.2 – A (non-exhaustive) methodology for proving the uniqueness of a Nash equilibrium in strategic form games. The meaning of the used acronyms is as follows: CG? (Is the game concave?; Rosen, [8]), PG (potential games; Monderer and Shapley, [2]), SG (games with standard best responses; Yates, [9]), BRs? (Are the best-responses known?).

positive parameters. Define the pseudogradient of the function $w_r = r^\top \times u$ by $\gamma_{w_r}(a) = \left[r_1 \frac{\partial u_1}{\partial a_1}(a), \dots, r_N \frac{\partial u_N}{\partial a_N}(a) \right]^\top$. If the following condition holds for some $r > 0$:

$$\forall (a, a') \in \mathcal{A}^2, a \neq a' : (a - a') (\gamma_{w_r}(a') - \gamma_{w_r}(a))^\top > 0, \quad (2.11)$$

then the game Γ has a unique NE.

The condition (2.11) is called the diagonally strict concavity (DSC) condition.

To conclude this section, we will mention a trivial but useful theorem for potential games, which is a direct consequence of Monderer [2] (e.g., see [33]).

Theorem 2.6. Uniqueness in Potential Games

If the potential function is strictly concave and the strategy spaces are non-empty, compact, and convex, then there is a unique pure Nash equilibrium.

2.1.4.2 Explicit Best Responses

If the best responses (BRs) of every player can be explicitly expressed, it is possible to analyze their properties, and for some classes of functions (or correspondences) to characterize the number of intersection points between them. A noteworthy class of BRs is the class of standard BRs. Standard functions have been introduced by Yates [9]; they are defined as follows:

Definition 2.9. Standard Functions

A vector function $g : \mathbb{R}_+^N \rightarrow \mathbb{R}_+^N$ is said to be standard if it has the two following properties:

1. Monotonicity:

$$\forall (x, x') \in \mathbb{R}_+^{2N}, x \leq x' \implies g(x) \leq g(x'); \quad (2.12)$$

2. Scalability:

$$\forall \alpha > 1, \forall x \in \mathbb{R}_+^N, g(\alpha x) < \alpha g(x). \quad (2.13)$$

In Yates [9], it is shown that if a function is standard, then it has a unique fixed point. Applying this result to the best responses of a game gives the following:

Theorem 2.7. Yates (1995)

If the best responses of the strategic-form game Γ are standard, then the game has a unique Nash Equilibrium (NE).

As mentioned at the end of Subsection 2.1.2.2, if the BRs are available and reasonably simple to exploit, it might be possible to find their intersection points. The number of intersection points corresponds to the number of possible equilibria. A well-known game that exemplifies this kind of approach is the Cournot duopoly [34], for which the BRs of the two players are affine and intersect at a single point. A counterpart of the Cournot duopoly is treated, for instance, in Belmega et al. [35].

In the study of game theory, "dominated action" is a key idea that helps us understand how players make decisions within a game. It refers to actions or plans that consistently yield better results than others, regardless of the choices made by other players. This concept underlies strategic decision-making, encouraging players to avoid dominated actions and instead choose better alternatives.

Definition 2.10. For any player $n \in \mathcal{N}$ and two distinct actions a_n and a'_n belonging to the action set \mathcal{A}_n for player n .

- Action a_n is said to weakly dominate action a'_n if the following conditions hold:

1. For any $a_{-n} \in \mathcal{A}_{-n}$,

$$u_n(a_n, a_{-n}) \geq u_n(a'_n, a_{-n}) \quad (\text{in the case of maximization convention}) \quad (2.14a)$$

$$u_n(a_n, a_{-n}) \leq u_n(a'_n, a_{-n}) \quad (\text{in the case of minimization convention}). \quad (2.14b)$$

2. There exists at least one strategy profile $a_{-n} \in \mathcal{A}_{-n}$ for which the inequality is strict,

$$u_n(a_n, a_{-n}) > u_n(a'_n, a_{-n}) \quad (\text{in the case of maximization convention}) \quad (2.15a)$$

$$u_n(a_n, a_{-n}) < u_n(a'_n, a_{-n}) \quad (\text{in the case of minimization convention}). \quad (2.15b)$$

- Action a_n is said to strictly dominate action a'_n if the inequality is always strict for any $a_{-n} \in \mathcal{A}_{-n}$.

The concept of dominance in game theory extends beyond individual strategies to encompass the comparison of entire action profiles. While dominance focuses on the superiority of one strategy over another for an individual player, Pareto dominance broadens the scope to evaluate the overall effectiveness and efficiency of action profiles in a multi-player setting.

Definition 2.11. Action profile u^p is said to Pareto dominate action profile u if and only if the following conditions hold simultaneously:

1. For every player $n \in \mathcal{N}$, a_n^p weakly dominates a_n .
2. There exists at least one player $n \in \mathcal{N}$, such that a_n^p strictly dominates a_n .

Pareto dominance (or Pareto optimality) captures the notion of improvement for at least one player without worsening the situation for any other player. It represents a powerful criterion for assessing the desirability of action profiles, as it identifies outcomes that are collectively preferable and non-inferior to alternative outcomes. In the literature of non-cooperative games, the term "social welfare" (SW) introduced by Arrow in [36] allows for a comprehensive evaluation of system performance, considering the contributions of each entity in decentralized decision-making environments.

Definition 2.12. The social welfare of a game is defined as the sum of the utilities (or cost function) of all players:

$$SW = \sum_{n=1}^N u_n. \quad (2.16)$$

A social optimum represents the action that corresponds to the highest level of social welfare (or minimal level, depending on the convention). It is Pareto optimal as it ensures

that no alternative strategy can improve the welfare of any individual without worsening the welfare of others [37]. However, it should be noted that the converse is not always guaranteed and holds only under certain conditions. Specifically, if the feasible range of utilities forms a convex region, then the converse holds true [38]. In practical scenarios, social welfare can serve as a useful measure when the players experience similar conditions, leading to comparable utilities after averaging. However, if the players encounter significantly different conditions, the use of social welfare as a performance measure can be debatable and may even result in unfair outcomes, as a player's utility might not be maximized while the sum of utilities is maximized.

2.1.5 Comments on the concept of Nash Bargaining solution

Nash equilibria can not only be interpreted as possible outcomes of interactive situations involving rational players but also as a result of some evolutionary process. A second groundbreaking contribution by Nash is the Nash bargaining solution. Different bargaining solutions exist, but only the one by Nash is discussed here. One important idea of bargaining theory, initiated by the Nash bargaining solution for two-player games, is that solution concepts of cooperative games (which are generally based on an abstract approach to the problem) can be re-interpreted as non-cooperative games (involving much more detail and based on the idea of individual utility maximization from the knowledge/belief of the players). The concern of the corresponding literature is to construct non-cooperative bargaining games that sustain various cooperative solution concepts as their equilibrium outcomes. Mainly based on the idea of having a solution that is fair and more efficient than the one obtained without bargaining or agreement, the Nash bargaining solution is characterized by a set of axioms regarding the desired outcome:

1. Pareto optimality (PO);
2. Individual rationality (IR);
3. Invariance to positive affine transformations (IPAT);
4. Independence of irrelevant alternatives (IIA);
5. Symmetry (S).

To define these axioms, some definitions are in order.

Definition 2.13. Bargaining Problem

A bargaining problem is a pair (F, v) where $F \subseteq \mathbb{R}^2$ is a closed and convex set, $v = (v_1, v_2) \in \mathbb{R}^2$, and such that the set $F \cap \{(u_1, u_2) \in \mathbb{R}^2 : u_1 \geq v_1, u_2 \geq v_2\}$ is bounded and non-empty.

Following the terminology used by Nash, F represents the set of deals available to the bargainers, while v is a point representing the status quo.

Definition 2.14. Bargaining Solution

Let \mathcal{B} be the set of all bargaining problems. A bargaining solution is an application which assigns to each bargaining problem a unique solution that is:

$$\mu : \begin{cases} \mathcal{B} & \longrightarrow \mathbb{R}^2 \\ (F, v) & \longmapsto \mu(F, v) = (\mu_1, \mu_2). \end{cases} \quad (2.17)$$

Definition 2.15. Symmetric Bargaining Problem

A bargaining problem (F, v) is symmetric if the following conditions are satisfied:

- $v_1 = v_2$;
- $(u_1, u_2) \in F \Rightarrow (u_2, u_1) \in F$.

From these definitions, the above axioms can be expressed mathematically as follows:

1. **PO:** $\forall u \in F, u > \mu(F, v) \Rightarrow u = \mu(F, v)$;
2. **IR:** $\mu(F, v) \geq v$;
3. **IPAT:** Let the pair (F', v') be defined as the result of an affine transformation applied to $(F, v) : F' = \{(u'_1, u'_2) \in \mathbb{R}^2 : u'_i = \alpha'_i u_i + \beta_i, (u_1, u_2) \in F, \alpha_i > 0, \beta_i \in \mathbb{R}\}$. Then, $\mu(F', v') = (\alpha_1 u_1 + \beta_1, \alpha_2 u_2 + \beta_2)$;
4. **IIA:** Let $F' \subseteq F$. Then, $\mu(F, v) \in F' \Rightarrow \mu(F', v) = \mu(F, v)$;
5. **S:** If (F, v) is symmetric, then $\mu_1 = \mu_2$.

The following theorem exploits these axioms.

Theorem 2.8. Nash Bargaining Solution

There exists a unique bargaining solution verifying axioms 1-5. This solution is the unique Pareto-optimal utility profile, satisfying:

$$\mu(F, v) \in \arg \max_{\{u \in F, u \geq v\}} (u_1 - v_1)(u_2 - v_2). \quad (2.18)$$

This solution concept can be extended to K -players for $K \geq 2$, but it does not take into account the fact that some coalitions of players can form and deviate from the solution.

Problem Formulation

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Abstract - *The necessity to confront the complex issue of global warming is outlined in the Conference of Parties (COP), which seeks to form agreements aimed at mitigating emissions. However, the efficacy of current strategies is blocked by their vulnerability to limitations, often arising from predominant short-term economic considerations that weaken collective efforts for emission reductions. In this chapter, we go through the literature, doing a survey of climate and game theoretical frameworks that have been instrumental in informing the structure of our devised game model. We start with an analysis of climate models, approximations inspired by the projections of the Intergovernmental Panel on Climate Change (IPCC). Using these models, we aim to clarify the complex circumstances under which nations may exhibit more environmentally conscientious behavior. Afterward, we illustrate key assumptions that exist in the existing literature, providing a critical foundation for the theoretical support of our research.*

3.1 Introduction

Recently, summers are getting hotter, marked by devastating wildfires, people’s attention to scientific works related to climate, energy, and the environment was more focused than ever. A natural question arose: How can our community contribute to addressing these challenges? This chapter addresses this by utilizing our methodological expertise to help understand and solve the challenges at hand.

Specifically, we intend to demonstrate how tools from game theory, optimization, dynamic systems, and numerical analysis can contribute to decision-making regarding the choice of CO₂ emissions levels at the national level. Game theory naturally comes into play as each country remains largely sovereign in this decision, and analyzing a Nash equilibrium of a game equates to considering a decarbonization solution that each country has an interest in following.

While developing emission reduction strategies has become imperative, a review of the literature on decarbonization strategies reveals that the majority of climato-socio-economic analysis provide empirical or ad hoc decarbonization strategies (e.g., [39–41]). Formally, the literature is often lacking. Among formal works, [42] studies the coalition problem without considering the dynamics of atmospheric and oceanic temperatures in economic and social damage functions. [43] presents a theoretical game framework where damage depends on temperature, but no game analysis is provided. [44] studies coalition stability among countries in the context of climate change but does not use game theory tools or analyze equilibriums. In contrast, [15] presents an economic-ecological model, studying the internal stability of coalitions and addressing the existence and uniqueness of Nash equilibrium in certain cases.

In conclusion, the presented works claim several notable contributions compared to the existing literature. While the literature considers very generic utility functions for countries, we propose utility functions structured by a dynamic geophysical model [45]. This avoids unrealistic conclusions and allows the construction of decarbonization strategies that countries realistically adhere to. Due to the nonlinear dynamics of atmospheric temperature concerning the global CO₂ emission quantity, analyzing the uniqueness of Nash equilibrium in the game proves non-trivial, as demonstrated in this chapter for the quadratic case.

This chapter aims to provide valuable insights into the intersection of game theory and climate policy, addressing gaps in existing formal analysis and proposing a more realistic approach to decarbonization strategy formulation.

3.2 State of the art

The journey of climate science started in the 1960s with the development of Global Climate Models (GCMs), driven by the need to comprehend, predict, and control weather patterns, influenced by the pioneering work of John von Neumann [46]. The trajectory continued with the creation of the Intergovernmental Panel on Climate Change (IPCC) and its report in 1990 [47]. The evolution of climate science, concluding in the release of the sixth IPCC report in 2022 [48], consistently reveals evidence of anthropogenic influences on climate change.

Despite the global participation of governments in COPs, the stark reality is that greenhouse gas (GHG) emissions, particularly CO₂, from major emitting countries have either increased or experienced minimal reductions (excluding the COVID-19 pandemic period). Most nations continue on a Business-As-Usual trajectory, with a significant part of their production still carbon-based [49]. Governments often adopt minimal policies to reduce their emissions, aligning with their national economic interests but falling short of global welfare ideals.

Various climate models and projections, approximating or simplifying the IPCC models [50–56], coexist in the literature.

Integrated Assessment Models (IAMs) serve as tools to simulate global economic dynamics. While these models aim to capture the complexity of the global economy, not all IAMs incorporate climate considerations into their equations. In contrast to the complex climate models, economists commonly use Simple Climate Models (SCMs) to simulate and estimate the impact of human CO₂ emissions on both climate and the economy.

Among the limited IAMs extended to address large-scale, discontinuous changes in the climate system, W. Nordhaus's DICE model [57] [58] stands out, often critiqued but widely used for projecting trends and climate policy impacts. Other notable IAMs include FAIR [51] [53], FUND [59] [60], PAGE [61] [62], and others. In the economic studies, works such as [63–65], [66, 67] contribute valuable insights. Discussions on damage functions, especially in [68, 69], involve economists comparing models to Weitzman's studies as seen in [70].

From a game theoretical perspective, various climate games are modeled as prisoners' dilemmas without simulations as seen in [71], and some involve coalition games, as explored in [25, 72–74], [44, 75–77], [78–81], [82–84].

Essentially, a vast literature exists on the dynamics of the Earth's climate system and the global economics of our current global market. However, there is a noticeable gap in research regarding the methodology for conceiving decentralized decarbonization strategies that take into account the competitiveness of economic agents responding to each other.

The limitation of the current strategies adopted by major actors (governments) is their empirical nature, lacking sufficient strategic-economic modeling that aligns with reality. Consequently, the proposed strategies lack coordination to achieve global objectives, such as those put forth by the IPCC, like RCP 2.6. This highlights the need to use tools from game theory, designed to understand phenomena arising from interactions among decision-makers. Coupling this with an understanding of climate modeling will enable the development of sustainable decarbonization strategies from a competitive perspective. While the question of game theory analysis of climate decisions has been explored in the literature [73], [44], [15, 85], and [64], there is a notable absence of modelization using economic

models in these analysis. Additionally, these studies often overlook the temperature variation in their assessments.

3.3 Simple Climate Model (SCM)

We will present in this section the mathematical climate modelization with the Simple Climate Models. The SCM is a simplified representation, including a radiative forcing equation and two climate system equations. Radiative forcing quantifies the change in energy balance in the Earth's atmosphere caused by various factors, such as concentrations of greenhouse gases, aerosols, and changes in solar radiation. In more technical terms, it is "the change in the net, downward minus upward, radiative flux (expressed in W/m²) due to a change in an external driver of climate change." Both of the equations come from the environment and climate literature. What is important to note is that the way we calculate the CO₂ is detached from the way we calculate the temperature. The literature of modelization of the carbon cycle is vast where there are some main models used, but the initial state is known as the current measurement of carbon in the atmosphere. There is a consensus on the way to calculate the radiative forcing but the value used changes with the authors with respect to their objective in terms of global temperature. For the temperature model, there is a main one that is the closest to the IPCC prediction.

Note that we only present a mathematical model based on deterministic state transition whereas institutions such as IPCC use a stochastic approximation of models CMIP5 and CMIP6, which, even if they are more precise, are not practical for building a first-game theoretical approach on. What is used usually by IPCC is what they call Earth system models (ESMs) which include the Carbon Cycle and much more, we can read [48] Chapters 2, 3, and 5 for more information. Still, since it is a first approach, we will leave it aside and we will focus on the simpler modelization that is SCMs.

We will use the survey made by [45] which explains the well-known and common different ways of modeling carbon cycles and warming models used by economists. In the first subsection, we will present well-known carbon cycle models, we will then explain the radiative forcing equation and, after that, we will present temperature models.

3.3.1 Models of carbon cycle

Carbon cycle modelization is a main component of the decarbonization strategies building. Indeed, it is the entry point of human emissions and exogenous emissions into the modelization of change in temperature. The literature is large in this area. We will first give the framework of how linear carbon cycle models work and provide a few well-known models with their values as examples, for sure, the following list is non-exhaustive.

3.3.1.1 Linear models of carbon cycle

A linear carbon cycle is described by m different equations, where $C(t)$ is a m -vector whose elements contain the amount of carbon in each box, (we will use reservoir, and layer to express the same idea) at time $t \in \mathbb{N}$. m is the number of virtual boxes, that represent containers of carbone. \mathbf{A}_C is a square matrix of constants that represent the flows between the various boxes and b_C is a column vector. Let d_C be the vector that maps the contents of the various boxes into the stock of atmospheric carbon C_{AT} , i.e.

$$C_{AT}(t) = d_C^\top C(t), \quad \forall t \in \mathbb{N}, \quad (3.1)$$

with for all $t \in \mathbb{N}$,

$$C(t) = \mathbf{A}_C C(t-1) + b_C E(t) \quad (3.2)$$

$$= \mathbf{A}_C^t C(0) + \sum_{s=1}^t \mathbf{A}_C^{t-s} b_C E(s), \quad (3.3)$$

where $E(t)$ is the global CO_2 emissions at time t . The first equation is a recursive representation of how CO_2 concentrations at the current time $C(t)$ depend on the concentrations at the previous time $C(t-1)$ and the impact of current emissions on the different layers $b_C E(t)$.

The second equation is an alternative form that expresses $C(t)$ explicitly in terms of the initial concentrations $C(0)$ and the cumulative impact of emissions over time. The term $\mathbf{A}_C^t C(0)$ represents the contribution from the initial conditions, and the summation term represent the accumulated effect of emissions from each previous time step.

Let us note that the boxes do not necessarily have a physical interpretation, it could, as we will see for the DICE model, but it is not mandatory. Usually, those boxes are a way to take into account, the carbon sink present on Earth, or the lifetime of this carbon.

3.3.1.2 The FUND model

The FUND carbon cycle model, which is based on [86], has 5 boxes with shares of emissions flowing into each of them. It is a model with time steps equal to 1 year. The specific value of \mathbf{A}_C , b_C and d_C are

$$\mathbf{A}_C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & e^{-1/363} & 0 & 0 & 0 \\ 0 & 0 & e^{-1/74} & 0 & 0 \\ 0 & 0 & 0 & e^{-1/17} & 0 \\ 0 & 0 & 0 & 0 & e^{-1/2} \end{bmatrix}, \quad b_C = \zeta \begin{bmatrix} 0.13 \\ 0.20 \\ 0.32 \\ 0.25 \\ 0.1 \end{bmatrix}, \quad \text{and} \quad d_C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

In this case, those boxes are theoretical boxes in which a proportion of the global CO₂ emissions are stocked, each box has a specific lifetime. The coefficient in \mathbf{A}_C corresponds to those lifetimes here ∞ , 363, 74, 17 and 2 years respectively. We note that $\zeta = 0.000471$ corresponds to the conversion from carbon to ppmCO₂.

3.3.1.3 DICE-2016

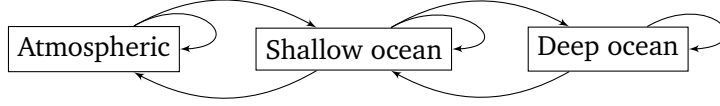


FIGURE 3.1 – DICE Carbon cycle: 3 boxes.

The DICE 2016 carbon cycle of Nordhaus [57], see Fig. 3.1, has three boxes: the atmosphere, the upper oceans and biosphere, and the lower/deep oceans. It is a model with time steps equal to 5 years. The diffusion matrix is

$$\mathbf{A}_C = \begin{bmatrix} 0.88 & 0.196 & 0 \\ 0.12 & 0.797 & 0.001465 \\ 0 & 0.007 & 0.998535 \end{bmatrix}, \quad \text{and} \quad b_C = d_C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

This matrix is by design a stochastic one (row sums equal 1) which means that no CO₂ leaves the system. The rate of uptake by the biosphere and oceans is independent of the amount of carbon stored in each box, so positive feedback between atmospheric CO₂ and CO₂ uptake is ruled out. There is no direct interchange of carbon between non-adjacent layers. The lower/deep oceans can store a large amount of carbon, but the rate of diffusion into the lower/deep oceans is really slow.

3.3.1.4 The Joos et al. (2013) carbon cycle model

Joos et al. (2013) [55] use a continuous-time model with one permanent and three transitory boxes. This model was designed to fit the impulse response functions to a set of ESMS simulations done by the CMIP5 model of IPCC. They have the following values

$$\mathbf{A}_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9975 & 0 & 0 \\ 0 & 0 & 0.9730 & 0 \\ 0 & 0 & 0 & 0.7927 \end{bmatrix}, \quad b_C = \begin{bmatrix} 0.2173 \\ 0.2240 \\ 0.2824 \\ 0.2763 \end{bmatrix} \quad \text{and} \quad d_C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

3.3.1.5 Other models

There are obviously a lot of other models that are more or less close to the CMIP5 prediction. We could for example cite model FAIR from [53] [51] which is a well-known model in the literature due to the fact that it can fit previous CO₂ emissions path which makes it a great tool for initializing models, like Joos's model for example.

3.3.2 Radiative forcing

The radiative forcing equation calculates the impact of greenhouse gas accumulation on the global radiation balance. The climate equation calculates the average surface temperature of the earth and the average deep-sea temperature at each time step.

The accumulation of greenhouse gases causes the earth's surface to warm by increasing radiative forcing. The relationship between the accumulation of greenhouse gases and the increase in radiative forcing is derived from empirical measurements and climate models, as we can see in the following equation:

$$F(t) = F_{2 \times CO_2} \log_2 \left(\frac{C_{AT}(t)}{C_{AT,ref}} \right) + F_{nonCO_2}(t), \quad \forall t \in \mathbb{R}^+ \quad (3.4)$$

where the first term is the forcing caused by CO₂, $F_{2 \times CO_2}$ is a parameter representing the radiative forcing resulting from 2 times atmospheric CO₂ and $F_{nonCO_2}(t)$ is an exogenous forcing, and the first term is forcing caused by CO₂. In [65], for the DICE modelization, Nordhaus consider the exogenous forcing as the following:

$$F_{nonCO_2}(t) = f_0 + \min \left\{ f_1 - f_0, \frac{f_1 - f_0}{t_f} (t - 1) \right\}, \quad \forall t \in \mathbb{R}^+. \quad (3.5)$$

Indeed, $F(t)$ is the change in total radiative forcing of greenhouse gases from man-made sources such as CO₂ since 1750. As a pre-industrial reference, the estimated carbon in different reservoirs in 1750 is used, sometimes in the literature, some radiative forcing can be taken with 1900 CO₂ estimation instead of 1750.

Also, the issue with the equation (3.5) is that the formulation is underestimating the radiative forcing from other GHGs, the article [87] makes a survey of how IAMs often get this part wrong. For doing so, the authors included MAGICC, a much more complex and stand-alone SCM used by most, as for example IPCC [48]. Again, as an introduction, we will keep the exogenous forcing of other GHGs as a linear increasing function of time, given by the expression of $F_{nonCO_2}(t)$ in equation (3.5).

The concave, logarithmic relationship between radiative forcing and atmospheric CO₂

captures the fact that the absorption of radiation in a box of CO₂ becomes progressively saturated.

3.3.3 Temperature Dynamic Models

The final part of an SCM is a Temperature Dynamic Model. This allows us to calculate from specific radiative forcing the evolution of the global atmospheric temperature. As before, the literature is vast on this topic. We will, again, explain the basic framework for linear temperature dynamic systems and provide a few well-known models of the literature, again the following list is non-exhaustive.

3.3.3.1 Linear model of temperature dynamic

The linear temperature dynamic is described by m' different equations, where $\theta(t)$ is a m' -vector whose elements contain the temperature of one of the m' boxes at time t , \mathbf{A}_θ is a square matrix of constants which represent the flows between the various boxes and b_θ is a column vector that maps the radiative forcing to the temperature boxes. We highlight that the number of boxes, which is the number of equations, is not necessarily the same in both carbon cycle and temperature dynamics. Let d_θ be the vector that maps the temperatures of the various layers to the atmospheric one denoted by θ_{AT} , i.e.

$$\theta_{\text{AT}}(t) = d_\theta^\top \theta(t), \quad \forall t \in \mathbb{N}, \quad (3.6)$$

with for all $t \in \mathbb{N}$,

$$\theta(t) = \mathbf{A}_\theta \theta(t-1) + b_\theta F(t) \quad (3.7)$$

$$= \mathbf{A}_\theta^t \theta(0) + \sum_{s=1}^t \mathbf{A}_\theta^{t-s} b_\theta F(s), \quad (3.8)$$

where $F(t)$ stands for the radiative forcing at time t . The first equation represents the temperature $\theta(t)$ at time $t \in \mathbb{N}$ as a linear combination of the temperature at the previous time step $\theta(t-1)$ and the radiative forcing $F(t)$ at the current time step.

The second equation provides a recursive expression for the temperature at time t , expressing it as a function of the initial temperature $\theta(0)$, and the cumulative effect of radiative forcing over time.

As we can see, the structure is exactly the same as the one of the Carbon Cycle model.

3.3.3.2 Geoffroy et al. (2013)

Geoffroy et al. (2013) [50] have a two-layer model for temperature dynamics in continuous time, the atmospheric temperature θ_{AT} , and the temperature in the lower ocean level

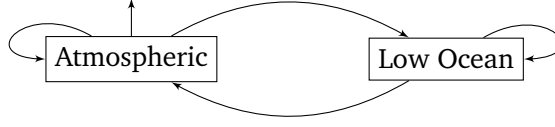


FIGURE 3.2 – Geoffroy et al. temperature model: 2 boxes.

θ_{LO} . The dynamics are given by

$$\dot{\theta}_{AT} = \frac{1}{c} [F - \iota \theta_{AT} - \varrho (\theta_{AT} - \theta_{LO})] \quad (3.9)$$

and

$$\dot{\theta}_{LO} = \frac{\varrho}{c_0} (\theta - \theta_{LO}) \quad (3.10)$$

where $c = 7.3 \text{ W.yr.m}^{-2}.\text{K}^{-1}$ is the effective heat capacity of the upper/mixed ocean layer, $c_0 = 106 \text{ W.yr.m}^{-2}.\text{K}^{-1}$ is the effective heat capacity of the deep oceans, $\iota = 1.13 \text{ W.m}^{-2}.\text{K}^{-1}$ and $\varrho = 0.73 \text{ W.m}^{-2}.\text{K}^{-1}$. Those values are the ones that fit best the multimodel mean of the CMIP5 set, which is the model used by IPCC [45]. The Equilibrium Climate Sensitivity (ECS) is defined as the steady-state atmospheric temperature arising from a doubling of atmospheric carbon.

The temperature transition matrix and the vectors are

$$\mathbf{A}_\theta = \begin{bmatrix} 1 - (\iota + \varrho)/c & \varrho/c \\ \varrho/c_0 & 1 - \varrho/c_0 \end{bmatrix}, \quad \mathbf{b}_\theta = \begin{bmatrix} 1/c \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{d}_\theta = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (3.11)$$

3.3.3.3 DICE 2016

DICE 2016 is formulated in discrete time with a time unit of 5 years due to the coupling with the carbon model and, like the model of Geoffroy et al. (2013), has two heat boxes, one for the temperature of the atmosphere, land and upper oceans, and one for the temperature of the deep oceans:

$$\theta_{AT}(t) = \theta_{AT}(t-1) - \frac{1}{c_{UP}} \left[F(t) - \frac{3.6813}{ECS} \theta_{AT}(t-1) - 0.088 [\theta_{AT}(t-1) - \theta_{LO}(t-1)] \right], \quad (3.12)$$

and

$$\theta_{LO}(t) = \theta_{LO}(t-1) + \frac{0.088}{c_{LO}} [\theta_{AT}(t-1) - \theta_{LO}(t-1)], \quad (3.13)$$

where $c_{UP} = 1/0.1005 \text{ W.yr.m}^{-2}.\text{K}^{-1}$ and $c_{LO} = 1/0.025 \text{ W.yr.m}^{-2}.\text{K}^{-1}$ are the effective heat capacities of the upper and lower oceans respectively, and 0.088 and 0.025 are the coefficients of heat exchange between the upper and deep oceans respectively.

The steady state temperature is $\theta_{AT}(t) = \theta_{LO}(t) = ECS \times F(t)/3.6813 = 0.842F(t)$, where ECS is set to 3.1 K.

The state transition matrix and the vectors are given as follows:

$$\mathbf{A}_\theta = \begin{bmatrix} 0.8718 & 0.0088 \\ 0.025 & 0.975 \end{bmatrix}, \quad b_\theta = \begin{bmatrix} 1/c_{UP} \\ 0 \end{bmatrix}, \quad \text{and} \quad d_\theta = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (3.14)$$

For a more complete explanation of DICE [65] through a mathematical perspective, see Section 3.4.

3.3.3.4 Golosov et al. (2014)

Golosov et al. model (2014) [88] has no temperature lag with a time step of 10 years. Indeed, they use the following temperature dynamics

$$\theta_{AT}(t) = 0.842F(t). \quad (3.15)$$

3.3.3.5 Gerlagh and Liski (2018),

Gerlagh and Liski model [89] has also a time step of 10 years, but have a simple lag with partial adjustment of 0.183 per decade (or 2% per year). Indeed, they use the following temperature dynamics

$$\theta_{AT}(t) = \theta_{AT}(t-1) + 0.183 [0.842F(t) - \theta_{AT}(t-1)]. \quad (3.16)$$

Although this long lag is in line with the scientific evidence of some time ago[90], it is not consistent with more recent scientific evidence [50], for more details, see [45].

3.3.3.6 Other models

There are other models for the dynamic of the temperature in the literature which, coupled with a carbon cycle, more or less fit the CMIP5 prediction in terms of temperature variation. Again, we can cite the FAIR model, which uses a similar temperature dynamic as Geoffroy's one [51, 53, 64]. It is important to highlight that in our analysis, we do not use the values of the models. Nevertheless, for the numerical analysis, these values become essential.

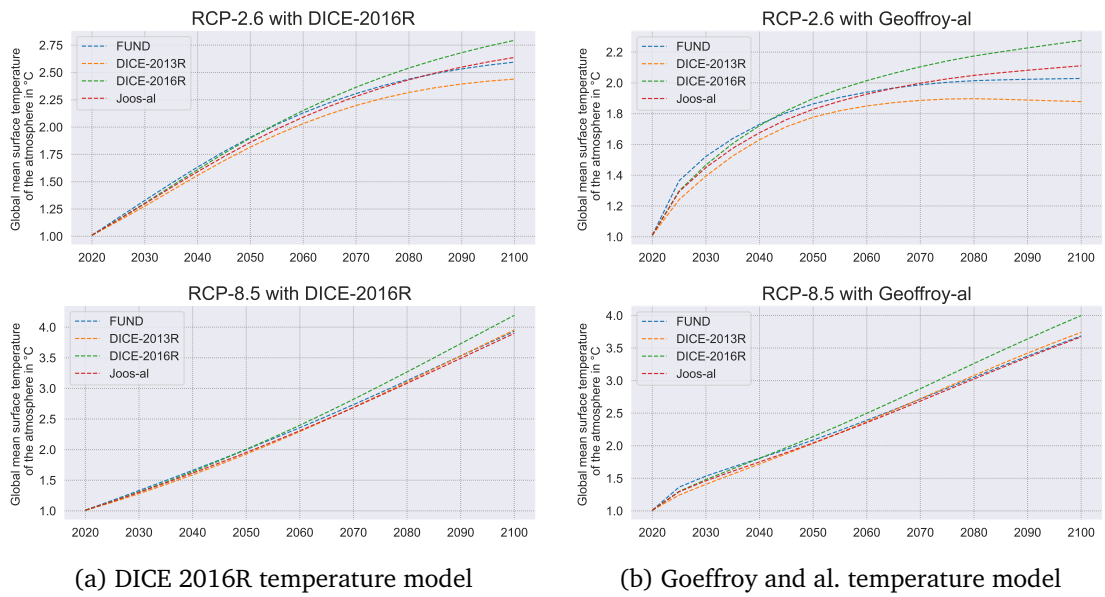


FIGURE 3.3 – Comparison of possible combination of Carbon cycle model with two different models of dynamic temperature model.

3.3.4 Comparison between model combinations

The natural question arising from an observation of these models is: "Does the selection of models significantly impact a game theory analysis?" We have plotted different combinations of carbon cycle models and temperature dynamic models for two carbon emission paths, RCP2.6 and RCP8.5, in Figure 3.3. As illustrated in the figure, while there are some variations between the models, they exhibit similar curve shapes and behaviors. This uniformity is primarily attributed to the structure shared by these models. Therefore, it is reasonable to deduce that a game theoretical analysis conducted on one of these models should yield essentially the same insights for another model or combination of models.

However, it is crucial to note that: while the models exhibit similar responses to "small" quantities of CO₂ over an extended period, they display substantial differences when subjected to, for instance, injecting 100 GtC into the atmosphere at the initial time and no further emissions thereafter, as detailed in [45].

3.4 DICE Model as Integrated Assessment Model

3.4.1 Introduction

The DICE model (Dynamic Integrated Model of Climate and Economy) is a simplified empirical model that encompasses economic, policy, and scientific aspects of climate change

on a global scale.

The DICE model adopts the perspective of neoclassical economic growth theory, specifically the Solow-Swan Growth Model, to analyze the economic forces influencing climate change. It extends this model by introducing "natural capital" representing the climate system. Greenhouse gas concentration is viewed as negative natural capital, and emission reduction is considered an investment that enhances natural capital, thus mitigating climate change impacts.

3.4.2 Objectives of Integrated Assessment Models (IAMs)

Integrated Assessment Models (IAMs) can be categorized into policy optimization and policy evaluation models. The DICE model falls into the former category, aiming to maximize an objective function representing economic welfare, often measured by utility functions or consumer and producer surplus in economic structures.

3.4.3 Detailed Equations of the DICE-2013R Model

3.4.3.1 Preferences and the Objective Function

Within the DICE model, each region or entity has well-defined preferences represented by a social welfare function to maximize. For each time period $t \in \mathbb{R}_+$, the social welfare function is given by:

$$W = \sum_{t=1}^{T_{\max}} U(c(t), L(t))R(t), \quad (3.17)$$

where $R(t)$ is the discount factor, and the utility function $U[c(t), L(t)]$ is defined as:

$$U[c(t), L(t)] = \frac{L(t)c(t)^{1-\alpha}}{1-\alpha}. \quad (3.18)$$

The utility function U is assumed to be defined and strictly concave in the consumption $c(t)$. This is the so-called power function. Many authors, including Nordhaus, select this function. In earlier versions of the DICE model, [91, 92] chooses $\alpha = 2$. See Kelly and Kolstad [93] and Stern [94] for the case where $\alpha = 1$. In the 2016 version of the DICE model $\alpha = 1.45$. Using the equations (3.17), and (3.18), leads to the overall welfare equation given by the following:

$$W = \sum_{t=1}^{T_{\max}} \frac{L(t)R(t)c(t)^{1-\alpha}}{1-\alpha}. \quad (3.19)$$

The elasticity parameter α in the utility function represents the elasticity of marginal utility of consumption. The discount factor $R(t)$ is determined by the pure rate of social time

preference ρ :

$$R(t) = (1 + \rho)^{-t}. \quad (3.20)$$

This reflects the refund on the financial well-being of future generations.

The DICE model supports policies optimizing the investment and total consumption over time, assuming a constant elasticity of the marginal utility of consumption. The discount factor accounts for the welfare weights on the utilities of different generations.

The model projects the growth of major economic and environmental variables, considering current climate change policies. The potential advancements in world welfare from effective climate change approaches are evaluated within the existing distribution of income and investments over space and time.

3.4.3.1.1 Economic Variables Given the long time frame over which climate change models are analyzed and the lack of evidence in economic literature, most projections and assumptions are based on limited data, primarily from countries such as the United States, China, the EU, and India.

The DICE model treats all countries collectively, considering an initial stock of capital, labor, and technology. Capital stocks and outputs are aggregated using PPP (purchasing power parity) exchange rates. The evolution of world output is described by logistic-type equations (3.21), where population and labor force are exogenous:

$$\begin{aligned} L(t) &= L(t-1)[1 + g_L(t)], \\ \text{with: } g_L(t) &= \frac{g_L(t-1)}{1 + \delta_L}. \end{aligned} \quad (3.21)$$

Here, g_L represents the growth rate of labor, with a reference of $g_L(2015)$ at 13.4% per 5 years to align with UN 2050 projections. Output is produced using a Cobb-Douglas production function in capital, labor, and energy, which can be carbon-based fuels or non-carbon-based technologies. Technological change in the DICE model can be economy-wide or carbon-saving. The Total Factor Productivity (TFP) denoted by $A(t)$ is defined as:

$$\begin{aligned} A(t) &= A(t-1)[1 + g_A(t)], \\ \text{with: } g_A(t) &= \frac{g_A(t-1)}{1 + \delta_A}. \end{aligned} \quad (3.22)$$

Here, g_A represents the growth in consumption per capita, and δ_A is the decline rate of TFP per 5 years. With a reference of $g_A(2015) = 7.9\%$ per 5 years and $\delta_A = 0.6\%$ per 5 years, the growth in consumption per capita is 1.9% per year from 2010 to 2100 and 0.9% from 2100 to 2200.

Carbon-saving technological change aims to reduce CO₂ emissions, and in its current form, carbon constraints are not imposed in the base case. The shift from carbon to non-

carbon fuels occurs gradually over time as carbon-based sources become costlier. In a partial convergence model, regional outputs are projected, resulting in the global output given by the following equations:

$$Q(t) = [1 - \Lambda(t)] \frac{A(t)K(t)^\gamma L(t)^{1-\gamma}}{1 + \Omega(t)}, \quad (3.23a)$$

$$\Omega(t) = \beta \theta_{AT}(t) + \delta [\theta_{AT}(t)]^2, \quad (3.23b)$$

$$\Lambda(t) = \vartheta_1(t) \mu(t)^{\vartheta_2}, \quad (3.23c)$$

where:

$$\left\{ \begin{array}{l} Q(t): \text{output net of damages and abatement,} \\ \Lambda(t): \text{ratio of abatement cost to output,} \\ A(t): \text{total factor productivity,} \\ K(t): \text{capital stocks and services,} \\ L(t): \text{population and labor force,} \\ \gamma: \text{abatement costs,} \\ \Omega(t): \text{climate damages,} \\ \beta: \text{damage intercept,} \\ \delta: \text{damage quadratic term,} \\ \mu(t): \text{emissions reduction rate,} \\ \vartheta_1: \text{adjusted cost for the backstop,} \\ \vartheta_2: \text{exponent of control cost function.} \end{array} \right.$$

Costs of emission reductions are a function of the emission reduction rate, and damages are a quadratic function of temperature change. The damage does not include sharp thresholds or tipping points, consistent with the survey by [95]. Damage estimates range from 0 to 3°C, and the functional form ensures that the output cannot be exceeded, imposing constraints on catastrophic climate change. The cost function Λ , is estimated to be highly convex, indicating that the marginal cost of reductions rises from zero more than linearly with the reductions rate.

It is important to note that placing climate damage in the denominator ensures compliance with production limits. In cases where catastrophic climate change is considered, as a very high temperature increase, or when the climate system is forced to cross some threshold, triggering a transition to a new state, the damage function must be redefined.

In the full local model, the cost of reducing emissions ranges from \$230 to \$540 per ton of CO₂ depending on the region (2005 prices) to replace 100% of carbon emissions. For the global DICE-2013R model, the 2010 cost of the backstop technology, which is a

technology that can replace all fossil fuels, is \$344 per ton of CO₂ for 100% removal. Indeed, it is a technology that can replace all fossil fuels. The backstop technology could be one that removes carbon from the atmosphere or an all-purpose environmentally benign zero-carbon energy technology. It might be solar power, or carbon-eating trees or windmills, or some as-yet undiscovered source. The backstop price is assumed to be initially high and to decline over time with carbon-saving technological change. The cost of the backstop technology is expected to decrease by 0.5% each year. This technology is introduced into the model by setting the time path of the constraints in the reduction cost equation (3.23a) such that the minimum cost of reduction at a control rate of 100% is equal to the backstop price for a given year.

Next, the economic accounting equations are presented. Equation (3.24a) outlines the dynamics of capital stocks using the perpetual inventory method with exponential depreciation rates:

$$Q(t) = C(t) + I(t), \quad (3.24a)$$

$$c(t) = \frac{C(t)}{L(t)}, \quad (3.24b)$$

$$K(t) = I(t) - \delta_K K(t-1), \quad (3.24c)$$

where:

$$\left\{ \begin{array}{l} C(t): \text{consumption,} \\ I(t): \text{gross investment,} \\ c(t): \text{per capita consumption,} \\ \delta_K: \text{depreciation rate.} \end{array} \right.$$

CO₂ emissions are considered as total output and are controlled by a function that varies with time:

$$E_{\text{Ind}}(t) = \sigma(t)[1 - \mu(t)]A(t)K(t)^\gamma L(t)^{1-\gamma}, \quad (3.25)$$

where $\sigma(t)$ is the carbon intensity level, considered exogenous and congested from region estimates, that are a logistics-type equations, given by the following:

$$\begin{aligned} \sigma(t) &= \sigma(t-1)[1 + g_\sigma(t)], \\ \text{with: } g_\sigma(t) &= \frac{g_\sigma(t-1)}{1 + \delta_\sigma}. \end{aligned} \quad (3.26)$$

Here, $\sigma(2015)$ represents carbon intensity in 2010 at 0.549 tons of CO₂ per \$1000 of GDP, $g_\sigma(2015) = -1.0\%$ per year, and $\delta_\sigma = -1.0\%$ per five years. The carbon intensity changes by -0.95% per year between 2010 and 2100 and -0.87% per year between 2100 and 2200.

Finally, the economic section includes the resource constraint on carbon fuels, denoted by CCum, and given by the following:

$$\text{CCum} \geq \sum_{t=1}^{T_{\max}} E_{\text{Ind}}(t). \quad (3.27)$$

In the DICE-2013R model, the limit is set at 6000 tons of carbon content. Cumulative carbon emissions from 2010 to 2100 in the DICE-2013R model are projected to be 1870 GtC, and for the entire time frame, 4800 GtC. These estimates for 2100 are slightly higher than those found in models examined in the IPCC Fifth Assessment Report, Science (2013).

3.4.3.1.2 Geophysical Sectors The DICE-2013R model integrates geophysical components that establish connections between the economy and various factors influencing climate change. These components encompass the carbon cycle, a radiative constraint condition, climate change conditions, and a climate-damage relationship. A key characteristic of IAMs is the coordinated operation of modules, avoiding the assumption of factors as exogenous inputs from other models or assumptions.

In the DICE-2013R model, the primary greenhouse gas (GHG) subject to control is industrial CO₂. This indicates that CO₂ is the major contributor to global warming, and other GHGs are likely to be regulated differently (analogous to the control of chlorofluorocarbons through the Montreal Protocol). Other GHGs are incorporated as exogenous trends in radiative forcing. These include primarily CO₂ emissions from land-use changes, other GHGs, and aerosols. Total emissions are then the sum of industrial emissions and land-use emissions, represented as:

$$E(t) = E_{\text{Ind}}(t) + E_{\text{Land}}(t). \quad (3.28)$$

The carbon cycle (3.29) is based on a three-reservoir model aligned with existing carbon-cycle models and historical data. It assumes three reservoirs for carbon: $C_{\text{AT}}(t)$ for carbon in the atmosphere, $C_{\text{UP}}(t)$ for carbon in a rapidly mixing reservoir in the upper oceans and biosphere, and $C_{\text{LO}}(t)$ for carbon in the deep oceans. Carbon flows between these reservoirs, with mixing between the deep seas and other reservoirs occurring slowly. Here, ϕ_{ij} are the flow parameters between reservoirs.

$$\begin{aligned} C_{\text{AT}}(t) &= \phi_{11}C_{\text{AT}}(t-1) + \phi_{21}C_{\text{UP}}(t-1) + E(t), \\ C_{\text{UP}}(t) &= \phi_{12}C_{\text{AT}}(t-1) + \phi_{22}C_{\text{UP}}(t-1) + \phi_{32}C_{\text{LO}}(t-1), \\ C_{\text{LO}}(t) &= \phi_{23}C_{\text{UP}}(t-1) + \phi_{33}C_{\text{LO}}(t-1). \end{aligned} \quad (3.29)$$

The climate equation involves a simplified representation, including a radiative forcing

equation and two climate system equations. The radiative forcing equation calculates the impact of greenhouse gas accumulation on the global radiation balance. The climate equation computes the average surface temperature of the Earth and the average deep sea temperature at each time step.

The accumulation of greenhouse gases warms the Earth's surface by increasing radiative forcing. The relationship between the accumulation of greenhouse gases and the increase in radiative forcing is derived from empirical measurements and climate models, expressed in the following equation:

$$F(t) = \eta \log \frac{C_{\text{AT}}(t)}{C_{\text{AT,ref}}} + F_{\text{nonCO}_2}(t). \quad (3.30)$$

Here, $F_{\text{nonCO}_2}(t)$ is an exogenous forcing, and the equation captures the change in total radiative forcing of greenhouse gases from man-made sources like CO_2 since 1750. The temperature equation (3.31) is represented by:

$$\begin{aligned} \theta_{\text{AT}}(t) &= \theta_{\text{AT}}(t-1) + \xi_1 \{F(t) - \xi_2 \theta_{\text{AT}}(t-1) - \xi_3 [\theta_{\text{AT}}(t-1) - \theta_{\text{LO}}(t-1)]\}, \\ \theta_{\text{LO}}(t) &= \theta_{\text{LO}}(t-1) + \xi_4 \{\theta_{\text{AT}}(t-1) - \theta_{\text{LO}}(t-1)\}. \end{aligned} \quad (3.31)$$

Here, $\theta_{\text{AT}}(t)$ is the mean surface temperature, and $\theta_{\text{LO}}(t)$ is the temperature of the deep oceans.

Later in our study, the dynamics for all $t \in \mathbb{R}$ will be presented as:

$$\begin{cases} \theta(t+1) = \mathbf{A}_\theta \theta(t) + b_\theta F(t+1), \\ C(t+1) = \mathbf{A}_C C(t) + b_C E(t+1), \end{cases} \quad (3.32)$$

where

$$\mathbf{A}_\theta = \begin{bmatrix} 1 - \xi_1 \xi_2 - \xi_1 \xi_3 & \xi_1 \xi_3 \\ \xi_4 & 1 - \xi_4 \end{bmatrix}, b_\theta = \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix}, \mathbf{A}_C = \begin{bmatrix} \phi_{11} & \phi_{21} & 0 \\ \phi_{12} & \phi_{22} & \phi_{32} \\ 0 & \phi_{23} & \phi_{33} \end{bmatrix}, b_C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

3.4.4 DICE Formulas

In this part, we summarize the formulas of the DICE model in the following tables.

3.4. DICE Model as Integrated Assessment Model

Parameter	Parameter name	Formula	Unit/Domain
W	Welfare	$\sum_{t=1}^{T_{\max}} \frac{L(t)R(t)c(t)^{1-\alpha}}{1-\alpha}$	\$
$R(t)$	Discount factor	$(1+\rho)^{-t}$	%
$L(t)$	Population and labor force	$\frac{L(t-1)}{1+g_L(t)} = \prod_{i=0}^t \left[1 + \frac{g_L(0)}{(1+\delta_L)^{t-i}} \right]$	Millions
$g_L(t)$	Growth rate of population	$\frac{g_L(t-1)}{1+\delta_L}$	%
$c(t)$	Consumption per capita	$\frac{C(t)}{L(t)}$	Thousands 2005\$ per year
$Q(t)$	Net output of damages and abatement	$[1-\Lambda(t)] \frac{A(t)K(t)^\gamma L(t)^{1-\gamma}}{1+\Omega(t)}$ $= C(t) + I(t)$	Trillions 2005 \$ per year $0 < \gamma < 1$
$A(t)$	Level of total factor productivity (TFP)	$\frac{A(t-1)}{1+g_A(t)} = \prod_{i=0}^t \left[1 + \frac{g_A(0)}{(1+\delta_A)^{t-i}} \right]$	%
$g_A(t)$	Growth rate of productivity	$\frac{g_A(t-1)}{1+\delta_A}$	%
$K(t)$	Capital stocks and services	$I(t) - \delta_K K(t-1) = \sum_{i=0}^t (-1)^i \delta_K^i I(t-i)$	Trillions 2005 \$ per year
$\Lambda(t)$	Marginal abatement cost	$\vartheta_1(t) \mu^{\vartheta_2(t)}$	2005 \$ per CO ₂ ton
$\vartheta_1(t)$	Backstop price	$344(1-0.025)^{t-1}$	2005 \$ per CO ₂ ton $0 < \vartheta_1(t) < 1$
$\Omega(t)$	Climate damage	$\beta \theta_{AT}(t) + \delta \theta_{AT}^2$	Trillions 2005 \$ per year
$\theta_{AT}(t)$	Atmospheric temperature	$\frac{1}{\xi_1} b_\theta \mathbf{A}_\theta \theta_{AT}(t-1) + \xi_1 F(t)$	°C
$F(t)$	Radiative forcing	$\eta \log \frac{C_{AT}(t)}{C_{AT,ref}} + F_{nonCO_2}(t)$	$W.m^{-2}$
$F_{nonCO_2}(t)$	Exogenous forcing for other GHG's	$f_0 + \min \{ f_1 - f_0; \frac{f_1 - f_0}{t_f} (t-1) \}$	$W.m^{-2}$
$C_{AT}(t)$	CO ₂ atmospheric concentration	$b_C \mathbf{A}_C C(t-1) + E(t)$	GtC : Gega ton of carbon
$E(t)$	Total emissions	$E_{Ind}(t) + E_{Land}(t)$	GtCO ₂ per year
$E_{Ind}(t)$	Industrial CO ₂ emissions	$\sigma(t-1) \left[1 + g_\sigma(t) \right]$	GtCO ₂ per year
$g_\sigma(t)$	Cumulative improvement of energy efficiency	$\frac{g_\sigma(t-1)}{1+\delta_\sigma}$	GtCO ₂ per year

where some of the parameters are given as in the following table.

Parameter	Parameter name	Unit/Domain
α	Elasticity of marginal utility consumption	$\alpha > 0$
ρ	Discount rate	% $0 < \rho < 1$
δ_L	Decline rate of population	%
δ_A	Decline rate of TFP	%
$I(t)$	Investment	Trillions 2005 \$ per year %
δ_K	Depreciation rate of capital	$0 < \delta_K < 1,$ $I(t) \geq 0$
$\mu(t)$	Emission control rate GHG's	$0 \leq \mu(t) \leq 1, \forall t < 30,$ $0 \leq \mu(t) \leq 1.2, \forall t \geq 30.$
ϑ_2	Exponent of control cost function	$\vartheta_2 > 1$
f_1	Forcing of other GHG's in 2100	$W.m^{-2}$
f_0	Forcing of other GHG's in 2010	$W.m^{-2}$
ξ_1	Climate coefficient for upper level	$W.m^{-2}$
η	Forcing of equilibrium CO ₂ doubling	$W.m^{-2}$
A_C	CO ₂ exchanges between reservoirs	No unit
A_θ	Temperature exchanges between reservoirs	No unit
$E_{Land}(t)$	Land use emissions	Gt CO ₂ per year
$\sigma(t-1)$	CO ₂ equivalent emissions output ratio	%
δ_σ	Decline rate of decarbonization per period	%

3.5 FINUS Model

3.5.1 Introduction

In recent years, the game theory analysis of international environmental issues has gained increasing attention. This is not surprising, as game theory provides a framework to analyze the interaction between agents, make assumptions about their behavior, and predict the final outcomes. Therefore, game theory is particularly well-suited for examining the incentive structure of international environmental issues. The central question that can be explored through this approach is: under what conditions will the International Environmental Agreement (IEA) be signed and approved?

Some critics argue that important aspects of the international pollution problem have been neglected, making the results less generalizable and, consequently, less suitable for policy analysis and recommendations. Finus, in his book [72], seeks to address this criticism by outlining basic assumptions and key results, restricting the analysis to global environmental problems.

3.5.2 Finus game analysis

Consider N countries, and for $i \in \{1, \dots, N\}$, let the benefit from emissions be denoted by $\beta_i(e_i)$, damages caused by global emissions denoted by $\Phi_i\left(\sum_{i=1}^N e_i\right)$, and the welfare of country i denoted by π_i , given by:

$$\pi_i = \beta_i(e_i) - \Phi_i\left(\sum_{i=1}^N e_i\right). \quad (3.33)$$

Emissions can be viewed both as benefits from the production and consumption standpoint and as environmental damage to one's own and other countries. This explains why the damage function depends on the aggregate emissions, $\sum_{i=1}^N e_i$.

In this game, the strategy space for player $i \in \{1, \dots, N\}$ is given by $[0, e_i^{\max}]$, where $e_i^{\max} > 0$ is the sufficiently large upper bound on emissions, representing the maximum emissions of country i . The benefit and damage functions are assumed to be continuous and twice differentiable. Additionally, the benefit function is assumed to be strictly increasing and strictly concave, while the damage function is assumed to be strictly increasing and convex.

$$\begin{cases} \beta'_i > 0 & (e_i \in]0, e_i^{\max}[), \\ \beta''_i < 0 & (e_i \in]0, e_i^{\max}[), \\ \Phi'_i > 0 & \left(\sum_{i=1}^N e_i \in]0, \sum_{i=1}^N e_i^{\max}[\right), \\ \Phi''_i \geq 0. \end{cases} \quad (3.34)$$

To interpret e_i^{\max} from an economic perspective, we assume that

$$\beta'_i \begin{cases} = 0 & \text{if } e_i = e_i^{\max}, \\ < 0 & \text{if } e_i > e_i^{\max}. \end{cases} \quad (3.35)$$

This implies that a country would prefer to emit to the maximum if it perceives no damage. In this case, e_i^{\max} is the unique solution to $\arg \max \beta_i(e_i) \iff \beta'_i(e_i^{\max}) = 0$.

3.5.2.1 Individual emission optimization

In a non-cooperative scenario, each country maximizes its welfare with respect to its own emissions without considering the damage from others' emissions. This is expressed in the

maximization problem:

$$\max_{e_i} \pi_i \implies \max_{e_i} \beta_i(e_i) - \Phi_i \left(\sum_{i=1}^N e_i \right) \implies \beta'_i(e_i) = \Phi'_i \left(\sum_{i=1}^N e_i \right). \quad (3.36)$$

This determines the best response function for country i , denoted by $R_i := e_i = e_i(e_{-i})$. The optimal choice is unique due to the assumptions over benefit and damage functions (3.34). The best response function is decreasing:

$$\begin{aligned} \pi_i &= \beta_i(e_i) - \Phi_i \left(\sum_{j=1}^N e_j \right), \\ \beta'_i(e_i) &= \Phi'_i \left(e_i + \sum_{j=1, j \neq i}^N e_j \right), \\ \beta''_i de_i &= \Phi''_i de_i + \Phi''_i de_j, \\ \beta''_i \frac{de_i}{de_j} &= \Phi''_i \frac{de_i}{de_j} + \Phi''_i, \\ \frac{de_i}{de_j} &= \frac{\Phi''_i}{\beta''_i - \Phi''_i}. \end{aligned} \quad (3.37)$$

In conclusion, in non-cooperative case, the equilibrium $e^N = (e_1^N, \dots, e_N^N)$ is unique if (3.36) holds simultaneously for all countries. It follows that $e_i^{\max} > e_i^N > 0$, indicating that countries tend to reduce their emissions compared to a no-abatement policy where all countries emit to the maximum: $e^{\max} = (e_1^{\max}, \dots, e_N^{\max})$.

It is important to note that in the Nash equilibrium, though governments behave non-cooperatively, they reduce some emissions (namely, $e_i^{\max} - e_i^N$) compared to a "no-abatement policy" where country 1 would emit e_1^{\max} and country 2 e_2^{\max} . This is at least true as long as governments recognize damages at all, i.e., $\Phi_i(\sum e_j > 0) > 0$. If this were not the case, then a government's optimal non-cooperative emission level would be e_i^{\max} , i.e., $e_i^N = e_i^{\max}$. Indeed, e_i^{\max} follows from $\arg \max \pi_i \beta_i(e_i) \implies \pi_i \beta'_i(e_i) = 0$. That represents, environmental damages are not considered in a government's objective function. The assumption of strictly convex payoff functions in policy levels suggests that countries maintain an inherent interest in collaborating on global pollution control. However, realizing such cooperation proves to be a formidable challenge, primarily due to the absence of a supranational institution provided with global authority to enforce cooperation. From a game theoretical point of view, the International Court of Justice could potentially address disputes, but its intervention

is conditional on the voluntary agreement of the accused party. In the following part, we present the analysis of Finus for a cooperative scenario, where the objective is to maximize the sum of welfares.

3.5.2.2 Global emission optimization

In a cooperative scenario, each country maximizes world welfare with respect to the sum of emissions:

$$\max_{e_1, \dots, e_N} \sum_{i=1}^N \pi_i \implies \max_{e_1, \dots, e_N} \sum_{i=1}^N \beta_i(e_i) - \sum_{i=1}^N \Phi_i \left(\sum_{j=1}^N e_j \right) \implies \beta'_i(e_i) = \sum_{k=1}^N \Phi'_k \left(\sum_{j=1}^N e_j \right). \quad (3.38)$$

In a fully cooperative game, the optimal scenario is that marginal benefits are equal for all countries: $\beta'_i(e_i) = \beta'_j(e_j), \forall i \neq j$.

3.5.2.2.1 Problems of Cooperation

- In this example, countries are supposed to reduce emissions concerning the emissions in the non-cooperative equilibrium. The benefit for a country in cooperation is higher welfare, see [72] for the proofs.
- Countries may experience lower welfare if they have heterogeneous welfare functions, with full or partial cooperation. It is explained in [96] that there is no efficient emission vector that guarantees a higher payoff than the NE for each country.

3.6 Problem Formulation

After introducing several climate models from the literature, introducing the economic climate model of Nordhaus, and introducing the game studied by Finus, we start this section by presenting the climate model that we are considering, before presenting the game that we are studying in this thesis.

3.6.1 Climate Model (CM)

Economic models are often associated with climate models and are utilized as integrated assessment models (IAMs). We incorporate the climate structure of IAMs into our game. Typically, the model integrates a carbon cycle (CC), a radiative forcing relationship, and temperature dynamics (TD). The global system state is denoted by $x = (C, \theta)$, where C is

the state of the CC, and θ is the state of the TD. The modeling is linear, as described in [45]:

$$(CC) \begin{cases} C(t+1) = \mathbf{A}_C C(t) + d_C E(t), \\ C_{AT}(t+1) = b_C^\top C(t+1), \end{cases} \quad \forall t \in \mathbb{N}. \quad (3.39)$$

$$(TD) \begin{cases} \theta(t+1) = \mathbf{A}_\theta \theta(t) + d_\theta F(t+1), \\ \theta_{AT}(t+1) = b_\theta^\top \theta(t+1), \end{cases} \quad \forall t \in \mathbb{N}. \quad (3.40)$$

Here, $C(t) \in \mathbb{R}^{m_1}$ is the carbon equivalent vector in m_1 boxes of the CC dynamics at time t , and $\theta(t) \in \mathbb{R}^{m_2}$ is the temperature vector in m_2 boxes of the TD at time t . Matrices \mathbf{A}_C and \mathbf{A}_θ represent the flows between the CC and TD boxes, respectively, and d_C, d_θ are vectors in $\mathbb{R}^{m_1}, \mathbb{R}^{m_2}$, respectively. Vectors $b_C \in \mathbb{R}^{m_1}$ and $b_\theta \in \mathbb{R}^{m_2}$ are output extraction vectors for C_{AT} (atmospheric carbon equivalent quantity) and θ_{AT} (global average atmospheric temperature), respectively. The input for the carbon cycle is $E(t) \in \mathbb{R}$, representing emissions in GtCO₂, while radiative forcing $F(t) \in \mathbb{R}$, in W.m⁻², is the input for TD. The latter is an approximation of the impact of greenhouse gas accumulation on the global radiative balance [54]. The relationship between greenhouse gas accumulation and radiative forcing increase is derived from empirical measurements and climate models, as shown in the following equation:

$$F(t) = F_{2 \times CO_2} \log_2 \left[\frac{C_{AT}(t)}{C_{AT,ref}} \right] + F_{nonCO_2}(t), \quad \forall t \in \mathbb{N},$$

where $C_{AT,ref}$ is the reference atmospheric carbon quantity, often set to 592.14 GtC, corresponding to the year 1750 as the start of the industrial era. To align with the IPCC Earth climate model, we use the common parameter $F_{2 \times CO_2}$ representing the radiative forcing resulting from 2 times atmospheric CO₂ (typically $F_{2 \times CO_2} = 3.7$ W.m⁻²). Exogenous forcings are represented by the function of time $F_{nonCO_2}(t)$.

3.6.2 Proposed Game Model

Similar to [15], we consider a global economy composed of N countries. A country $n \in \mathcal{N} = \{1, \dots, N\}$ is an economic agent. Its profit, denoted by B_n , represents its gross domestic product (GDP), which depends on its greenhouse gas emissions (GHGs) expressed in terms of CO₂ emissions. For the player's action, we use a_n , which is nothing but its emissions. In [15], a player n 's utility function is the difference between a profit that depends on the player's action and individual damage D_n , a function of the game state and the sum of actions of all players, *i.e.*, $\sum_{n=1}^N a_n$, representing economic losses.

In this thesis, we consider that the damage depends on the projection of the global average atmospheric temperature. It is important to note that the utility of each player depends

on the actions played and the output $\theta_{\text{AT}}(t + 1)$ of the climate model (3.39)-(3.40). We denote that $\theta_{\text{AT}}(t + 1)$ is a function of the players' actions a and the game state x , at time t . Indeed, we omit the time t since we consider a static game, and we consider θ_{AT} as a function $\theta_{\text{AT}}(x, a)$. In the numerical analysis, we will repeat this static game to see the evolution of the strategies of the players, which means that we consider that the players decide about the amount of emissions once for the five coming years, hence the reason for the game being static.

To establish a connection between climate modeling and the game, we denote the atmospheric temperature by θ_{AT} , which is given by the following:

$$\theta_{\text{AT}}(x, a) = \psi_{\theta}(x) + \tilde{b}_{\theta} \ln \left(\psi_C(x) + \tilde{b}_C \sum_{n=1}^N a_n \right),$$

where \tilde{b}_{θ} , \tilde{b}_C are positive parameters from (3.39)-(3.40), and ψ_{θ} , ψ_C are positive functions of the state x , given by:

$$\psi_{\theta}(x) = b_{\theta}^{\top} \theta + d_{\theta} F_{\text{nonCO}_2} - \frac{d_{\theta} F_{2 \times \text{CO}_2}}{\ln 2} \ln C_{\text{AT,ref}}, \quad (3.41)$$

$$\tilde{b}_{\theta} = \frac{d_{\theta} F_{2 \times \text{CO}_2}}{\ln 2}, \quad (3.42)$$

$$\psi_C(x) = b_C^{\top} \mathbf{A}_C C, \quad (3.43)$$

$$\tilde{b}_C = b_C^{\top} d_C. \quad (3.44)$$

Due to the specific form of θ_{AT} in the literature, u_n is not quasi-concave, making the theoretical analysis of the game less immediate.

For the mathematical formulation of the problem, we set the action set for player $n \in \mathcal{N}$ as $\mathcal{A}_n = [e_n^{\min}, e_n^{\max}] \subset \mathbb{R}$. The set of possible actions is denoted by $\mathcal{A} = \prod_{n=1}^N \mathcal{A}_n$, and \mathcal{A}_{-n} is the set of possible collective actions of all agents other than n . An action of player n is given by $a_n \in \mathcal{A}_n$. For an action profile $a = (a_1, a_2, \dots, a_N) \in \mathcal{A}$, let a_{-n} be the action profile of agents other than agent n , i.e., $a_{-n} = (a_1, \dots, a_{n-1}, a_{n+1}, \dots, a_N) \in \mathcal{A}_{-n}$. With this notation, we sometimes write an action profile a as (a_n, a_{-n}) . The set of geophysical states x is denoted by $\mathcal{X} \subset \mathbb{R}^{m_1 + m_2}$. The utility of a player $n \in \mathcal{N}$, is a trade-off between a benefit function that depends on its own emissions, minus a weighted global damage function, which is a function of the projection of the atmospheric temperature. The utility function is defined in the following definition.

Definition 3.1. For all $x \in \mathcal{X}$, $(a_n, a_{-n}) \in \mathcal{A}$, we define the utility function for player $n \in \mathcal{N}$,

$u_n : \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$, as a profit function $B_n : \mathcal{A}_n \rightarrow \mathbb{R}$ minus a damage positively weighted by w_n , $D : \mathbb{R} \rightarrow \mathbb{R}$, as follows:

$$u_n(x, a_n, a_{-n}) = B_n(a_n) - w_n D(\theta_{AT}(x, a)). \quad (3.45)$$

Note that, unlike [15], individual damages D_n are expressed in terms of collective damage; we assume that $D_n = w_n D$. This assumption will be useful in the following chapters of the thesis. It is worth mentioning that in the literature, one can find such shared damage functions, as in Nordhaus' RICE model [97].

3.6.3 Assumptions on Benefit and Damage Functions

3.6.3.1 Benefit Functions

Assumption 3.1. We assume that the benefit functions are characterized by strict monotonicity, continuity, and twice differentiability with respect to player n 's action, where $n \in \mathcal{N}$. This implies that the derivative of B_n must consistently exhibit strict positivity:

$$\frac{\partial B_n}{\partial a_n}(a_n) > 0, \quad \forall n \in \mathcal{N}, \forall a_n \in \mathcal{A}_n.$$

Remark 3.1. This assumption aligns with real-world observations, illustrating the correlation between GDP per Capita and CO₂ per Capita. These dynamics include unmodeled factors such as investments in energy efficiency, transitions to alternative energy sources, and other relevant variables.

The significance of this hypothesis is its ability to encompass a spectrum of benefit function shapes, as illustrated in Figure 3.4.

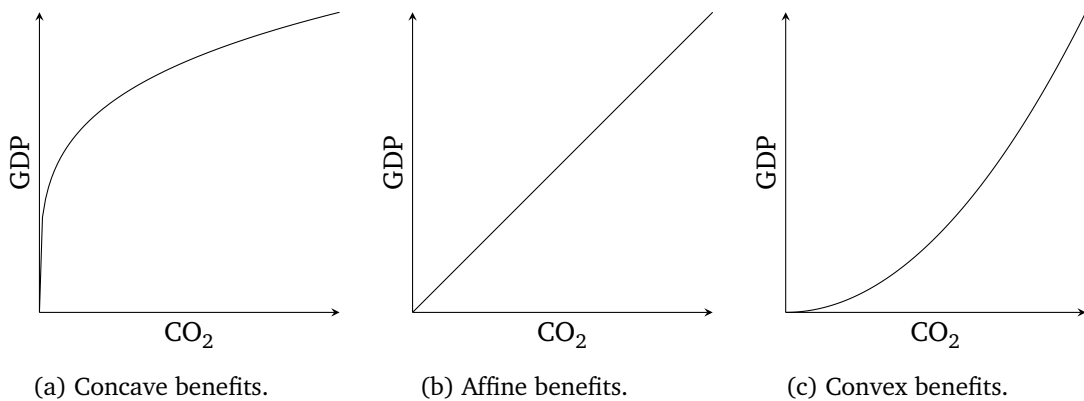


FIGURE 3.4 – Different shapes of benefit functions found in the literature.

3.6.3.2 Damage Function

Assumption 3.2. *We assume that the damage function exhibits strict monotonicity, continuity, and twice differentiability with respect to θ_{AT} , the atmospheric temperature. This implies that the derivative of D must be strictly positive:*

$$D'(\theta_{AT}(x, a)) > 0, \quad \forall x \in \mathcal{X}, \forall a \in \mathcal{A}.$$

The first assumption describes the fact that an increase in profit is achieved through an increase in consumption (energy), reflecting an increase in emissions. The second assumption emphasizes that a higher atmospheric temperature will have a more significant impact on the economy of each player. Convexity implies that this phenomenon increases for a higher temperature. These assumptions are reasonable and common in the literature.

3.7 Conclusion

In conclusion, this chapter introduces our exploration of global emissions reduction strategies in the context of climate change. We began by highlighting the imperative addressed by the COP to tackle the pressing issue of global warming through international agreements. However, we underscored the limitations of current strategies, primarily influenced by short-term economic considerations, frustrating collaborative efforts for emission reductions.

We continued with a comprehensive survey that includes climate and game theoretical frameworks, pivotal in informing the structure of our devised game model. Drawing inspiration from climate models, we aimed to clarify the complex circumstances influencing nations' behavior in emissions reduction. This critical examination serves as fundamentals to our analysis in the following chapters, where we propose and study static and imitation game models.

We showed the DICE economic model and the game proposed by Finus, which were our motivations for formulating our game. We presented our model and highlighted our contributions. Indeed, we proposed a game inspired by Finus, but using a weighted global damage function, as a function of the temperature, which was considered in the DICE model. Finus did not consider the atmospheric temperature in his model, and Nordhaus did not study a game in his economic DICE model.

As we transition to the following chapters, we carry forward the motivation illustrated in this chapter. The exploration of static and imitation game models will provide valuable insights into the circumstances under which countries exhibit environmentally conscientious behavior. Using game theory, we aim to contribute to the discussion on sustainable solutions for addressing the urgent and complex challenge of global warming.

Game of non-influenced players

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Abstract - *The COP aims to address the global warming problem through agreements for reducing emissions. However, the current strategies fall short due to minimal efforts for emission reductions driven by short-term economic considerations. To better understand under which circumstances countries may have more virtuous behavior, we analyze a static game model inspired from the literature, where the players are not influenced by the other players. In this model, the players are represented by governments, and their actions correspond to emissions levels. The utilities are a trade-off between economic benefits and damage caused by climate change. A key feature of the game is that it is parameterized by the state of the climate dynamics. We conduct the Nash equilibrium analysis. In the numerical analysis, we assess the impact of the damage function on the behavior of the governments.*

4.1 Introduction

In this chapter, we will study the static game formulated in Chapter 3, and where the players are not connected to the other players. We will start by recalling the atmospheric temperature, since using it in the damage function is an important contribution of this thesis.

Atmospheric temperature In this thesis, we consider that the damage depends on the projection of the global average atmospheric temperature. It is important to note that the utility of each player depends on the actions played and the output $\theta_{AT}(t + 1)$ of the climate model. We denote that $\theta_{AT}(t + 1)$ is a function of the players' actions a and the game state x , at time t . Indeed, we omit the time t since the game is static, and we consider θ_{AT} as a function $\theta_{AT}(x, a)$.

To make the notations easier for the reader, we will replace \tilde{b}_C , and \tilde{b}_θ , given in the previous chapter by b_C , and b_θ respectively. To establish a connection between climate modeling and the game, the atmospheric temperature can be expressed as follows:

$$\theta_{AT}(x, a) = \psi_\theta(x) + b_\theta \ln \left(\psi_C(x) + b_C \sum_{n=1}^N a_n \right), \quad (4.1)$$

In this study, we make the following key contributions to the field of environmental game theory:

- We propose a novel static potential game and where the utility function is a function of the geophysical state and the players' actions.
- We provide the expression of the unique pure Nash equilibrium, in some sufficient conditions, for the case with quadratic functions.
- We assess numerically, the effects of the economic damage function due to climate change modeling on the behavior of the countries in terms of CO₂ emissions.

The rest of the chapter is organized as follows. The problem analyzed in this work is formulated and the subsequent game-theoretical analysis is provided in Section 4.2, where we study the existence and the uniqueness of the Nash equilibrium in a specific case. Numerical simulations illustrate our results in Section 4.3 and provide several insights and societal interest. We conclude the chapter and give some perspectives in Section 4.4.

4.2 Game-theoretic analysis

This chapter is the analysis of the game proposed in Chapter 3, where we will prove the existence of a pure Nash equilibrium, and we will provide a sufficient condition for the uniqueness of the pure Nash equilibrium, in the general case. Moreover, we consider a case

study where the benefit function of player n is quadratic in his action a_n , and the damage function is quadratic in the atmospheric temperature. We start by recalling the utility function with the case study that we are proposing in the following definition.

Definition 4.1. *The utility function for the n^{th} player is chosen to be a difference between an individual benefit function B_n and a (weighted) global cost/damage function D :*

$$u_n(x, a) = \sum_{i=0}^2 \beta_{i,n} a_n^i - w_n \sum_{i=0}^2 \gamma_i \theta_{\text{AT}}^i(x, a) := B_n(a_n) - w_n D(\theta_{\text{AT}}(x, a)), \quad (4.2)$$

where w_n is a positive weight that measures the economic impact of climate change on the player n . In the sequel we denote by $(u_n)_{n \in \mathcal{N}}$, the family of utility functions which defines the strategic form of the static game under study.

The following analysis will highlight the potentiality of this game, leading to the existence of a pure Nash Equilibrium. We begin by recalling the definition of a weighted potential game [2]. Then, we prove that the proposed game is a weighted potential game in the following proposition.

4.2.1 Existence and uniqueness of a pure Nash equilibrium

An important property for a game is precisely to know whether it possesses a pure Nash equilibrium. It turns out that, by construction, the game under study has always a pure Nash equilibrium. This is because it belongs to the class of weighted potential games as defined by Monderer and Shapley [2].

4.2.1.1 Existence of a pure Nash equilibrium

Using the definition of a weighted potential game recalled in the preliminaries chapter, we prove that the game that we proposed is weighted potential, in the following proposition.

Proposition 4.1. *Our game is a weighted potential game for any $B_n, n \in \mathcal{N}$, D , θ_{AT} and for any positive weights $w_n, n \in \mathcal{N}$, with the following potential function, for all $a \in \mathcal{A}, x \in \mathcal{X}$,*

$$\phi(x, a) = \sum_{n=1}^N \frac{1}{w_n} B_n(a_n) - D(\theta_{\text{AT}}(x, a)). \quad (4.3)$$

Proof. To prove that the function ϕ is a potential for the game Γ , consider for all $a_n, a'_n \in$

$\mathcal{A}_n, a_{-n} \in \mathcal{A}_{-n}$ where $a_n \neq a'_n$ and for some $w_n > 0, n \in \mathcal{N}$,

$$\begin{aligned}
 & \phi(x, a) - \phi(x, a'_n, a_{-n}) \\
 = & \sum_{n=1}^N \frac{1}{w_n} B_n(a_n) - D(\theta_{\text{AT}}(x, a)) - \sum_{\substack{m=1 \\ m \neq n}}^N \frac{1}{w_m} B_m(a_m) - \frac{1}{w_n} B_n(a'_n) + D(\theta_{\text{AT}}(x, a'_n, a_{-n})) \\
 = & \frac{B_n(a_n) - B_n(a'_n)}{w_n} + D(\theta_{\text{AT}}(x, a'_n, a_{-n})) - D(\theta_{\text{AT}}(x, a)) \\
 = & \frac{B_n(a_n) - w_n D(\theta_{\text{AT}}(x, a))}{w_n} - \frac{B_n(a'_n) - w_n D(\theta_{\text{AT}}(x, a'_n, a_{-n}))}{w_n} \\
 = & \frac{u_n(x, a) - u_n(x, a'_n, a_{-n})}{w_n}.
 \end{aligned} \tag{4.4}$$

Thus, the game Γ is a weighted potential game with a potential function ϕ and positive weights w_n , for all $n \in \mathcal{N}$. ■

The previous results yield the potentiality of the game with general utility function, given by the difference between a benefit part and a weighted global damage part, without considering a specific B_n , nor a specific D . Indeed, we will use the following lemma from Lã et al.[98], to conclude the existence of a pure Nash equilibrium when the potential function is continuous in $a \in \mathcal{A}$.

Lemma 4.1. Consider $\Gamma = (\mathcal{N}, \mathcal{A}, (u_n)_{n \in \mathcal{N}})$ a weighted potential game of potential function ϕ , and positive weights w_n , and the game of identical utility functions $\hat{\Gamma} = (\mathcal{N}, \mathcal{A}, (\phi)_{n \in \mathcal{N}})$. The sets of Nash equilibria of both games Γ and $\hat{\Gamma}$ coincide.

$$NE(\Gamma) = NE(\hat{\Gamma}).$$

Proposition 4.2. If the benefit and damage functions B_n and $D \circ \theta_{\text{AT}}$ are continuous with respect to the actions $a \in \mathcal{A}$, then there exists at least one pure Nash equilibrium of the game Γ .

Proof. Considering that $\mathcal{A}_n = [e_n^{\min}, e_n^{\max}]$ are compact sets, then the finite product \mathcal{A} is compact as well. Moreover assuming B_n and $D \circ \theta_{\text{AT}}$ are continuous with respect to a , ϕ is continuous as well. Then, ϕ reaches its maximum by the extreme value theorem. This leads to the game $\hat{\Gamma}$ admitting at least one pure Nash equilibrium. Finally, from Lemma 4.1, the game Γ admits also at least one pure Nash equilibrium. ■

Remark 4.1. Even if we do not have the continuity assumption, the existence of a NE still holds if the action sets are finite (this case could appear in some simulations). Since range of ϕ is a finite, ordered, and bounded subset of \mathbb{R} .

4.2.1.2 Uniqueness of pure Nash equilibrium

We study, in this subsection, the uniqueness of a pure Nash equilibrium, depending on the concavity of the potential function. We start by studying the convex case, then we go into details in the case of the concave potential function.

4.2.1.2.1 Potential function strictly convex In this subsection, we aim to establish that there exist at most 2^N Nash equilibria, and the set of these points is defined by:

$$\text{NE}_\Gamma = \{(a_1, \dots, a_N) \mid a_n \in \{e_n^{\min}, e_n^{\max}\} \quad \forall n \in \mathcal{N}\}. \quad (4.5)$$

The proof of this assertion relies on convex set properties, with \mathcal{A} having a specific form and the strict convexity of ϕ . To start, we recall the following lemma:

Lemma 4.2. *Let A and $B \subset \mathbb{R}^N$ be convex sets such that $\text{int}A \cap \text{int}B \neq \emptyset$. Then,*

$$\text{Fr}A \cap \text{int}B = \emptyset \Leftrightarrow B \subset A. \quad (4.6)$$

Proof. (\Rightarrow) By contraposition. Assume there exists $y \in B \setminus A$. Let $x \in \text{int}A \cap \text{int}B$, and let $[x, y[= \{z = (1 - \lambda)x + \lambda y \mid 0 \leq \lambda < 1\}$. Since B is convex and $x \in \text{int}A$, we have that $\forall z \in [x, y[, z \in \text{int}B$. Moreover, since $x \in \text{int}A$ and $y \in B \setminus A$, we have $\emptyset \neq ([x, y[\cap \partial A) \subset \text{int}B$. Then, $\partial A \cap \text{int}B \neq \emptyset$.

(\Leftarrow) $B \subset A \Rightarrow \text{int}B \subset \text{int}A \Rightarrow \partial A \cap \text{int}B = \emptyset$. ■

The previous lemma is instrumental in proving the following proposition.

Proposition 4.3. *Let f be a convex continuous function with domain D . If there exists $x^0 \in \text{int}D$ such that $f(x^0) < f(\bar{x})$, then*

$$\bar{x} \in \arg \max\{f(x) \mid x \in D\} \Leftrightarrow \{x \in \mathbb{R}^n \mid f(x) = f(\bar{x})\} \cap \text{int}D = \emptyset. \quad (4.7)$$

Proof. Since f is convex and continuous on \mathbb{R}^n , it is lower-semicontinuous and upper-semicontinuous. Lower-semi-continuity implies that $A = \{x \in \mathbb{R}^n \mid f(x) \leq f(\bar{x})\}$ is closed, and upper-semi-continuity with $f(x^0) < f(\bar{x})$ implies that $\emptyset \neq \{x \in \mathbb{R}^n \mid f(x) < f(\bar{x})\} \subset \text{int}A$ is open. Moreover, since $\partial A = A \setminus \text{int}A$, we have that $\{x \in \mathbb{R}^n \mid f(x) = f(\bar{x})\} \supset \partial A$; but by definition, we have $\{x \in \mathbb{R}^n \mid f(x) = f(\bar{x})\} \subset \partial A$ and then the equality. Now replacing B in the lemma by D leads to (4.7). ■

With the preceding proposition, we conclude that if ϕ is strictly convex, there are at most 2^N Nash equilibria. One sufficient condition for uniqueness is that for a given $a_{-n} \in \mathcal{A}_{-n}$, the

functions $\psi_n(\cdot; a_{-n}) : \mathcal{A}_n \rightarrow \mathbb{R}$ defined by $\psi_n(a_n, a_{-n}) = \phi(a)$ do not reach their minimum on the interior of \mathcal{A}_n for all $n \in \mathcal{N}$ (for example, having the derivative never equal to 0 over \mathcal{A}_n).

4.2.1.2.2 Potential function strictly concave Next, we will provide a necessary condition for uniqueness, in the case where the potential function is concave. We consider a concave benefit function and a convex damage function since the connection between GDP and emissions is frequently represented through a concave function. Also, a quadratic damage convex function is mostly used in the economic literature focusing on the consequences of climate change.

Proposition 4.4. *Supposing that $\gamma_2 > 0$ and for all $n \in \mathcal{N}$, if*

$$\frac{\gamma_2 b_\theta^2 b_C w_n}{e_n^{\max}} \exp\left(\frac{\gamma_1 + 2\gamma_2 \psi_\theta(x)}{2\gamma_2 b_\theta} - 1\right) - \frac{\beta_{1,n}}{2e_n^{\max}} < \beta_{2,n} < 0. \quad (4.8)$$

Then the pure Nash equilibrium is unique, and it corresponds to all players emitting to the maximum, i.e., $a_n = e_n^{\max}$, $\forall n \in \mathcal{N}$.

Proof. If the following condition is satisfied,

$$\min_{a_n \in \mathcal{A}_n} \frac{B'_n(a_n)}{w_n} > \max_{a_n \in \mathcal{A}_n} \frac{\partial [D(\theta_{AT}(x, a))]}{\partial a_n}, \quad (4.9)$$

then one has a unique pure Nash equilibrium. Indeed, in this case, the utility functions will be strictly increasing, and then the maximum is attained when all players emit the maximum of possible emissions. Straightforward computation shows that:

$$\min_{a_n \in \mathcal{A}_n} B'_n(a_n) = \min_{a_n \in \mathcal{A}_n} [\beta_{1,n} + 2\beta_{2,n}a_n] \stackrel{\text{if } \beta_{2,n} < 0}{=} \beta_{1,n} + 2\beta_{2,n}e_n^{\max}. \quad (4.10)$$

On the other hand, the first derivative of the damage with respect to a_n is independent of $n \in \mathcal{N}$. Indeed, $\frac{\partial [D(\theta_{AT}(x, a))]}{\partial a_n} = \frac{\partial [D(\theta_{AT}(x, a))]}{\partial a_m}$, for all $n, m \in \mathcal{N}$, and it is given by the following:

$$\frac{\partial [D(\theta_{AT}(x, a))]}{\partial a_n} = \frac{b_\theta b_C \left[\gamma_1 + 2\gamma_2 \left[\psi_\theta(x) + b_\theta \ln \left(\psi_C(x) + b_C \sum_{n=1}^N a_n \right) \right] \right]}{\psi_C(x) + b_C \sum_{n=1}^N a_n}. \quad (4.11)$$

In this part of the proof, we will use the parameters $a, b, c, d, k \in \mathbb{R}$ that are constants, and independent of the problem formulated before. They are used to ease the presentation of the variation of the function. To find the maximum of (4.11), we consider the case where $\gamma_2 > 0$. Let us find the maximum of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ for $a, b, c, k \in \mathbb{R}_+^*$, $d \in \mathbb{R}$, given for all $z \in \mathbb{R}$ by:

$$f(z) = \frac{k(d + c \ln(a + bz))}{a + bz}.$$

When differentiating f with respect to $z \in \mathbb{R}$, we find the unique root of f' given by $z_0 = (e^{(1-d/c)} - a)/b$. Then simply computing $f'((e^{-d/c} - a)/b) = kbc/e^{(4-2d/c)} > 0$, and $f'((e^{(2-d/c)} - a)/b) = -kbc/e^{(4-2d/c)} < 0$ provides that f' is strictly decreasing. We can conclude that the function f is strictly concave and reaches its maximum at z_0 , given by $f(z_0) = kc \exp(d/c - 1)$. Now by using f with $a = \psi_C(x) + b_C \sum_{m=1, m \neq n}^N a_m$, $b = b_C$, $c = 2\gamma_2 b_\theta$, $d = \gamma_1 + 2\gamma_2 \psi_\theta(x)$ and $k = b_\theta b_C$, we conclude that the maximum of D' is given by:

$$\max_{a_n \in \mathcal{A}_n} \frac{\partial [D(\theta_{AT}(x, a))]}{\partial a_n} = 2\gamma_2 b_\theta^2 b_C \exp\left(\frac{\gamma_1 + 2\gamma_2 \psi_\theta(x)}{2\gamma_2 b_\theta} - 1\right).$$

After minimizing the benefit variations and maximizing the damage variations we get that $\min_{a_n \in \mathcal{A}_n} \frac{B'_n(a_n)}{w_n} > \max_{a_n \in \mathcal{A}_n} \frac{\partial [D(\theta_{AT}(x, a))]}{\partial a_n}$ is equivalent to:

$$\frac{\beta_{1,n} + 2\beta_{2,n} e_n^{\max}}{w_n} > 2\gamma_2 b_\theta^2 b_C \exp\left(\frac{\gamma_1 + 2\gamma_2 \psi_\theta(x)}{2\gamma_2 b_\theta} - 1\right),$$

which is equivalent to (4.8). □ ■

Proposition 4.4 basically states that looking at the short term, all the countries will emit as much as possible *i.e.* $a_n = e_n^{\max}, \forall n \in \mathcal{N}$, as long as the damage function does not have a sufficiently large impact. This can be changed either by considering less optimistic damage functions or looking at the long-term behavior when the atmospheric temperature is higher which will lead to larger damages.

4.2.2 Expression of the pure Nash equilibrium

In this section, the goal is to express the Nash equilibrium actions for the player. The motivation for this is twofold; it makes interpretations much easier (e.g., the impact of radiative forcing or the damage severity level on the behavior of the countries) and it renders the problem of computing the equilibrium very simple to solve. To express the NE, let us

assume from now on that ϕ is strictly concave. The pure NE is denoted by $a^* = (a_1^*, \dots, a_N^*)$ where either there exists $n \in \mathcal{N}$ such that $a_n^* \in \{e_n^{\min}, e_n^{\max}\}$, or a^* is an interior NE. In the latter case, the players will tend to reduce their emissions.

We recall that the potential function is defined by (4.3) with the atmospheric temperature θ_{AT} given by:

$$\theta_{\text{AT}}(x, a) = \psi_{\theta}(x) + b_{\theta} \ln \left(\psi_C(x) + b_C \sum_{n=1}^N a_n \right). \quad (4.12)$$

Proposition 4.5. *If ϕ is strictly concave and differentiable, the Nash is $a^* = (a_1^*, \dots, a_N^*)$ that satisfies, for all $n \in \mathcal{N}$, the following N equations*

$$\frac{1}{w_n} B'_n(a_n) - \frac{\partial [D(\theta_{\text{AT}}(x, a))]}{\partial a_n} = \bar{\lambda}_n - \underline{\lambda}_n. \quad (\text{KKT})$$

with $\bar{\lambda}_n, \underline{\lambda}_n \geq 0$, $\underline{\lambda}_n^*(a_n^* - e_n^{\min}) = 0$, and $\bar{\lambda}_n^*(a_n^* - e_n^{\max}) = 0$ being the KKT multipliers with associated constraints.

Proof. The proof is straightforward:

ϕ is continuous over \mathcal{A} , then there exists a NE, a^* . Moreover, if ϕ is strictly concave, then the NE is unique. Since the constraints are linear, we can apply the KKT conditions.

■

In the proposition below, we provide sufficient conditions to express the NE. To do so, we will need the following lemma on the zeros of the equation $rs^2 + ps + q = k \ln(rs + v)$.

Lemma 4.3. *For $k, p, q \in \mathbb{R}$, $r, v \in \mathbb{R}_+^*$, and for all $s \in \mathbb{R}$ such that $rs + v > 0$, the following equation in s :*

$$rs^2 + ps + q = k \ln(rs + v), \quad (4.13)$$

- has at most one solution if $rk + (2v - p)^2/8 < 0$,
- has at most two solutions if $rk + (2v - p)^2/8 = 0$,
- has at most three solutions if $rk + (2v - p)^2/8 > 0$.

Proof. To find the possible number of solutions of the equation (4.13), we define the

set S as $S := \left[\sum_{n=1}^N e_n^{\min}, \sum_{n=1}^N e_n^{\max} \right]$. Let the function f be given by the following:

$$\begin{aligned} f : S &\longrightarrow \mathbb{R} \\ s &\longrightarrow f(s) = rs^2 + ps + q - k \ln(rs + v) \end{aligned} \quad (4.14)$$

By finding the roots of the derivative f' , we can discuss the possible number of solutions of the equation (4.13). Indeed, for all $s \in S$, we have:

$$\begin{aligned}
f'(s) = 0 &\iff 2rs + p - \frac{rk}{rs+v} = 0 \\
&\iff 2r^2s^2 + 2rvs + rps + pv = rk \\
&\iff 2r^2s^2 + (p + 2v)rs + pv = rk \\
&\iff \left(r\sqrt{2} + \frac{p+2v}{2\sqrt{2}}\right)^2 - \left(\frac{p+2v}{2\sqrt{2}}\right)^2 + pv = rk \\
&\iff \left(r\sqrt{2} + \frac{p+2v}{2\sqrt{2}}\right)^2 = \frac{p^2}{8} + \frac{4pv}{8} + \frac{4v^2}{8} + rk - pv \\
&\iff \left(r\sqrt{2} + \frac{p+2v}{2\sqrt{2}}\right)^2 = \frac{p^2}{8} - \frac{pv}{2} + \frac{v^2}{2} + rk \\
&\iff \left(r\sqrt{2} + \frac{p+2v}{2\sqrt{2}}\right)^2 = \frac{(2v-p)^2}{8} + rk
\end{aligned} \tag{4.15}$$

Then, the sign of $rk + \frac{(2v-p)^2}{8}$, will decide about the number of roots of the derivative f' . We have three cases given as follows:

1. If $rk + \frac{(2v-p)^2}{8} < 0$, then f' has no root, which leads to conclude that f is monotone. Therefore, if $rk + \frac{(2v-p)^2}{8} < 0$, the equation (4.13) has at most one solution.
2. If $rk + \frac{(2v-p)^2}{8} = 0$, then f' has one root, which leads to conclude that f changes of sign once. Therefore, if $rk + \frac{(2v-p)^2}{8} = 0$, the equation (4.13) has at most two solutions.
3. If $rk + \frac{(2v-p)^2}{8} > 0$, then f' has two roots, which leads to conclude that f changes of sign twice. Therefore, if $rk + \frac{(2v-p)^2}{8} > 0$, the equation (4.13) has at most three solutions.

■

When ϕ is strictly concave, we can apply the KKT conditions to find the unique pure NE. Let us assume that ϕ is continuous on \mathcal{A} , so there exists a unique pure NE, denoted by a^* . Applying Proposition 4.5, we have for every $n \in \mathcal{N}$, a^* verifies (KKT). This leads to the sufficient condition to get a unique interior NE given in the following proposition.

This condition highlights the case where the benefits are concave *i.e.* $\beta_{2,n} < 0$, and the damage is convex *i.e.* $\gamma_2 > 0$. In this case, when γ_2 is large enough, which means that the damages are severe, the sufficient condition that we are proposing is satisfied, and the players will tend to reduce their emissions, thus they will not tend to emit to the maximum.

Proposition 4.6. *Assuming that ϕ is strictly concave with $\beta_{2,n} \neq 0, \forall n \in \mathcal{N}$, and denoting*

$\Lambda_n := \frac{w_n (\underline{\lambda}_n - \bar{\lambda}_n) - \beta_{1,n}}{2\beta_{2,n}}$. If the following condition is satisfied,

$$b_\theta^2 b_C^2 \gamma_2 \sum_{n=1}^N \frac{w_n}{\beta_{2,n}} + \frac{\left(\psi_C(x) + b_C \sum_{n=1}^N \Lambda_n \right)^2}{8} < 0, \quad (4.16)$$

then the unique pure NE of the game Γ is given by $a^* = (a_1^*, \dots, a_N^*)$, where for all $n \in \mathcal{N}$,

$$a_n^* = \Lambda_n + \frac{w_n b_\theta b_C [\gamma_1 + 2\gamma_2 \psi_\theta(x) + 2\gamma_2 b_\theta \ln(\psi_C(x) + b_C \tilde{S})]}{2\beta_{2,n} (\psi_C(x) + b_C \tilde{S})}, \quad (4.17)$$

where \tilde{S} is the unique solution of the equation $rs^2 + ps + q = k \ln(rs + v)$, with

$$r = b_C, \quad (4.18)$$

$$p = \psi_C(x) - b_C \sum_{n=1}^N \Lambda_n, \quad (4.19)$$

$$q = -\psi_C(x) \sum_{n=1}^N \Lambda_n - \sum_{n=1}^N w_n \frac{b_\theta b_C (\gamma_1 + 2\gamma_2 \psi_\theta)}{2\beta_{2,n}}, \quad (4.20)$$

$$k = b_\theta^2 b_C \gamma_2 \sum_{n=1}^N \frac{\beta_{2,n}}{w_n}, \quad (4.21)$$

$$v = \psi_C(x). \quad (4.22)$$

Proof. Using Proposition 4.5, for all $n \in \mathcal{N}$ one has

$$\frac{1}{w_n} (\beta_{1,n} + 2\beta_{2,n} a_n^*) - [\gamma_1 + 2\gamma_2 \theta_{\text{AT}}(x, a^*)] \frac{\partial \theta_{\text{AT}}}{\partial a_n}(x, a^*) = \underline{\lambda}_n - \bar{\lambda}_n.$$

Dividing by $\beta_{2,n} \neq 0$ and using the notation Λ_n introduced in the statement, we get that $\forall n \in \mathcal{N}$,

$$a_n^* = \Lambda_n + \frac{w_n b_\theta b_C \left[\gamma_1 + 2\gamma_2 \psi_\theta(x) + 2\gamma_2 b_\theta \ln \left(\psi_C(x) + b_C \sum_{n=1}^N a_n^* \right) \right]}{2\beta_{2,n} \left(\psi_C(x) + b_C \sum_{n=1}^N a_n^* \right)}. \quad (4.23)$$

Let us introduce the notation $s := \sum_{n=1}^N a_n^*$. Summing (4.23) over $n \in \mathcal{N}$ yields:

$$s = \sum_{n=1}^N \Lambda_n + \sum_{n=1}^N \frac{w_n}{2\beta_{2,n}} \frac{b_\theta b_C [\gamma_1 + 2\gamma_2 \psi_\theta(x) + 2\gamma_2 b_\theta \ln(\psi_C(x) + b_C s)]}{\psi_C(x) + b_C s},$$

which can be re-written as $rs^2 + ps + q = k \ln(rs + v)$ with r, p, q, v , and k given in the statement above. By using the results of the lemma 4.3, we conclude that if (4.16) is verified, then there exists at most one solution \tilde{S} of the equation $rs^2 + ps + q = k \ln(rs + v)$ in $\left[\sum_{n=1}^N e_n^{\min}, \sum_{n=1}^N e_n^{\max} \right]$. Thus, the unique pure NE is given by the profile of actions $a^* = (a_1^*, \dots, a_N^*)$, where for all $n \in \mathcal{N}$, a_n^* satisfies Equation (4.23). ■

4.3 Numerical analysis

We will illustrate the previous theoretical results and the behavior of the proposed model. We have implemented a code that allows us to play with the parameters, whether they are from the SCM or the benefit and damage functions. We set $N = 6$ with the parameters specified in Table 4.1, where AOC refers to all other countries. For the presented graphs, we will use the CC from [55] and the TD from [50], as they are often considered the closest to the IPCC results [99, 100].

Player	e_n^{\max} (GtCO ₂ /y)	GDP _n ^{max} (10 ⁹ \$)	w_n
China	11	14630	1.1847
USA	7	19290	1.1941
EU	4	13890	1.1248
India	3	2500	0.9074
Russia	2	1420	1.2866
AOC	5	11640	1.1847

TABLE 4.1 – Specific values for each player of the static game in 2020.

In this section, we will study two cases, in the first one, we consider a quadratic benefit function in player n 's emissions and quadratic global damage in the projection of the atmospheric temperature. Afterward, we consider the case with sigmoid benefit function in player n 's emissions and quadratic global damages. In both cases, we illustrate the power that we can give to the damage function, by a power α .

4.3.1 Quadratic benefits and damages

We illustrate the case where the benefit function is quadratic in a_n , and the damages are quadratic and re-scaled, *i.e.* ,

$$u_n(x, a) = \text{GDP}_n^{\max} \left(2 \frac{a_n}{e_n^{\max}} - \left(\frac{a_n}{e_n^{\max}} \right)^2 - w_n [D(\theta_{AT}(x, a))]^\alpha \right), \quad (4.24)$$

where α represents the power of the damages and measures the severity level of climate change on the economics. The static game is played repetitively every five years until 2100 while updating e_n^{\max} and GDP_n^{\max} at each iteration of the game. Indeed, the game in the simulations is considered to be a repeated game where the players decide their strategy at every step of the game, independently of the history and the next steps. For more information, refer to [101].

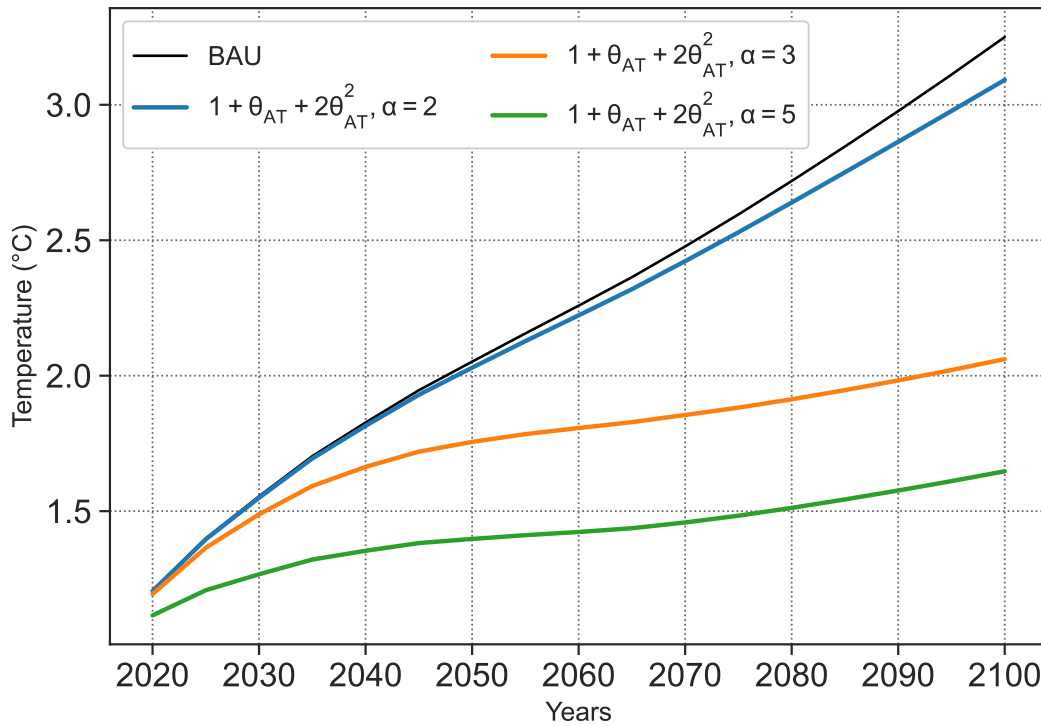


FIGURE 4.1 – The increase of the forecast temperature due to the CO₂ emissions at NE in different scenarios.

It is interesting to note that higher α induces higher damages and consequently lower

CO₂ emissions and smaller increases in the temperature. For large α (e.g., $\alpha = 5$), China, the USA, the EU, and AOC reduce their emissions until they completely stop emitting (see Figure 4.2). The temperatures in 2100 range from around +3.2°C for low damages, resembling a Business-as-Usual (BAU) scenario, to +1.6°C for high damages. These temperature levels are in line with the projections of the IPCC [99] and correspond to the emission trajectories of the countries. In order to prevent the over-warming of the planet by 2100 we need to revise the modeling of the economic damages and change the strategies accordingly. The idea is that the players are not able to see the real damage caused by their emissions, since the damage functions are under-estimating it, and this is the reason why the players are not interested in stopping emitting. In short, if the damages are not significant, the players will continue to emit excessively, even emit at maximum, so the atmospheric temperature will continue to rise rapidly.

Since quadratic damages, which are commonly used in the literature, are too optimistic and yield lower temperatures than those currently observed, the question arises: what level of damages would induce the players to stop emitting? The Table 4.2 presents the time at which countries stop emissions based on different values of α .

Player	China	USA	EU	India	Russia	AOC
$\alpha = 1$	/	/	/	/	/	/
$\alpha = 2$	/	/	/	/	/	/
$\alpha = 3$	2065	2075	/	/	/	2080
$\alpha = 4$	2020	2020	2045	/	/	2025
$\alpha = 5$	2020	2020	2020	/	/	2020
$\alpha = 6$	2020	2020	2020	/	/	2020
$\alpha = 7$	2020	2020	2020	/	/	2020
$\alpha = 8$	2020	2020	2020	2095	/	2020
$\alpha = 9$	2020	2020	2020	2085	/	2020
$\alpha = 10$	2020	2020	2020	2080	/	2020

TABLE 4.2 – Time at which the countries stop emitting versus α (which measures the economic damage due to climate change). The symbol / means no stopping.

Table 4.2 shows that if the damages are not significant, *i.e.*, α is small, the CO₂ emissions of the players will not stop before 2100. Low damage hampers the cooperation recommended by the IPCC. When α is large enough, the NE strategies of the players are to stop emitting as soon as possible. Except for Russia which continues to emit no matter how big is the damage, and for India which stops emitting only when $\alpha \geq 8$. This can be explained by the fact that the benefits of India and Russia are still very big compared with the corresponding loss. The product $w_n \text{GDP}_n^{\max}$ has to be increased for these countries in order to stop their

emissions. This is also because Russia has the smallest maximum emissions, so when other countries reduce emissions, the damage decreases and becomes less than their benefits, thus increasing Russia's utility. Therefore, as player n 's emissions affect other countries, being a nation with high emissions, player n can incentivize countries with low maximum emissions to continue emitting when player n reduce its emissions, as they will incur lower damages.

4.3.2 Sigmoid benefit and quadratic damage case

We illustrate the case where the benefit function is a sigmoid between 0 and PIB_n^{\max} , and the damages are quadratic and rescaled, *i.e.*,

$$u_n(x, a) = B_n(a_n) - w_n PIB_n^{\max} [D(\theta_{AT}(x, a))]^\alpha, \quad (4.25)$$

with α as the power of damage. In this part of the simulations, EU, India, and Russia exhibit convex benefits curves, while the other players have concave ones, though reaching their maximum benefits at different points in time. China and AOC reach their maximum benefits much earlier than the USA, which only reaches it for their maximum emissions. For realism, we update e_n^{\max} and PIB_n^{\max} at each iteration of the game. We consider that our static game, is played repetitively every five years until 2100. For more information, see the code [101].

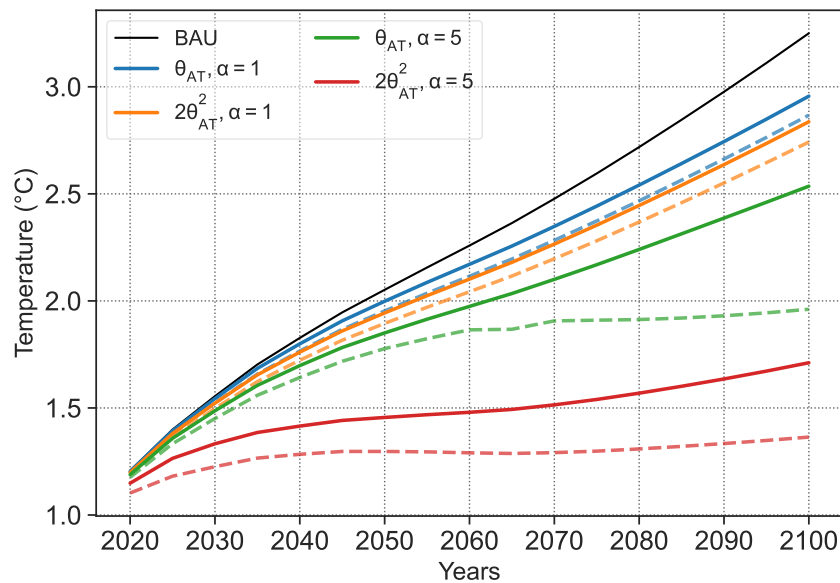


FIGURE 4.2 – Increase in θ_{AT} in $^{\circ}C$. Solid and dashed lines represent NE and social optimum (SO), respectively.

It is interesting to note the importance of the temperature trajectory achieved by players

in the NE and SO frameworks. This is because the variation in damages is small compared to that of benefits. Especially for small α values (e.g., $\alpha = 1$), players have no incentive to reduce their emissions, as clear in Figure 4.3. For large α values (e.g., $\alpha = 5$), China, the USA, and the EU reduce their emissions until they stop emitting. In Figure 4.2, the curves represent different damages. The temperatures in 2100 are close to $+3^\circ\text{C}$ for low damages, resembling a Business As Usual (BAU) strategy, while for high damages, it decreases to $+1.6^\circ\text{C}$. These temperatures are consistent with the IPCC projections [99] and correspond to the different emission trajectories of the countries, illustrated in Figure 4.3. To be able to see the real damage caused by the actual emissions, the modeling of economic damages must, therefore, be a top priority to change economic actors' strategies and prevent them from leading us to a $+3^\circ\text{C}$ world by 2100. Several studies have been conducted, but there is still too much uncertainty on this topic.

Indeed, with sigmoidal benefits, temperatures also hover around 1.5°C for higher values of α . Furthermore, considering a social optimum demonstrates that we can achieve less than 1.5°C , which is the limit set in the Paris Agreement, it shows that the cooperation between the players will lead to better results.

Furthermore, we note in Figure 4.3 that if damages are not high, the NE strategies of the players are close to SO strategies. This is a barrier to cooperation recommended by the IPCC. This can be explained by the lack of planning, which, as we know, makes the strategy space infinitely larger. Indeed, if players do not plan over a long time scale, as is the case here (only over 5 years), players miss future damages, and therefore, any possibility of agreement. This phenomenon can be observed in Figure 4.3, where only players with a large action space like China, the USA, and AOC reduce their emissions, while India and Russia cannot measure the impact of their emissions on the climate. So, we were able to show the effect of the choice of the benefit function and that of the damage function on the players' behavior.

4.4 Conclusion

In this chapter, we have introduced and analyzed a static game that provides some insights into the strategic behavior of non-influenced governments in terms of carbon emissions with respect to climate change. Each government is assumed to implement a trade-off between a benefit due to emitting and damage due to climate change. The Nash equilibrium of the game being a suitable solution concept, we have studied the existence and uniqueness of the equilibrium and also addressed the problem of expression of the actions at equilibrium. The numerical analysis provides several insights into the carbon emission problem. For instance, it is seen that to reach the Paris Agreement on climate (namely, maintain the temperature excess below 2 degrees), the damage from climate change has to be significant

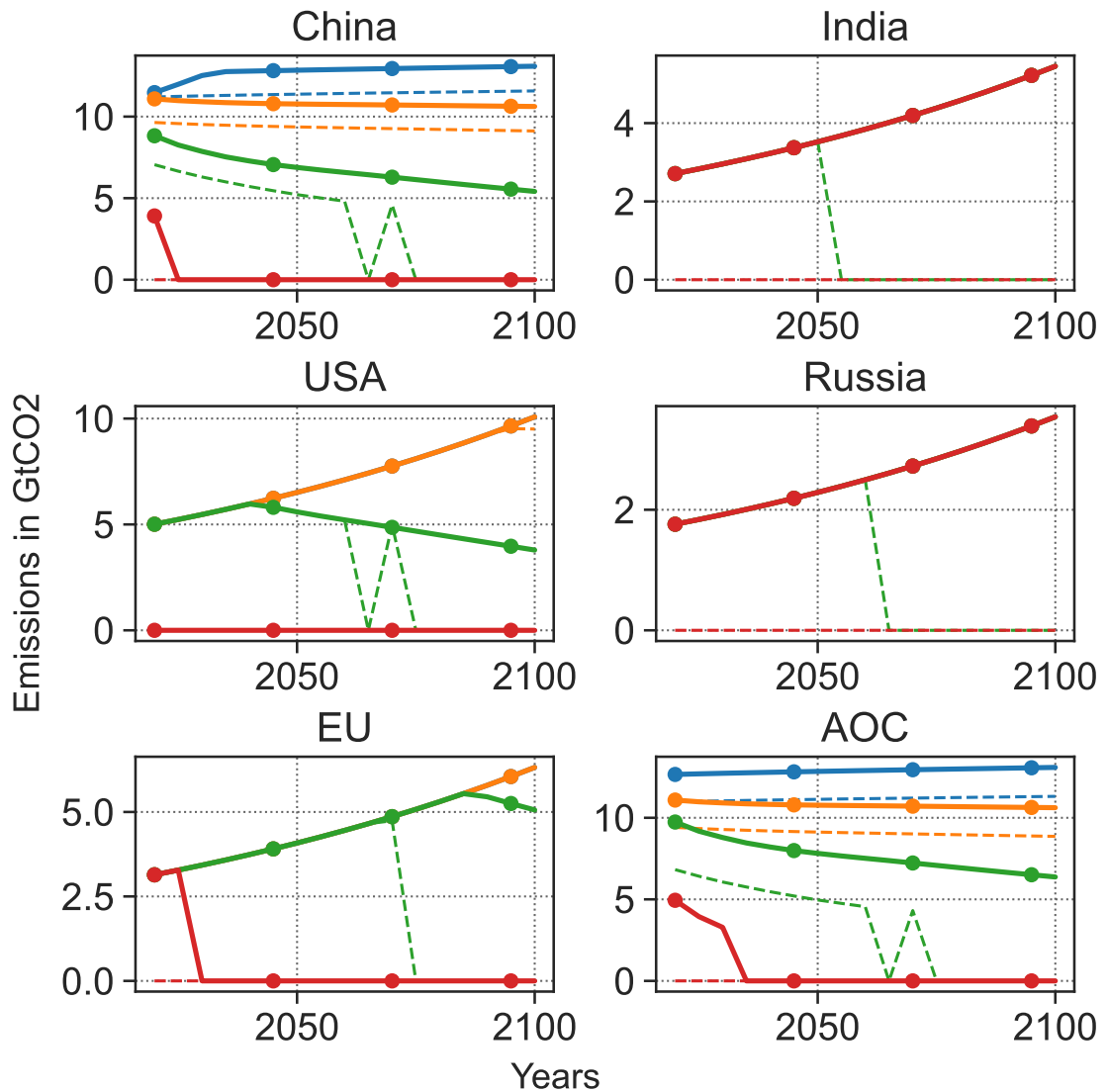


FIGURE 4.3 – Player actions in GtCO₂ at NE, where curves correspond to the chosen damage functions for Figure 4.2.

enough. This constitutes a sufficient condition under which governments will spontaneously reduce their emissions. Depending on the severity level of the damage (which is measured by the exponent α), governments are incited to stop emitting CO₂ and it is shown to be possible to (roughly) forecast a time at which a country stops emitting. The obtained times are typically higher than values claimed publicly (e.g., 2050).

To conclude this chapter, we would like to mention several extensions of the present work. First, the present problem formulation might be enriched by considering the planning

aspect for which countries consider long-term utilities instead of short-term ones. Second, the emergence of cooperation might be studied by considering other solution concepts, such as the social optimum or Nash bargaining solution. Thirdly, the damage functions might be more individualized while maintaining the potential structure of the game. At last, the present work can be seen as the first necessary step to be taken to study formally repeated interactions between the players, e.g., through a repeated or stochastic game model. In such a framework, it might be assumed that the action and state of a country are not perfectly observed by the others, which defines a non-trivial observation graph to be taken into account in the equilibrium analysis.

Imitation Game

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Abstract - *In this chapter, we study the climate change problem as an imitation game, where the utility of each country integrates a term that models the influence of other countries in terms of emitting CO₂. This game is an extension of the game analysed in the previous chapter, to study the effect of the influence on the actions of the players. Players are countries and their utility implements a trade-off between the individual socio-economic benefit of emitting, the global damage induced by climate change, and an imitation term. We conduct the Nash equilibrium analysis of this game to exhibit the possible behaviors of the countries in the presence of climate change awareness and the influence of other countries. We explore the role of different key*

parameters such as the weight of the imitation on the NE expression. Our results highlight the importance of coordination and cooperation for mitigating climate change.

5.1 Introduction

The climate is described by a complex geophysical model that interconnects many non-linear dynamics (CO₂ in the atmosphere and oceans, temperatures in the atmosphere and oceans, radiative forcing, etc). Climate change is one of the most challenging problems that humanity is facing today. It may potentially impact the ecosystems, economies, and societies worldwide. Mitigating climate change requires a coordinated effort from all countries, as greenhouse gas emissions from one country affect the global climate dynamics. It is noteworthy that the benefits and costs of mitigation efforts are not evenly distributed among countries. The strategy of each country to reduce global warming is the result of a trade-off between the short-term economic loss induced by the ecological policies, the damage induced by the increased CO₂ emissions, and the political pressure of the economic partners. This leads to a problem where actors need to make interdependent decisions. A natural mathematical framework to study these interactive decision scenarios is game theory.

Studying climate change from a game theoretical point of view has already been considered in the literature [25, 42, 74] where the authors are imposing convexity properties on the utility functions in a static game. The agreement of the countries in a non-cooperative game was analyzed in [102]. Also in [103] the authors present a dynamical model that effectively simulates the negotiation process leading to the 2015 Paris Agreement on climate change. This work presents the consensus-building process and the key role of leadership in the agreement process.

Another line of research investigates opinion dynamics and social interactions in the context of climate change. In [104], the authors introduce a climate model coupled with a Hegselmann-Krause opinion dynamics model. An evolutionary game to understand the interconnectedness of climate dynamics and social networks, especially real-world inequalities, is presented in [105]. Public debates shaped by incomplete scientific data, highlighting the role of inflexible/stubborn agents in influencing public opinion, are studied in [106]. The influence of media on climate communication is another area of exploration. The analysis of how various media influence public perception of climate change through their coverage, agenda setting, and framing is done in [107]. Furthermore, several studies have explored the dynamics of climate change agreements and the cooperation among countries in addressing this global challenge, e.g., [108].

In this work, we propose a nonlinear climate dynamics controlled by strategies designed to maximize individual utility functions comprising three terms: an individual benefit term,

a weighted global damage term, and a term capturing the effect of imitation/agreement with the other players. It is useful to notice that the introduced imitation term can also be interpreted as a contract for emissions reduction or as a penalty for countries that do not align their emissions with a common strategy. This results in the study of a complex network between the players and the climate, yielding a certain trajectory of global warming. Our key contributions are the following:

1. We propose a novel static game containing an imitation term in the players' utilities. We provide a sufficient condition that ensures the existence of at least one pure Nash equilibrium. We also show the uniqueness of the pure NE by proving that the utility functions are diagonally strictly concave.
2. We provide the expression of the corresponding unique pure Nash equilibrium, under a sufficient condition, when the utility functions are quadratic.
3. We assess numerically, the effects of the imitation function due to the influence of the players, connected in a complete graph, on each other.

The rest of the chapter is organized as follows. Section 5.2 is dedicated to the problem formulation. The game-theoretical analysis is provided in Section 5.3, where we study the existence and the uniqueness of the Nash equilibrium in a specific case. Numerical simulations illustrate our results in Section 5.4 and provide several insights. Conclusions and some perspectives are given in Section 5.5.

5.2 Problem formulation

We consider a static imitation game Γ between N different countries that exchange information through a fixed communication network (namely, a graph). We denote by \mathcal{V}_n the set of neighbors of player n , which is the set of players that are influencing the player n . Each player aims to maximize its own utility function, as a trade-off between its benefits, and a weighted global damage, where a part of this trade-off is the imitation cost. Indeed, when the players have the same emission behavior as their neighbors, they are not penalized, while they are whenever they deviate. Moreover, the player will not tend to emit more, because they will be penalized by the climate, from the damage function, which will get them to reduce emissions. We denote by \mathcal{N} the set of countries, and the social network is represented by a fixed weighted directed graph $(\mathcal{N}, \mathcal{E}, P)$ with \mathcal{E} and \mathcal{P} respectively representing the set of edges of the graph and the matrix of corresponding weights. To a country $n \in \mathcal{N}$, we assign an action a_n , that represents their opinion, which is a scalar in the actions set $\mathcal{A}_n = [e_n^{\min}, e_n^{\max}] \subset \mathbb{R}$. Also, to denote the bounds of emission that players can emit all together, we write $\mathbb{A} = [E^{\min}, E^{\max}]$ where E^{\min} and E^{\max} are respectively the sum of e_n^{\min}

and e_n^{\max} for all players. The geophysical state is given by a vector $x \in \mathcal{X} \subset \mathbb{R}^p$ where p depends on the CM (climate model) chosen. The state is a vector of average global atmospheric temperature given in some boxes and CO_2 concentrations in some boxes. We use the CM dynamics used in [12].

We choose the global atmospheric temperature given for $x \in \mathcal{X}$, and $a \in \mathcal{A} = \prod_{n \in \mathcal{N}} \mathcal{A}_n$, by:

$$\theta_{\text{AT}}(x, a) = \psi_\theta(x) + b_\theta \ln \left(\psi_C(x) + b_C \sum_{n=1}^N a_n \right), \quad (5.1)$$

where $\psi_\theta(x)$, and $\psi_C(x)$, are functions of the climate state, and b_θ , and b_C are constants of the climate model, that can be found in [12].

Definition 5.1. *The utility function of a player $n \in \mathcal{N}$ is defined as the difference between their benefits B_n and the sum of the weighted global damage $w_n D$ and a weighted imitation term $\delta_n \text{Imi}_n$. Notice that B_n depends on the action of player n , $w_n D$ represents a projection of the global damage induced by the increase of the atmospheric temperature, and the imitation function Imi_n depends on the overall vector of actions. We also note that the weights w_n and δ_n are positive for all $n \in \mathcal{N}$. For all $n \in \mathcal{N}$ the utility function is mathematically described by :*

$$u_n(x, a) := B_n(a_n) - w_n D(\theta_{\text{AT}}(x, a)) - \delta_n \text{Imi}_n(a). \quad (5.2)$$

Note that, strictly speaking, the utility u_n should only depend on the action profile a . However, because of the presence of the climate dynamics, we parameterize u_n by the dynamics state x and use a piecewise static analysis of the dynamical problem at hand. Although a dynamical game model would be more mathematically general, governments would not implement such complex decision-making. Modeling governments' behavior by a piecewise static model is thus considered much more reasonable. In the rest of the chapter, we will mention the use of a quadratic case, where the benefits are quadratic in player n 's action, and the damages are quadratic in the atmospheric temperature. We denote by a_n^i , and θ_{AT}^i the action of the player n to the power i and the atmospheric temperature to the power i . In this case, for all $n \in \mathcal{N}$, the utility function is given by the following:

$$u_n(x, a) = \sum_{i=0}^2 \beta_{i,n} a_n^i - w_n \sum_{i=0}^2 \gamma_i \theta_{\text{AT}}^i(x, a) - \delta_n e_n^{\max} \sum_{m \in \mathcal{Y}_n} e_m^{\max} \left(\frac{a_n}{e_n^{\max}} - \frac{a_m}{e_m^{\max}} \right)^2. \quad (5.3)$$

In the following, we denote by $\xi_n = a_n/e_n^{\max}$, the ratio between the player n 's action and their maximal emissions. Note that the imitation term is minimal (maximizes the utility) when all the neighbors have the same emission ratio: $\xi_n = \xi_m, \forall n, m \in \mathcal{N}$.

5.3 Game analysis

In this section, we analyze the game proposed in the preceding section. The Nash equilibrium is a reasonable solution concept for the proposed game, since governments have the freedom to choose their emission strategies. Therefore, the existence and uniqueness of an equilibrium scenario is a key problem to be investigated.

5.3.1 Imitation game scenario

We start the analysis by providing a sufficient condition related to the convexity of global damages. This condition guarantees the existence of at least one pure Nash equilibrium. Moving forward, we introduce additional sufficient conditions ensuring the uniqueness of the pure Nash equilibrium. Precisely, we will focus on the concavity of the benefits. Finally, we assume a complete interaction graph among players and, in this context, we provide a closed-form expression of the unique interior pure Nash equilibrium. This equilibrium represents a scenario where players strategically optimize emissions, avoiding emission levels at their maximum. The comprehensive exploration of these subsections contributes to a coherent understanding of equilibrium properties in the game.

5.3.1.1 Existence of a pure Nash equilibrium

Similarly to the static game where the players are not influenced, studied in the Chapter 4, the players aim at maximizing their individual utilities while being aware that their decisions significantly influence one another. To prove the existence of a pure Nash equilibrium, we provide in the following proposition, a sufficient condition for the concavity of the utility functions with respect to a_n (see e.g., [109] for continuous quasi-concave games), a concept introduced by [17]. The next result proves that if one has enough severe damage as the temperature rises, then there exists a pure NE.

Proposition 5.1. *Assume that: the benefit function is twice differentiable and concave with respect to a_n ; the damage function is twice differentiable with respect to θ_{AT} ; the imitation function is convex with respect to a_n . If the following condition is satisfied,*

$$\frac{D'(\theta_{AT}(x, a))}{D''(\theta_{AT}(x, a))} < b_\theta, \quad (5.4)$$

then there exists at least one pure NE for the game Γ .

Proof. First, we prove that the utility function u_n of player $n \in \mathcal{N}$, is concave with respect to a_n . Consequently, we calculate the second derivative of u_n with respect to a_n by using the expressions of u_n in Equation (5.2) and of the atmospheric temperature in

Equation (5.1). For all $n \in \mathcal{N}$ one obtains

$$\frac{\partial^2 u_n}{\partial a_n^2}(x, a) = B_n''(a_n) - \delta_n \frac{\partial^2 \text{Imi}_n}{\partial a_n^2}(a) - \frac{w_n b_\theta b_C^2 [b_\theta D''(\theta_{AT}(x, a)) - D'(\theta_{AT}(x, a))]}{\left(\psi_C(x) + b_C \sum_{n=1}^N a_n\right)^2}.$$

Since we assume that the benefit function is concave with respect to a_n , the damage function is convex with respect to θ_{AT} , and the imitation function is convex with respect to a_n , from Equation (5.4) we conclude that the utility function u_n is concave with respect to a_n . Moreover, for all $n \in \mathcal{N}$, the action set \mathcal{A}_n is compact and convex and u_n is continuous in the profile of actions $a \in \mathcal{A}$. Thus, using Debreu's theorem from [17], there exists at least one pure NE of the game Γ . ■

5.3.1.2 Uniqueness of the pure Nash equilibrium

In this section, our focus is on examining the game's structure to establish the uniqueness of the pure Nash equilibrium. In the following, the Jacobian of $U(x, a) := (u_1(x, a), \dots, u_N(x, a))$ is $\nabla U(x, a) := [\nabla_1 u_1(x, a), \dots, \nabla_N u_N(x, a)]^\top$ where $\nabla_m u_n(x, a)$ represents the gradient vector of u_n with respect to a_m , for all $n, m \in \mathcal{N}$. The Hessian matrix $H(x, a)$ is defined as:

$$H(x, a) = \begin{bmatrix} \frac{\partial^2 u_1}{\partial a_1^2}(x, a) & \frac{\partial^2 u_1}{\partial a_1 \partial a_2}(x, a) & \dots \\ \frac{\partial^2 u_1}{\partial a_2 \partial a_1}(x, a) & \ddots & \\ \vdots & & \ddots \end{bmatrix}.$$

It is worth noting that $H(x, a)$ is Hermitian because $\frac{\partial^2 u_n}{\partial a_i \partial a_j}(x, a) = \frac{\partial^2 u_n}{\partial a_j \partial a_i}(x, a)$, holds for all $n, i, j \in \mathcal{N}$. To demonstrate the uniqueness of the pure Nash equilibrium, we start by recalling the definition of the Diagonally Strict Concavity (DSC) condition as follows.

Definition 5.2. Assume that: $u_n(a)$ is a continuous function in $a \in \mathcal{A}$, and concave in a_n . Let $r = (r_1, \dots, r_N)^\top$ be an arbitrary vector of fixed parameters. Define the pseudogradient by $\gamma_r(a) = \left[r_1 \frac{\partial u_1}{\partial a_1}(a), \dots, r_N \frac{\partial u_N}{\partial a_N}(a) \right]^\top$. The following condition is called the diagonally strict concavity (DSC) condition. There exists $r > 0$ such that:

$$\forall (a, a') \in \mathcal{A}^2, a \neq a' : (a - a') (\gamma_r(a') - \gamma_r(a))^\top > 0. \quad (5.5)$$

Indeed, to prove the uniqueness of the pure NE, we use the following lemma from [8] to find a sufficient condition for ensuring the uniqueness of the pure Nash equilibrium.

Lemma 5.1. *If for all $a \in \mathcal{A}$, one has*

$$y^\top (H(x, a) + H^\top(x, a))y < 0, \quad \forall y \neq 0, \quad (5.6)$$

then, the utility functions (u_1, \dots, u_N) are diagonally strictly concave for $a \in \mathcal{A}$.

To establish a sufficient condition for the uniqueness of the pure Nash equilibrium we provide an instrumental result presented in the following Lemma.

Lemma 5.2. *Let $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^N$. Then, the maximal eigenvalue of the matrix $M = \mathbf{v}_1 \mathbf{v}_2^\top + (\mathbf{v}_1 \mathbf{v}_2^\top)^\top$ is $\mu = \mathbf{v}_1^\top \mathbf{v}_2 + \|\mathbf{v}_1\| \|\mathbf{v}_2\|$.*

Proof. Since $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^N$, the matrix $\mathbf{v}_1 \mathbf{v}_2^\top$ is of rank 1 which implies that M is at most a rank 2 matrix. Let us note the non-zero eigenvalues of M by μ_1 and μ_2 . We have the following,

$$\mu_1 + \mu_2 = \text{Tr}(M) = 2\mathbf{v}_1^\top \mathbf{v}_2,$$

and

$$\begin{aligned} \mu_1^2 + \mu_2^2 &= \text{Tr}(M^2) = \text{Tr}\left((\mathbf{v}_1 \mathbf{v}_2^\top)^2 + (\mathbf{v}_2 \mathbf{v}_1^\top)^2 + \mathbf{v}_1 \mathbf{v}_2^\top \mathbf{v}_2 \mathbf{v}_1^\top + \mathbf{v}_2 \mathbf{v}_1^\top \mathbf{v}_1 \mathbf{v}_2^\top\right) \\ &= 2\text{Tr}\left((\mathbf{v}_1 \mathbf{v}_2^\top)^2\right) + (\mathbf{v}_2^\top \mathbf{v}_2)\text{Tr}(\mathbf{v}_1 \mathbf{v}_1^\top) + (\mathbf{v}_1^\top \mathbf{v}_1)\text{Tr}(\mathbf{v}_2 \mathbf{v}_2^\top) \\ &= 2\text{Tr}(\mathbf{v}_1 \mathbf{v}_2^\top)^2 + 2(\mathbf{v}_1^\top \mathbf{v}_1)(\mathbf{v}_2^\top \mathbf{v}_2) = 2\left((\mathbf{v}_1^\top \mathbf{v}_2)^2 + (\mathbf{v}_1^\top \mathbf{v}_1)(\mathbf{v}_2^\top \mathbf{v}_2)\right). \end{aligned}$$

Moreover, we have also,

$$\mu_1 \mu_2 = \frac{(\mu_1 + \mu_2)^2 - (\mu_1^2 + \mu_2^2)}{2} = (\mathbf{v}_1^\top \mathbf{v}_2)^2 - (\mathbf{v}_1^\top \mathbf{v}_1)(\mathbf{v}_2^\top \mathbf{v}_2).$$

Then by injecting this into the characteristics polynomial of M given by:

$$P_M(X) = (-X)^{N-2}(\mu_1 - X)(\mu_2 - X).$$

We notice that μ_1 and μ_2 are also the roots of

$$Q(X) = X^2 - 2(\mathbf{v}_1^\top \mathbf{v}_2)X + (\mathbf{v}_1^\top \mathbf{v}_2)^2 - (\mathbf{v}_1^\top \mathbf{v}_1)(\mathbf{v}_2^\top \mathbf{v}_2).$$

And finally, the biggest root of this polynomial $\mu := \max(\mu_1, \mu_2)$, is given by

$$\begin{aligned}\mu &= \frac{2(\mathbf{v}_1^\top \mathbf{v}_2) + \sqrt{4(\mathbf{v}_1^\top \mathbf{v}_2)^2 - 4((\mathbf{v}_1^\top \mathbf{v}_2)^2 + (\mathbf{v}_1^\top \mathbf{v}_1)(\mathbf{v}_2^\top \mathbf{v}_2))}}{2} \\ &= \mathbf{v}_1^\top \mathbf{v}_2 + \sqrt{(\mathbf{v}_1^\top \mathbf{v}_1)(\mathbf{v}_2^\top \mathbf{v}_2)} = \mathbf{v}_1^\top \mathbf{v}_2 + \|\mathbf{v}_1\| \|\mathbf{v}_2\|.\end{aligned}$$

■

The next result proves that in addition to the condition of severe damage for the existence, a sufficient condition is given on the benefits, to ensure the uniqueness of the pure NE. Indeed, the right-hand side of the condition in Equation (5.8), is a simple combination of the damage and imitation weights. This limit is shown to be positive in the numerical examples that we took, and then by simply considering a concave benefit function, which is assumed to be true, this condition is satisfied. Eventually, this condition can be interpreted as a threshold on the concavity of the benefit function.

Proposition 5.2. *Let the benefit functions B_n be twice differentiable with respect to a_n for all $n \in \mathcal{N}$ and the damage function be twice differentiable with respect to $\theta_{\text{AT}} \in \mathbb{R}$. If the two following conditions are satisfied, then the game has a unique pure NE.*

- The damage function must verify

$$\frac{D'(\theta_{\text{AT}}(x, a))}{D''(\theta_{\text{AT}}(x, a))} < b_\theta. \quad (5.7)$$

- The benefit functions must verify

$$B''_{\underline{n}}(a_{\underline{n}}) < 2 \left(\sum_{m \in \mathcal{Y}_{\underline{n}}} \frac{e_m^{\max}}{e_{\underline{n}}^{\max}} - 1 \right) \delta_{\underline{n}} - \sum_{n=1}^N \delta_n - N \text{sign}(H_{2, \bar{n}}(x, a)) \delta_{\bar{n}}, \quad \forall n \in \mathcal{N} \quad (5.8)$$

with the notation $\underline{n} = \arg \max_{n \in \mathcal{N}} H_{1,n}(a_n)$, $\bar{n} = \arg \max_{n \in \mathcal{N}} |H_{2,n}(x, a)|$ and $H_{2,n}$ given in the Equation (5.10).

Moreover, if for all $n \in \mathcal{N}$, $\text{sign}(H_{2,n}(x, a)) < 0$, the following additional condition over the damage weights, must be verified

$$w_{\bar{n}} < \frac{1}{N} \sum_{n=1}^N w_n. \quad (5.9)$$

Proof. For all $a \in \mathcal{A}, x \in \mathcal{X}$ let us introduce

$$H_{1,n}(a_n) = B_n''(a_n) - 2\delta_n \left(\sum_{m \in \mathcal{Y}_n} \frac{e_m^{\max}}{e_n^{\max}} - 1 \right)$$

and

$$H_{2,n}(x, a) = 2\delta_n + \frac{b_\theta b_C^2 w_n (D'(\theta_{AT}(x, a)) - b_\theta D''(\theta_{AT}(x, a)))}{\psi_C(x) + b_C \sum_{n=1}^N a_n}. \quad (5.10)$$

With this notation we built sequentially $H_1(a) = \text{diag}(H_{1,1}(a_1), \dots, H_{1,N}(a_N))$, $H_2(x, a) = (H_{2,1}(x, a), \dots, H_{2,N}(x, a))^\top$, the matrix

$$H(x, a) = H_1(a) + H_2(x, a) \mathbb{1}_N^\top,$$

and the Hermitian matrix $M := H(x, a) + H^\top(x, a)$. Applying Weyl's inequality to the largest eigenvalue of M , will lead to the DSC. This will allow us to conclude the uniqueness of the pure Nash equilibrium.

Indeed, $M = 2H_1(a) + H_2(x, a) \mathbb{1}_N^\top + \mathbb{1}_N H_2^\top(x, a)$, we recall that the eigenvalues of $H_1(a)$ are $H_{1,n}(a_n)$, for all $n \in \mathcal{N}$. Using Lemma 5.2, for $v_1 = H_2(x, a)$, and $v_2 = \mathbb{1}_N$, we get that the maximum eigenvalue of $H_2(x, a) \mathbb{1}_N^\top + \mathbb{1}_N H_2^\top(x, a)$, is given by $\mu = H_2^\top(x, a) \mathbb{1}_N + \|H_2(x, a)\| \|\mathbb{1}_N\|$. Knowing that $\|x\| \leq \sqrt{N} \|x\|_\infty = \sqrt{N} \max_{n \in \mathcal{N}} |x_n|, \forall x \in \mathbb{R}^N$, it suffices to use the infinity norm instead of the Euclidean norm. Indeed, the infinity norm of $H_2(x, a)$ is given by $\|H_2(x, a)\|_\infty = \max_{n \in \mathcal{N}} |H_{2,n}(x, a)|$, and $\|\mathbb{1}_N\|_\infty = 1$. To find a sufficient condition for the uniqueness of the NE, it suffices to show that:

$$2 \max_{n \in \mathcal{N}} H_{1,n}(a_n) + \sum_{n=1}^N H_{2,n}(x, a) + N \max_{n \in \mathcal{N}} |H_{2,n}(x, a)| < 0.$$

Let $H_{2,n}(x, a)$ be a vector for each $n \in \mathcal{N}$. The index \bar{n} is defined as the unique element in \mathcal{N} that maximizes the absolute value of $H_{2,n}(x, a)$, *i.e.*,

$$\bar{n} = \arg \max_{n \in \mathcal{N}} |H_{2,n}(x, a)|.$$

This notation denotes that \bar{n} is the index for which $|H_{2,n}(x, a)|$ is maximal among all indices n in the set \mathcal{N} . Since we are interested in the largest eigenvalue of H_1 , we

denote by \underline{n} the index of the largest $H_{1,n}$:

$$\underline{n} = \arg \max_{n \in \mathcal{N}} H_{1,n}(a_n).$$

Hence,

$$\begin{aligned} & 2H_{1,\underline{n}}(a_{\underline{n}}) + \sum_{n=1}^N H_{2,n}(x, a) + N \max_{n \in \mathcal{N}} |H_{2,n}(x, a)| \\ &= 2B''_{\underline{n}}(a_{\underline{n}}) - 4\delta_{\underline{n}} \left(\sum_{m \in \mathcal{Y}_{\underline{n}}} \frac{e_m^{\max}}{e_{\underline{n}}^{\max}} - 1 \right) + \sum_{n=1}^N w_n \frac{b_{\theta} b_C^2 [D'(\theta_{AT}(x, a)) - b_{\theta} D''(\theta_{AT}(x, a))]}{\psi_C(x) + b_C \sum_{n=1}^N a_n} \\ & \quad + 2 \sum_{n=1}^N \delta_n + N \text{sign}(H_{2,\bar{n}}(x, a)) \left[2\delta_{\bar{n}} + w_{\bar{n}} \frac{b_{\theta} b_C^2 [D'(\theta_{AT}(x, a)) - b_{\theta} D''(\theta_{AT}(x, a))]}{\psi_C(x) + b_C \sum_{n=1}^N a_n} \right] \\ &= 2B''_{\underline{n}}(a_{\underline{n}}) - 4\delta_{\underline{n}} \left(\sum_{m \in \mathcal{Y}_{\underline{n}}} \frac{e_m^{\max}}{e_{\underline{n}}^{\max}} - 1 \right) + 2 \sum_{n=1}^N \delta_n + 2N \text{sign}(H_{2,\bar{n}}(x, a)) \delta_{\bar{n}} \\ & \quad + \left[\sum_{n=1}^N w_n + N \text{sign}(H_{2,\bar{n}}(x, a)) w_{\bar{n}} \right] \frac{b_{\theta} b_C^2 [D'(\theta_{AT}(x, a)) - b_{\theta} D''(\theta_{AT}(x, a))]}{\psi_C(x) + b_C \sum_{n=1}^N a_n} \end{aligned}$$

In conclusion, the sufficient conditions are given by the following,

$$\frac{D'(\theta_{AT}(x, a))}{D''(\theta_{AT}(x, a))} < b_{\theta},$$

$$B''_{\underline{n}}(a_{\underline{n}}) < 2 \left(\sum_{m \in \mathcal{Y}_{\underline{n}}} \frac{e_m^{\max}}{e_{\underline{n}}^{\max}} - 1 \right) \delta_{\underline{n}} - \sum_{n=1}^N \delta_n - N \text{sign}(H_{2,\bar{n}}(x, a)) \delta_{\bar{n}}, \quad \forall n \in \mathcal{N}.$$

In addition, if $H_{2,\bar{n}}(x, a) < 0$, the additional condition that has to be satisfied is $w_{\bar{n}} < \frac{1}{N} \sum_{n=1}^N w_n$. Then, using Lemma 5.1, we conclude that the payoff functions are diagonally strictly concave. Hence, the game has a unique pure Nash equilibrium. ■

5.3.1.3 Expression of the pure Nash equilibrium

In this section, we will give the expression of the unique pure NE of the game when the utility functions are quadratic strictly concave, given in the Equation (5.3). Let us denote by $a^* = (a_1^*, \dots, a_N^*)$ the Nash equilibrium profile of actions for all the players. We emphasize that the actions of the players are the amount of emissions they emit to maximize their utility. The Nash equilibrium of the game under consideration may be either on the boundary, *i.e.*, $a_n^* \in \{e_n^{\min}, e_n^{\max}\}$, for some $n \in \mathcal{N}$, or an interior point $a_n^* \in (e_n^{\min}, e_n^{\max})$, $\forall n \in \mathcal{N}$. Our goal is

to characterize the interior solution because this means that the emissions are not maximal. We are also interested in pointing out the impact of different ingredients: the imitation term, the economic coefficients (benefit coefficients and damage coefficients), the impact of the CM parameters, and the radiative forcing.

Assuming that the utility functions (u_1, \dots, u_N) verify the DSC condition in Equation (5.5), we get that the Nash equilibrium is unique. We emphasize that the Nash equilibrium is the solution to the optimization problem given by the following, for all $n \in \mathcal{N}$,

$$\begin{aligned} \max \quad & u_n(x, a) \\ \text{s.t.} \quad & a_n \in \mathcal{A}_n. \end{aligned}$$

Indeed the NE can be found by applying the KKT conditions given as follows:

$$\frac{\partial u_n}{\partial a_n}(x, a) = \bar{\lambda}_n - \underline{\lambda}_n =: \lambda_n, \quad \forall n \in \mathcal{N}, \quad (\text{KKT})$$

with $\bar{\lambda}_n, \underline{\lambda}_n \geq 0$ being the KKT multipliers that satisfy at equilibrium the following $\underline{\lambda}_n^*(a_n^* - e_n^{\min}) = 0$, and $\bar{\lambda}_n^*(a_n^* - e_n^{\max}) = 0$.

To simplify the statement of the next proposition, we will define the following notations. We recall that $s = \sum_{n \in \mathcal{N}} a_n$ and since for a given state $x \in \mathcal{X}$, the atmospheric temperature can be expressed as a function of the sum of actions, i.e., $\theta_{\text{AT}}(x, a) := \theta_{\text{AT}}(s)$. For all $n \in \mathcal{N}$, let $z_n = 2(\beta_{2,n} - \delta_n E^{\max}/e_n^{\max})$ and $\sigma_n = \beta_{1,n} - \lambda_n$ with $\beta_{1,n}, \beta_{2,n}$ and δ_n from Equation (5.3) and λ_n from Equation (KKT). Last, we define the function h defined for $o \in \mathbb{R}^N$ by

$$h(o) = \frac{\sum_{n=1}^N (o_n/z_n)}{1 + 2 \sum_{n=1}^N (\delta_n/z_n)}.$$

Proposition 5.3. *Let the interaction graph among players be complete, i.e. everyone is everyone's neighbor. For all $n \in \mathcal{N}$, let u_n be strictly concave and $w_n > 0$. Using the above notations, we assume that $z_n \neq 0$ and $1 + 2 \sum_{n=1}^N \delta_n/z_n \neq 0$. If the following holds*

$$2\gamma_2 b_\theta^2 b_C^2 h(w) + \frac{(\psi_C(x) + b_C h(\sigma))^2}{8} < 0, \quad (5.11)$$

then the unique pure NE of the game $a^* = (a_1^*, \dots, a_N^*)$ is expressed by the following:

$$a_n^* = \frac{w_n b_\theta b_C (\gamma_1 + 2\gamma_2 \theta_{AT}(\tilde{s}))}{z_n \psi_C(x) + b_C \tilde{s}} - \frac{2\delta_n \tilde{s}}{z_n} - \frac{\sigma_n}{z_n}, \quad (5.12)$$

with \tilde{s} being the unique solution of the equation $rs^2 + ps + q = k \ln(rs + v)$, where $r = b_C$, $p = \psi_C + b_C h(\sigma)$, $q = -b_\theta b_C (\gamma_1 + 2\gamma_2 \psi_\theta(x)) h(w)$, $k = 2\gamma_2 b_\theta^2 b_C h(w)$, and $v = \psi_C$.

Proof. We recall the utility function, where the benefit, damage, and imitation functions are quadratic. Using the KKT conditions, we get for all $n \in \mathcal{N}$,

$$\beta_{1,n} + 2\beta_{2,n} a_n^* - w_n (\gamma_1 + 2\gamma_2 \theta_{AT}(x, a^*)) \frac{\partial \theta_{AT}}{\partial a_n}(x, a^*) - \frac{2\delta_n}{e_n^{\max}} \sum_{m \in \mathcal{Y}_n} (e_m^{\max} a_n^* - e_n^{\max} a_m^*) = \lambda_n^*.$$

Assuming a complete graph, the expression can be further simplified by using:

$$\begin{aligned} & \sum_{m \in \mathcal{Y}_n} (e_m^{\max} a_n^* - e_n^{\max} a_m^*) + e_n^{\max} a_n^* - e_n^{\max} a_n^* \\ &= \sum_{m \in \mathcal{N}, m \neq n} (e_m^{\max} a_n^* - e_n^{\max} a_m^*) + e_n^{\max} a_n^* - e_n^{\max} a_n^* \\ &= a_n^* \sum_{n=1}^N e_n^{\max} - e_n^{\max} \sum_{n=1}^N a_n^*. \end{aligned}$$

The KKT condition for each $n \in \mathcal{N}$ takes the form:

$$z_n a_n^* + 2\delta_n s + \sigma_n - w_n \frac{b_\theta b_C (\gamma_1 + 2\gamma_2 \theta_{AT}(s))}{\psi_C(x) + b_C s} = 0. \quad (5.13)$$

Dividing by $z_n = 2(\beta_{2,n} - \delta_n E^{\max}/e_n^{\max}) \neq 0$ for all $n \in \mathcal{N}$, and summing over n yields:

$$\left(1 + 2 \sum_{n=1}^N \frac{\delta_n}{z_n}\right) s + \sum_{n=1}^N \frac{\sigma_n}{z_n} = \sum_{n=1}^N \frac{w_n b_\theta b_C (\gamma_1 + 2\gamma_2 \theta_{AT}(s))}{z_n \psi_C(x) + b_C s}. \quad (5.14)$$

Furthermore, dividing by $1 + 2 \sum_{n=1}^N (\delta_n/z_n) \neq 0$, the KKT conditions can be expressed as:

$$s + h(\sigma) - h(w) \frac{b_\theta b_C (\gamma_1 + 2\gamma_2 \theta_{AT}(s))}{\psi_C(x) + b_C s} = 0. \quad (5.15)$$

Using the expression of θ_{AT} given in Equation (5.1), the condition given in Equation (5.15) can be rewritten as $rs^2 + ps + q = k \ln(rs + v)$. By using the results of [11, Lemma 1], we conclude that if the condition in Equation (5.11) is verified, then there exists at most

one solution \tilde{s} to the equation $rs^2 + ps + q = k \ln(rs + v)$ in $\mathbb{A} = [E^{\min}, E^{\max}]$. Using the Proposition 5.1, and assuming that for all $n \in \mathcal{N}$, u_n is concave, then we know that there exists at least one pure NE. Thus, the NE is unique and is given explicitly by the Equation (5.12). ■

5.3.1.3.1 Remark on the NE expression. In this context, it is of interest to exhibit several limiting cases for which the expression of the NE is particularly simple. Using the notations provided in Proposition 5.3 and $z_n = 2(\beta_{2,n} - \delta_n E^{\max}/e_n^{\max})$, we point out that:

$$\lim_{\delta_n \rightarrow +\infty} z_n = -\infty, \quad \lim_{\delta_n \rightarrow +\infty} \frac{-\delta_n}{z_n} = \frac{e_n^{\max}}{2E^{\max}}. \quad (5.16)$$

In a straightforward manner, one obtains that:

$$\lim_{\delta_n \rightarrow +\infty} a_n^* = \frac{e_n^{\max}}{E^{\max}} \tilde{s}, \quad \text{for all } n \in \mathcal{N}, \quad (5.17)$$

where \tilde{s} is the unique solution of the equation $rs^2 + ps + q = k \ln(rs + v)$. When δ_n is large enough, one can see that players' actions at the equilibrium represent a proportion of the total maximum emissions, that is proportional to their maximal emissions. They will tend to stop emitting when δ_n tends to infinity. Indeed, $\lim_{\delta_n \rightarrow +\infty} h(o) = 0$, for all $o_n < +\infty$. In this case, the equation is rewritten as: $b_C s^2 + \psi_C(x)s = 0$, and $\tilde{s} \in \mathbb{A}$, with $e_n^{\min} = 0$, for all $n \in \mathcal{N}$. In other words, the only solution of this equation is $\tilde{s} = 0$. Thus, at the NE, players stop emitting when faced with sufficiently considerable imitation weights or significant benefit coefficients.

5.4 Numerical analysis

In this section, we aim to numerically illustrate our theoretical findings, emphasizing the role played by the imitation term in the proposed model. We go even further with the numerical analysis by considering scenarios that have not been theoretically addressed. It is noteworthy that the numerical analysis can be further improved by using the flexible code that we have developed. It enables us to manipulate various parameters such as climate model attributes, benefit, damage, and imitation functions. Our simulations involve a setting with $N = 6$ players, and the parameter details are outlined in Table 5.1, where "AOC" collectively refers to all other countries. The visualizations are derived using the carbon cycle data from [55] and temperature dynamics from [50], recognized for their proximity to IPCC results [99, 100].

We specifically focus on a scenario where the benefit function adopts a sigmoid shape,

Player	e_n^{\max} (GtCO ₂ /y)	GDP _{<i>n</i>} ^{max} (10 ⁹ \$)	w_n	r_n	e_n
China	22	14630	1.1847	0.7	3
USA	14	19290	1.1941	1	-1
EU	8	13890	1.1248	2	-5
India	6	2500	0.9074	0.5	-4
Russia	4	1420	1.2866	1.25	-5
AOC	10	11640	1.1847	0.45	2.5

TABLE 5.1 – Specific values for each player of the imitation game in 2020.

and damages follow a quadratic and rescaled pattern, *i.e.* ,

$$u_n(x, a) = \text{GDP}_n^{\max} (B_n(a_n) - w_n [D(\theta_{\text{AT}}(x, a))]^\alpha) - \delta_n \text{GDP}_n^{\max} e_n^{\max} \sum_{m \in \mathcal{V}_n} e_m^{\max} (\xi_n - \xi_m)^2, \quad (5.18)$$

where the sigmoid benefit function is given by :

$$B_n(a_n) = \frac{f_n(a_n) - f_n(e_n^{\min})}{f_n(e_n^{\max}) - f_n(e_n^{\min})}, \quad (5.19)$$

with $e_n^{\min} = 0, \forall n \in \mathcal{N}$, the logistic function is given by :

$$f_n(a_n) = \frac{1}{1 + e^{-r_n(a_n - e_n)}}.$$

The global damage, quadratic function, is given by :

$$D(\theta_{\text{AT}}(x, a)) = 1 + \theta_{\text{AT}}(x, a) + 2\theta_{\text{AT}}^2(x, a), \quad (5.20)$$

and α represents the power of the damages and measures the severity level of climate change on the economics, we used $\alpha = 2.5$.

The utility function, denoted as $u_n(x, a)$, incorporates an imitation term influenced by the actions of neighboring players. We consider that the imitation weight is a function of time, represented by $\delta_n(t)$, that exponentially increases with respect to time. This illustrates the influence of neighboring countries on the utility optimization of each player. We highlight that the influence weights are given for a player $n \in \mathcal{N}$ and for a neighbour $m \in \mathcal{V}_n$, by $\frac{1}{e_n^{\max} e_m^{\max}}$.

5.4.1 Network structure: global vs grouped influence

The static game model for climate change mitigation is recurrently applied every five years from 2020 until 2100. We consider two types of network structure: a complete graph and a network containing two groups of countries that do not influence each other. In both configurations, we make a numerical analysis by varying the influence weights. In the second scenario, we assume that EU, USA, and AOC form one group while China, Russia, and India form the other. The interconnection topology is directed: China does not receive any influence, while AOC does not influence any player. The associated network structures are plotted in Figure 5.1.

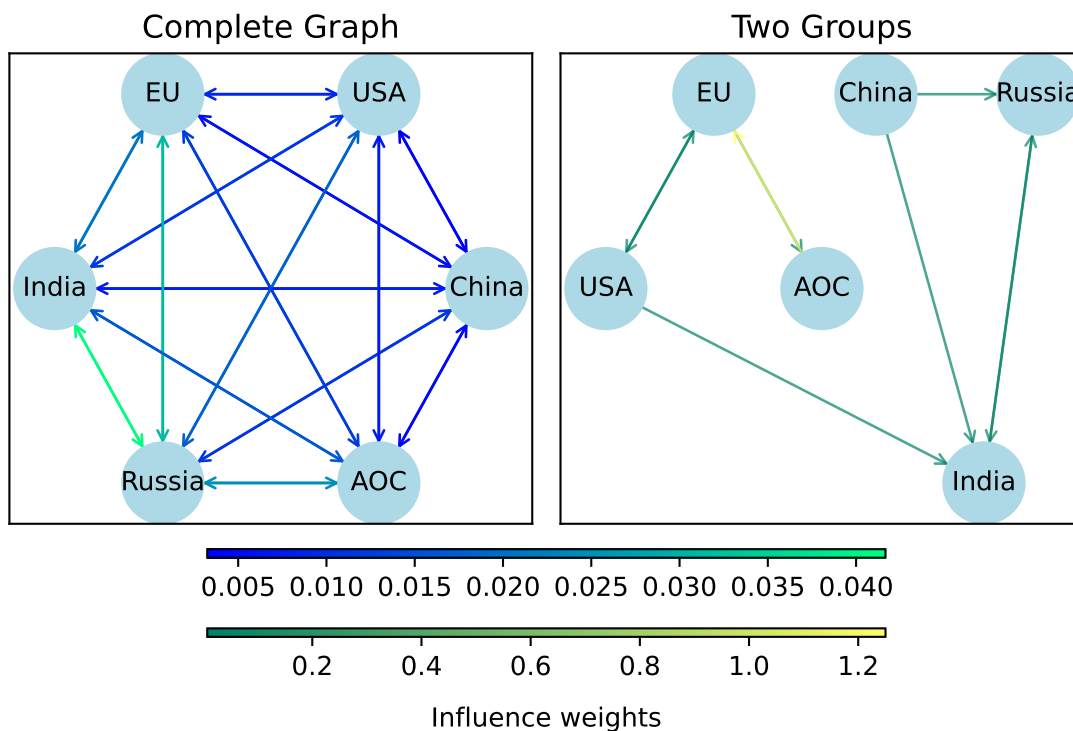


FIGURE 5.1 – Illustration of two assumed influence/imitation network topologies among players.

Our goal is to numerically compare the players' strategies and the temperature increase in the two scenarios. In Figure 5.2, the solid line is related to the first scenario (complete graph) while the dashed line is related to the second scenario (network containing two groups that do not influence each other). In Figure 5.2 we plot the temperature increase and observe that global cooperation leads to better behavior.

In the following, we address the scenario in which EU progressively ceases emissions,

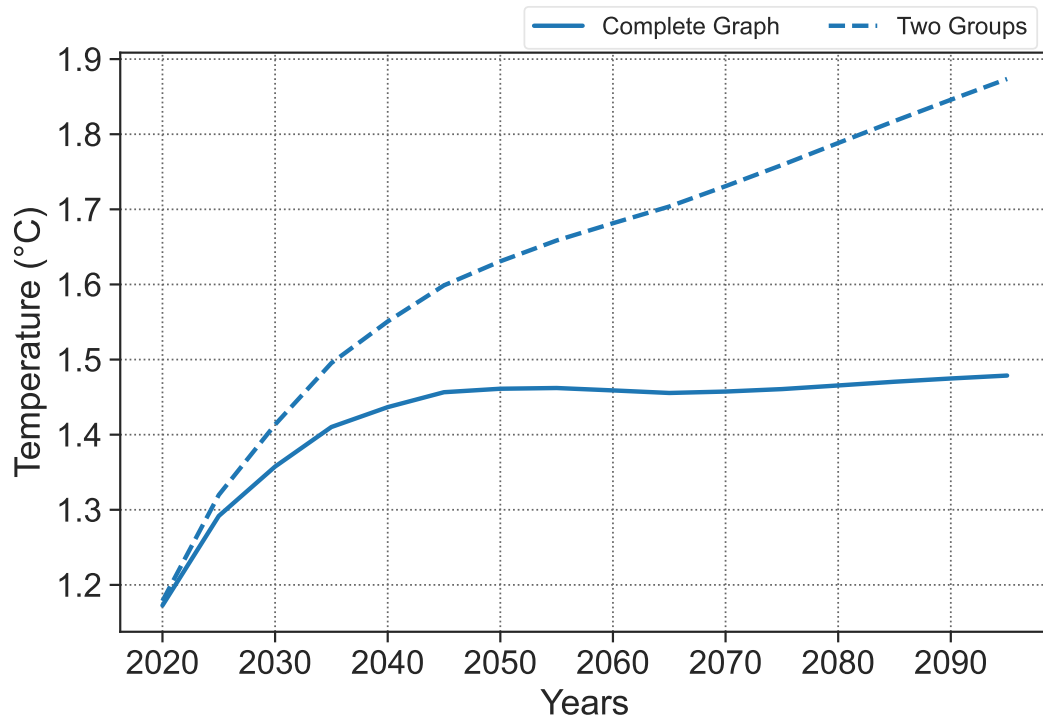


FIGURE 5.2 – Temperature increase due to CO₂ emissions at Nash Equilibrium under different influence/imitation graph structures. The dashed line represents a two-groups network, while the solid line represents a complete graph scenario.

modeled by an exponentially decreasing function, reaching zero CO₂ emissions in 2050. In response, other players adjust their strategies influenced by EU decision through the imitation term. The subsequent figures illustrate the trajectories followed by the players' CO₂ emissions under each of the two different network topologies introduced in Figure 5.1. We can notice the influence of the imitation term in mitigating the temperature increases. This influence is more important in the case of the complete graph, where all players influence each other. We note that in the first scenario, all the players have an exponential decrease of the CO₂ emissions (Figure 5.3 top). In this scenario, all the players emit less than 2 GtCO₂ by 2055.

In the second scenario one can see a high improvement in the emissions of the AOC which is only and directly influenced by EU, the USA roughly preserves the same behaviour while the other 3 players perform worse due to the influence of China that keeps constant its CO₂ emission level to 10 GtCO₂ (Figure 5.3 bottom). It is noteworthy that all the players that

are directly or indirectly influenced by EU decrease their emissions to less than 2.5 GtCO₂.

Another important insight revealed by these simulations is that the imitation term significantly contributes to mitigating the short-term utility increase resulting from emissions. Indeed, in [11] we have shown that the short-term benefits increase faster than the damages as long as the atmospheric temperature is not very high. Adding the imitation term helps to better compensate for this short-term increase in the benefits. In addition, When forming groups, the decision will be with the one who is not influenced, and if we choose to act virtuously, we can change the behavior of countries that we can influence.

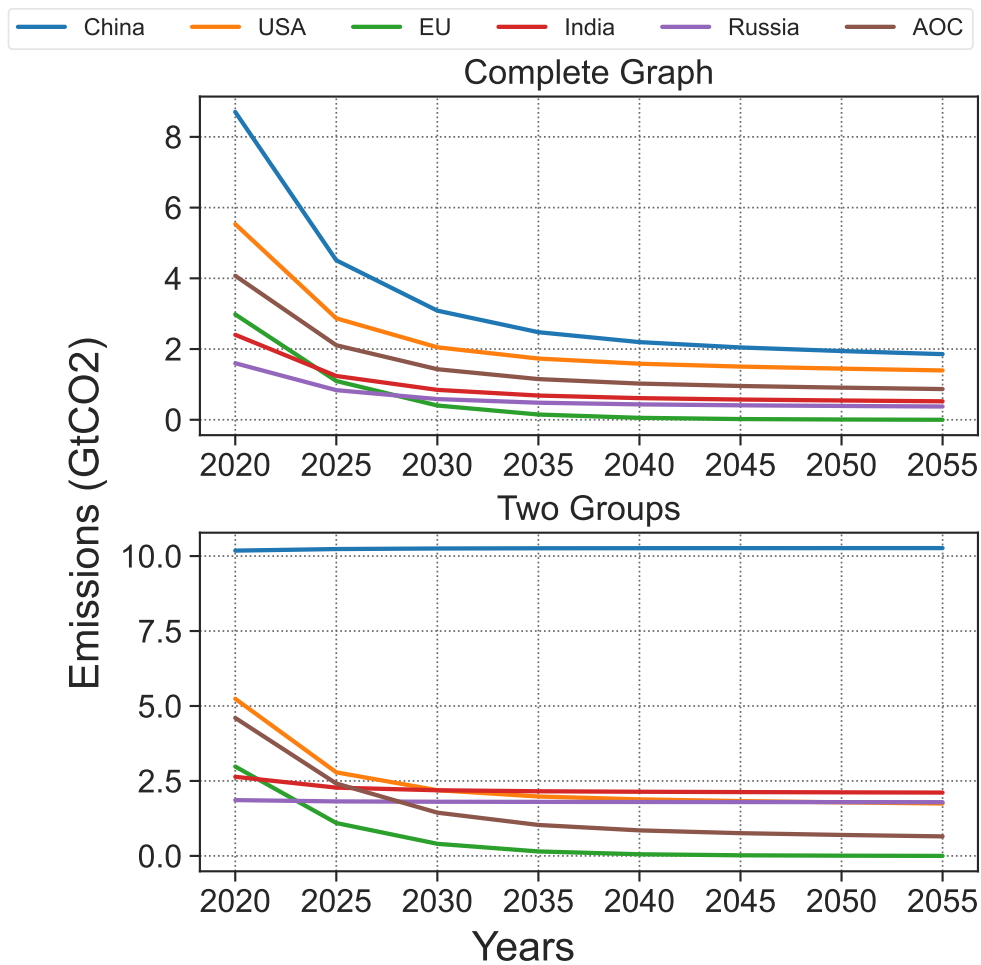


FIGURE 5.3 – CO₂ emissions of players at Nash Equilibrium in different scenarios. Colors and line shapes are explained in Figure 5.2.

Table 5.2 emphasizes the synchronization of emission ratios at Nash Equilibrium with players' maximal emissions. It is another way to highlight the impact of the imitation term

on emission reductions across all players.

	Complete Graph			Two Groups		
	2025	2050	2100	2025	2050	2100
China	0.4	0.1	0.06	0.46	0.47	0.47
USA	0.4	0.1	0.08	0.37	0.13	0.11
EU	0.4	0.003	0	0.37	0.003	0
India	0.4	0.1	0.06	0.44	0.35	0.34
Russia	0.4	0.1	0.07	0.47	0.45	0.44
AOC	0.4	0.1	0.06	0.46	0.08	0.04

TABLE 5.2 – Synchronization of the ratio of players' emissions at the NE over their maximal emissions, *i.e.*, $\frac{a_n^*}{e_n^{\max}}$.

It is noteworthy that in the complete graph case, the players will roughly synchronize at the same ratio. On the other hand, when the graph is directed and not strongly connected, the differences in the ratios become apparent. The presence of groups can be observed through the different behavior of the corresponding ratio δ_n . Precisely, the players in the second group preserve relatively high ratios while the ones in the first group have a clear decrease in it. In summary, our study underscores the crucial role of imitation in influencing player behavior and favoring cooperative emission reduction strategies, thereby contributing to global climate change mitigation. We notice that a country between two coalitions will be influenced by a virtuous player, even if in the second group, it is encouraged to continue emitting.

5.4.2 Network structure: EU and Russia as independent influencers

In this subsection, we explore an alternative network structure to further investigate the dynamics of climate change mitigation strategies. Unlike the previously considered scenarios, we configure the network such that EU and Russia act as independent influencers. Specifically, these two countries do not receive any influence from other players, but they influence all the remaining countries. Meanwhile, the other players continue to influence each other. Moreover, EU's strategy remains the same: they decrease their emissions exponentially to reach zero emissions by 2050 and stop emitting thereafter.

The configuration of this network is represented in Figure 5.4. The directed edges from EU and Russia to the remaining players indicate their influential roles, while the interconnections among the other countries represent the influence they exert on each other.

5.4.2.1 Network Topology

The network topology is designed as follows:

- The USA and EU do not receive any influence from other players.
- The USA and EU influence all other countries in the network.
- The remaining players (China, India, Russia, AOC) influence each other.

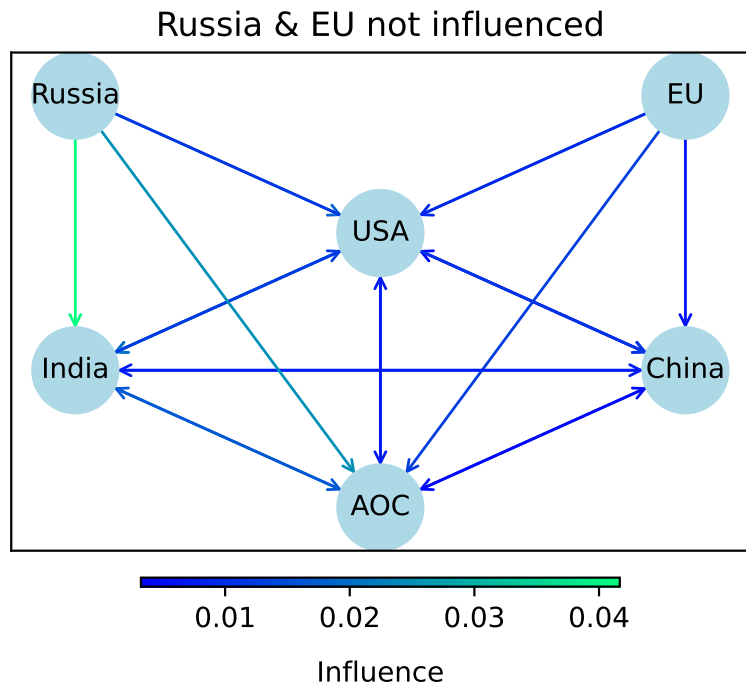


FIGURE 5.4 – Illustration of a network where USA and Russia are independent influencers.

To evaluate the impact of this alternative network structure on climate change mitigation, we are varying the influence weights among the players, we represent three different functions that describe the variation of the weights with time. Our focus remains on understanding the strategies adopted by each player and the resulting temperature increase. Especially that, EU continues to pursue its decarbonization strategy until achieving zero emissions by 2050, while Russia intends to continue emitting at the same level as in 2020. We model the dynamic of having two opposing forces: one prioritizing climate concerns and the other seeking to emit and increase profits without considering climate damage, to observe their effects on countries' behaviors in this scenario. In other words, we consider two influencers: the first advocates for climate action, aiming to halt emissions in line with IPCC and COP recommendations, while the second prioritizes profit maximization.

In Figure 5.5, we present the outcomes of our numerical simulations in this case. The solid line corresponds to the temperature increase in the scenario with the alternative network

structure, while the dashed line represents the baseline scenario with a complete graph. One can see that as the influence weights become important, the temperature rise is limited. Furthermore, when countries are interconnected, the temperature remains lower compared to the scenario where the USA and Russia act as independent influencers. In this case, the temperature reaches 1.5°C for $\delta_n(t) = e_n^{\max} \exp(t/20)$. This highlights the effectiveness of global cooperation, by being all connected, compared to the scenario where the USA and Russia play distinct roles. Indeed, in the connected network, we naturally achieve better results, as the country aiming to emit to the maximum, like Russia here, won't be able to do so in the complete graph due to the influence of EU. Even with exponential influence weights, the outcomes in the complete graph are superior.

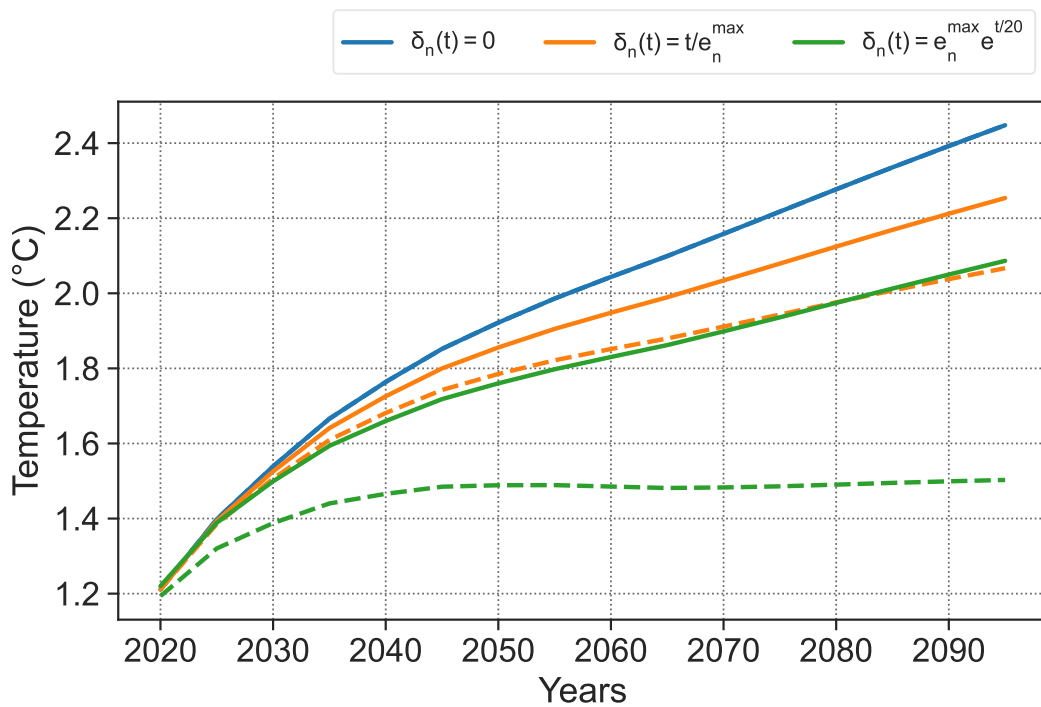


FIGURE 5.5 – Temperature increase due to CO_2 emissions at Nash equilibrium under different influence/imitation graph structures. The dashed line represents the complete graph scenario, while the solid line represents the case where USA and Russia are independent influencers.

Next, we explore the trajectories of CO_2 emissions for each player under this alternative network structure. We focus on the behaviors of EU and Russia, which, being independent

influencers, might exhibit distinct emission patterns compared to other players.

This alternative scenario provides valuable insights into the role of key players and the effectiveness of different network structures in mitigating climate change. In Figure 5.6, we represent, for each player, the ratio of emissions at Nash Equilibrium over their maximal emissions, considering $\delta_n(t) = t/e_n^{\max}$. The ratio of EU decreases exponentially, reaching 0 by 2040. Meanwhile, Russia maintain constant ratio of 1. The ratios of China, USA, India, and AOC converge to the ratio of 0.4. Since EU and Russia do not have an imitation part in their utility, their ratios result from the maximization of their trade-off between benefits and weighted global damage. Moreover, China, USA, India, and AOC are more influenced by EU because EU has the largest maximal emissions, and the imitation/influence weights depend on the influencer's maximal emissions. The synchronization of ratios occurs between those of the two leading countries. The followers' ratios lie between the two, aiming to mitigate the penalty caused by the influence of the two opposing powers. It follows that as long as players are strongly influenced by others, they are forced to follow the result of the average between actions weighted by maximum emissions.

In the following Table 5.3, we provide additional data for various weight functions, supporting our analysis of the synchronization of ratios.

In this table, t denotes the number of repetitions of the game. Indeed, when $t = 0$, it corresponds to the initial iteration of the game, yielding the Nash equilibrium in 2025. As t increments by 5, each step represents an additional shot of the game, reaching the Nash equilibrium in 2050 when $t = 5$, and finally, reaching the Nash equilibrium in 2100 when $t = 15$. Moreover, the primary three columns specify the influence weight functions used in this study.

	$\delta_n(t) = 0$			$\delta_n(t) = \frac{t}{e_n^{\max}}$			$\delta_n(t) = e_n^{\max} \exp(t/20)$		
	2025	2050	2100	2025	2050	2100	2025	2050	2100
Chine	0.459	0.46	0.46	0.459	0.461	0.419	0.671	0.349	0.342
USA	0.394	0.396	0.396	0.394	0.45	0.41	0.671	0.349	0.342
UE	0.5	0.003	0	0.5	0.003	0	0.5	0.003	0
Inde	1	1	1	1	0.49	0.423	0.674	0.353	0.344
Russie	1	1	1	1	1	1	1	1	1
RDM	1	1	1	1	0.609	0.481	0.677	0.359	0.348

TABLE 5.3 – Synchronization of the ratio of players' emissions at the NE over their maximal emissions, *i.e.*, $\frac{a_n^*}{e_n^{\max}}$, when USA and Russia act as independent influencers.

To summarize, we present the Table 5.3 showing different scenarios with various imitation weights. The first three columns present the ratios in the case of the game without

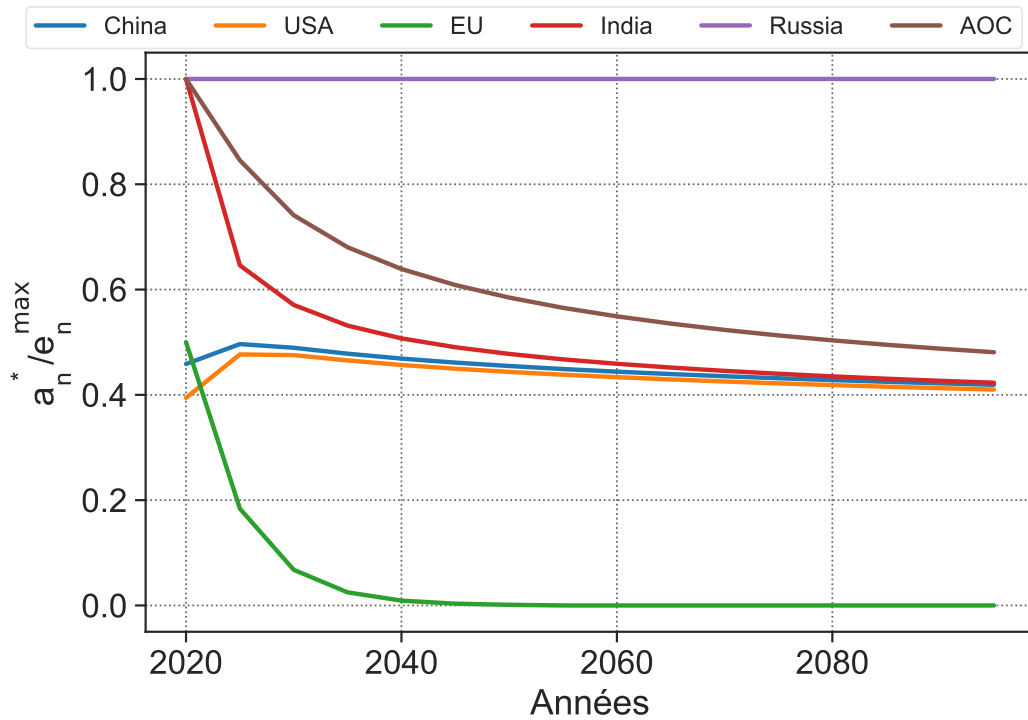


FIGURE 5.6 – Synchronization of players’ emission ratios at the Nash equilibrium over their maximum emissions. The strategy of EU is an exponential decrease in emissions, reaching zero by 2050. USA and Russia act as independent influencers in the network, considering $\delta_n(t) = t/e_n^{\max}$.

imitation, where all players maintain the same ratios except for EU, which decides to reduce its emissions. For linear weights over time, the ratios synchronize and converge to 0.4, and for heavier weights, such as exponential ones, synchronization occurs around 0.3. Naturally, EU and Russia maintain the same ratios for different weight functions because they are not influenced. This suggests that even within a group where one player emits at maximum and encourages others to do the same, a consensus towards a smaller and more favorable ratio is reached compared to the coalition network consensus involving both groups.

5.5 Conclusion

We did a complete Nash equilibrium analysis of a static game model designed to mathematically model the behavior of governments as far as their carbon emissions are concerned. We have considered a game in which the players maximize their utility, a tradeoff between socio-economic benefits for emitting, economic damage due to climate change, and imita-

tion among countries. In contrast with the existing literature, the game integrates climate dynamics, which is nonlinear. The choice of Nash equilibrium as the game solution concept illustrates the importance of aligning individual interests with collective decarbonization strategies.

Our investigation went through the existence and uniqueness of the equilibrium, highlighting the evolution of carbon emissions by major emitters from 2020 to 2100. We exploited this model by using data provided by the IPCC. Our simulations revealed a promising trend: players tend to reduce emissions when playing across multiple stages using imitation terms in the utility functions. We used utility functions that encompass general parameters like GDP and gauged the influence of atmospheric temperature on emission patterns, in addition to the influence of players on each other.

The observed synchronization in emissions refers to a notable alignment or coordination in the emission-related actions taken by different players in the modeled scenario. This coordination is influenced by the network of connections among the players. In simpler terms, the decisions made by one player regarding emission reduction have a domino effect on its neighboring players.

Specifically, when a player unilaterally decides to reduce its emissions, the model suggests that this decision triggers a response from the surrounding players. These neighboring players, influenced by various factors such as mutual influence or strategic considerations, tend to follow a similar path of reducing emissions. This observed behavior highlights the interconnected nature of the decision-making process in the context of emissions reduction strategies, where individual choices can affect the network of players.

As an extension of this chapter, introducing the opinion dynamics of players becomes a pivotal consideration. The incorporation of opinion dynamics into the game framework has the potential to significantly impact the weights within the connected network. This addition not only enhances the complexity of the possible planning game but also introduces a dynamic element that makes the study more challenging and inherently interesting. The evolving nature of opinions can contribute to a richer analysis of the game's dynamics, providing insights into the adaptive strategies of players over time.

Conclusion and Future Work

6.1 Overview

Throughout this thesis, we explored the domain of decision-making in the context of climate change. By adopting a game-theoretic perspective, we clarified the complex dynamics that constitute the basis of global initiatives dedicated to confronting environmental crises. We studied the challenges of climate change from both theoretical and practical perspectives by doing some simulations about the strategic interactions between the nations.

The theoretical framework combines geophysical aspects and strategic considerations into a unified model. Indeed, we proposed two static games, within the climate change context, where the potentiality of the first game ensures the existence of a pure Nash equilibrium, and we rigorously proposed sufficient conditions for the existence and uniqueness of a pure Nash equilibrium in the imitation game. This highlighted the strategic decision-making processes of interconnected countries, where they aim to find a trade-off between individual benefits, weighted global damage, and imitation behaviors in shaping emission strategies.

In the wider context of our thesis, these efforts contribute to both the foundational aspects of theoretical game theory and offer practical insights for climate change decision-makers. The findings emphasize the complexity of decision-making, where considerations of external factors play pivotal roles. As we conclude through the theoretical and practical aspects of climate change decision-making, we highlight the importance of our insights in developing effective strategies to deal with the challenges posed by climate change on a global scale.

In Chapter 3, we established a robust theoretical framework for our game-theoretic analysis, modeling the complex connections between climate dynamics and strategic decision-making by nations. The key innovation lies in integrating geophysical aspects and imitation behavior into the model, unveiling the interconnected nature of global climate efforts. This chapter forms the foundation for our analyses, combining geophysical processes and strategic decision-making to guide policymakers towards sustainable and equitable climate change solutions.

In Chapter 4, we studied the proposed static game within the complex context of climate change, ensuring the existence and uniqueness of at least one pure Nash equilibrium. This theoretical exploration, deeply rooted in game theory, provides valuable insights into the complexities of emission strategies, balancing economic interests and ecological impacts on a global scale. The chapter's practical implications underscore the need for informed climate change policies in the face of intricate global dynamics, bridging theoretical depth with real-world relevance.

In Chapter 5, our focus shifted to the imitation game within the climate change context. Rigorously establishing conditions for the existence and uniqueness of a pure Nash equilibrium, we delved into the strategic interactions among interconnected countries. Theoretical investigations into emission strategies, coupled with numerical simulations, contribute significantly to game theory foundations and offer tangible representations of strategic behaviors. This fusion of theoretical precision and practical relevance strengthens our understanding of the proposed model and its applications in climate change decision-making.

In summary, the interdependent relationship between theoretical foundations and practical insights highlights the importance of considering both aspects in the pursuit of effective climate change mitigation strategies. This study anticipates continued research, encouraging further exploration and application of game theory principles in environmental decision-making.

6.2 Perspectives

Having thoroughly examined the contributions presented in each chapter, it is imperative to direct our focus toward potential avenues for future research. The insights gained from this work lay the foundation for continued exploration and expansion of our understanding in the respective domains. In this section, we outline potential directions for future investigations, filling in the current knowledge gaps and proposing areas where further research could yield valuable contributions.

6.2.1 Extension to dynamic games

One promising continuation for future research involves extending the current static game model to a dynamic framework. Introducing a temporal dimension to the game could enhance our understanding of how strategic decisions evolve over time in response to changing climate conditions, policy interventions, and technological advancements. Incorporating dynamic elements could provide more realistic insights into the long-term implications of climate change mitigation strategies.

6.2.2 Integration of Opinion Dynamics

The influence of public opinion on climate change decisions is a crucial factor that has yet to be fully explored in our current model. Future research could delve into the integration of opinion dynamics, capturing how public perceptions and attitudes toward climate change evolve. Understanding the interplay between strategic decision-making and public opinion could contribute significantly to the development of more effective and socially acceptable climate policies.

6.2.3 Data Analysis and Empirical Validation

To enhance the practical relevance of our theoretical framework, future work could involve extensive data analysis and empirical validation. Using real-world data on emissions, policy implementations, and environmental indicators would provide an opportunity to test the model against historical events and current trends. This empirical validation could offer valuable insights into the applicability and accuracy of the game-theoretic approach in predicting and explaining environmental phenomena.

6.2.4 Interdisciplinary Collaborations

Given the complexity of climate change and its multifaceted nature, fostering interdisciplinary collaborations could open new frontiers for research. Collaborating with experts in fields such as environmental science, economics, and policy-making could enrich the theoretical foundation of the game and facilitate the development of more holistic and impactful climate change mitigation strategies.

These future perspectives outline potential directions for extending and improving the current research, and prepare for a more comprehensive and practically relevant understanding of the strategic dynamics involved in climate change decision-making.

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Résumé étendu

La pandémie de COVID-19 en 2019 a entraîné une réduction notable des émissions, soulignant une prise de conscience croissante chez les populations quant à leur rôle dans la lutte contre le réchauffement climatique. Cette période a influencé l'ensemble de l'Anthropocène, incitant les dirigeants du monde entier à imaginer un avenir transformé, marqué par la durabilité environnementale, une connectivité sociale accrue et une plus grande équité. Ces engagements, issus de déclarations officielles des gouvernements, ont émergé naturellement en réponse à une prise de conscience accrue des inégalités et des divers défis rencontrés pendant la crise. Ce moment charnière a offert une unique occasion de réflexion collective, incitant l'humanité à réévaluer le monde et à poser les bases d'un nouveau chemin en prévision de la crise imminente : le changement climatique.

Cependant, l'état actuel révèle une divergence nette par rapport aux engagements promis, le monde semblant revenir à des schémas familiers, succombant à l'attrait du scénario "Business-As-Usual" (BAU). Les émissions de carbone de la plupart des pays ont rebondi à des niveaux pré-pandémiques, soulignant la réalité préoccupante que les visions transformatrices articulées après le COVID-19 sont supplantées par un retour aux normes établies. Cette régression est particulièrement inquiétante dans le contexte d'un été qui a annoncé que des catastrophes naturelles attendent l'humanité si des mesures décisives pour la crise climatique ne sont pas prises rapidement.

Les événements catastrophiques récents témoignent de l'urgence de lutter contre le changement climatique, avec des inondations dévastatrices submergeant un tiers du Pakistan, des pénuries d'eau frappant les villes américaines après des inondations, et des vagues de chaleur record brûlant des régions du Moyen-Orient, d'Afrique, d'Asie et d'Europe.

Cette situation paradoxale soulève une question fondamentale : « Si les gouvernements connaissent l'imminence de la crise climatique et son impact potentiel, pourquoi persistent-ils dans des actions "Business-As-Usual" ? » Pour répondre à cette énigme, nous nous tournons

vers les outils de la théorie des jeux pour obtenir quelques éclaircissements sur les stratégies de décarbonation.

Dans le **Chapitre 1**, je donne une introduction bien détaillée pour aider le lecteur à mieux comprendre le sujet du problème climatique que j'ai traité dans ma thèse. Ensuite dans **Chapitre 2**, je donne des notions préliminaires que j'utilise dans l'analyse des problèmes que je pose dans les chapitres 3, 4 et 5.

Chapitre 3

Les étés deviennent de plus en plus chauds, accentués par des incendies dévastateurs, ce qui focalise l'attention sur les travaux scientifiques liés au climat, à l'énergie et à l'environnement. Une question naturelle se pose : comment notre communauté peut-elle contribuer à relever ces défis ? Ce chapitre aborde cette question mathématiquement pour comprendre et résoudre les défis actuels.

Nous démontrons comment les outils de la théorie des jeux, de l'optimisation, des systèmes dynamiques et de l'analyse numérique peuvent contribuer à la prise de décision concernant les niveaux d'émissions de CO₂ au niveau national. La théorie des jeux s'impose naturellement car chaque pays reste largement souverain dans cette décision, et analyser un équilibre de Nash revient à envisager une solution de décarbonation que chaque pays a intérêt à suivre.

Bien que le développement de stratégies de réduction des émissions soit devenu impératif, une revue de la littérature sur les stratégies de décarbonation révèle que la majorité des analyses climato-socio-économiques proposent des stratégies empiriques ou ad hoc [39–41]. Les travaux formels sont rares. Parmi eux, certains étudient les problèmes de coalition sans considérer la dynamique des températures atmosphériques et océaniques dans les fonctions de dommages économiques et sociaux [42, 43]. En conclusion, nous proposons des fonctions d'utilité structurées par un modèle géophysique dynamique, évitant des conclusions irréalistes et permettant la construction de stratégies de décarbonation réalistes. Analyser l'unicité de l'équilibre de Nash dans ce jeu prouve être non trivial, comme démontré dans ce chapitre pour le cas quadratique.

Ce chapitre vise à fournir des perspectives précieuses sur l'intersection de la théorie des jeux et de la politique climatique, comblant les lacunes dans l'analyse formelle existante et proposant une approche plus réaliste de la formulation des stratégies de décarbonation.

La science du climat a débuté dans les années 1960 avec le développement des Modèles Climatiques Globaux (GCM), motivée par la nécessité de comprendre, prédire et contrôler les phénomènes météorologiques, influencée par les travaux pionniers de John von Neumann [46]. L'évolution a continué avec la création du Groupe d'experts intergouvernemental sur

l'évolution du climat (GIEC) et son rapport de 1990 [47], culminant dans la publication du sixième rapport du GIEC en 2022 [48], qui démontre constamment l'influence anthropique sur le changement climatique.

Malgré la participation globale des gouvernements aux Conférences des Parties (COP), les émissions de gaz à effet de serre (GES), en particulier le CO₂, des principaux pays émetteurs ont soit augmenté, soit connu des réductions minimales (à l'exception de la période de la pandémie de COVID-19) [49]. La plupart des nations continuent sur une trajectoire de statu quo, avec une part significative de leur production encore basée sur le carbone [50]. Les gouvernements adoptent souvent des politiques minimales pour réduire leurs émissions, alignées sur leurs intérêts économiques nationaux mais insuffisantes pour le bien-être mondial.

Divers modèles climatiques et projections, approximant ou simplifiant les modèles du GIEC, coexistent dans la littérature [51–55]. Les Modèles d'Évaluation Intégrée (IAM) servent à simuler les dynamiques économiques globales. Cependant, tous les IAM n'intègrent pas les considérations climatiques dans leurs équations. Les Modèles Climatiques Simples (SCM) sont couramment utilisés par les économistes pour simuler et estimer l'impact des émissions humaines de CO₂ sur le climat et l'économie.

Parmi les IAM limités étendus pour aborder les changements climatiques à grande échelle, le modèle DICE de W. Nordhaus se distingue, souvent critiqué mais largement utilisé pour projeter les tendances et les impacts des politiques climatiques [57, 58]. D'autres IAM notables incluent FAIR [51], FUND [59, 60], PAGE [61, 62]. Dans les études économiques, des travaux tels que [68–70] apportent des perspectives précieuses. Les discussions sur les fonctions de dommage, notamment dans [71], voient des économistes comparer les modèles aux études de Weitzman, comme vu dans [68].

Du point de vue de la théorie des jeux, divers jeux climatiques sont modélisés comme des dilemmes du prisonnier sans simulations [64], et certains impliquent des jeux de coalition [25, 72–74].

En somme, une vaste littérature existe sur les dynamiques du système climatique terrestre et l'économie globale de notre marché mondial actuel. Cependant, il y a une lacune notable dans la recherche sur la méthodologie pour concevoir des stratégies de décarbonation décentralisées tenant compte de la compétitivité des agents économiques interagissant entre eux.

Les limitations des stratégies actuelles adoptées par les principaux acteurs (gouvernements) résident dans leur nature empirique, manquant de modélisation stratégique-économique suffisante alignée sur la réalité. En conséquence, les stratégies proposées manquent de coordination pour atteindre les objectifs globaux, tels que ceux proposés par le GIEC, comme

le RCP 2.6. Cela souligne la nécessité d'utiliser des outils de la théorie des jeux, conçus pour comprendre les phénomènes résultant des interactions entre les décideurs. Coupler cela avec une compréhension de la modélisation climatique permettra de développer des stratégies de décarbonation durables d'un point de vue compétitif. Bien que la question de l'analyse de la théorie des jeux des décisions climatiques ait été explorée dans la littérature [44, 75–85], il y a une absence notable de modélisation utilisant des modèles économiques dans ces analyses. De plus, ces études négligent souvent la variation de la température dans leurs évaluations.

Chapitre 4

Ce chapitre formule le problème de la réduction des émissions de CO₂ en utilisant la théorie des jeux. Nous modélisons un jeu statique non coopératif où chaque joueur, représentant un pays, maximise son utilité en tenant compte des coûts économiques liés aux émissions de CO₂. Le modèle tient compte des bénéfices d'un pays liés à ses émissions de CO₂ et des dommages causés par l'augmentation de la température atmosphérique due aux émissions globales.

Nous avons démontré l'existence d'un équilibre de Nash pur, où aucun joueur n'a d'intérêt à changer unilatéralement sa stratégie d'émission. De plus, nous avons montré que cet équilibre est unique sous certaines conditions de convexité et concavité des fonctions de bénéfice et de dommage. Toujours sous des conditions suffisantes, je donne l'expression de l'unique équilibre de Nash qui modélise le cas où les joueurs ont tendance à réduire leurs émissions et donc ils n'émettent pas au maximum.

Pour illustrer nos résultats théoriques, nous avons utilisé des données réelles des émissions de CO₂ pour analyser comment les différentes formes de la fonction de dommage (linéaire, quadratique, exponentielle) affectent les stratégies d'émission des joueurs. Nos analyses numériques montrent que plus la fonction de dommage est convexe, plus les joueurs réduisent rapidement leurs émissions en réponse à l'augmentation de la température atmosphérique.

En conclusion, cette étude montre que la prise en compte des coûts économiques liés aux émissions de CO₂ incite les pays à réduire leurs émissions, contribuant ainsi à la lutte contre le changement climatique. Des recherches futures pourraient explorer des modèles dynamiques pour mieux comprendre l'évolution des stratégies d'émission au fil du temps et l'impact de politiques climatiques spécifiques sur les comportements des pays.

Chapitre 5

Ce chapitre étudie le problème du changement climatique en tant que jeu d'imitation, où l'utilité de chaque pays intègre un terme modélisant l'influence des autres pays en termes

d'émissions de CO₂. Ce jeu est une extension de celui analysé dans le chapitre 3, visant à étudier l'effet de l'influence sur les actions des joueurs. Les joueurs sont des pays et leur utilité est un compromis entre le bénéfice socio-économique individuel des émissions, les dommages globaux induits par le changement climatique, et un terme d'imitation. Nous effectuons l'analyse de l'équilibre de Nash de ce jeu pour montrer les comportements possibles des pays en présence de la sensibilisation au changement climatique et de l'influence des autres pays. Nous explorons le rôle de différents paramètres clés tels que le poids de l'imitation sur l'expression de l'équilibre de Nash. Nos résultats soulignent l'importance de la coordination et de la coopération pour atténuer le changement climatique.

En effet, Le climat est décrit par un modèle géophysique complexe qui interconnecte de nombreuses dynamiques non linéaires (CO₂ dans l'atmosphère et les océans, températures dans l'atmosphère et les océans, forçage radiatif, etc.). Le changement climatique est l'un des problèmes les plus difficiles auxquels l'humanité est confrontée aujourd'hui. Il peut potentiellement affecter les écosystèmes, les économies et les sociétés du monde entier. Atténuer le changement climatique nécessite un effort coordonné de tous les pays, car les émissions de gaz à effet de serre d'un pays affectent la dynamique climatique globale. Les stratégies de chaque pays pour réduire le réchauffement climatique résultent d'un compromis entre les pertes économiques à court terme induites par les politiques écologiques, les dommages causés par l'augmentation des émissions de CO₂, et la pression politique des partenaires économiques. Cela conduit à un problème où les acteurs doivent prendre des décisions interdépendantes. Un cadre mathématique naturel pour étudier ces scénarios de décision interactive est la théorie des jeux.

L'étude du changement climatique sous un angle théorique des jeux a déjà été envisagée dans la littérature, imposant des propriétés de convexité sur les fonctions d'utilité dans un jeu statique. L'accord des pays dans un jeu non coopératif a été analysé. De plus, un modèle dynamique simulant le processus de négociation menant à l'Accord de Paris de 2015 sur le changement climatique a été présenté, mettant en lumière l'importance du leadership dans le processus d'accord.

Une autre ligne de recherche examine la dynamique des opinions et les interactions sociales dans le contexte du changement climatique. Un modèle couplant un modèle climatique et un modèle de dynamique d'opinion de Hegselmann-Krause a été introduit. Un jeu évolutif pour comprendre l'interconnexion des dynamiques climatiques et des réseaux sociaux, notamment les inégalités dans le monde réel, a été présenté. Les débats publics influencés par des données scientifiques incomplètes et le rôle des agents inflexibles dans l'influence de l'opinion publique ont été étudiés. L'influence des médias sur la communication climatique est une autre zone d'exploration, analysant comment divers médias influencent la percep-

tion publique du changement climatique par leur couverture, leur agenda et leur cadrage. Plusieurs études ont exploré les dynamiques des accords climatiques et la coopération entre pays pour relever ce défi mondial.

Dans le chapitre 5 comme dans cette thèse, nous proposons une dynamique climatique non linéaire contrôlée par des stratégies visant à maximiser des fonctions d'utilité individuelles comprenant trois termes : un terme de bénéfice individuel, un terme de dommage global pondéré, et un terme capturant l'effet d'imitation/accord avec les autres joueurs. Il est utile de noter que le terme d'imitation introduit peut également être interprété comme un contrat de réduction des émissions ou comme une pénalité pour les pays qui n'alignent pas leurs émissions sur une stratégie commune. Cela résulte en l'étude d'un réseau complexe entre les joueurs et le climat, produisant une certaine trajectoire de réchauffement climatique.

Contributions clés

1. Nous proposons un jeu statique novateur contenant un terme d'imitation dans les utilités des joueurs. Nous fournissons une condition suffisante garantissant l'existence d'au moins un équilibre de Nash pur. Nous montrons également l'unicité de l'équilibre de Nash pur en prouvant que les fonctions d'utilité sont strictement concaves diagonalement.
2. Nous fournissons l'expression de l'unique équilibre de Nash pur correspondant, sous une condition suffisante, lorsque les fonctions d'utilité sont quadratiques.
3. Nous évaluons numériquement les effets de la fonction d'imitation due à l'influence des joueurs, connectés dans un graphe complet, les uns sur les autres.

Dans le **Chapitre 6**, je conclus ma thèse et je donne quelques perspectives avec des idées qui m'intéressent et suscitent pour continuer mes recherches sur ce sujet assez intéressant et d'actualité.